Deformation of an Expanding Void in Redshift Space

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Abstract

We investigate the dynamics of a spherical void in the FLRW universe and discuss its observational consequences in redshift space. We show that a void is prolonged in the line of sight and that the deformation ratio depends on the dynamics of the background universe. Our results together with a future deep sky survey of galaxy distribution will confirm the existence of a cosmological constant.

1 Introduction

The existence of local voids with a radius larger than $100h^{-1}$Mpc is suggested by recent observational and theoretical studies. Granett et al. measured hot and cold spots on the cosmic microwave background associated with superclusters and supervoids, which were identified in the Sloan Digital Sky Survey Luminous Red Galaxy catalog [1]. Subsequently, Inoue et al. estimated the integrated Sachs-Wolfe effect due to spherically symmetric super-structures and found that observed cold and hot spots respectively imply presence of nonlinear voids and clusters with a radius $100 \sim 200h^{-1}$Mpc [2].

In this paper, we investigate the dynamics of a single spherical void embedded in the Friedmann-Lemaître-Robertson-Walker (FLRW) universe and discuss its observational consequences in redshift space. Especially, we focus on whether we can judge the existence of a cosmological constant by observing voids in redshift space. We expect that an expanding void is deformed, which is analogous but in the opposite way to the "fingers of God" effect or the Kaiser effect for a cluster in redshift space [3].

2 Dynamics of a Void

Let us consider a spherical shell that defines the boundary of a void. We assume that both the inner region (−) and the outer region (+) are described by the FLRW metrics. Both spacetimes satisfy the Friedmann equation with dust and a cosmological constant,

$$H_\pm^2 + \frac{k_\pm}{a_\pm^2} = \frac{8\pi G}{3}\rho_\pm, \qquad H_\pm = \frac{\dot{a}_\pm}{a_\pm}, \qquad \rho_\pm = \rho_{\rm vac} + \rho_\pm^{(m)}. \tag{1}$$

The equations of motion of the shell were originally derived by Maeda and Sato [4], and later they were rewritten in a more convenient expression [5] that we use here. To integrate them, we should give the initial values of the radius of a void, its velocity, the ratio of the inner expansion rate to the outer one, the density contrast, and the density parameter, and the dimensionless cosmological constant,

$$R, \quad v, \quad \frac{H^-}{H_+}, \quad \delta \equiv \frac{\rho_-^{(m)}}{\rho_+^{(m)}} - 1, \quad \Omega \equiv \frac{8\pi G\rho^{(m)+}}{3H_+^2}, \quad \text{and} \quad \lambda \equiv \frac{\Lambda}{3H_+^2}, \tag{2}$$

at the initial time t_i (or the corresponding initial redshift z_i). Here we choose $z_i = 100$, $v_i = 0$ and $H_i^-/H_i^+ = 1$; however, the dynamics does not depend on these values as long as the "decaying mode" becomes negligibly small. In order to discuss comparison between our theoretical calculation and observation, the remaining model parameters Ω, λ, δ and R should be given at the present time t_0, i.e., Ω_0, λ_0, δ_0 and R_0. Hereafter we omit the subscript +.

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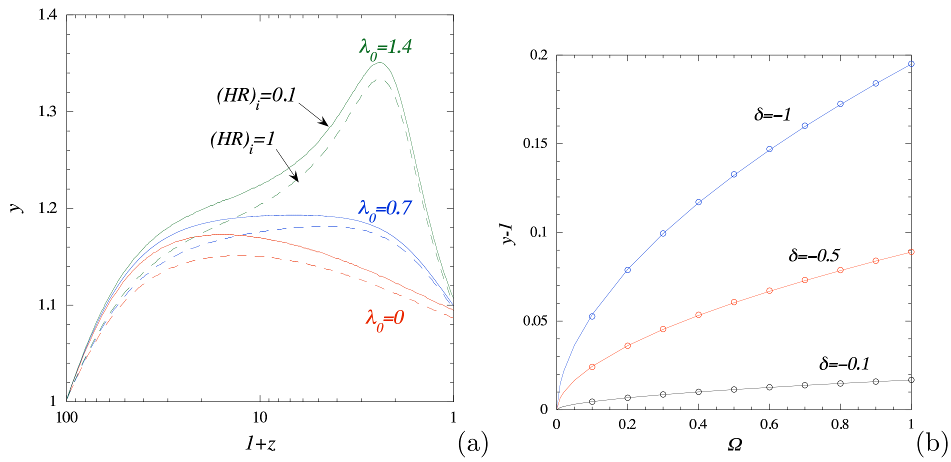

