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Review

Topological tensor invariants and the current algebra approach: analysis of 196 nonleptonic two-body decays of single and double charm baryons – a review

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Abstract We shed new light on the standard current algebra approach to the nonleptonic two-body decays of single and double heavy charm baryons. By making use of the completeness relation for the flavor wave functions of the ground state baryon **20**$'$ representation we are able to rewrite the results of the current algebra approach in terms of the seven topological tensor invariants describing the decays. The representation of the current algebra results in terms of topological tensor invariants depends only on the initial and final state of the process. The summation over intermediate states inherent to the current algebra result is automatically taken care of in the tensor representation. In this way one arrives at a new, quick and very compact assessment of the results of the current algebra/pole model calculation. For example, one can quickly identify the decays in which the p.v. S-wave amplitude is predicted to vanish implying zero polarization asymmetries in these decays. We provide tables of the values of the seven topological tensor invariants for all Cabibbo favored and singly and doubly Cabibbo suppressed nonleptonic single and double charm baryon decays. In total we treat 196 charm baryon decays. We also discuss the charm preserving $\Delta C = 0$ decays of single charm baryons and the usual hyperon decays.

1 Introduction

The experimental landscape of charm baryon decays has considerably changed in 2015 through the detection of Λ_c^+ pair production by the BESIII collaboration at the Beijing e^+e^- collider BEPCII [1]. Not only was it now possible to determine the absolute branching ratios of various nonleptonic Λ_c decays through tagging techniques (see also Ref. [2]), but the BESIII collaboration could also determine the asymmetry parameters of the nonleptonic two-body decays $\Lambda_c^+ \to pK_S^0,\ \Lambda^0\pi^+,\ \Sigma^+\pi^0$ and $\Sigma^0\pi^+$ [3] (see also the review [4]). The decay asymmetry parameter of the decay $\Xi_c^0 \to \Xi^-\pi^+$ was recently measured by the Belle collaboration with the result $\alpha_{\Xi_c^0\to\Xi^-\pi^+} = -0.60 \pm 0.04$ [5].

Both the absolute branching ratios and the asymmetry parameters provide essential input for attempts to theoretically understand the nonleptonic two-body decays of charm baryons. The first Cabibbo suppressed decay $\Lambda_c \to p\,\phi$ was measured by the CLEO collaboration in 1995 [6], followed by more precise measurements of this mode later on [7,8]. At a later stage the BESIII collaboration measured the singly Cabibbo suppressed decay $\Lambda_c^+ \to p\,\eta$ [9], later on made more precise in Ref. [10] including an improved upper limit on the Cabibbo suppressed decay $\Lambda_c^+ \to p\,\pi^0$. By using double tag techniques, the BESIII collaboration also identified the decay $\Lambda_c^+ \to nK_s^0\pi^+$ involving a neutron [11] which raises the hope that final states including a neutron can be measured in the future. A considerably larger data sample of Λ_c^+ pairs is expected in the near future at BEPCII. The rate of Λ_c^+ pairs is to be increased by a factor of 16 from a total of about 10^5 to 1.6×10^6 Λ_c^+ pairs [12] which will considerably enlarge the data sample of Λ_c^+ decays. At the same time an approved energy upgrade will increase the energy of BEPCII to 4.9 GeV, opening the possibility of Σ_c pair production. If the energy of the BEPCII could be further increased to a little above 4.95 GeV one could also detect the production of Ξ_c^+ and Ξ_c^0 pairs.

Absolute values of the branching fractions of the Ξ_c^+ and Ξ_c^0 have recently become available again from the BELLE collaboration through tagging techniques in the decays $B^- \to \bar\Lambda_c^-\,\Xi_c^0$ and $B^0 \to \bar\Lambda_c^-\,\Xi_c^+$ [13,14]. Using similar techniques one could also determine the absolute val-

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ues of the branching fractions of the decays of the Ω_c^0 e.g. in the kinematically accessible decay $B^- \to \Omega_c^0\,\overline{\Xi_c^+}$ [15]. This would be quite welcome since the absolute branching ratios of the decays of the Ω_c^0 are experimentally unknown at present.

The LHCb collaboration has shown that it is now even possible to detect doubly Cabibbo suppressed (DCS) nonleptonic charm baryon decays by their measurement of the decay $\Xi_c^+ \to p\phi$ [16]. Furthermore, the LHCb collaboration recently provided results on a branching ratio measurement of the charm conserving $\Delta C = 0$ singly Cabibbo suppressed (SCS) decay $\Xi_c^0 \to \Lambda_c^+\pi^-$ [17]. Finally, the LHCb collaboration has extended the scope of possible charm baryon decay measurements by presenting results on the nonleptonic double charm baryon two-body decay $\Xi_{cc}^{++} \to \Xi_c^+\pi^+$ [18].

It is apparent that the experimentalists have provided us with a rich sample of results on exclusive charm baryons decays with the hope of much more to come. This has led to an upsurge of interest in charm baryon decays in the theoretical community documented by the many recent papers on this subject.

Basic to the current–current (more precisely coined current×current) quark model description of nonleptonic charm baryon two-body decays are the five generic quark diagrams depicted in Fig. 1 which are also termed topological diagrams. Diagrams Ia and Ib are usually referred to as factorizable diagrams or tree diagrams while diagrams IIa, IIb and III are referred to as nonfactorizable or W-exchange diagrams. In this paper we concentrate on the structure of the W-exchange diagrams IIa, IIb and III which we relate to the s- and u-channel contributions of the current algebra approach.

The theoretical papers on exclusive two-body charm baryons decays roughly fall into three classes which differ by their dynamical input. In the simplest case one exploits SU(3) symmetry to relate the amplitudes of different charm baryon decays [19–29]. An open issue is whether to apply SU(3) symmetry to the invariant or to the helicity amplitudes of these decays. This will make a big difference for decays involving the η and η' mesons. The SU(3) approach has been extended to include SU(3) symmetry breaking effects [20,27]. Even more insight can be gained by incorporating the topological diagram approach [30–33] into the SU(3) analysis where the topological diagrams are characterized by the five diagrams Fig. 1 described above. It is, however, not sufficient to simply associate a given decay with its accompanying topological diagrams, but one has to calculate the appropriate weight factors for each decay. The SU(3) fit in Ref. [34] using topological diagrams must therefore be considered to be invalid.

In the constituent quark model approach one attempts to calculate the nonleptonic transitions represented by Fig. 1 in terms of constituent quark model transitions. While diagrams Ia, Ib and IIb are directly accessible to a constituent quark model calculation [35,36], diagrams IIa and III involve the creation of a quark pair from the vacuum in addition to the constituent quark model transitions. In the covariantized constituent quark model (CCQM) calculations [30,37] the quark pair creation from the vacuum is effectively described by the $^3P_0$ model where the strength of the $^3P_0$ interaction is fixed by a fit to the data. Both of the above papers are based on an approach developed much earlier by Hussain and Rotelli [38] and by Körner and Gudehus [39] for studying nonleptonic hyperon decays. In the covariant confined quark model calculations of Refs. [40–42] the five diagrams in Fig. 1 are interpreted as multiloop Feynman diagrams with nonlocal interactions describing the three- and four-point hadron-quark vertices.

Even more dynamics is added in the standard current algebra plus pole model approach [21,43–54]. This approach has its roots in the description of nonleptonic hyperon decays developed in the 1960s [55–58] which is based on current algebra and the soft pion theorem. Even if the criterion of a soft pseudoscalar meson is not always satisfied in the nonleptonic decays of single and double charm baryons (in exception are the $\Delta C = 0$ single charm baryon decays), the current algebra approach to charm baryon decays provides a definite calculational framework which can be tested against experiment. In a recent series of papers Cheng and collaborators presented a detailed analysis of the nonleptonic decays of single and double charm baryons using a variant of the current algebra approach in which the purported current algebra contributions of the topological diagram III in Fig. 1 are dropped [59–62]. The authors of Ref. [36] make use of the current algebra approach in addition to calculating the diagram I and IIb transitions in the constituent quark model. This possibly amounts to a double counting of the contribution from the W-exchange diagram IIb.

Our paper is structured as follows. In Sect. 2 we write down the effective Hamiltonian for the nonleptonic current–current transition. In Sect. 3 we discuss issues of SU(4) and SU(3) relevant to the nonleptonic single and double charm baryon decays. In particular, we introduce the 20 ground state baryons comprising the $\mathbf{20'}$ representation of SU(4) and write down the third rank flavor tensors that are associated with them. We present the orthonormality and completeness relation of the third rank flavor tensors of the $\mathbf{20'}$ representation. We introduce a minimal set of seven topological tensor invariants in flavor space, built from the tensor representations of the ground state baryons, the ground state pseudoscalar mesons, and the effective Hamiltonian. The seven tensor invariants are each associated with one of the corresponding topological quark diagrams.

In Sect. 4 we present tables of the values of the topological tensor invariants for the antitriplet and sextet single charm

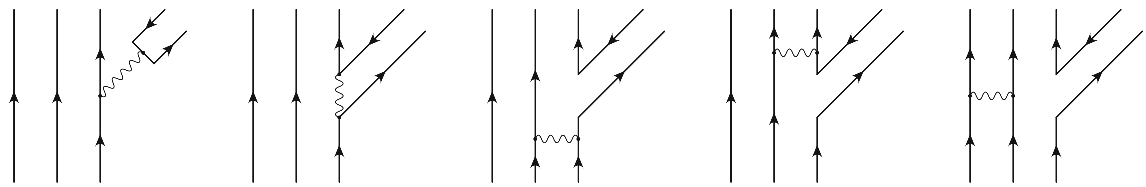


Fig. 1 The topological diagrams contributing to the nonleptonic charm baryon decays

baryon decays, the $\Delta C = 0$ single charm baryon decays, the triplet double charm baryon decays to the antitriplet and sextet single charm baryons, and the double charm baryon decays to the light baryon octet and heavy D, D_s mesons, each for the Cabibbo favored (CF), singly Cabibbo suppressed (SCS) and doubly Cabibbo suppressed (DCS) decays. Altogether we list the values of the topological tensor invariants for 196 single and double nonleptonic charm baryon decays. For each class of decays we explicate the linear relations among the associated tensor invariants. For completeness we also list the values of the tensor invariants for the ordinary nonleptonic hyperon decays.

In Sect. 5 we briefly recapitulate the current algebra plus pole model approach to nonleptonic charm baryon decays where we closely follow the presentation and notation of Refs. [59–62]. Using the completeness relation for the ground state baryons we explicitly derive the relation of the so-called s-channel and u-channel contributions of the current algebra approach to the topological tensor invariants. The representation of the current algebra results in terms of topological tensor invariants depends only on the initial and final state of the process. The summation over intermediate states inherent to the current algebra result is automatically taken care of in the tensor representation. In Sect. 6 we discuss several explicit examples, leading to the exposure of general features of the topological tensor and current algebra approaches in Sect. 7. Sect. 8 contains our summary and outlook, while technical issues are treated in five Appendices.

In Appendix A we list explicit values of the tensor components of the ground state baryons, the ground state pseudoscalar mesons, and the effective Hamiltonian. Appendix B contains the derivation of the completeness relation for the members of the **20**$'$ representation. In Appendix C we define a set of flavor tensor invariants describing the strong and weak transitions $\langle B_f M|B_i\rangle$ and $\langle B_f|\mathcal{H}_{\rm eff}|B_i\rangle$ involving the charm baryon states, the values of which are needed for the comparison of the current algebra and the topological diagram approaches. We provide tables of the values of the strong and weak tensor invariants needed for the example cases discussed in the main text. In Appendix D we clarify the implications and shortcomings of the variant of the current algebra approach introduced in Refs. [59–62]. In Appendix E we are dealing with the spin kinematics.

2 The effective weak current–current Hamiltonian

The $\Delta C = 1$ effective Hamiltonian for the Cabibbo favored (CF) decays is given by

$$\mathcal{H}_{\rm eff} = \frac{G_F}{2\sqrt{2}} V_{cs} V_{ud}^* \left(c_+ \mathcal{O}_+ + c_- \mathcal{O}_-\right) + H.c. \tag{1}$$

where the four-quark operators read

$$\mathcal{O}_\pm = (\bar{s}c)(\bar{u}d) \pm (\bar{u}c)(\bar{s}d) \tag{2}$$

with $(\bar{q}_1 q_2) = \bar{q}_1 \gamma_\mu (1-\gamma_5) q_2$.

The $\Delta C = 1$ singly Cabibbo suppressed (SCS) decays are induced by two distinct effective Hamiltonians which are labeled by a and b. They read

$$\mathcal{H}_{\rm eff}(a) = \frac{G_F}{2\sqrt{2}} V_{cs} V_{us}^* \left(c_+ \mathcal{O}_+(a) + c_- \mathcal{O}_-(a)\right) + H.c. \tag{3}$$

$$\mathcal{H}_{\rm eff}(b) = \frac{G_F}{2\sqrt{2}} V_{cd} V_{ud}^* \left(c_+ \mathcal{O}_+(b) + c_- \mathcal{O}_-(b)\right) + H.c. \tag{4}$$

with $\mathcal{O}_\pm(a) = (\bar{s}c)(\bar{u}s) \pm (\bar{u}c)(\bar{s}s)$ and $\mathcal{O}_\pm(b) = (\bar{d}c)(\bar{u}d) \pm (\bar{u}c)(\bar{d}d)$, respectively. Using e.g. the Wolfenstein parametrization one finds that $V_{cs}V_{us}^* = -V_{cd}V_{ud}^* + \mathcal{O}(\lambda^4)$ from unitarity, i.e. to a very good approximation one has $V_{cs}V_{us}^* = -V_{cd}V_{ud}^*$ which we shall always use.

The $\Delta C = 1$ doubly Cabibbo suppressed decays (DCS) are induced by the effective Hamiltonian

$$\mathcal{H}_{\rm eff}(c) = \frac{G_F}{2\sqrt{2}} V_{cd} V_{us}^* \left(c_+ \mathcal{O}_+(c) + c_- \mathcal{O}_-(c)\right) + H.c. \tag{5}$$

where $\mathcal{O}_\pm(c) = (\bar{d}c)(\bar{u}s) \pm (\bar{u}c)(\bar{d}s)$.

Finally, the $\Delta C = 0$ charm baryon decays are induced by the two SCS effective Hamiltonians

$$\mathcal{H}_{\rm eff}(a') = \frac{G_F}{2\sqrt{2}} V_{us} V_{ud}^* \left(c_+ \mathcal{O}_+(a') + c_- \mathcal{O}_-(a')\right) + H.c. \tag{6}$$

$$\mathcal{H}_{\rm eff}(b') = \frac{G_F}{2\sqrt{2}} V_{cd} V_{cs}^* \left(c_+ \mathcal{O}_+(b') + c_- \mathcal{O}_-(b')\right) + H.c. \tag{7}$$

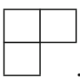
with $\mathcal{O}_\pm(a') = (\bar{u}s)(\bar{d}u) \pm (\bar{d}s)(\bar{u}u)$ and $\mathcal{O}_\pm(b') = (\bar{d}c)(\bar{c}s) \pm (\bar{c}c)(\bar{d}s)$, respectively. From unitarity one has $V_{cd}V_{cs}^* = -V_{us}V_{ud}^*$ which again is true up to $\mathcal{O}(\lambda^4)$ in the Wolfenstein parametrization.

