MAXIMAL SUPER YANG-MILLS AND SUPERGRAVITY IN FULL SUPERSPACE

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1. Introduction

In order to be able to use explicitly the ultra-violet divergence cancellation potentiality of supersymmetric field theories, especially for N= 4 SYM and N= 8 SGR we must construct them in superspace. We immediately meet the N= 3 barrier [1] which arises because it is increasingly difficult to re-arrange the 2^{2N-1} fermionic degrees of freedom (or more) into only 2N propagating physical modes (for N-SYM) and auxiliary fields; a similar difficulty arises for N-SGR. To penetrate the N= 3 barrier up to N= 4 for SYM or N= 8 for SGR we need to reduce the dimension of spinors by a factor of 2. This can be achieved either by losing explicit lorentz covariance, as in the light-cone gauge approach, or by losing explicit N-SUSY by working in terms of N/2 superfields. One of the two currently known ways of penetrating the N= 3 barrier preserving the maximal symmetries in a linear realisation is through the use of central charges, and I wish to present results obtained up to date in that programme here. The more recent technique of harmonic superspace has also proved of value in constructing N= 3 super Yang-Mills theory in N= 3 superspace, (as described by E. Sokatchev in the previous talk), though this has not yet been extended to N= 4 theories. The central charge avenue on the other hand, has allowed the construction of up to, and including, N= 8 supergravity in the appropriate full superspace, as I will now try to explain.

2. Central Charges

These are operators Z^{ij} $(1\leq i,j\leq N)$ defined by the anticommutator of two chiral SUSY generators

$$[S^i_\alpha, S^j_\beta]_+ = \varepsilon_{\alpha\beta} Z^{ij} \tag{1}$$

The resulting N-SUSY algebra $\mathcal{S}^Z_N = \{J_{\mu\nu}, P_\lambda, S^i_\alpha, \bar{S}_{\dot\alpha j}, Z^{ij}\}$ has a Casimir which extends that of the Poincare group in the form $W_\mu W^\mu$ (where W_μ is the Pauli-Lubanski vector) by replacing W_μ in that expression by

$$C_\mu = W_\mu - \tfrac{1}{8}\bar{S}^\ell \gamma_\mu (S_m + \not{p}^{-1}\bar{S}^n Z^{*mn})(1 - p^{-2}Z^*Z)^{-1}{}_\ell{}^m + \tag{2}$$

The irreps of $\mathcal{S}^Z_N$ can thus be divided into two classes:

(1) non-degenerate, with $p^2 \neq Z^*Z$. These may be shown not to be helpful in trying to broach the N = 3 barrier.

(2) degenerate, with $p^2 = Z^*Z$. These require that the numerator in the factor (2) also vanish. This is a Dirac-type of equation which allows $\bar{S}_{\dot\beta j}$ to be rewritten in terms of S^i_α:

$$\bar{S}_{\dot\beta j} = -\not{p}^{-1}{}_{\dot\beta}{}^{\alpha} Z^{*ji} S^i_\alpha \tag{3}$$

We see that given (3) and the relation

$$p^2 \delta^i_j = Z^{ik} Z^{*kj} \tag{4}$$

we may deduce the correct commutation relation between two oppositely chiral SUSY generators. There are therefore only, say, the S^i_α as independent fermionic elements of $\mathcal{S}^Z_N$ on degenerate irreps. It is precisely this feature that allows us to pass through the N = 3 barrier. This is because the dimension of the spinors defining representations of $\mathcal{S}^Z_N$ have been reduced by a factor of one half by eliminating $\bar{S}_{\dot\alpha i}$, so that N has been effectively reduced to N/2 in the analysis of the No-Go theorems (without associated loss of internal $SU(N)$ symmetry, however).

We may represent the Z^{ij} as $\partial/\partial z_{ij}$, where z_{ij} are bosonic co-ordinates. There are enough of these to construct the 'full' super-space measure $d^4x\, d^{4N}\theta\, d^{2(N-1)}z$, of dimension L^2, by integrating over a 2(N-1)-dimensional subspace of the z_{ij}. This latter can be chosen naturally for N= 4 as the set of self-dual z_{ij}, and for N= 8 as defined by the Dirac matrices in seven dimensions.