Figure 1: (a) Time-variation of $y(z)$ for an empty void with $\Omega_0 = 0.3$. (b) y versus Ω and δ for $\Omega + \lambda = 1$. The circles and the continuous lines correspond respectively to our numerical results and to Eq. (4).

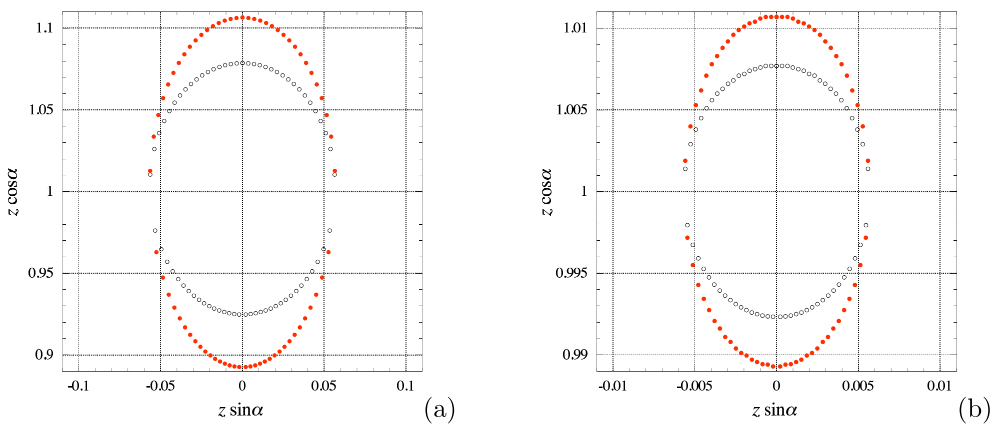

Figure 2: The image of a void at $z_{\rm V} = 1$ in redshift space of the universe with $\Omega_0 = 0.3$ and $\lambda_0 = 0.7$. The observer is located at the origin. We show two voids with different scales, i.e., (a) $R_0 = 0.1 H_0^{-1}$ and (b) $0.01 H_0^{-1}$. The red dots and the black circles denote the image of the expanding shell and the reference image of the r =constant surface, respectively.

In order to compare the present general relativistic results with the Newtonian approximation [6], we define a "proper kinematics" of the shell as

$$y \equiv \frac{dR/dt}{HR} = 1 + \frac{f'v}{HR}, \quad f' = \sqrt{1 - \frac{kR^2}{a^2}}, \tag{3}$$

and present our numerical result in Fig. 1(a). We fix $\Omega_0 = 0.3$ and see how the dynamics depends on λ_0 and R_i. The result for the sub-horizon void confirms the previous result in the Newtonian approximation. The difference in dynamics between the sub-horizon void and the horizon-scale void is small; that is, the relativistic effect is small.

Because real voids may not be empty, it is also important to investigate how the shell velocity depends on the density contrast δ. We survey the shell velocity in the flat universe ($\Omega + \lambda = 1$), and show the numerical results by circles in Fig. 1(b). From this figure, we find that the velocity y highly depend on δ as well as Ω. For a practical purpose, we present a fitting formula,

$$y - 1 = \frac{v}{HR} = \frac{\Omega^{0.56}}{6}(|\delta| + 0.1|\delta|^2 + 0.07|\delta|^3), \quad \text{for} \quad \Omega + \lambda = 1, \tag{4}$$

which is shown by the solid curves in Fig. 1(b). They approximate our numerical data very well. The maximal difference between the numerical data and the formula (4) is less than one percent.