In this paper we will mostly be concerned with the so-called W-exchange contributions which, according to the Körner–Pati–Woo (KPW) theorem [63,64], are related to the operators $\mathcal{O}_- = (\bar{q}_1q_2)(\bar{q}_3q_4) - (\bar{q}_3q_2)(\bar{q}_1q_4)$ only, i.e. induced by the effective Hamiltonian

$$\mathcal{H}_{\rm eff}(\mathcal{O}_-) = \frac{G_F}{2\sqrt{2}} V_{q_2q_1} V_{q_3q_4}^* \left(c_-\mathcal{O}_-\right) + H.c. \tag{8}$$

3 SU(4), SU(3) and topological tensor invariants

The 20 ground state baryons with spin parity quantum numbers $J^P = 1/2^+$ make up the **20'** representation of SU(4) associated with the Young tableaux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In the $C = 0$ sector one has the usual light baryon octet comprised of the eight light baryons (p, n, Λ^0, Σ^+, Σ^0, Σ^-, Ξ^+, Ξ^0). The $C = 1$ single charm sector contains the antitriplet charm baryons (Λ_c^+, Ξ_c^+, Ξ_c^0) and the sextet charm baryons (Σ_c^+, Σ_c^0, Σ_c^-, $\Xi_c'^+$, $\Xi_c'^0$, Ω_c^0).

In HQET (or in the spectator quark model) the nine $C = 1$ antitriplet and sextet single charm baryons can be viewed as the 21 members of the **21** representation of the approximate light spin–flavor symmetry group SU(6) with the subgroup SU(2)×SU(3). The decomposition reads

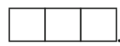
$$\mathbf{21} \subset \mathbf{1} \otimes \bar{\mathbf{3}} \oplus \mathbf{3} \otimes \mathbf{6} \tag{9}$$

where **1** and **3** denote the $J = 0$ and $J = 1$ representations of spin SU(2) with dimensions $2J + 1$. The SU(6) spin–flavor wave functions of the antitriplet and sextet single charm baryons can be found e.g. in Ref. [60].

Finally, the $C = 2$ double charm baryons are made up by the triplet of the double heavy charm baryons (Ξ_{cc}^{++}, Ξ_{cc}^+, Ω_{cc}^+). In Table 1 we list the quark content, the quantum numbers and, when available, the experimental mass values [65] of the 12 charm baryon states. The masses of the double charm states Ξ_{cc}^+ and Ω_{cc}^+ have not been measured yet. For Ξ_{cc}^+ we assume equality of the mass value with Ξ_{cc}^{++}, dropping, however, the fifth digit and the error specification. For Ω_{cc}^+ we list the mass prediction of Refs. [37,66] based on the one-gluon exchange model of de Rujula, Georgi and Glashow which features a Breit–Fermi spin–spin interaction term [67]. In as much as the model prediction (dated from 1992) for the mass of the Ξ_{cc} iso-doublet of 3621 MeV is very close to the measured value of 3621.2 ± 0.7 MeV [65], we feel quite confident about the predicted mass of the Ω_{cc}^+ listed in Table 1 based on the same model calculation [37,66].

In the charm sector the single charm baryons $\Sigma_c^{++,+,0}$ decay dominantly via one-pion emission while the dominant decay modes of the $\Xi_c'^+$ and $\Xi_c'^0$ are the one-photon emission modes. The remaining seven single and double charm baryons (Λ_c^+, Ξ_c^+, Ξ_c^0, Ω_c^0, Ξ_{cc}^{++}, Ξ_{cc}^+, Ω_{cc}^+) decay via weak interactions. An important class of these weak charm baryon decays are their nonleptonic two-body decays into a ground state baryon with $J^P = 1/2^+$ and a pseudoscalar meson with $J^P = 0^-$. The two-body nonleptonic decays of the seven single and double charm baryons (Λ_c^+, Ξ_c^+, Ξ_c^0, Ω_c^0, Ξ_{cc}^{++}, Ξ_{cc}^+, Ω_{cc}^+) are the subject of this paper.

In this paper we will not be concerned with the 20 $C = 0, 1, 2, 3$ $J^P = 3/2^+$ ground state baryons which, in SU(4), belong to the **20** representation with the associated Young tableaux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The properties of the **20** representation are discussed in Ref. [66]. In principle, the states of the **20** representation could contribute as intermediate states in the current algebra plus pole model approach discussed in Sect. 5 but are prevented from contributing as intermediate states by the KPW theorem [63,64] which states that the contraction of the flavor antisymmetric current–current operator with a flavor symmetric final state configuration is zero in the SU(3) limit.

It is quite illuminating to represent the baryon flavor wave functions as third rank tensors B_{abc} in flavor space instead of the second rank tensors B_a^b frequently used in the literature. In this way one can keep track of the quark content of the baryons. Moreover, in the transitions involving baryons one can identify the flavor flow in the transitions which directly leads to the concept of topological diagrams and the topological tensor invariants associated with them. Explicit representations of the third rank flavor tensors of the twenty $J^P = 1/2^+$ ground state baryons are given in Appendix A. For the ground state baryons and mesons we have used the phase conventions of Lichtenberg [68].

The flavor wave functions of the $J^P = 1/2^+$ ground state baryons satisfy a Jacobi-type identity which reads

$$B^\ell_{a[bc]} + B^\ell_{b[ca]} + B^\ell_{c[ab]} = 0, \tag{10}$$

for each of the flavor wave functions of the ground state multiplet, where $\ell = 1, \ldots, 8$ in SU(3) and $\ell = 1, \ldots, 20$ in SU(4). They are normalized and orthogonal according to

$$\sum_{k,m,n} B_\ell^{k[mn]} B^{\ell'}_{k[mn]} = \delta_\ell^{\ell'}. \tag{11}$$

Table 1 Ground state charm baryon states with $J^P = 1/2^+$. The square and curly brackets $[ab]$ and $\{ab\}$ denote antisymmetric and symmetric flavor label combinations

Notation	Quark content	SU(3)	(I, I_3)	S	C	Mass (MeV)
Λ_c^+	$c[ud]$	$\bar{\mathbf{3}}$	(0, 0)	0	1	2286.46 ± 0.14
Ξ_c^+	$c[su]$	$\bar{\mathbf{3}}$	(1/2, 1/2)	−1	1	2467.95 ± 0.19
Ξ_c^0	$c[sd]$	$\bar{\mathbf{3}}$	(1/2, −1/2)	−1	1	2470.99 ± 0.40
Σ_c^{++}	cuu	$\mathbf{6}$	(1, 1)	0	1	2453.97 ± 0.14
Σ_c^+	$\{cud\}$	$\mathbf{6}$	(1, 0)	0	1	2452.9 ± 0.4
Σ_c^0	cdd	$\mathbf{6}$	(1, −1)	0	1	2453.75 ± 0.14
$\Xi_c^{\prime +}$	$c\{su\}$	$\mathbf{6}$	(1/2, 1/2)	−1	1	2578.4 ± 0.5
$\Xi_c^{\prime 0}$	$c\{sd\}$	$\mathbf{6}$	(1/2, −1/2)	−1	1	2579.2 ± 0.5
Ω_c^0	css	$\mathbf{6}$	(0, 0)	−2	1	2695.2 ± 1.7
Ξ_{cc}^{++}	ccu	$\mathbf{3}$	(1/2, 1/2)	0	2	3621.2 ± 0.7
Ξ_{cc}^+	ccd	$\mathbf{3}$	(1/2, −1/2)	0	2	3621
Ω_{cc}^+	ccs	$\mathbf{3}$	(0, 0)	−1	2	3710

Furthermore, they satisfy the completeness relation

$$\sum_\ell B^\ell_{k[mn]} B_\ell^{b[cd]} = \frac{2}{6}(\delta^b_k \delta^c_m \delta^d_n - \delta^b_k \delta^d_m \delta^c_n) - \frac{1}{6}(\delta^b_m \delta^c_n \delta^d_k - \delta^b_m \delta^d_n \delta^c_k) - \frac{1}{6}(\delta^b_n \delta^c_k \delta^d_m - \delta^b_n \delta^d_k \delta^c_m). \quad (12)$$

Contracting the completeness relation (12) with $\delta^k_b\, \delta^m_c\, \delta^n_d$ one obtains the dimension $d = N(N^2-1)/3$ of the ground state representation in SU(N) which agrees with the dimension of the representation calculated according to the hook rule with the result

$$\dim\left(\begin{array}{|c|c|}\hline & \\ \hline & \multicolumn{1}{c}{} \\ \cline{1-1}\end{array}\right) = \frac{N(N+1)(N-1)}{3\cdot 1\cdot 1} = \frac{1}{3}N(N^2-1). \quad (13)$$

One thus obtains $d = 8$ and $d = 20$ for SU(3) and SU(4), respectively, as expected.

The completeness relation (12) is central to the chain of reasoning in Sect. 5 which, in the limit of SU(3) and the absence of hyperfine interactions, links the results of the current algebra approach to a linear superposition of topological reduced matrix elements with coefficients given by the topological tensor invariants. For the interested reader we provide an explicit proof of the completeness relation in Appendix B. Using the completeness relation we never invoke SU(4) symmetry but rather make use of the SU(3) symmetry of the separate $C = 0$, $C = 1$ and $C = 2$ sectors of the completeness relation (12). For a given flavor configuration defined by the in and out states, the sum over intermediate states ℓ in Eq. (12) extends either over one of the ground state baryons or, in the case of the flavor degenerate pairs of states (Λ^0, Σ^0), $(\Lambda_c^+, \Sigma_c^+)$, $(\Xi_c^0, \Xi_c^{\prime 0})$ and $(\Xi_c^+, \Xi_c^{\prime +})$, over maximally two ground state baryons. In most of the cases analyzed in this paper, due to symmetry and constituent quark model considerations only one state of the flavor degenerate pair of states contribute as intermediate state. When both flavor degenerate states contribute to a given intermediate state we assume mass degeneracy of the two states to sum the two contributions. More details can be found in Sect. 5.

In flavor space the ground state to ground state transitions $1/2^+ \to 1/2^+ + 0^-(1^-)$ induced by the antisymmetric flavor-changing tensor $H^{[q_2q_4]}_{[q_1q_3]}$ representing the effective Hamiltonian $\mathcal{H}_{\text{eff}}(\mathcal{O}_-)$ in Eq. (8) are represented by the seven topological tensor invariants

$$\begin{aligned}
I_1^-(\ell,\ell') &= B_\ell^{a[bc]} B^{\ell'}_{a[bc']} M^d_{d'} H^{[c'd']}_{[cd]}\\
I_2^-(\ell,\ell') &= B_\ell^{a[bc]} B^{\ell'}_{b[c'a]} M^d_{d'} H^{[c'd']}_{[cd]}\\
I_3(\ell,\ell') &= B_\ell^{a[bc]} B^{\ell'}_{a[b'c']} M^d_c H^{[c'b']}_{[db]}\\
I_4(\ell,\ell') &= B_\ell^{b[ca]} B^{\ell'}_{a[b'c']} M^d_c H^{[c'b']}_{[db]}\\
\hat{I}_3(\ell,\ell') &= B_\ell^{a[bc]} B^{\ell'}_{a[b'c']} M^{c'}_d H^{[db']}_{[cb]}\\
\hat{I}_4(\ell,\ell') &= B_\ell^{a[bc]} B^{\ell'}_{b'[c'a]} M^{c'}_d H^{[db']}_{[cb]}\\
I_5(\ell,\ell') &= B_\ell^{a[bc]} B^{\ell'}_{a'[b'c']} M^{c'}_c H^{[a'b']}_{[ab]}
\end{aligned} \quad (14)$$

for the baryonic transitions $\ell' \to \ell$, where the fourth rank flavor changing tensor $H^{[q_2q_4]}_{[q_1q_3]}$ takes the values ± 1 as specified in Appendix A. A summation over doubly occurring indices in Eq. (14) is implied. In the present application in which we consider various SU(3) subsector transitions, the summation runs over the three light quarks ($u = 1, d = 2, s = 3$) and the charm quark which we label as $c = 4$. After having used the Jacobi identity (10) and/or by rearranging tensor labels including the exchange of dummy summation indices, the set of seven tensor invariants comprise the minimal set of possible tensor contractions. For example, the tensor invariant $I'_5(\ell,\ell') = B^{a[bc]}_{\ell} B^{\ell'}_{b'[c'a']} M^{c'}_{c} H^{[a'b']}_{[ab]}$ is redundant since it can be seen to be equal to $I_5(\ell',\ell)$ by simply rearranging the tensor labels. The list (14) contains only connected tensor contractions. We have thus not included the tadpole-type unconnected tensor contraction $B^{a[bc]}_{\ell} B^{\ell'}_{a[bc]} M^{d}_{d'} H^{[cd']}_{[cd]}$ which vanishes for the $\Delta C = 1$ transitions since $H^{[cd']}_{[cd]} = 0$ (see Appendix A).

In Table 2 the SU(3) and SU(2) properties of the fourth rank flavor changing tensor are listed. Note that each of the tensors $H^{[su]}_{[cs]}$ and $H^{[du]}_{[cd]}$ inducing the SCS transition transforms separately as $\mathbf{6} \oplus \bar{\mathbf{3}}$ in SU(3). Only their difference $H^{[su]}_{[cs]} - H^{[du]}_{[cd]}$ transforms as the $\mathbf{6}$ representation in SU(3) as can be explicitly verified by using the Clebsch–Gordan tables of Kaeding [69]. In Fig. 2 we display the weight diagrams of the $\mathbf{6}$ and $\bar{\mathbf{3}}$ representations where we have marked the locations of the CF, SCSa, SCSb and DCS effective quark transitions in the two weight diagrams. When taking the difference of the two SCS contributions one remains with a U-spin triplet at the left boundary of the $\mathbf{6}$ weight diagram. We have somewhat dwelt on this point because we have not found an adequate discussion concerning this cancellation in the literature.

We mention that the reduction to a minimal set of tensor invariants has not been attained in Refs. [31,33]. As an example we take the Cabibbo enhanced decays of the antitriplet charm baryons. The authors of Ref. [33] introduce six and three topological tensor invariants each for the topology classes IIa and III where only two and one are needed (I_3, I_4 and I_5, resp.) while Kohara [31] defines four and one topological tensor invariants for the same topology classes IIa and III where, as before, only the two invariants I_3, I_4 and the invariant I_5 are needed. This makes it quite cumbersome to compare mutual results. Note that the distinction of whether the strange quark from the charm quark transition ends up in the meson or the baryon for the topology class IIa, as done in Ref. [33], is not warranted in SU(3).