We must now interpret full superspace actions

$$A = \int d^4x \int_\Gamma d^{2(N-1)}z \int d^{4N}\theta\, \mathcal{L}(\Phi(x,z,\theta),\ldots) \tag{5}$$

These are defined on superfields Φ satisfying (3) and (4), which may be used to determine how initial data, for a given value of the z_{ij}, propagate into the whole of z-space. This propagation can be made unambiguous if we require dependence on only one central charge, and this can be achieved by adding the further constraint

$$z^{ij} = Z\alpha^{ij} \tag{6}$$

where α^{ij} is a metric for USp(N). We remark also that without (6) the spectrum of the theories we can construct would be unsatisfactory[2].

In particular there are an infinite number of propagating particles in the spectrum if there are two central charges z_1, z_2 even if the spin reducing constraint (4) in the form $\Box = \partial^2/\partial z_1^2 + \partial^2/\partial z_2^2$ is satisfied. This can be seen from the fact that the fields in four-dimensions of this boundary-controlled theory are the sequence of derivatives w.r.t. z_1 and z_2 of the fields in z_1, z_2. Thus if $A(x, z_1, z_2)$ is the complete field, its four dimensional content is the sequence ($\partial_i = \partial/\partial z_i$) of derivatives evaluated at $z_1 = z_2 = 0$:

$$\{A, \partial_1 A, \partial_2 A, (\partial_1^2/\Box)A, (\partial_1\partial_2/\Box)A, \ldots, (\partial_1^{2n}/\Box^n)A, (\partial_1^{2n+1}/\Box^n)A, (\partial_1^{2n}\partial_2/\Box)A, (\partial_1^{2n+1}\partial_2/\Box^{n+1})A, \ldots\}$$

If A is a physical propagating field then the fields $(\partial_1^{2n}/\Box^n)A$, $(\partial_1^{2n-1}\partial_2/\Box^n)A$ are also propagating, thus giving the infinite sequence of such fields. Actions and field equations for this case have been analysed in detail[2].

The constraints (3), (4), (6) on (5) reduce the action to one controlled by the values of Φ and its derivatives on $\partial\Gamma$; the theory is thus a boundary control problem[3].

3. Examples

The simplest example is for N = 2, when $Z^{ij} = \varepsilon^{ij}Z$, $Z = \partial/\partial x^5 + i\partial/\partial x^6$. Then (4) becomes the 6-dimensional massless wave equation. Since the superspin Y and multiplicity values in the scalar S.F are $(o^2, \frac{1}{2})$ then the fundamental irrep (the N = 2 hypermultiplet or HYM) is contained in and is singled out by the constraint [4]

$$(D_\alpha^{(i}\Phi^{j)})_{\Box\Box} = (\bar{D}_{\dot\alpha}^{(i}\bar\Phi^{j)})_{\Box\Box} = o \tag{7}$$

Similarly the N = 4 is in the 5-plet W^{ij} of USp(4) with constraint [5]

$$(D_\alpha^{(i}W^{jk)})_{\text{⊞}} = (\bar{D}_{\dot\alpha}^{(j}W^{jk)})_{\text{⊞}} = o \tag{8}$$

whilst the N = 8 HYM is in the 42-plet $W^{ijk\ell}$ of USp(8) with constraint[5]

$$(D_\alpha^{(i}W^{jk\ell m)})_{\text{⊞}} = (\bar{D}_{\dot\alpha}^{(i}W^{jk\ell m)})_{\text{⊞}} = o \tag{9}$$

The linearised actions (5) have been evaluated for the N = 2, 4 and 8 fundamental representations by integrating over the Grassmann variables

In all those cases it has been shown[6][7] that the resultant quadratic actions are explicitly boundary controlled, being of the form

$$\int d^4x \int d^{2(N-1)}z\, \partial_z^{2N+2}(\Phi^*\Phi) \tag{10}$$

where Φ is the appropriate superfield Φ^i, W^{ij}, $W^{ijk\ell}$ satisfying the contraints (7), (8) or (9) repectively. The same is true of the centrally charged N = 2 SYM multiplet at linearised level[6].

4. Constrained Superspaces

In order to use the above irreps to construct fully covariant interacting field theories for N = 4 SYM or N = 8 SGR we may impose constraints on the curvatures and/or torsions available. For N = 4 SYM the geometry is based on the gauge potential B_A (with $A = a, \alpha_i, ij$) and covariant derivative $\mathcal{D}_A = D_A + iB_A$; the field strengths are defined by

$$[\mathcal{D}_A, \mathcal{D}_B] = -T^{(0)C}_{AB}\mathcal{D}_C - F_{AB} \tag{11}$$

where $T^{(0)C}_{AB}$ is the torsion of flat superspace. We impose the constraints [8]

$$F^{(ij)}_{\alpha\beta} = F^{i}_{\alpha\dot\beta j} = o, \quad \mathcal{D}_{ij} = \alpha_{ij}\mathcal{D}_z \tag{12}$$

and solve the Bianchi identities. All the F_{AB} are found to be functions of W^{ij} satisfying the gauge-covariant extension of (8). Further consistency checks [9] show this equation to have a non-trivial solution as the field-strength superfield.

