Thermal Model Approach for Study of Multi-Component Interacting Hadron Resonance Gas

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Introduction

The ideal hadron resonance gas (HRG) model of point-like non-interacting particles does not reproduce the ground state properties of the nuclear matter and no reasonable quark hadron phase transition is obtained with sufficiently large number of degrees of freedom in HRG [1-3]. Therefore in a realistic approach both the attractive and repulsive interactions should be taken into account. The repulsive interaction is proportional to number density (n) and vanishes for $n \cong 0$. At sufficiently high temperature (T), a large number of point-like hadronic resonances can be thermally excited (without repulsive interaction). Hence in a thermodynamical model assuming a first order quark-hadron phase transition, as a result of the large number of degrees of freedom, the hadronic pressure becomes larger than quark gluon plasma (QGP) pressure and the system reverts to the HRG phase. This contradicts the lattice quantum chromodynamics (LQCD) predictions that the phase transition occurs at the critical temperature around 160-180 MeV for vanishing net baryon number and the system remains in the QGP phase for further higher temperatures. The equation of state (EOS) is an input for the hydro-chemical model of nuclear collisions. We have used the EoS to obtain the hadronic yields using van der Waals type interaction which contain both attractive ($a > 0$) and repulsive ($b > 0$) parameters [4-5].

Model

A brief generalization of the Van der Waals equation of state (EoS) for the grand canonical ensemble (GCE) is presented in this section. We have taken both, the attractive and repulsive interactions among all the baryons (antibaryons). The van der Waals equation of state (EoS) governing the behaviour of baryonic matter can be derived from the grand partition function.

$$\mathcal{Z}(T,\mu,V) = \sum_{N=0}^{\infty} e^{\frac{\mu N}{T}} \mathcal{Z}(T,N,V) \qquad (1)$$

Here $\mathcal{Z}(T,N,V)$ represents the canonical partition function for N number of particles. The T and V are temperature and total physical volume of the system, respectively. The phenomenological grand partition function with both the attractive and repulsive interactions taken into account can be written as:

$$\mathcal{Z}^{int}(T,\mu,V) = \sum_{N=0}^{\infty} e^{\frac{\mu N}{T}} \mathcal{Z}(T,N,V - bN) e^{-\frac{\overline{U}}{T}} \qquad (2)$$

Here N is the number of particles in the system having attractive and repulsive interactions (i.e. $N = N^{int}$). Here bN represents the excluded volume due to baryons (antibaryons) having hard-core repulsive interaction [6-8]. For multi-component baryonic system the modified number density with interactions is given by:

$$n_j(\mu_j,T) = \frac{n_j^{id}(T,\mu_j^*)}{1+\sum_i b_i n_i^{id}(T,\mu_j^*)} \qquad (3)$$

Equation (3) represents the modified number density of the j^{th} hadronic specie for a multi component interacting HRG, using Van der Waals type equation of state (EoS). The excluded volume arising due to the j^{th} hadronic species considering it a hard sphere is ($b_j = \frac{16}{3}\pi r_j^3$), where r_j represents the radius of the j^{th} hadronic species. The summation over the index i in the denominator also includes j. The effect of the attractive and the repulsive hard-core interaction appears in the equation of state of the system through the "effective" baryon chemical potential (μ^*) in equation (4).

$$\mu_j^* = \mu_j - \frac{b_j n_j T}{1-\sum_i b_i n_i} + 2an_j \qquad (4)$$

The application of equation (4) to the anti-baryonic sector requires that the "effective" chemical potential for antibaryon (i.e. $\overline{\mu^*}$) be written as:

$$\overline{\mu^*} = \bar{\mu} - \frac{\bar{n}bT}{1-b\bar{n}} + 2a\bar{n} \text{ with } \bar{\mu} = -\mu$$

Where the quantities with bar indicate their values for the anti-baryons. For the anti-baryonic sector in the multi-component interacting HRG we will get:

$$n_{\bar{j}} = \frac{n_{\bar{j}}^{id}(T,\overline{\mu_j^*})}{1+\sum_i b_{\bar{\imath}}\, n_{\bar{\imath}}^{id}(T,\overline{\mu_j^*})} \qquad (5)$$

For a baryon free matter, $\overline{\mu_J} = -\mu_j = 0$. This also gives $\overline{\mu_J^*} = \mu_j^*$, thus providing $n_j = n_{\bar{j}}$ at any given temperature T, which is mandatory to maintain the baryon–antibaryon symmetry in the system when $\mu_j = 0$. In the following we present results of our numerical calculations and discussion.