Let us briefly pause to discuss the charge conjugation properties of the seven tensor invariants (14). In flavor space the charge conjugation operation raises or lowers the tensor indices, i.e. one has $\bar{B}^{a[bc]}_{\ell} = B^{\ell}_{a[bc]}$, $\bar{B}^{\ell}_{a[bc]} = B^{a[bc]}_{\ell}$, $\bar{M}^{b}_{a} = M^{a}_{b}$, and $\bar{H}^{[cd]}_{[ab]} = H^{[ab]}_{[cd]}$. As an example we consider the charge conjugation operation on the tensor invariant I_3. One has

$$\bar{I}_3(\ell,\ell') = B^{\ell}_{a[bc]} B^{a[b'c']}_{\ell'} M^{c}_{d} H^{[db]}_{[c'b']} = \hat{I}_3(\ell',\ell). \tag{15}$$

Similarly one has $\bar{I}_4(\ell,\ell') = \hat{I}_4(\ell',\ell)$, $\bar{I}^{-}_{1,2}(\ell,\ell') = I^{-}_{1,2}(\ell',\ell)$, and $\bar{I}_5(\ell,\ell') = I_5(\ell',\ell)$. Therefore, there are two antisymmetric and five symmetric tensor combinations under the flavor charge conjugation operation. The two antisymmetric combinations are

$$(\bar{I}_3 - \bar{\hat{I}}_3)(\ell,\ell') = -(I_3 - \hat{I}_3)(\ell',\ell),$$
$$(\bar{I}_4 - \bar{\hat{I}}_4)(\ell,\ell') = -(I_4 - \hat{I}_4)(\ell',\ell). \tag{16}$$

They would contribute to the parity violating (p.v.) amplitude A of Sect. 5. The five symmetric combinations read

$$\begin{aligned}
(\bar{I}_3 + \bar{\hat{I}}_3)(\ell,\ell') &= +(I_3 + \hat{I}_3)(\ell',\ell)\\
(\bar{I}_4 + \bar{\hat{I}}_4)(\ell,\ell') &= +(I_4 + \hat{I}_4)(\ell',\ell)\\
\bar{I}^{-}_1(\ell,\ell') &= I^{-}_1(\ell',\ell)\\
\bar{I}^{-}_2(\ell,\ell') &= I^{-}_2(\ell',\ell)\\
\bar{I}_5(\ell,\ell') &= I_5(\ell',\ell).
\end{aligned} \tag{17}$$

They would contribute to the parity conserving (p.c.) amplitude B of Sect. 5. Note, however, that the dynamics of the tree diagram contributions leads to an extra factor of $(m_1 - m_2)$ in the case of the p.v. amplitude A which implies that the tree diagram invariants also contribute to the amplitude A.

Returning to the topological analysis one can associate each of the tensor invariants (14) with one of the topological diagrams in Fig. 1 by following the flavor flow in the diagrams. It is important to realize that it is not sufficient to just associate a given decay with a given set of topological diagrams. Instead, one needs to calculate the projection of that given decay onto the topological diagrams in terms of the topological tensor invariants. The association reads

$$\begin{aligned}
(I^{-}_1, I^{-}_2) &\longleftrightarrow \text{diagrams Ia, Ib}\\
(I_3, I_4) &\longleftrightarrow \text{diagram IIa}
\end{aligned} \tag{18}$$

$$\begin{aligned}
(\hat{I}_3, \hat{I}_4) &\longleftrightarrow \text{diagram IIb}\\
I_5 &\longleftrightarrow \text{diagram III.}
\end{aligned} \tag{19}$$

Note that this relation is not one-to-one. For example, from the flavor flow the decay $\Lambda^+_c \to \Xi^0 K^+$ can be seen to be contributed to by the topology IIa. However, from an explicit calculation one finds that only the topological tensor invariant I_4 becomes populated. This has important ramifications for the structure of the decay as will be discussed in Sect. 4.

To conclude, in flavor space a general amplitude $\mathcal{A}_{fki}$ describing the decay $B_i \to B_f + M_k$ can be expanded along seven topological reduced amplitudes $\mathcal{T}_j$ with coefficients given by the seven topological tensor invariants $I^{-}_1, \ldots, I_5$. One has

$$\mathcal{A}(B_i \xrightarrow{H} B_f M_k) = \mathcal{A}_{fki} = \sum_j I^{j}_{fki}\, \mathcal{T}_j$$
$$j = 1^-, 2^-, 3, 4, \hat{3}, \hat{4}, 5 \tag{20}$$

where the B_i and B_f belong to some given SU(3) representation. The $\mathcal{T}_j$ are the topological invariant amplitudes of the process while the tensor invariants I^{j}_{fki} project onto these invariant amplitudes. Put in a different language, the

Table 2 SU(3) and SU(2) properties of the effective $\Delta C = 1$ and $\Delta C = 0$ Hamiltonian $\mathcal{H}_{\text{eff}}(\mathcal{O}_-)$ in Eq. (8), represented in flavor space by the fourth rank tensor $H^{[q_2q_4]}_{[q_1q_3]}$

		Y	I_3	$SU(3)$	ΔI	ΔU	ΔU_3	ΔV
CF	$H^{[su]}_{[cd]}$	2/3	−1	**6**	1	1	1	0
SCS	$H^{[su]}_{[cs]}$	−1/3	−1/2	$\mathbf{6}\oplus\bar{\mathbf{3}}$	1/2	1, 0	0, 0	1/2
	$H^{[du]}_{[cd]}$	−1/3	−1/2	$\mathbf{6}\oplus\bar{\mathbf{3}}$	1/2	1, 0	0, 0	1/2
	$H^{[su]}_{[cs]} - H^{[du]}_{[cd]}$	−1/3	−1/2	**6**	1/2	1	0	1/2
DCS	$H^{[du]}_{[cs]}$	−4/3	0	**6**	0	1	−1	1
$\Delta C = 0$	$H^{[ud]}_{[su]}$	−1	1/2	**8**	1/2	1	−1	1/2
	$H^{[dc]}_{[cs]}$	−1	1/2	**8**	1/2	1	−1	1/2

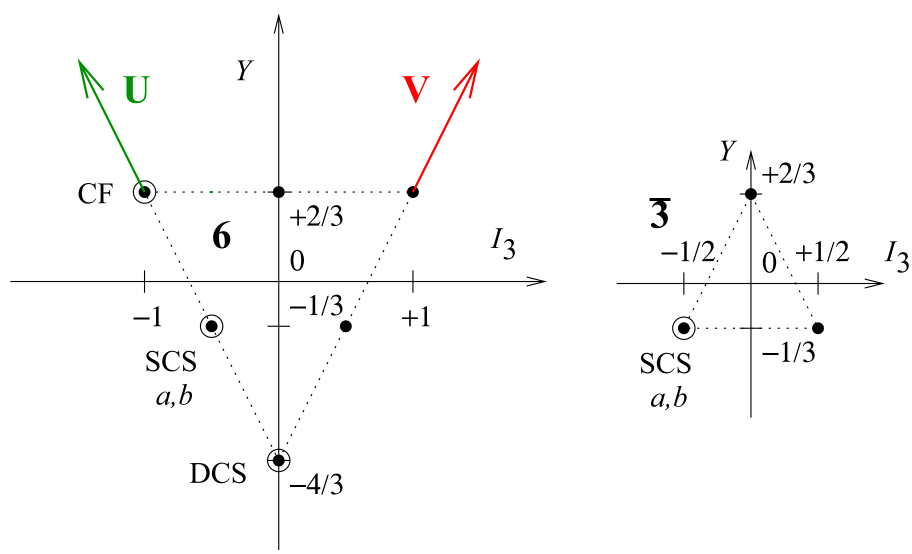

Fig. 2 Weight diagrams of the sextet (left) and antitriplet (right) representation of the effective weak Hamiltonian. The locations of the CF, SCS and DCS transitions are marked by a circle dot symbol ⊙

topological tensor invariants I^j_{fki} would correspond to SU(3) Clebsch–Gordan coefficients while the reduced matrix elements $\mathcal{T}_j$ would correspond to the SU(3) invariant amplitudes. All SU(3) relations between given transition amplitudes including the subclasses of I, U and V sum rules are implicit in the expansion (20).

In the general case one is overcounting the number of tensor invariants in the expansion (20), i.e. the number of significant SU(3) invariants can generally be less than the number of seven SU(3) invariants in Eq. (20). In general, the rank of the coefficient matrix I^j_{fki} linking $\mathcal{A}_{fki}$ with $\mathcal{T}_j$ is less than seven. This implies that one cannot, in general, determine the reduced topological amplitudes $\mathcal{T}_j$ from a given set of experimentally measured amplitudes $\mathcal{A}_{fki}$. In such a case there will be a number of linear relations among the seven tensor invariants I^j_{fki} which will be commented on and written down for the different SU(3) transitions treated in this paper. Some of the linear relations involve tensor invariants of the same topology class. In such a case the linear relations can be obtained by combining the information on the symmetry or antisymmetry of the (u, d, s) light quark components of a given class of charm baryons with the use of the Jacobi identity. The remaining linear relations between tensor invariants involving different topology classes cannot be obtained by mere tensor manipulations. We conjecture that one needs Schouten type identities to derive these additional tensor identities. In this paper we use linear algebra methods to construct these additional linear relations explicitly. With the help of the linear relations between tensor invariants one can recombine the topological invariant amplitudes to a minimal set of significant (nontrivial) topological invariant amplitudes. However, it should be clear that this set of minimal reduced amplitudes is not unique.

The flavor space tensor contractions represent a convenient way of calculating Clebsch–Gordon coefficients entering in nonleptonic charm baryon decays. They were first introduced in Ref. [30] which also contains some supplementary background material. The values of the topological tensor invariants for all Cabibbo favored (CF), singly Cabibbo suppressed (SCS) and doubly Cabibbo suppressed (DCS) nonleptonic charm baryon decays needed in this paper are for instance given in Tables 3, 4, 5, 6 and 7.

The transition to the isoscalar states are written in terms of the ideally mixed states $\eta_\omega = \frac{1}{\sqrt{2}}(u\bar{u}+d\bar{d})$ and $\eta_\phi = s\bar{s}$. For each decay we have factored out the products of the denominator factors appearing in the flavor space wave functions, the inverse of which appear as factors multiplying the amplitude of the respective decays.

4 Tables of the topological tensor invariants

In this section we proceed to discuss the various classes of charm baryon decays. They are classified according to the SU(3) transitions that specify the decays. Even though we do not discuss the factorizable contributions, we also list the values of the tensor invariants I_1^- and I_2^- since they enter in the linear relations between the seven $\mathcal{O}^-$ induced tensor invariants. The corresponding tensor invariants I_1^+ and I_2^+ induced by the operator $\mathcal{O}^+$ can be easily obtained from I_1^- and I_2^- as described in Appendix A.

For the transitions involving the isospin zero neutral meson states we separately list results for the octet state $\eta_8 = (u\bar{u} + d\bar{d} - 2s\bar{s})/\sqrt{6}$ and the singlet state $\eta_1 = (u\bar{u} + d\bar{d} + s\bar{s})/\sqrt{3}$. The physical η and η' states are linear superpositions of these two states according to

$$\eta = \cos\theta\eta_8 - \sin\theta\eta_1 \qquad \eta' = \sin\theta\eta_8 + \cos\theta\eta_1 \tag{21}$$

with $\theta = -15.4°$ [70,71]. We also list the transitions into the ideally mixed states $\eta_\omega = (u\bar{u} + d\bar{d})/\sqrt{2}$ and $\eta_\phi = s\bar{s}$ with obvious applications to the corresponding $1/2^+ \to 1/2^+ + 1^-$ transitions involving the ideally mixed vector states ω and ϕ.

4.1 The charm baryon decays $B_c(\bar{\mathbf{3}}) \xrightarrow{H(\mathbf{6})} B(\mathbf{8}) + M(\mathbf{8},\mathbf{1})$

The nonleptonic two-body decays of the three antitriplet charm baryons (Λ_c^+, Ξ_c^+, Ξ_c^0) belong to this class of decays. The SU(3) decomposition of the transitions read (we use the notation of Kaeding for multiple SU(3) representations [69])

$$\bar{\mathbf{3}} \to \mathbf{6}\otimes\mathbf{8}\otimes\mathbf{8} = 3\cdot\bar{\mathbf{3}} \oplus 4\cdot\mathbf{6} \oplus 5\cdot\overline{\mathbf{15}} \oplus \overline{\mathbf{15}}' \oplus \overline{\mathbf{21}} \oplus \overline{\mathbf{24}} \oplus 2\cdot\overline{\mathbf{42}} \oplus \overline{\mathbf{60}}. \tag{22}$$

As explained in Sect. 3, the effective Hamiltonian inducing the W-exchange contributions transforms as a sextet in SU(3). For the SCS decays this comes about by a cancellation of the antitriplet contributions when the two effective Hamiltonians $\mathcal{H}_{\rm eff}(c \to s;\ s \to u)$ and $\mathcal{H}_{\rm eff}(c \to d;\ d \to u)$ are subtracted from one another. The $\bar{\mathbf{3}}$ representation appears three times in the decomposition. Therefore, there are three SU(3) invariant amplitudes. This in turn implies that there are four linear relations among the seven topological tensor invariants. They read

$$I_1^- = I_2^-, \qquad 2\hat{I}_3 + \hat{I}_4 = 0,$$
$$I_3 + I_4 = 2I_5, \qquad 2I_1^- = I_3 + \hat{I}_3. \tag{23}$$

The relation $I_1^- = I_2^-$ can be obtained by rewriting the tensor labels of the parent baryon in the tensor invariant I_2^- in the way $B^1_{b[c'a]} \to -B^1_{a[c'b]} \to B^1_{a[bc']}$ by realizing that the indices a and b refer to light quarks. Similarly one can proof $2\hat{I}_3 + \hat{I}_4 = 0$ by realizing that a and c' are the light quarks in the parent baryon $B^1_{b'[c'a]}$ and using the Jacobi identity $B^1_{b'[c'a]} + B^1_{c'[ab']} + B^1_{a[b'c']} = 0$.

A more direct access to a minimal set of independent tensor contractions is to switch to second rank tensor representations of the antitriplet charm baryons, the octet baryons and the effective Hamiltonian [19,29]. As a result one now has only three independent tensor contractions, the number of which agrees with the above number of SU(3) invariants. The linear relations (23) can be seen to be in agreement with the analysis of Jia et al. [29].

Equations (23) are very useful when checking the results for the topological tensor invariants $I_j(fki)$ that will be listed later on. Note in particular that the linear relations (23) do not separately hold for the contributions of the two transitions (a) $c \to s;\ s \to u$ and (b) $c \to d;\ d \to u$. They can be seen to hold true only for the sum of the two contributions. Note that the last equation does not hold for all decay processes listed in Tables 3, 4, 5, 6 and 7, in particular not for decays into η_1 and, related to that, into η_ω and η_ϕ, as those are members of (a mixture of $M(\mathbf{8})$ and) $M(\mathbf{1})$.

Therefore, we also need to discuss the transition $B_c(\bar{\mathbf{3}}) \xrightarrow{H(\mathbf{6})} B(\mathbf{8}) + M(\mathbf{1})$ involving the singlet meson state $M(\mathbf{1})$. In this case the SU(3) decomposition reads

$$\bar{\mathbf{3}} \to \mathbf{6}\otimes\mathbf{8}\otimes\mathbf{1} = \bar{\mathbf{3}} \oplus \mathbf{6} \oplus \overline{\mathbf{15}} \oplus \overline{\mathbf{24}}. \tag{24}$$

Thus there is only one SU(3) invariant amplitude describing the transition $B_c(\bar{\mathbf{3}}) \xrightarrow{H(\mathbf{6})} B(\mathbf{8}) + M(\mathbf{1})$ which means that all transitions involving a singlet meson state $M(\mathbf{1})$ are proportional to one another which is manifest in the decays into η_1. In the corresponding rows of Tables 3, 4, 5, 6 and 7 one has

$$I_1^- = I_2^- = 0, \qquad I_3 = -2I_4 = 2\hat{I}_3 = -\hat{I}_4 = 4I_5. \tag{25}$$

The vanishing of the tree diagram invariants can be argued again directly, as the single quark line transition $\mathbf{1} \to \mathbf{6}\otimes\mathbf{1}\otimes\mathbf{3} = \mathbf{8}\oplus\mathbf{10}$ into singlet meson states vanishes in SU(3).