We may then write down a full super-space action

$$A = \frac{1}{g^2}\int d^4x \int_\Gamma d^6z \quad d^{16}\theta \;\; \mathrm{Tr}(W^{ij}W_{ij}) \tag{13}$$

and the resulting equations of motion, derived by boundary control theory, are the simple ones[8][9]

$$\partial_z W_{ij} = o \tag{14}$$

For the case of N = 2 matter theory we may simply take the N = 2 hypermultiplet constraint (7) and construct the action

$$\int d^4x \int_\Gamma d^2z \int d^8\theta\; \Phi^{i*}\Phi^i \tag{15}$$

This may be put in interaction with an external N = 2 super-Yang-Mills gauge field by replacing the derivative D_α^i in (7) by the covariant derivative $\mathcal{D}_\alpha$ as for the N = 4 case. An action for the N = 2 super-Yang-Mills theory may be obtained in a similar fashion to the N = 4 case by imposing the constraints (12) (now with $i, j = 1, 2$) and the additional constraint

$$\mathcal{D}^{2ij} F^{\alpha}{}_{\underline{\alpha}} = 0 \qquad (16)$$

This latter removes the non-centrally-charged multiplet allowed by the other constraints. The action for this theory is now

$$\int d^4x \int_{\Gamma} d^2z \int d^8\theta \, (F^{\alpha}{}_{\underline{\alpha}})^2 \qquad (17)$$

The equations of motion for the appropriate field strength superfields are all of the simple form (14)

In fact three versions of N = 8 supergravity appear possible. The first two versions[10][11] have non-trivial torsions of dimension zero. Their full superspace actions

$$A = \frac{1}{\kappa^2} \int d^4x \int_{\Gamma} d^{14}z \int d^{32}\theta \det E \qquad (18)$$

have only been shown to give the equations of motion

$$\partial_z E_A{}^M = \partial_z \Omega_{AB}{}^C = 0 \qquad (19)$$

for the multi-bein $E_A{}^M$ and connection $\Omega_{AB}{}^C$ at the linearised level. The third model has no non-trivial dimension zero torsions and has the field equation (16) at the non-linear level. We are presently trying to show that (15) is the correct action for this theory[12].

5. Quantisation

We may quantise the above theories by introducing lagrange multipliers to relax the constraints. It is necessary to show that this approach is satisfactory even at the component level, where there it has proved difficult to quantise the non-Abelian component version of N = 4 super-Yang-Mills[13] due to the presence of a conservation constraint equation for an auxiliary vector V_μ:

$$\partial^\mu V_\mu + \text{non-linear terms} = 0 \qquad (20)$$

This constraint can only be solved non-locally, so that the resulting quantum field theory would seem to be unsatisfactory. However the use of lagrange multipliers allows a satisfactory perturbation theory to be developed[14], in which the lagrange multiplier for (20) becomes the propagating sixth scalar of the theory. We thus expect the superfield approach, using lagrange multiplier superfields, to relax the constraints (12), (16) etc., to give a satisfactory quantum field theory. The resulting supergraph rules have been developed[15] and lead to superpropagators with inverse powers of ∂_z. This leads to the exciting possibility of proving a non-renormalisation theorem for SYM and SGR theories. We base this on the fact that the equations of motion for N= 2 HYM, N= 2 and 4 SYM and N= 8 SGR are all of the form (14). It has been shown that after performing the integration over the Grassmann variables the N= 2 SYM action is a total derivative ∂_z[12]; we expect all the actions we are concerned with to become total derivatives $\partial_z{}^{2N}$. From the nature of the superpropagators we also expect radiative corrections to be total derivatives $\partial_z{}^r$ with $r < 2N$. The results of ultra-violet finiteness on- or off-shell for the various theories are as given in the table.

Table

Theory \ r	$0 < r < 2N$	$r = 0$	on- or off-shell
N SYM	$\Delta g = 0$	$\Delta g = 0$	off-shell
N SGR	Finite, $\Delta\kappa = 0$	Unknown, $\Delta\kappa = 0$	on-shell

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