3 A void in redshift space

Next, we discuss how expanding voids look in redshift space. Our analysis method is as follows.
(i) Solve our equations of motion of the void shell to find $r = r(z)$ and $v = v(z)$.
(ii) For a given angle of sight from an observer, α, find the intersecting points of the light path with the shell and the time z when the photon is emitted.
(iii) Calculate the redshift of the void shell, z_s, for each angle α.
(iv) Draw the shape, i.e. $(z_x, z_y) = (z_s(\alpha)\cos\alpha,\ z_s(\alpha)\sin\alpha)$, in redshift space.

Figure 2 shows the image of a void, whose center is located at $z_V = 1$, with two scales,$R_0 = 0.1H_0^{-1}$ and $0.01H_0^{-1}$. Note that the observed void size is given by $R_V(z_V) \equiv a_0 r(z_V) = R_0 \times r(z_V)/r(0) < R_0$, because the photons were emitted at z_V. A void shape is prolonged in the direction of the line of sight mainly by the Doppler effects, which is the opposite deformation to the Kaiser effect because of the opposite direction of the peculiar velocity. Though the deformation in redshift space is proportional to the void size as $\Delta z_s \sim v \sim 0.1HR$, the ratio of two radii $(\Delta z_{||}/\Delta z_{\perp})$ is almost independent of the void size.

The larger void with $R_0 = 0.1H_0^{-1}$ in the $\Omega_0 = 0.3$ universe in Fig. 2 (a) also showsa small front-back asymmetry. This is due to the evolutionary effect of the shell radius $r(t)$. On the other hand, for the smaller void with $R_0 \sim 0.01H_0^{-1}$ in Fig. 2 (b) the front-back asymmetry does not exist. It is because the evolutionary effect is negligible for the smaller void. This asymmetry could be used to test whether the observed void is an isolated one or not.

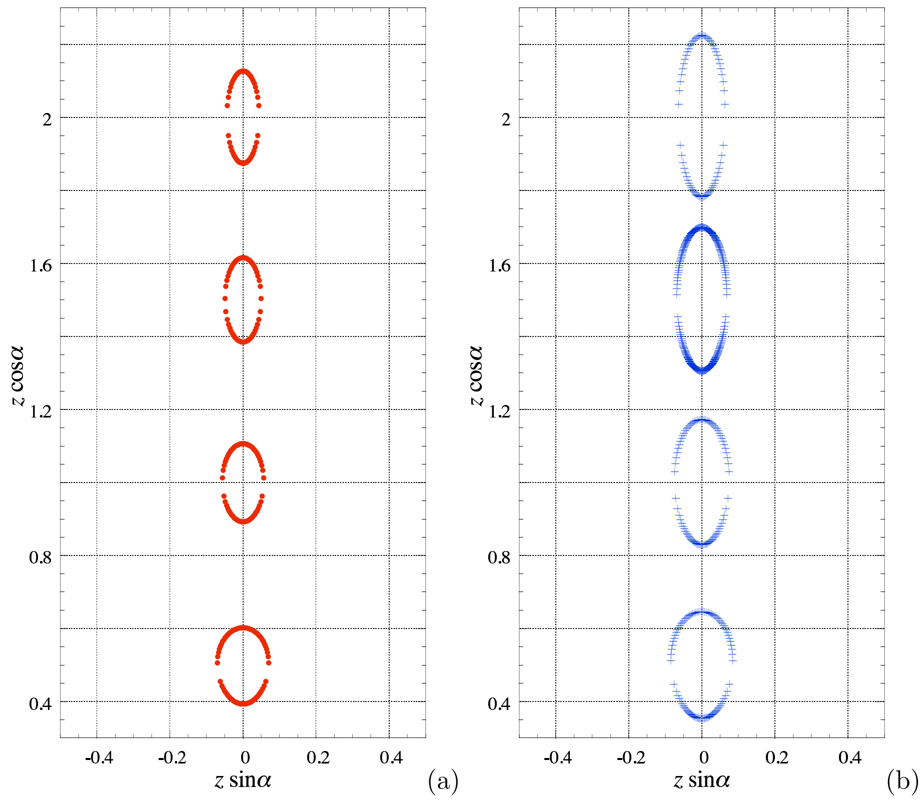

Figure 3: The images of four empty voids at $z_V = 0.5, 1, 1.5$ and 2 in redshift space for $\Omega_0 = 0.3$ and $\lambda_0 = 0.7$ (a) and those for t$\Omega_0 = 1$and $\lambda_0 = 0$ (b). We choose $R_0 = 0.1H_0^{-1}$.