We list the values of the seven topological tensor invariants separately for the Cabibbo favored (CF) (Table 3), singly Cabibbo suppressed (SCS) (Tables 4, 5 and 6), and doubly Cabibbo suppressed (DCS) (Table 7) antitriplet charm baryon decays. The SCS decays are induced by the two SCS transitions (a) $c \to s;\ s \to u$ and (b) $c \to d;\ d \to u$ which are listed separately in Tables 4 and 5. As explained in Sect. 2, in

Table 3 Values of the seven topological tensor invariants for the antitriplet charm baryon decays $B_c(\bar{\mathbf{3}}) \to B(\mathbf{8}) + M(\mathbf{8}, \mathbf{1})$ induced by the CF flavor transitions ($c \to s;\ d \to u$). We have always factored out the products of the normalizing denominator factors appearing in the flavor space quark model wave functions. The product of the denominator factors appear as overall factors of the process specification in column 2

				I_1^-	I_2^-	I_3	I_4	$\hat{I}_3$	$\hat{I}_4$	I_5
CF	$12\,\Lambda_c^+$	$\to$	$\Lambda^0\pi^+$	-2	-2	-2	$+4$	-2	$+4$	$+1$
	$4\sqrt{3}\,\Lambda_c^+$	$\to$	$\Sigma^0\pi^+$	0	0	$+2$	0	-2	$+4$	$+1$
	$4\sqrt{3}\,\Lambda_c^+$	$\to$	$\Sigma^+\pi^0$	0	0	-2	0	$+2$	-4	-1
	$4\sqrt{3}\,\Lambda_c^+$	$\to$	$\Sigma^+\eta_\omega$	0	0	-2	0	-2	$+4$	-1
	$2\sqrt{6}\Lambda_c^+$	$\to$	$\Sigma^+\eta_\phi$	0	0	-2	$+2$	0	0	0
	$12\Lambda_c^+$	$\to$	$\Sigma^+\eta_8$	0	0	$+2$	-4	-2	$+4$	-1
	$6\sqrt{2}\Lambda_c^+$	$\to$	$\Sigma^+\eta_1$	0	0	-4	$+2$	-2	$+4$	-1
	$2\sqrt{6}\,\Lambda_c^+$	$\to$	$p\bar{K}^0$	$+1$	$+1$	$+2$	-2	0	0	0
	$2\sqrt{6}\,\Lambda_c^+$	$\to$	Ξ^0K^+	0	0	0	-2	0	0	-1
	$2\sqrt{6}\,\Xi_c^+$	$\to$	$\Sigma^+\bar{K}^0$	$+1$	$+1$	0	0	$+2$	-4	0
	$2\sqrt{6}\,\Xi_c^+$	$\to$	$\Xi^0\pi^+$	-1	-1	0	0	-2	$+4$	0
	$12\,\Xi_c^0$	$\to$	$\Lambda^0\bar{K}^0$	-1	-1	-4	$+2$	$+2$	-4	-1
	$4\sqrt{3}\,\Xi_c^0$	$\to$	$\Sigma^0\bar{K}^0$	$+1$	$+1$	0	-2	$+2$	-4	-1
	$2\sqrt{6}\,\Xi_c^0$	$\to$	Σ^+K^-	0	0	0	$+2$	0	0	$+1$
	$4\sqrt{3}\,\Xi_c^0$	$\to$	$\Xi^0\pi^0$	0	0	$+2$	-2	-2	$+4$	0
	$4\sqrt{3}\,\Xi_c^0$	$\to$	$\Xi^0\eta_\omega$	0	0	$+2$	-2	$+2$	-4	0
	$2\sqrt{6}\,\Xi_c^0$	$\to$	$\Xi^0\eta_\phi$	0	0	$+2$	0	0	0	$+1$
	$12\,\Xi_c^0$	$\to$	$\Xi^0\eta_8$	0	0	-2	-2	$+2$	-4	-2
	$6\sqrt{2}\,\Xi_c^0$	$\to$	$\Xi^0\eta_1$	0	0	$+4$	-2	$+2$	-4	$+1$
	$2\sqrt{6}\,\Xi_c^0$	$\to$	$\Xi^-\pi^+$	-1	-1	-2	$+2$	0	0	0

the SU(3) limit dealt with in this paper one has to subtract the two contributions (a) and (b). The result of this subtraction is shown in Table 6.

As discussed in Sect. 3, Kohara has introduced four reduced matrix elements d_1, d_2, d_3, d_4 for the topology class IIa where only two are needed [31]. Using the results of e.g. Table 3 for the decays involving octet mesons one finds that the four topological reduced matrix elements of Kohara are related to our topological reduced matrix elements by $d_1 = 2\mathcal{T}_3, d_2 = -2(\mathcal{T}_3 - \mathcal{T}_4), d_3 = 2\mathcal{T}_4$, and $d_4 = 2\mathcal{T}_3 - \mathcal{T}_4$. This shows again that the set $d_1,\ d_2,\ d_3,\ d_4$ is redundant since one has $d_1 = d_3 + d_4$ and $d_2 = -d_4$. We do not agree with Ref. [31] on the contributions of the d_i to the decays involving the SU(3) singlet meson η_1.

4.2 The charm baryon decays $B_c(\mathbf{6}) \xrightarrow{H(\mathbf{6})} B(\mathbf{8}) + M(\mathbf{8}, \mathbf{1})$

The nonleptonic two-body decays of the charm baryon Ω_c^0 belong to this class of decays. The SU(3) decomposition of the direct product $\mathbf{6} \otimes \mathbf{8} \otimes \mathbf{8}$ has been written down before in Eq. (22). One notes that the $\mathbf{6}$ representation appears four times in the decomposition. The transition $\mathbf{6} \to \mathbf{6} \otimes \mathbf{8} \otimes \mathbf{8}$ is thus described by four SU(3) invariants. This implies that there are $7 - 4 = 3$ linear relations among the 7 tensor invariants. These are

$$I_1^- + I_2^- = 0, \qquad \hat{I}_4 = 0, \qquad 2I_1^- = I_3 + \hat{I}_3. \tag{26}$$

The first relation $I_1^- + I_2^- = 0$ follows from the Jacobi identity rewriting I_2^- as

$$\begin{aligned} I_2^-(\ell, \ell') &= B_\ell^{a[bc]} B_{b[c'a]}^{\ell'} M_{d'}^{d} H_{[cd]}^{[c'd']} \\ &= B_\ell^{a[bc]} (-B_{c'[ab]}^{\ell'} - B_{a[bc']}^{\ell'}) M_{d'}^{d} H_{[cd]}^{[c'd']} \end{aligned} \tag{27}$$

and noting that the first term on the r.h.s. of Eq. (27) vanishes due to the fact that c' must be the charm quark, and that the light quarks a, b are symmetric in the $\mathbf{6}$ representation. With the same reasoning one can show that $\hat{I}_4 = 0$. The relation $2I_1^- = I_3 + \hat{I}_3$ cannot be obtained by tensor label manipulations but can be derived from the values of the three respective tensor invariants in Table 8. In the case of the SCS transitions the suffixes a and b identify the origin of the respective flavor transitions (a) and (b) including the relative sign. Note that we always keep the (a)- and (b)-type contributions in the SCS contribution apart for two reasons:

Table 4 Values of the seven topological tensor invariants for the SCS antitriplet charm baryon decays $B_c(\bar{\mathbf{3}}) \to B(\mathbf{8}) + M(\mathbf{8}, \mathbf{1})$ induced by the flavor transition $(c \to s;\ s \to u)$

				I_1^-	I_2^-	I_3	I_4	$\hat{I}_3$	$\hat{I}_4$	I_5
SCSa	$12\,\Lambda_c^+$	$\to$	$\Lambda^0 K^+$	+2	+2	0	0	+2	−4	0
	$4\sqrt{3}\,\Lambda_c^+$	$\to$	$\Sigma^0 K^+$	0	0	0	0	+2	−4	0
	$2\sqrt{6}\,\Lambda_c^+$	$\to$	$\Sigma^+ K^0$	0	0	0	0	−2	+4	0
	$2\sqrt{6}\,\Lambda_c^+$	$\to$	$n\pi^+$	0	0	0	0	0	0	0
	$4\sqrt{3}\,\Lambda_c^+$	$\to$	$p\pi^0$	0	0	0	0	0	0	0
	$4\sqrt{3}\,\Lambda_c^+$	$\to$	$p\eta_\omega$	0	0	0	0	0	0	0
	$2\sqrt{6}\,\Lambda_c^+$	$\to$	$p\eta_\phi$	−1	−1	0	0	0	0	0
	$12\,\Lambda_c^+$	$\to$	$p\eta_8$	+2	+2	0	0	0	0	0
	$6\sqrt{2}\,\Lambda_c^+$	$\to$	$p\eta_1$	−1	−1	0	0	0	0	0
	$2\sqrt{6}\,\Xi_c^+$	$\to$	$\Xi^0 K^+$	+1	+1	0	−2	+2	−4	−1
	$4\sqrt{3}\,\Xi_c^+$	$\to$	$\Sigma^+ \pi^0$	0	0	−2	0	0	0	−1
	$4\sqrt{3}\,\Xi_c^+$	$\to$	$\Sigma^+ \eta_\omega$	0	0	−2	0	0	0	−1
	$2\sqrt{6}\,\Xi_c^+$	$\to$	$\Sigma^+ \eta_\phi$	−1	−1	−2	+2	−2	+4	0
	$12\,\Xi_c^+$	$\to$	$\Sigma^+ \eta_8$	+2	+2	+2	−4	+4	−8	−1
	$6\sqrt{2}\,\Xi_c^+$	$\to$	$\Sigma^+ \eta_1$	−1	−1	−4	+2	−2	+4	−1
	$2\sqrt{6}\,\Xi_c^+$	$\to$	$p\bar{K}^0$	0	0	+2	−2	0	0	0
	$12\,\Xi_c^+$	$\to$	$\Lambda^0 \pi^+$	0	0	−2	+4	0	0	+1
	$4\sqrt{3}\,\Xi_c^+$	$\to$	$\Sigma^0 \pi^+$	0	0	+2	0	0	0	+1
	$2\sqrt{6}\,\Xi_c^0$	$\to$	$\Xi^0 K^0$	0	0	0	0	+2	−4	−1
	$2\sqrt{6}\,\Xi_c^0$	$\to$	$\Xi^- K^+$	+1	+1	0	−2	0	0	0
	$2\sqrt{6}\,\Xi_c^0$	$\to$	$\Sigma^+ \pi^-$	0	0	0	0	0	0	−1
	$2\sqrt{6}\,\Xi_c^0$	$\to$	$\Sigma^- \pi^+$	0	0	+2	0	0	0	0
	$4\sqrt{6}\,\Xi_c^0$	$\to$	$\Sigma^0 \pi^0$	0	0	−2	0	0	0	+1
	$4\sqrt{6}\,\Xi_c^0$	$\to$	$\Sigma^0 \eta_\omega$	0	0	−2	0	0	0	−1
	$4\sqrt{3}\,\Xi_c^0$	$\to$	$\Sigma^0 \eta_\phi$	−1	−1	−2	+2	−2	+4	0
	$12\sqrt{2}\,\Xi_c^0$	$\to$	$\Sigma^0 \eta_8$	+2	+2	+2	−4	+4	−8	−1
	$12\,\Xi_c^0$	$\to$	$\Sigma^0 \eta_1$	−1	−1	−4	+2	−2	+4	−1
	$12\sqrt{2}\,\Xi_c^0$	$\to$	$\Lambda^0 \pi^0$	0	0	−2	+4	0	0	+1
	$12\sqrt{2}\,\Xi_c^0$	$\to$	$\Lambda^0 \eta_\omega$	0	0	−2	+4	0	0	−1
	$12\,\Xi_c^0$	$\to$	$\Lambda^0 \eta_\phi$	+1	+1	−2	−2	−2	+4	0
	$12\sqrt{6}\,\Xi_c^0$	$\to$	$\Lambda^0 \eta_8$	−2	−2	+2	+8	+4	−8	−1
	$12\sqrt{3}\,\Xi_c^0$	$\to$	$\Lambda^0 \eta_1$	+1	+1	−4	+2	−2	+4	−1
	$2\sqrt{6}\,\Xi_c^0$	$\to$	pK^-	0	0	0	−2	0	0	0
	$2\sqrt{6}\,\Xi_c^0$	$\to$	$n\bar{K}^0$	0	0	+2	0	0	0	0

in order to incorporate some SU(3) breaking effects in this way, and for comparison with parts of the literature in which the two contributions have been kept apart. Note that the tensor relation $2I_1^- = I_3 + \hat{I}_3$ does not hold separately for the transitions (a) and (b) but only for their sum, and it does not hold if the singlet meson state is involved.

The transitions involving the singlet meson state η_1 are again described by a single SU(3) invariant, as can be read off from the decomposition (24). For these transitions on finds $I_1^- = I_2^- = \hat{I}_4 = 0$ and $I_3 = -2I_4 = 2\hat{I}_3 = 4I_5$.

4.3 The $\Delta C = 0$ singly Cabibbo suppressed charm baryon decays $B_c(\bar{\mathbf{3}}) \xrightarrow{H(\mathbf{8})} B_c(\bar{\mathbf{3}}) + M(\mathbf{8})$ and $B_c(\mathbf{6}) \xrightarrow{H(\mathbf{8})} B_c(\bar{\mathbf{3}}) + M(\mathbf{8})$

Table 5 Values of the seven topological tensor invariants for the SCS antitriplet charm baryon decays $B_c(\bar{\mathbf{3}}) \rightarrow B(\mathbf{8}) + M(\mathbf{8}, \mathbf{1})$ induced by the flavor transition $(c \rightarrow d;\ d \rightarrow u)$