Results and Discussion

We have studied effect of the attractive and repulsive interaction strengths on the relative particle yields. In figure 1, we have shown the dependence of $\left(\frac{\bar{N}}{N}\right)$ ratio on temperature at fixed baryon chemical potential ($\mu_B = 300$ MeV) for various values of the attractive parameter "a" and the hard-core repulsion parameter "b". It can be easily seen that as the hard-core radii of (anti)baryons become less and less, the value of the ratio thus obtained from the VDW type EoS asymptotically reaches with the point-like baryon case. In order to test the validity of the VDW model in the limiting cases we calculate the $\left(\frac{\bar{N}}{N}\right)$ ratio using the VDW type EoS with a=b=0, which means neither with the attractive nor repulsive interactions present in the system. We find that the result thus obtained completely merges with point-like (non-interacting) case, which should be expected from any reasonable acceptable model. Next if we switch-off the repulsive part only (i.e. r_h=0) and keep only the attractive part i.e. non-zero value of a=329 MeVfm^3, we notice that the particle ratio gets suppressed more than the point like case. This is because there is a relatively faster increase in the nucleon density compared to the anti-nucleon density when the temperature is increased. This is essentially due to the absence of any repulsive hard core interaction which leads to an excluded volume effect. Further on the contrary, we find that (at higher temperatures) there is an enhancement in $\left(\frac{\bar{N}}{N}\right)$ ratio as the baryonic hard core radii (r_h) increases. In figure 2, the temperature dependence of nucleon and antinucleon density at μ_B=300 MeV have been show, we notice that at higher temperature nucleon and antinucleon density gets modified using VDW type Eos, as compared to point like case and thermodynamically inconsistent model [1].

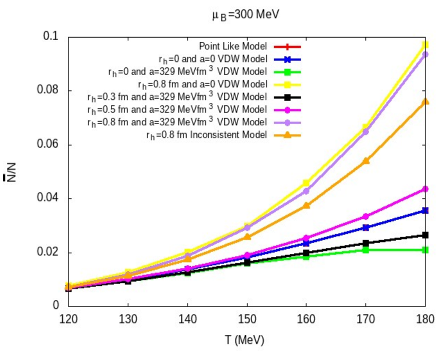

Fig. 1: Variation of particle ratio $\left(\frac{\bar{N}}{N}\right)$ for different values of hard core radii withtemperature at fixed baryon chemical potentials, $\mu_B = 300$ MeV.

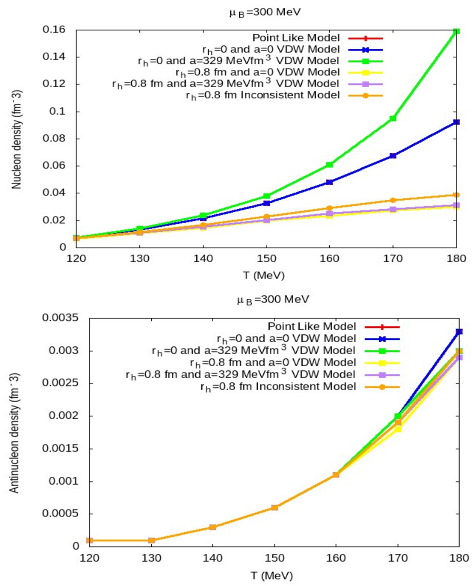

Fig. 2: Variation of nucleon and antinucleon number densities with temperature at fixed baryon chemical potentials, $\mu_B = 300$ MeV.

Conclusion:

We have provided a modified grand canonical ensemble formulation for a multi-component hadronic resonance gas system. We have considered the attractive as well as repulsive interaction among the constituent baryons (antibaryons). Using our formulation we have calculated nucleon to antinucleon ratio as well as nucleon (antinucleon) densities in the system. We find that the relative particle yields get modified if we incorporate the attractive and repulsive interactions in the system.

References

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