In Fig. 3 (a), we present how voids look like in redshift space by showing four equivalent voids at $z_V = 0.5, 1, 1.5$ and 2. The present size is chosen as $R_0 = 0.1H_0^{-1}$. We can easily see the change of the vhoid shapes in terms of the distance z_V. When we have several equivalent voids in the universe, the observed void size $R_V(z_V)$ gets smaller as the distance z_V increases. As a result, $\Delta z_{\perp}$, which corresponds to the observed void size in redshift space, decreases. However, we find that as z_V increases, $\Delta z_{||}$ increases at least in the large z_V region. This is because the Doppler effect overcomes the above evolutionary effect.

In Fig.3 (b), for reference, we show the shapes of four voids in the $\Omega_0 = 1$ universe. Comparing the shape change in two different cosmological models, we find the deformation ratio of a void in the $\Omega_0 = 1$ universe is larger than that in the $\Omega_0 = 0.3$. In order to show how much the shape is deformed quantitatively, we define the deformation ratio by $\mathcal{R} = \Delta z_{||}/\Delta z_{\perp}$, which is the ratio of the semi-major axis to the semi-minor axis of the approximate ellipse.

Figure 4 (a) confirms that, as z_V increases, $\Delta z_{\perp}$ decreases while $\Delta z_{||}$ increases. Only for neighbor voids in the universe with $\Omega_0 = 0.3$ and $\lambda_0 = 0.7$, $\Delta z_{||}$ as well as $\Delta z_{\perp}$ decrease due to the evolutionary effect. In Fig. 4 (b), we show the deformation ratio with respect to the distance z_V. The ratio $\mathcal{R}$ increases as the distance z_V increases. We also find the ratio $\mathcal{R}$ higly depends on the background cosmological model, which may give us some information on a cosmological constant.

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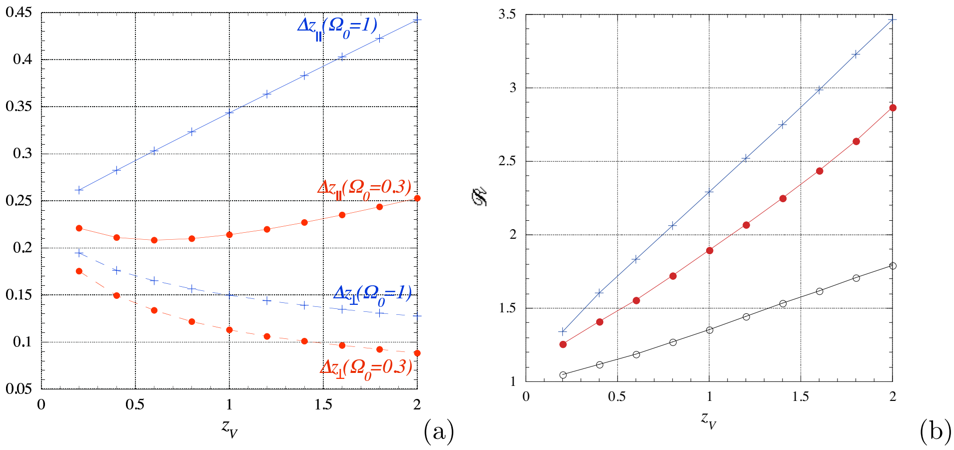

Figure 4: (a) The semi-major axis $\Delta z_{||}$ and the semi-minor axis $\Delta z_{\perp}$ in terms of the distance z_V. (b) The deformation ratio $\mathcal{R}$ of the shape of a void in the red shift space, which is defined by $\mathcal{R} = \Delta z_{||}/\Delta z_{\perp}$. The black line with circles represents the reference values of r = constant surface.

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