				I_1^-	I_2^-	I_3	I_4	$\hat{I}_3$	$\hat{I}_4$	I_5
SCSb	$12\,\Lambda_c^+$	$\rightarrow$	$\Lambda^0 K^+$	0	0	−2	−2	0	0	−2
	$4\sqrt{3}\,\Lambda_c^+$	$\rightarrow$	$\Sigma^0 K^+$	0	0	+2	−2	0	0	0
	$2\sqrt{6}\,\Lambda_c^+$	$\rightarrow$	$\Sigma^+ K^0$	0	0	−2	+2	0	0	0
	$2\sqrt{6}\,\Lambda_c^+$	$\rightarrow$	$n\pi^+$	−1	−1	0	+2	−2	+4	+1
	$4\sqrt{3}\,\Lambda_c^+$	$\rightarrow$	$p\pi^0$	+1	+1	0	−2	+2	−4	−1
	$4\sqrt{3}\,\Lambda_c^+$	$\rightarrow$	$p\eta_\omega$	−1	−1	−4	+2	−2	+4	−1
	$2\sqrt{6}\,\Lambda_c^+$	$\rightarrow$	$p\eta_\phi$	0	0	0	0	0	0	0
	$12\,\Lambda_c^+$	$\rightarrow$	$p\eta_8$	−1	−1	−4	+2	−2	+4	−1
	$6\sqrt{2}\,\Lambda_c^+$	$\rightarrow$	$p\eta_1$	−1	−1	−4	+2	−2	+4	−1
	$2\sqrt{6}\,\Xi_c^+$	$\rightarrow$	$\Xi^0 K^+$	0	0	0	0	0	0	0
	$4\sqrt{3}\,\Xi_c^+$	$\rightarrow$	$\Sigma^+\pi^0$	+1	+1	0	0	0	0	0
	$4\sqrt{3}\,\Xi_c^+$	$\rightarrow$	$\Sigma^+\eta_\omega$	−1	−1	0	0	0	0	0
	$2\sqrt{6}\,\Xi_c^+$	$\rightarrow$	$\Sigma^+\eta_\phi$	0	0	0	0	0	0	0
	$12\,\Xi_c^+$	$\rightarrow$	$\Sigma^+\eta_8$	−1	−1	0	0	0	0	0
	$6\sqrt{2}\,\Xi_c^+$	$\rightarrow$	$\Sigma^+\eta_1$	−1	−1	0	0	0	0	0
	$2\sqrt{6}\,\Xi_c^+$	$\rightarrow$	$p\bar{K}^0$	0	0	0	0	+2	−4	0
	$12\,\Xi_c^+$	$\rightarrow$	$\Lambda^0\pi^+$	−1	−1	0	0	−4	+8	0
	$4\sqrt{3}\,\Xi_c^+$	$\rightarrow$	$\Sigma^0\pi^+$	−1	−1	0	0	0	0	0
	$2\sqrt{6}\,\Xi_c^0$	$\rightarrow$	$\Xi^0 K^0$	0	0	+2	0	0	0	0
	$2\sqrt{6}\,\Xi_c^0$	$\rightarrow$	$\Xi^- K^+$	0	0	−2	0	0	0	0
	$2\sqrt{6}\,\Xi_c^0$	$\rightarrow$	$\Sigma^+\pi^-$	0	0	0	+2	0	0	0
	$2\sqrt{6}\,\Xi_c^0$	$\rightarrow$	$\Sigma^-\pi^+$	−1	−1	0	+2	0	0	0
	$4\sqrt{6}\,\Xi_c^0$	$\rightarrow$	$\Sigma^0\pi^0$	+1	+1	0	−4	0	0	0
	$4\sqrt{6}\,\Xi_c^0$	$\rightarrow$	$\Sigma^0\eta_\omega$	−1	−1	0	0	0	0	0
	$4\sqrt{3}\,\Xi_c^0$	$\rightarrow$	$\Sigma^0\eta_\phi$	0	0	0	0	0	0	0
	$12\sqrt{2}\,\Xi_c^0$	$\rightarrow$	$\Sigma^0\eta_8$	−1	−1	0	0	0	0	0
	$12\,\Xi_c^0$	$\rightarrow$	$\Sigma^0\eta_1$	−1	−1	0	0	0	0	0
	$12\sqrt{2}\,\Xi_c^0$	$\rightarrow$	$\Lambda^0\pi^0$	−1	−1	0	0	−4	+8	0
	$12\sqrt{2}\,\Xi_c^0$	$\rightarrow$	$\Lambda^0\eta_\omega$	+1	+1	+8	−4	+4	−8	0
	$12\,\Xi_c^0$	$\rightarrow$	$\Lambda^0\eta_\phi$	0	0	0	0	0	0	+2
	$12\sqrt{6}\,\Xi_c^0$	$\rightarrow$	$\Lambda^0\eta_8$	+1	+1	+8	−4	+4	−8	−4
	$12\sqrt{3}\,\Xi_c^0$	$\rightarrow$	$\Lambda^0\eta_1$	+1	+1	+8	−4	+4	−8	+2
	$2\sqrt{6}\,\Xi_c^0$	$\rightarrow$	pK^-	0	0	0	0	0	0	+1
	$2\sqrt{6}\,\Xi_c^0$	$\rightarrow$	$n\bar{K}^0$	0	0	0	0	+2	−4	−1

The $\Delta C = 0$ charm baryon decays are induced by the SCS quark flavor transitions $(a')\ s \rightarrow u;\ u \rightarrow d$ and $(b')\ c \rightarrow d;\ s \rightarrow c$. The kinematical constraints of the $\Delta C = 0$ decays only allow for the pionic modes. Therefore, there is no need to discuss the decays into the η and η' states or the K-meson states. As concerns the application of the current algebra approach to charm baryon decays, the $\Delta C = 0$ charm baryon decays are the favorites of all charm baryon decays since the emitted pion has very little energy, i.e. the pion satisfies the requirement of the soft pion theorem. The $\Delta C = 0$ charm baryon decays have been discussed before in Refs. [72–76].

There are two isospin decays in each the two classes $B_c(\bar{\mathbf{3}}) \xrightarrow{H(\mathbf{8})} B_c(\bar{\mathbf{3}}) + M(\mathbf{8})$ and $B_c(\mathbf{6}) \xrightarrow{H(\mathbf{8})} B_c(\bar{\mathbf{3}}) + M(\mathbf{8})$. The four kinematically accessible decays are listed in Table 9 together with the values of the topological tensor invariants. As mentioned in the Introduction, the decay $\Xi_c^0 \rightarrow \Lambda_c^+\pi^-$ has recently been observed [17].

Table 6 Values of the seven topological tensor invariants for the SCS antitriplet charm baryon decays $B_c(\bar{\mathbf{3}}) \to B(\mathbf{8}) + M(\mathbf{8}, \mathbf{1})$ induced by both flavor transitions (a) and (b)

				I_1^-	I_2^-	I_3	I_4	$\hat{I}_3$	$\hat{I}_4$	I_5
SCS	$12\,\Lambda_c^+$	$\to$	$\Lambda^0 K^+$	+2	+2	+2	+2	+2	−4	+2
	$4\sqrt{3}\,\Lambda_c^+$	$\to$	$\Sigma^0 K^+$	0	0	−2	+2	+2	−4	0
	$2\sqrt{6}\,\Lambda_c^+$	$\to$	$\Sigma^+ K^0$	0	0	+2	−2	−2	+4	0
	$2\sqrt{6}\,\Lambda_c^+$	$\to$	$n\pi^+$	+1	+1	0	−2	+2	−4	−1
	$4\sqrt{3}\,\Lambda_c^+$	$\to$	$p\pi^0$	−1	−1	0	+2	−2	+4	+1
	$4\sqrt{3}\,\Lambda_c^+$	$\to$	$p\eta_\omega$	+1	+1	+4	−2	+2	−4	+1
	$2\sqrt{6}\,\Lambda_c^+$	$\to$	$p\eta_\phi$	−1	−1	0	0	0	0	0
	$12\,\Lambda_c^+$	$\to$	$p\eta_8$	+3	+3	+4	−2	+2	−4	+1
	$6\sqrt{2}\,\Lambda_c^+$	$\to$	$p\eta_1$	0	0	+4	−2	+2	−4	+1
	$2\sqrt{6}\,\Xi_c^+$	$\to$	$\Xi^0 K^+$	+1	+1	0	−2	+2	−4	−1
	$4\sqrt{3}\,\Xi_c^+$	$\to$	$\Sigma^+\pi^0$	−1	−1	−2	0	0	0	−1
	$4\sqrt{3}\,\Xi_c^+$	$\to$	$\Sigma^+\eta_\omega$	+1	+1	−2	0	0	0	−1
	$2\sqrt{6}\,\Xi_c^+$	$\to$	$\Sigma^+\eta_\phi$	−1	−1	−2	+2	−2	+4	0
	$12\,\Xi_c^+$	$\to$	$\Sigma^+\eta_8$	+3	+3	+2	−4	+4	−8	−1
	$6\sqrt{2}\,\Xi_c^+$	$\to$	$\Sigma^+\eta_1$	0	0	−4	+2	−2	+4	−1
	$2\sqrt{6}\,\Xi_c^+$	$\to$	$p\bar{K}^0$	0	0	+2	−2	−2	+4	0
	$12\,\Xi_c^+$	$\to$	$\Lambda^0\pi^+$	+1	+1	−2	+4	+4	−8	+1
	$4\sqrt{3}\,\Xi_c^+$	$\to$	$\Sigma^0\pi^+$	+1	+1	+2	0	0	0	+1
	$2\sqrt{6}\,\Xi_c^0$	$\to$	$\Xi^0 K^0$	0	0	−2	0	+2	−4	−1
	$2\sqrt{6}\,\Xi_c^0$	$\to$	$\Xi^- K^+$	+1	+1	+2	−2	0	0	0
	$2\sqrt{6}\,\Xi_c^0$	$\to$	$\Sigma^+\pi^-$	0	0	0	−2	0	0	−1
	$2\sqrt{6}\,\Xi_c^0$	$\to$	$\Sigma^-\pi^+$	+1	+1	+2	−2	0	0	0
	$4\sqrt{6}\,\Xi_c^0$	$\to$	$\Sigma^0\pi^0$	−1	−1	−2	+4	0	0	+1
	$4\sqrt{6}\,\Xi_c^0$	$\to$	$\Sigma^0\eta_\omega$	+1	+1	−2	0	0	0	−1
	$4\sqrt{3}\,\Xi_c^0$	$\to$	$\Sigma^0\eta_\phi$	−1	−1	−2	+2	−2	+4	0
	$12\sqrt{2}\,\Xi_c^0$	$\to$	$\Sigma^0\eta_8$	+3	+3	+2	−4	+4	−8	−1
	$12\,\Xi_c^0$	$\to$	$\Sigma^0\eta_1$	0	0	−4	+2	−2	+4	−1
	$12\sqrt{2}\,\Xi_c^0$	$\to$	$\Lambda^0\pi^0$	+1	+1	−2	+4	+4	−8	+1
	$12\sqrt{2}\,\Xi_c^0$	$\to$	$\Lambda^0\eta_\omega$	−1	−1	−10	+8	−4	+8	−1
	$12\,\Xi_c^0$	$\to$	$\Lambda^0\eta_\phi$	+1	+1	−2	−2	−2	+4	−2
	$12\sqrt{6}\,\Xi_c^0$	$\to$	$\Lambda^0\eta_8$	−3	−3	−6	+12	0	0	+3
	$12\sqrt{3}\,\Xi_c^0$	$\to$	$\Lambda^0\eta_1$	0	0	−12	+6	−6	+12	−3
	$2\sqrt{6}\,\Xi_c^0$	$\to$	pK^-	0	0	0	−2	0	0	−1
	$2\sqrt{6}\,\Xi_c^0$	$\to$	$n\bar{K}^0$	0	0	+2	0	−2	+4	+1

For the $\Xi_c^{+,0}$ and Ω^0 decays the SU(3) decomposition reads

$$\bar{\mathbf{3}} \to \mathbf{8} \otimes \bar{\mathbf{3}} \otimes \mathbf{8} = 3 \cdot \bar{\mathbf{3}} \oplus 3 \cdot \mathbf{6} \oplus 4 \cdot \overline{\mathbf{15}} \oplus \overline{\mathbf{15}}' \oplus 2 \cdot \overline{\mathbf{24}} \oplus \overline{\mathbf{42}} \tag{28}$$

$$\mathbf{6} \to \mathbf{8} \otimes \bar{\mathbf{3}} \otimes \mathbf{8} = 3 \cdot \bar{\mathbf{6}} \oplus 3 \cdot \mathbf{6} \oplus 4 \cdot \overline{\mathbf{15}} \oplus \overline{\mathbf{15}}' \oplus 2 \cdot \overline{\mathbf{24}} \oplus \overline{\mathbf{42}} \tag{29}$$

One concludes that one should have four linear relations each among the seven tensor invariants. With the help of tensor manipulations alone one finds the relations $2I_3 = -I_4$ for both transitions $\bar{\mathbf{3}} \xrightarrow{H(\mathbf{8})} \bar{\mathbf{3}} + \mathbf{8}$ and $\mathbf{6} \xrightarrow{H(\mathbf{8})} \bar{\mathbf{3}} + \mathbf{8}$. The data base in Table 9 is not large enough to identify the remaining tensor identities. From a group theoretical point of view one could, of course, enlarge the data base by including also off-shell decays. We have not pursued this possible avenue.

While there are three SU(3) reduced matrix elements each for the two classes of decays there is only one isospin SU(2) reduced matrix element for each of the two classes of decays.

Table 7 Values of the seven topological tensor invariants for the DCS antitriplet charm baryon decays $B_c(\bar{\mathbf{3}}) \to B(\mathbf{8}) + M(\mathbf{8}, \mathbf{1})$ induced by the flavor transition $(c \to d;\ s \to u)$

				I_1^-	I_2^-	I_3	I_4	$\hat{I}_3$	$\hat{I}_4$	I_5
DCS	$2\sqrt{6}\,\Lambda_c^+$	$\to$	pK^0	−1	−1	0	0	−2	+4	0
	$2\sqrt{6}\,\Lambda_c^+$	$\to$	nK^+	+1	+1	0	0	+2	−4	0
	$12\,\Xi_c^+$	$\to$	$\Lambda^0 K^+$	+1	+1	−2	−2	+4	−8	−2
	$4\sqrt{3}\,\Xi_c^+$	$\to$	$\Sigma^0 K^+$	+1	+1	+2	−2	0	0	0
	$2\sqrt{6}\,\Xi_c^+$	$\to$	$\Sigma^+ K^0$	−1	−1	−2	+2	0	0	0
	$4\sqrt{3}\,\Xi_c^+$	$\to$	$p\pi^0$	0	0	0	−2	0	0	−1
	$2\sqrt{6}\,\Xi_c^+$	$\to$	$n\pi^+$	0	0	0	+2	0	0	+1
	$4\sqrt{3}\,\Xi_c^+$	$\to$	$p\eta_\omega$	0	0	−4	+2	0	0	−1
	$2\sqrt{6}\,\Xi_c^+$	$\to$	$p\eta_\phi$	0	0	0	0	−2	+4	0
	$12\,\Xi_c^+$	$\to$	$p\eta_8$	0	0	−4	+2	+4	−8	−1
	$6\sqrt{2}\,\Xi_c^+$	$\to$	$p\eta_1$	0	0	−4	+2	−2	+4	−1
	$12\,\Xi_c^0$	$\to$	$\Lambda^0 K^0$	+1	+1	−2	−2	+4	−8	−2
	$4\sqrt{3}\,\Xi_c^0$	$\to$	$\Sigma^0 K^0$	−1	−1	−2	+2	0	0	0
	$2\sqrt{6}\,\Xi_c^0$	$\to$	$\Sigma^- K^+$	+1	+1	+2	−2	0	0	0
	$2\sqrt{6}\,\Xi_c^0$	$\to$	$p\pi^-$	0	0	0	−2	0	0	−1
	$4\sqrt{3}\,\Xi_c^0$	$\to$	$n\pi^0$	0	0	0	+2	0	0	+1
	$4\sqrt{3}\,\Xi_c^0$	$\to$	$n\eta_\omega$	0	0	−4	+2	0	0	−1
	$2\sqrt{6}\Xi_c^0$	$\to$	$n\eta_\phi$	0	0	0	0	−2	+4	0
	$12\,\Xi_c^0$	$\to$	$n\eta_8$	0	0	−4	+2	+4	−8	−1
	$6\sqrt{2}\,\Xi_c^0$	$\to$	$n\eta_1$	0	0	−4	+2	−2	+4	−1

This becomes evident when doing the same exercise as in Eq. (28) but now for isospin SU(2). One finds

$$\begin{aligned}
&\mathbf{2} \to \mathbf{2} \otimes \mathbf{1} \otimes \mathbf{3} = \mathbf{2} \oplus \mathbf{4} \\
&\quad (\text{in spin notation } \tfrac{1}{2} \to \tfrac{1}{2} \otimes 0 \otimes 1 = \tfrac{1}{2} \oplus \tfrac{3}{2}), \\
&\quad \mathbf{1} \to \mathbf{2} \otimes \mathbf{2} \otimes \mathbf{3} = \mathbf{1} \oplus 2 \cdot \mathbf{3} \oplus \mathbf{5} \\
&\quad (0 \to \tfrac{1}{2} \otimes \tfrac{1}{2} \otimes 1 = 0 \oplus 2 \cdot 1 \oplus 2). \qquad (30)
\end{aligned}$$

This leads to the $\Delta I = 1/2$ isospin relations

$$\sqrt{2}\, M(\Xi_c^+ \to \Lambda_c^+ \pi^0) = M(\Xi_c^0 \to \Lambda_c^+ \pi^-) \qquad (31)$$

$$\sqrt{2}\, M(\Omega_c^0 \to \Xi_c^0 \pi^0) = -M(\Omega_c^0 \to \Xi_c^+ \pi^-) \qquad (32)$$

in agreement with the entries in Table 9.

4.4 The double charm baryon decays $B_{cc}(\mathbf{3}) \xrightarrow{H(\mathbf{6})} B_c(\bar{\mathbf{3}}) + M(\mathbf{8},\ \mathbf{1})$

The nonleptonic two-body decays of the triplet of double charm states Ξ_{cc}^{++}, Ξ_{cc}^{+} and Ω_{cc}^{+} into the antitriplet single charm baryons belong to this class of decays. Of the many possible two body decays only the decay $\Xi_{cc}^{++} \to \Xi_c^+ \pi^+$ has been identified to date [18].

The $\mathbf{3}$ representation occurs twice in the decomposition

$$\mathbf{3} \to \mathbf{6} \otimes \bar{\mathbf{3}} \otimes \mathbf{8} = 2 \cdot \mathbf{3} \oplus 2 \cdot \bar{\mathbf{6}} \oplus 3 \cdot \mathbf{15} \oplus \mathbf{15}' \oplus \mathbf{24} \oplus \mathbf{42} \qquad (33)$$

There are thus $7 - 2 = 5$ relations for the tensor invariants which read

$$\begin{aligned}
&I_1^- + 2I_2^- = 0, \qquad I_3 + 2I_4 = 0, \\
&\hat{I}_3 + \hat{I}_4 = 0, \qquad I_5 = 0, \qquad 2I_1^- = I_3 + \hat{I}_3
\end{aligned} \qquad (34)$$

As before, the last relation does not hold for the decay involving the octet singlet state. For the transitions involving the singlet meson state one finds

$$\mathbf{3} \to \mathbf{6} \otimes \bar{\mathbf{3}} \otimes \mathbf{1} = \mathbf{3} \oplus \mathbf{15} \qquad (35)$$

which implies that all transitions involving the singlet meson are proportional to each other with $I_1^- = I_2^- = I_5 = 0$ and $I_3 = -2I_4 = 2\hat{I}_3 = -2\hat{I}_4$. In Table 10 we list the values of the tensor invariants for all the decays of this class.

4.5 The double charm baryon decays $B_{cc}(\mathbf{3}) \xrightarrow{H(\mathbf{6})} B_c(\mathbf{6}) + M(\mathbf{8},\ \mathbf{1})$

The nonleptonic two-body decays of the double charm states Ξ_{cc}^{++}, Ξ_{cc}^{+} and Ω_{cc}^{+} into the sextet single charm baryons belong to this class of decays. Of the many possible decay modes none has been identified to date.

Table 8 Values of the seven topological tensor invariants for the non-leptonic CF, SCS and DCS decays of the sextet charm baryon state Ω_c^0 belonging to the class of decays $B_c(\mathbf{6}) \to B(\mathbf{8}) + M(\mathbf{8}, \mathbf{1})$. The SCS decays are induced by the flavor transition (a) $(c \to s;\ s \to u)$ and (b) $(c \to d;\ d \to u)$. The subscripts a and b of the tensor invariants identify the origin of the respective flavor transitions where the relative sign of the two transitions has been accounted for

				I_1^-	I_2^-	I_3	I_4	$\hat{I}_3$	$\hat{I}_4$	I_5
CF	$2\Omega_c^0$	$\to$	$\Xi^0 \bar{K}^0$	-1	$+1$	0	0	-2	0	0
SCS	$2\sqrt{2}\Omega_c^0$	$\to$	$\Xi^0 \pi^0$	$+1_b$	-1_b	$+2_a$	-2_a	0	0	0
	$2\sqrt{2}\Omega_c^0$	$\to$	$\Xi^0 \eta_\omega$	-1_b	$+1_b$	$+2_a$	-2_a	0	0	0
	$2\Omega_c^0$	$\to$	$\Xi^0 \eta_\phi$	$+1_a$	-1_a	$+2_a$	0	$+2_a$	0	$+1_a$
	$2\sqrt{6}\Omega_c^0$	$\to$	$\Xi^0 \eta_8$	$-2_a - 1_b$	$+2_a + 1_b$	-2_a	-2_a	-4_a	0	-2_a
	$2\sqrt{3}\Omega_c^0$	$\to$	$\Xi^0 \eta_1$	$+1_a - 1_b$	$-1_a + 1_b$	$+4_a$	-2_a	$+2_a$	0	$+1_a$
	$2\sqrt{6}\Omega_c^0$	$\to$	$\Lambda^0 \bar{K}^0$	0	0	-4_a	$+2_a$	$+4_b$	0	-1_a
	$2\sqrt{2}\Omega_c^0$	$\to$	$\Sigma^0 \bar{K}^0$	0	0	0	-2_a	0	0	-1_a
	$2\Omega_c^0$	$\to$	$\Sigma^+ K^-$	0	0	0	$+2_a$	0	0	$+1_a$
	$2\Omega_c^0$	$\to$	$\Xi^- \pi^+$	-1_b	$+1_b$	-2_a	$+2_a$	0	0	0
DCS	$2\Omega_c^0$	$\to$	$p K^-$	0	0	0	0	0	0	$+1$
	$2\Omega_c^0$	$\to$	$n \bar{K}^0$	0	0	0	0	0	0	-1
	$4\sqrt{3}\Omega_c^0$	$\to$	$\Lambda^0 \pi^0$	0	0	0	0	0	0	0
	$4\sqrt{3}\Omega_c^0$	$\to$	$\Lambda^0 \eta_\omega$	0	0	$+8$	-4	0	0	0
	$2\sqrt{6}\Omega_c^0$	$\to$	$\Lambda^0 \eta_\phi$	0	0	0	0	$+4$	0	$+2$
	$12\Omega_c^0$	$\to$	$\Lambda^0 \eta_8$	0	0	$+8$	-4	-8	0	-4
	$6\sqrt{2}\Omega_c^0$	$\to$	$\Lambda^0 \eta_1$	0	0	$+8$	-4	$+4$	0	$+2$
	$4\Omega_c^0$	$\to$	$\Sigma^0 \pi^0$	0	0	0	-4	0	0	0
	$4\Omega_c^0$	$\to$	$\Sigma^0 \eta_\omega$	0	0	0	0	0	0	0
	$2\sqrt{2}\Omega_c^0$	$\to$	$\Sigma^0 \eta_\phi$	0	0	0	0	0	0	0
	$4\sqrt{3}\Omega_c^0$	$\to$	$\Sigma^0 \eta_8$	0	0	0	0	0	0	0
	$2\sqrt{6}\Omega_c^0$	$\to$	$\Sigma^0 \eta_1$	0	0	0	0	0	0	0
	$2\Omega_c^0$	$\to$	$\Sigma^+ \pi^-$	0	0	0	$+2$	0	0	0
	$2\Omega_c^0$	$\to$	$\Sigma^- \pi^+$	0	0	0	$+2$	0	0	0
	$2\Omega_c^0$	$\to$	$\Xi^- K^+$	-1	$+1$	-2	0	0	0	0
	$2\Omega_c^0$	$\to$	$\Xi^0 K^0$	$+1$	-1	$+2$	0	0	0	0

The **3** representation occurs twice in the decomposition

$$\mathbf{3} \to \mathbf{6} \otimes \mathbf{6} \ \ \otimes \mathbf{8} = 2 \cdot \mathbf{3} \oplus 2 \cdot \overline{\mathbf{6}} \oplus 4 \cdot \mathbf{15} \oplus 2 \cdot \mathbf{15}' \oplus 2 \cdot \mathbf{24} \oplus 2 \cdot \mathbf{42} \oplus \mathbf{48}. \qquad (36)$$

Therefore, there are $7 - 2 = 5$ relations among the seven tensor invariants which can be derived by either tracking the charm quark flavor flow in the respective topological diagrams or by taking into account that the single charm baryons $B_c(\mathbf{6})$ are symmetric in the light quark indices. The relations read

$$I_1^- = I_3 = \hat{I}_3 = \hat{I}_4 = I_5 = 0, \qquad (37)$$

i.e. the only nonvanishing tensor invariants are I_2^- and I_4. In particular, the tensor invariants $\hat{I}_3$ and $\hat{I}_4$ are zero because of the KPW theorem. The KPW theorem holds only in the SU(3) limit. Violations of the KPW theorem due to the SU(3) constituent mass breaking effect $m_s \neq m_u$ were calculated in Ref. [77] for the decays $\Xi_{cc}^{++} \to \Xi_c^{\prime +} \pi^+$ and $\Omega_{cc}^+ \to \Xi_c^{\prime +} \bar{K}^0$ and turned out to be of the order of $(1-4)\%$. The authors of Ref. [59] obtained a mass breaking effect of the KPW theorem of the order of 2% in their bag model calculation of the same decays.

The singlet meson state **1** does not contribute to this class of decays as can be seen from the decomposition

$$\mathbf{3} \to \mathbf{6} \otimes \mathbf{6} \otimes \mathbf{1} = \overline{\mathbf{6}} \oplus \mathbf{15} \oplus \mathbf{15}'. \qquad (38)$$

In Table 11 we list the values of the seven topological flavor invariants.

Table 9 Values of the seven tensor invariants for the $\Delta C = 0$ SCS decays of the single charm baryons Ξ_c^+, Ξ_c^0, and Ω_c^0 into antitriplet charm baryons induced by the flavor transitions $(a')\ s \to u;\ u \to d$ and $(b')\ c \to d;\ s \to c$. The subscripts a' and b' of the tensor invariants identify the origin of the respective flavor transitions where the relative sign of the two transitions has been accounted for

			I_1^-	I_2^-	I_3	I_4	$\hat{I}_3$	$\hat{I}_4$	I_5
$12\sqrt{2}\Xi_c^+$	$\to$	$\Lambda_c^+\pi^0$	$-5_{a'}$	$+4_{a'}$	$-2_{b'}$	$4_{b'}$	$-8_{a'}$	$4_{a'}$	$1_{b'}$
$12\Xi_c^0$	$\to$	$\Lambda_c^+\pi^-$	$-5_{a'}$	$+4_{a'}$	$-2_{b'}$	$4_{b'}$	$-8_{a'}$	$4_{a'}$	$1_{b'}$
$2\sqrt{6}\,\Omega_c^0$	$\to$	$\Xi_c^+\pi^-$	$1_{a'}$	$-2_{a'}$	$+2_{b'}$	$-4_{b'}$	0	0	0
$4\sqrt{3}\,\Omega_c^0$	$\to$	$\Xi_c^0\pi^0$	$-1_{a'}$	$+2_{a'}$	$-2_{b'}$	$+4_{b'}$	0	0	0

Table 10 Values of the seven topological tensor invariants for the CF, SCS and DCS decays of the triplet double charm baryon states Ξ_{cc}^{++}, Ξ_{cc}^+ and Ω_{cc}^+ into the antitriplet single charm baryon states Λ_c^+, Ξ_c^+ and Ξ_c^0 belonging to the class of decays $B_{cc}(\mathbf{3}) \to B_c(\bar{\mathbf{3}}) + M(\mathbf{8}, \mathbf{1})$. The notation of the subscripts a and b is explained in Table 8

				I_1^-	I_2^-	I_3	I_4	$\hat{I}_3$	$\hat{I}_4$	I_5
CF	$2\sqrt{6}\,\Xi_{cc}^{++}$	$\to$	$\Xi_c^+\pi^+$	-2	$+1$	0	0	-4	$+4$	0
	$2\sqrt{6}\,\Xi_{cc}^+$	$\to$	$\Xi_c^0\pi^+$	-2	$+1$	-4	$+2$	0	0	0
	$4\sqrt{3}\,\Xi_{cc}^+$	$\to$	$\Xi_c^+\pi^0$	0	0	$+4$	-2	-4	$+4$	0
	$4\sqrt{3}\,\Xi_{cc}^+$	$\to$	$\Xi_c^+\eta_\omega$	0	0	$+4$	-2	$+4$	-4	0
	$2\sqrt{6}\,\Xi_{cc}^+$	$\to$	$\Xi_c^+\eta_\phi$	0	0	$+4$	-2	0	0	0
	$12\,\Xi_{cc}^+$	$\to$	$\Xi_c^+\eta_8$	0	0	-4	$+2$	$+4$	-4	0
	$6\sqrt{2}\,\Xi_{cc}^+$	$\to$	$\Xi_c^+\eta_1$	0	0	$+8$	-4	$+4$	-4	0
	$2\sqrt{6}\,\Xi_{cc}^+$	$\to$	$\Lambda_c^+\bar{K}^0$	-2	$+1$	-4	$+2$	0	0	0
	$2\sqrt{6}\,\Omega_{cc}^+$	$\to$	$\Xi_c^+\bar{K}^0$	-2	$+1$	0	0	-4	$+4$	0
SCS	$2\sqrt{6}\,\Xi_{cc}^{++}$	$\to$	$\Xi_c^+K^+$	$+2_a$	-1_a	0	0	$+4_a$	-4_a	0
	$2\sqrt{6}\,\Xi_{cc}^{++}$	$\to$	$\Lambda_c^+\pi^+$	$+2_b$	-1_b	0	0	$+4_b$	-4_b	0
	$2\sqrt{6}\,\Xi_{cc}^+$	$\to$	$\Xi_c^0K^+$	$+2_a$	-1_a	$+4_b$	-2_b	0	0	0
	$4\sqrt{3}\,\Xi_{cc}^+$	$\to$	$\Lambda_c^+\pi^0$	$+2_b$	-1_b	0	0	$+4_b$	-4_b	0
	$4\sqrt{3}\,\Xi_{cc}^+$	$\to$	$\Lambda_c^+\eta_\omega$	-2_b	$+1_b$	-8_b	$+4_b$	-4_b	$+4_b$	0
	$2\sqrt{6}\,\Xi_{cc}^+$	$\to$	$\Lambda_c^+\eta_\phi$	$+2_a$	-1_a	0	0	0	0	0
	$12\,\Xi_{cc}^+$	$\to$	$\Lambda_c^+\eta_8$	$-4_a - 2_b$	$+2_a + 1_b$	-8_b	$+4_b$	-4_b	$+4_b$	0
	$6\sqrt{2}\,\Xi_{cc}^+$	$\to$	$\Lambda_c^+\eta_1$	$+2_a - 2_b$	$-1_a + 1_b$	-8_b	$+4_b$	-4_b	$+4_b$	0
	$4\sqrt{3}\,\Omega_{cc}^+$	$\to$	$\Xi_c^+\pi^0$	$+2_b$	-1_b	$+4_a$	-2_a	0	0	0
	$4\sqrt{3}\,\Omega_{cc}^+$	$\to$	$\Xi_c^+\eta_\omega$	-2_b	$+1_b$	$+4_a$	-2_a	0	0	0
	$2\sqrt{6}\,\Omega_{cc}^+$	$\to$	$\Xi_c^+\eta_\phi$	$+2_a$	-1_a	$+4_a$	-2_a	$+4_a$	-4_a	0
	$12\,\Omega_{cc}^+$	$\to$	$\Xi_c^+\eta_8$	$-4_a - 2_b$	$+2_a + 1_b$	-4_a	$+2_a$	-8_a	$+8_a$	0
	$6\sqrt{2}\,\Omega_{cc}^+$	$\to$	$\Xi_c^+\eta_1$	$+2_a - 2_b$	$-1_a + 1_b$	$+8_a$	-4_a	$+4_a$	-4_a	0
	$2\sqrt{6}\,\Omega_{cc}^+$	$\to$	$\Xi_c^0\pi^+$	-2_b	$+1_b$	-4_a	$+2_a$	0	0	0
	$2\sqrt{6}\,\Omega_{cc}^+$	$\to$	$\Lambda_c^+\bar{K}^0$	0	0	-4_a	$+2_a$	$+4_b$	-4_b	0
DCS	$2\sqrt{6}\,\Xi_{cc}^{++}$	$\to$	$\Lambda_c^+K^+$	$+2$	-1	0	0	$+4$	-4	0
	$2\sqrt{6}\,\Xi_{cc}^+$	$\to$	$\Lambda_c^+K^0$	$+2$	-1	0	0	$+4$	-4	0
	$2\sqrt{6}\,\Omega_{cc}^+$	$\to$	$\Xi_c^0K^+$	-2	$+1$	-4	$+2$	0	0	0
	$2\sqrt{6}\Omega_{cc}^+$	$\to$	$\Xi_c^+K^0$	$+2$	-1	$+4$	-2	0	0	0
	$4\sqrt{3}\,\Omega_{cc}^+$	$\to$	$\Lambda_c^+\pi^0$	0	0	0	0	0	0	0
	$4\sqrt{3}\,\Omega_{cc}^+$	$\to$	$\Lambda_c^+\eta_\omega$	0	0	$+8$	-4	0	0	0
	$2\sqrt{6}\,\Omega_{cc}^+$	$\to$	$\Lambda_c^+\eta_\phi$	0	0	0	0	$+4$	-4	0
	$12\,\Omega_{cc}^+$	$\to$	$\Lambda_c^+\eta_8$	0	0	$+8$	-4	-8	$+8$	0
	$6\sqrt{2}\,\Omega_{cc}^+$	$\to$	$\Lambda_c^+\eta_1$	0	0	$+8$	-4	$+4$	-4	0

Table 11 Values of the seven tensor invariants for the CF, SCS and DCS decays of the triplet double charm baryon states Ξ_{cc}^{++}, Ξ_{cc}^{+}, and Ω_{cc}^{+} into sextet single baryon states, $B_{cc}(\mathbf{3}) \to B_c(\mathbf{6}) + M(\mathbf{8})$. The subscripts a and b for the SCS decays has been explained in Table 8. Decays into η_1 states are not listed since they vanish identically

				I_1^-	I_2^-	I_3	I_4	$\hat{I}_3$	$\hat{I}_4$	I_5
CF	$2\sqrt{2}\,\Xi_{cc}^{++}$	$\to$	$\Xi_c^{\prime +}\pi^+$	0	−1	0	0	0	0	0
	$2\,\Xi_{cc}^{++}$	$\to$	$\Sigma_c^{++}\bar{K}^0$	0	+1	0	0	0	0	0
	$2\sqrt{2}\,\Xi_{cc}^{+}$	$\to$	$\Xi_c^{\prime 0}\pi^+$	0	−1	0	+2	0	0	0
	$4\,\Xi_{cc}^{+}$	$\to$	$\Xi_c^{\prime +}\pi^0$	0	0	0	−2	0	0	0
	$4\,\Xi_{cc}^{+}$	$\to$	$\Xi_c^{\prime +}\eta_\omega$	0	0	0	−2	0	0	0
	$2\sqrt{2}\,\Xi_{cc}^{+}$	$\to$	$\Xi_c^{\prime +}\eta_\phi$	0	0	0	+2	0	0	0
	$4\sqrt{3}\,\Xi_{cc}^{+}$	$\to$	$\Xi_c^{\prime +}\eta_8$	0	0	0	−6	0	0	0
	$2\,\Xi_{cc}^{+}$	$\to$	$\Sigma_c^{++}K^-$	0	0	0	+2	0	0	0
	$2\sqrt{2}\,\Xi_{cc}^{+}$	$\to$	$\Sigma_c^{+}\bar{K}^0$	0	+1	0	−2	0	0	0
	$2\,\Xi_{cc}^{+}$	$\to$	$\Omega_c^{0}K^+$	0	0	0	−2	0	0	0
	$2\sqrt{2}\,\Omega_{cc}^{+}$	$\to$	$\Xi_c^{\prime +}\bar{K}^0$	0	+1	0	0	0	0	0
	$2\,\Omega_{cc}^{+}$	$\to$	$\Omega_c^{0}\pi^+$	0	−1	0	0	0	0	0
SCS	$2\sqrt{2}\,\Xi_{cc}^{++}$	$\to$	$\Xi_c^{\prime +}K^+$	0	$+1_a$	0	0	0	0	0
	$2\sqrt{2}\,\Xi_{cc}^{++}$	$\to$	$\Sigma_c^{+}\pi^+$	0	$+1_b$	0	0	0	0	0
	$2\sqrt{2}\,\Xi_{cc}^{++}$	$\to$	$\Sigma_c^{++}\pi^0$	0	-1_b	0	0	0	0	0
	$2\sqrt{2}\,\Xi_{cc}^{++}$	$\to$	$\Sigma_c^{++}\eta_\omega$	0	$+1_b$	0	0	0	0	0
	$2\,\Xi_{cc}^{++}$	$\to$	$\Sigma_c^{++}\eta_\phi$	0	-1_a	0	0	0	0	0
	$2\sqrt{6}\,\Xi_{cc}^{++}$	$\to$	$\Sigma_c^{++}\eta_8$	0	$+2_a+1_b$	0	0	0	0	0
	$2\sqrt{2}\,\Xi_{cc}^{+}$	$\to$	$\Xi_c^{\prime 0}K^+$	0	$+1_a$	0	$+2_b$	0	0	0
	$2\,\Xi_{cc}^{+}$	$\to$	$\Sigma_c^{0}\pi^+$	0	$+1_b$	0	-2_b	0	0	0
	$4\,\Xi_{cc}^{+}$	$\to$	$\Sigma_c^{+}\pi^0$	0	-1_b	0	$+4_b$	0	0	0
	$4\,\Xi_{cc}^{+}$	$\to$	$\Sigma_c^{+}\eta_\omega$	0	$+1_b$	0	0	0	0	0
	$2\sqrt{2}\,\Xi_{cc}^{+}$	$\to$	$\Sigma_c^{+}\eta_\phi$	0	-1_a	0	0	0	0	0
	$4\sqrt{3}\,\Xi_{cc}^{+}$	$\to$	$\Sigma_c^{+}\eta_8$	0	$+2_a+1_b$	0	0	0	0	0
	$2\sqrt{2}\,\Omega_{cc}^{+}$	$\to$	$\Sigma_c^{+}\bar{K}^0$	0	0	0	-2_a	0	0	0
	$2\sqrt{2}\,\Omega_{cc}^{+}$	$\to$	$\Xi_c^{\prime 0}\pi^+$	0	$+1_b$	0	$+2_a$	0	0	0
	$2\,\Omega_{cc}^{+}$	$\to$	$\Omega_c^{0}K^+$	0	$+1_a$	0	-2_a	0	0	0
DCS	$2\sqrt{2}\,\Xi_{cc}^{++}$	$\to$	$\Sigma_c^{+}K^+$	0	+1	0	0	0	0	0
	$2\,\Xi_{cc}^{++}$	$\to$	$\Sigma_c^{++}K^0$	0	−1	0	0	0	0	0
	$2\,\Xi_{cc}^{+}$	$\to$	$\Sigma_c^{0}K^+$	0	+1	0	0	0	0	0
	$2\sqrt{2}\,\Xi_{cc}^{+}$	$\to$	$\Sigma_c^{+}K^0$	0	−1	0	0	0	0	0
	$2\sqrt{2}\,\Omega_{cc}^{+}$	$\to$	$\Xi_c^{\prime 0}K^+$	0	+1	0	−2	0	0	0
	$2\sqrt{2}\,\Omega_{cc}^{+}$	$\to$	$\Xi_c^{\prime +}K^0$	0	−1	0	+2	0	0	0
	$4\,\Omega_{cc}^{+}$	$\to$	$\Sigma_c^{+}\pi^0$	0	0	0	−4	0	0	0
	$4\,\Omega_{cc}^{+}$	$\to$	$\Sigma_c^{+}\eta_\omega$	0	0	0	0	0	0	0
	$2\sqrt{2}\,\Omega_{cc}^{+}$	$\to$	$\Sigma_c^{+}\eta_\phi$	0	0	0	0	0	0	0
	$4\sqrt{3}\,\Omega_{cc}^{+}$	$\to$	$\Sigma_c^{+}\eta_8$	0	0	0	0	0	0	0

Table 12 Values of the seven topological tensor invariants for the non-leptonic CF, SCS and DCS decays $B_{cc}(\mathbf{3}) \to B(\mathbf{8}) + M(\bar{\mathbf{3}})$ of the triplet double charm baryon states Ξ_{cc}^{++}, Ξ_{cc}^{+} and Ω_{cc}^{+} into the light baryon octet and the charm mesons D^+, D^0 and D_s^+. The notation for the SCS decays is explained in the caption of Table 8

				I_1^-	I_2^-	I_3	I_4	$\hat{I}_3$	$\hat{I}_4$	I_5
CF	$2\Xi_{cc}^{++}$	$\to$	$\Sigma^+ D^+$	0	0	0	0	0	-2	0
	$2\Xi_{cc}^{+}$	$\to$	$\Sigma^+ D^0$	0	0	0	0	0	0	$+1$
	$2\sqrt{2}\Xi_{cc}^{+}$	$\to$	$\Sigma^0 D^+$	0	0	0	0	0	-2	-1
	$2\sqrt{6}\Xi_{cc}^{+}$	$\to$	$\Lambda^0 D^+$	0	0	0	0	0	-2	$+1$
	$2\Xi_{cc}^{+}$	$\to$	$\Xi^0 D_s^+$	0	0	0	0	0	0	$+1$
	$2\Omega_{cc}^{+}$	$\to$	$\Xi^0 D^+$	0	0	0	0	0	-2	0
SCS	$2\Xi_{cc}^{++}$	$\to$	$p D^+$	0	0	0	0	0	$+2_b$	0
	$2\Xi_{cc}^{+}$	$\to$	$p D^0$	0	0	0	0	0	0	-1_b
	$2\Xi_{cc}^{+}$	$\to$	$n D^+$	0	0	0	0	0	$+2_b$	$+1_b$
	$2\Xi_{cc}^{++}$	$\to$	$\Sigma^+ D_s^+$	0	0	0	0	0	$+2_a$	0
	$\sqrt{2}\Xi_{cc}^{+}$	$\to$	$\Sigma^0 D_s^+$	0	0	0	0	0	$+2_a$	0
	$2\sqrt{6}\Xi_{cc}^{+}$	$\to$	$\Lambda^0 D_s^+$	0	0	0	0	0	$+2_a$	-2_b
	$2\Omega_{cc}^{+}$	$\to$	$\Sigma^+ D^0$	0	0	0	0	0	0	$+2_a$
	$2\sqrt{2}\Omega_{cc}^{+}$	$\to$	$\Sigma^0 D^+$	0	0	0	0	0	0	-1_a
	$2\sqrt{6}\Omega_{cc}^{+}$	$\to$	$\Lambda^0 D^+$	0	0	0	0	0	$+4_b$	-1_a
	$2\Omega_{cc}^{+}$	$\to$	$\Xi^0 D_s^+$	0	0	0	0	0	$+2_a$	$+1_a$
DCS	$2\Xi_{cc}^{++}$	$\to$	$p D_s^+$	0	0	0	0	0	$+2$	0
	$2\Xi_{cc}^{+}$	$\to$	$n D_s^+$	0	0	0	0	0	$+2$	0
	$2\Omega_{cc}^{+}$	$\to$	$p D^0$	0	0	0	0	0	0	$+1$
	$2\Omega_{cc}^{+}$	$\to$	$n D^+$	0	0	0	0	0	0	$+1$
	$2\sqrt{2}\Omega_{cc}^{+}$	$\to$	$\Sigma^0 D_s^+$	0	0	0	0	0	0	0
	$2\sqrt{6}\Omega_{cc}^{+}$	$\to$	$\Lambda^0 D_s^+$	0	0	0	0	0	$+4$	$+2$

4.6 The double charm baryon decays $B_{cc}(\mathbf{3}) \stackrel{H(\mathbf{6})}{\longrightarrow} B(\mathbf{8}) + M(\bar{\mathbf{3}})$

None of this class of decays into a light baryon and a charm meson in the final state has been observed. The final states in these decays have a very distinct signature which could lead to their discovery in the not too distant future. To our knowledge the only theoretical paper dealing with these decays is Ref. [78]. As a seed for the $B_{cc}(\mathbf{3}) \to B(\mathbf{8}) + M(\bar{\mathbf{3}})$ decay, Li et al. first consider the short distance decay of a double charm baryon state into a single charm baryon and a $C = 0$ meson [78]. The charm quantum number is then transferred from the baryon to the meson by long distance final state interactions. The authors of Ref. [78] estimate that the CF decays of this class of decays could be of the order of 1% with the SCS and DCS decays suppressed by the respective CKM suppression factors.

For the SU(3) decomposition one obtains

$$\mathbf{3} \to \mathbf{6} \otimes \mathbf{8} \otimes \bar{\mathbf{3}} = 2 \cdot \mathbf{3} \oplus 2 \cdot \bar{\mathbf{6}} \oplus 3 \cdot \mathbf{15} \oplus \mathbf{15}' \oplus \mathbf{24} \oplus \mathbf{42} \tag{39}$$

The decomposition shows that there are two independent SU(3) invariants. Therefore, one expects $7 - 2 = 5$ relations among the seven topological tensor invariants. By visual inspection of the five topologies one finds that $I_1^- = I_2^- = I_3 = I_4 = 0$. The topological invariant $\hat{I}_3(\ell, \ell') = B_\ell^{a[bc]} B^{\ell'}_{a[b'c']} M_d^{c'} H^{[c'b']}_{[db]}$ vanishes since b' and c' are heavy charm quark labels which are necessarily symmetric such that the initial state tensor $B^{\ell'}_{a[b'c']}$ vanishes. Altogether one has

$$I_1^- = I_2^- = I_3 = I_4 = \hat{I}_3 = 0. \tag{40}$$

The only nonvanishing topological tensor invariants are $\hat{I}_4$ and I_5. In Table 12 we list the values of the nonzero tensor invariants $\hat{I}_4$ and I_5 for the altogether 22 CF, SCS and DCS double charm decays $B_{cc} \to B\, D^{+/0}$, $B\, D_s^+$. Note that the decay $\Omega_{cc}^+ \to \Sigma^0 D_s^+$ is forbidden in the SU(3) limit because of the KPW theorem.

4.7 The hyperon decays $B(8) \stackrel{H(8)}{\longrightarrow} B(8) + M(8)$

In complete the picture obtained so far, we also consider the decays of hyperons into light baryons. The decomposition

Table 13 Values of the seven SU(3) tensor invariants for the nonleptonic hyperon decays induced by the transitions ($s \to u;\ u \to d$)

			I_1^-	I_2^-	I_3	I_4	$\hat{I}_3$	$\hat{I}_4$	$I_3 - \hat{I}_3$	$I_4 - \hat{I}_4$	I_5
$2\sqrt{6}\,\Lambda^0$	$\to$	$p\pi^-$	+1	+1	0	−2	+2	−4	−2	+2	−1
$4\sqrt{3}\,\Lambda^0$	$\to$	$n\pi^0$	−1	−1	0	+2	−2	+4	+2	−2	+1
$2\sqrt{2}\,\Sigma^+$	$\to$	$p\pi^0$	−1	+1	0	−2	−2	0	+2	−2	−1
$2\,\Sigma^+$	$\to$	$n\pi^+$	0	0	0	+2	0	0	0	+2	+1
$2\,\Sigma^-$	$\to$	$n\pi^-$	−1	+1	0	0	−2	0	+2	0	0
$2\sqrt{6}\,\Xi^-$	$\to$	$\Lambda^0\pi^-$	−2	+1	0	0	−4	+4	+4	−4	0
$4\sqrt{3}\,\Xi^0$	$\to$	$\Lambda^0\pi^0$	−2	+1	0	0	−4	+4	+4	−4	0

$$\mathbf{8} \to \mathbf{8} \otimes \mathbf{8} \otimes \mathbf{8} = 2 \cdot \mathbf{1} \oplus 8 \cdot \mathbf{8} \oplus 4 \cdot \mathbf{10} \oplus 4 \cdot \overline{\mathbf{10}} \oplus 6 \cdot \mathbf{27} \oplus 2 \cdot \mathbf{35} \oplus 2 \cdot \overline{\mathbf{35}} \oplus \mathbf{64} \tag{41}$$

suggests that there are eight SU(3) invariant couplings which outnumber the seven invariant tensor couplings by one. However, one with the help of a Fierz-type identity one can show that the eight SU(3) couplings are not independent.

To proof this, consider the usual second rank tensor baryon representation

$$B = \begin{pmatrix} -\frac{\Lambda^0}{\sqrt{6}} + \frac{\Sigma^0}{\sqrt{2}} & -\Sigma^+ & p \\ \Sigma^- & -\frac{\Lambda^0}{\sqrt{6}} - \frac{\Sigma^0}{\sqrt{2}} & n \\ \Xi^- & -\Xi^0 & \frac{2\Lambda^0}{\sqrt{6}} \end{pmatrix}, \tag{42}$$

together with the representations for mesons and the weak transition tensor

$$\bar{M} = \begin{pmatrix} \frac{\eta_8}{\sqrt{6}} + \frac{\pi^0}{\sqrt{2}} & \pi^- & K^- \\ -\pi^+ & \frac{\eta_8}{\sqrt{6}} - \frac{\pi^0}{\sqrt{2}} & -\bar{K}^0 \\ K^+ & K^0 & -\frac{2\eta_8}{\sqrt{6}} \end{pmatrix}, \qquad S = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}. \tag{43}$$

In SU(3) a Levi-Civita tensor with four indices in three flavor dimensions vanishes. Therefore, one can write down a Fierz-type identity which reads

$$\epsilon_{abcd}\epsilon^{efgh} = \begin{vmatrix} \delta_a^e & \delta_a^f & \delta_a^g & \delta_a^h \\ \delta_b^e & \delta_b^f & \delta_b^g & \delta_b^h \\ \delta_c^e & \delta_c^f & \delta_c^g & \delta_c^h \\ \delta_d^e & \delta_d^f & \delta_d^g & \delta_d^h \end{vmatrix} = 0. \tag{44}$$

When one contracts the Fierz-type identity with $B_e^a \bar{B}_f^b \bar{M}_g^c S_h^d$ one obtains the relation

$$\begin{aligned} 0 = &\ \mathrm{tr}(B\bar{B}\bar{M}S) + \mathrm{tr}(B\bar{B}S\bar{M}) + \mathrm{tr}(B\bar{M}S\bar{B}) \\ &+ \mathrm{tr}(B\bar{M}\bar{B}S) + \mathrm{tr}(BS\bar{M}\bar{B}) + \\ &+ \mathrm{tr}(BS\bar{B}\bar{M}) - \mathrm{tr}(B\bar{B})\,\mathrm{tr}(\bar{M}S) \\ &- \mathrm{tr}(B\bar{M})\,\mathrm{tr}(\bar{B}S) - \mathrm{tr}(BS)\,\mathrm{tr}(\bar{B}\bar{M}). \end{aligned} \tag{45}$$

We conjecture that the use of this Fierz-type identity would provide for the missing linear relations between tensor invariants of differing topologies. In Table 13 one can identify the $\Delta I = 1/2$ (or octet) rules [79,80]

$$\mathcal{A}(\Lambda^0 \to p\pi^-) = -\sqrt{2}\mathcal{A}(\Lambda^0 \to n\pi^0) \tag{46}$$

$$\sqrt{2}\mathcal{A}(\Sigma^+ \to p\pi^0) = -\mathcal{A}(\Sigma^+ \to n\pi^+) + \mathcal{A}(\Sigma^- \to n\pi^-) \tag{47}$$

$$\mathcal{A}(\Xi^- \to \Lambda^0\pi^-) = \sqrt{2}\mathcal{A}(\Xi^0 \to \Lambda^0\pi^0). \tag{48}$$

However, for some reason the Lee–Sugawara relation [81,82] is not satisfied,

$$2A(\Xi^- \to \Lambda^0\pi^-) + A(\Lambda^0 \to p\pi^-) \neq \sqrt{3}A(\Sigma^+ \to p\pi^0) \tag{49}$$

Topological tensor invariants for this class of processes have been considered in Ref. [83].

5 The current algebra description of nonleptonic charm baryon decays

In the second part of this general survey we add elements of the current algebra approach to the information contained in the tables. The current algebra approach was originally developed for the description of nonleptonic hyperon decays (see Refs. [84–87] and references therein) and later applied to nonleptonic decays of charmed baryons [88,89]. As will be described in the following it turns out that the flavor invariants of the W-exchange contributions to the current algebra amplitudes can be expressed in terms of the topological flavor invariants I_i $(i = 3, 4, 5)$ and $\hat{I}_i$ $(i = 3, 4)$ introduced in Sect. 3. We briefly recapitulate the standard current algebra plus soft pion approach to nonleptonic two-body charm baryon decays $B(1/2^+) \to B(1/2^+) + M(0^-)$ where we will stay quite close to the presentation and notation of Refs. [54,59–62].

We define S- and P-wave amplitudes A_{fki} and B_{fki}, resp., by writing

$$\langle B_f M_k | \mathcal{H}_{\mathrm{eff}} | B_i \rangle = \bar{u}_f (A_{fki} - B_{fki}\gamma_5) u_i. \tag{50}$$

We follow the convention of Refs. [54,59–62] in that A_{fki} and B_{fki} are defined with a relative minus sign in Eq. (50). Note that the choice of this sign will affect the sign of the calculated asymmetry parameter of the nonleptonic two-body decays.

Again in the notation of Refs. [54,59–62] the p.v. and p.c. amplitudes A and B are given by

$$A_{fki} = A^{\rm fac}_{fki} + A^{\rm pole}_{fki} + A^{\rm com}_{fki},$$
$$B_{fki} = B^{\rm fac}_{fki} + B^{\rm pole}_{fki} + B^{\rm com}_{fki}. \quad (51)$$

The factorizing contributions $A^{\rm fac}_{fki}$ and $B^{\rm fac}_{fki}$ related to the topological invariants I^-_1 and I^-_2 are complemented by the nonfactorizing contributions, consisting of a pole and a commutator part.

5.1 The parity violating S-wave amplitude $A^{\rm com}_{fki}$

For the commutator contribution one obtains

$$A^{\rm com}_{fki} = \frac{\sqrt{2}}{f_k}\langle B_f|[M_k, \mathcal{H}^{\rm pc}_{\rm eff}]|B_i\rangle, \quad (52)$$

where $\mathcal{H}^{\rm pc}_{\rm eff}$ is the p.c. part of the effective Hamiltonian, M_k is the SU(3) vector charge associated with the pseudoscalar meson k, and where the f_k are the pseudoscalar coupling constants, with e.g. $f_\pi = 0.95\, m_\pi$. One then introduces a sum over intermediate baryon states $\sum_\ell |B_\ell\rangle\langle B_\ell|$ and $\sum_{\ell'} |B_{\ell'}\rangle\langle B_{\ell'}|$ to rewrite Eq. (52) as

$$A^{\rm com}_{fki} = \frac{\sqrt{2}}{f_k}\Big(\sum_\ell \langle B_f|M_k|B_\ell\rangle\langle B_\ell|\mathcal{H}^{\rm pc}_{\rm eff}|B_i\rangle - \sum_{\ell'}\langle B_f|\mathcal{H}^{\rm pc}_{\rm eff}|B_{\ell'}\rangle\langle B_{\ell'}|M_k|B_i\rangle\Big)$$
$$= A^{\rm com}_{fki}(s) - A^{\rm com}_{fki}(u) \quad (53)$$

Since M_k is a conserved vector charge operator, the sum over intermediate states extends only over the twenty $J^P = 1/2^+$ ground state baryons of the SU(4) **20′** representation.

The notation "s" and "u" stands for the s- and u-channel contributions in the current algebra approach where the s-channel contribution refers to the process where one has a weak transition at the first stage followed by meson emission and vice-versa for the u-channel contribution, as depicted in Fig. 3. One can associate the topological diagrams with the s-channel and u-channel contributions by cutting the diagrams IIa, IIb and III in Fig. 1 at the appropriate places. Diagrams IIa and IIb are clearly associated with the s- and u-channel contributions, respectively, whereas diagram III can be seen to be associated with both s- and u-channel contributions depending on where diagram III is cut. As it turns out, diagram III contributes in equal amounts to the s- and u-channel parts for both S- and P-wave transitions. Altogether the s-channel contribution correspond to the contribution of diagram IIa

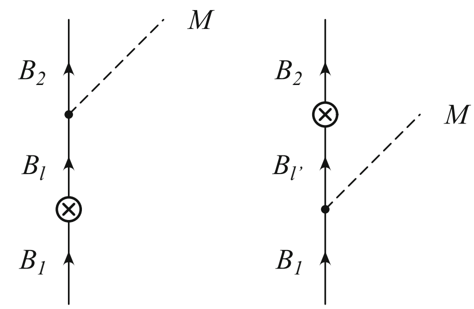

Fig. 3 s-channel contribution (left) and u-channel contribution (right)

and one half of diagram III, while the u-channel contribution corresponds to diagram IIb and one half of diagram III.

Our aim is to proof that the commutator contributions $A^{\rm com}_{fki}(s)$ and $A^{\rm com}_{fki}(u)$ can be rewritten in terms of a linear superposition of the topological tensor invariants I_3, I_4, $\hat{I}_3$, $\hat{I}_4$ and I_5 introduced in Sect. 3. This is achieved by making use of the completeness relation (12). The same can be done for the pole term contributions $A^{\rm pole}_{kfi}(s)$ and $A^{\rm pole}_{fki}(u)$ in the absence of hyperfine mass splittings for the flavor degenerate members of the **20′** representation, as will be written down later.

In Appendix C we demonstrate that the f-type baryon matrix element $\langle B_f|M_k|B_\ell\rangle$ of the conserved vector charge can be expressed in terms of two basic SU(3) contractions $\tilde{I}_1$ and $\tilde{I}_2$. One has

$$\langle B_f|M_k|B_\ell\rangle := (I^f)_{fk\ell} = 4(\tilde{I}_1)_{fk\ell} + 2(\tilde{I}_2)_{fk\ell} \quad (54)$$

and similarly for $\langle B_{\ell'}|M_k|B_i\rangle$. The two basic tensor contractions $\tilde{I}_1$ and $\tilde{I}_2$ are given by

$$(\tilde{I}_1)_{fk\ell} = B^{a[bc]}_f B^\ell_{a[bc']}(M_k)^{c'}_c$$
$$(\tilde{I}_2)_{fk\ell} = B^{a[bc]}_f B^\ell_{b[c'a]}(M_k)^{c'}_c. \quad (55)$$

The matrix element $\langle B_\ell|\mathcal{H}^{\rm pc}_{\rm eff}|B_i\rangle$ of the effective Hamiltonian (called $a_{\ell\, i}$ in Ref. [60]) splits into a dynamical piece and a symmetry factor. In the bag model calculation the factors $a_{\ell\, i}$ can be expressed in terms of a single tensor contraction. In the normalization of Ref. [60] one writes

$$\langle B_\ell|\mathcal{H}^{\rm pc}_{\rm eff}|B_{\ell'}\rangle := a_{\ell\ell'} = 6\bar{X}_2(4\pi)(I^{\rm pc})_{\ell\ell'} \quad (56)$$

with

$$(I^{\rm pc})_{\ell\ell'} = B^{a[bc]}_\ell B^{\ell'}_{a[b'c']} H^{[b'c']}_{[bc]}. \quad (57)$$

In addition, one has

$$A^{\rm com}_{fki}(s) - A^{\rm com}_{fki}(u)$$
$$= \frac{1}{f_k} 6\bar{X}_2(4\pi)\left(\hat{A}^{\rm com}_{fki}(s) - \hat{A}^{\rm com}_{fki}(u)\right) \quad (58)$$

with

$$\hat{A}^{\rm com}_{fki}(s) - \hat{A}^{\rm com}_{fki}(u) = \sum_{\ell}(I^f)_{fk\ell}(I^{\rm pc})_{\ell i} - \sum_{\ell'}(I^{\rm pc})_{f\ell'}(I^f)_{\ell' ki}. \tag{59}$$

In Ref. [60] the values of the bag integrals as e.g. X_2 are given as dimensionless numbers. This is puzzling at first sight since the amplitude $A^{\rm com}_{fki}$ is dimensionless. This conundrum is resolved after a literature search. As stated some time ago in Ref. [43], the values of the bag integrals are given in units of $c_- G_F$ GeV^3 which should be augmented by the CKM factor $C_{\rm CKM}$.

The relation to the topological tensor invariants is obtained by using the completeness relation (12) in order to perform the sum over the intermediate baryon states in Eq. (53). We begin with the s-channel contribution $A^{\rm com}_{fki}(s)$ for which we go through the derivation step by step. As discussed in Appendix C, the flavor invariant contribution I^f can be expressed as a linear superposition of the two building blocks $\tilde{I}_1$ and $\tilde{I}_2$ in the form $I^f = 4\tilde{I}_1 + 2\tilde{I}_2$. Let us write out the contributions of the two building blocks $\tilde{I}_1$ and $\tilde{I}_2$ to the sum over intermediate states in $A^{\rm com}_{fki}(s)$. The result of taking the sum over the intermediate states via the completeness relation (12) can be expressed in terms of the topological tensor invariants I_3, I_4 and I_5. One has

$$\begin{aligned}\sum_{\ell}(\tilde{I}_1)_{fk\ell}I^{\rm pc}_{\ell i} &= B^{a[bc]}_f (M_k)^{c'}_c \times\Big(\sum_{\ell} B^{\ell}_{a[bc']}B^{r[st]}_{\ell}\Big)B^i_{r[a'b']}H^{[a'b']}_{[st]} \\ &= \tfrac{2}{3}I_3 - \tfrac{1}{3}I_4 + \tfrac{2}{3}I_5, \\ \sum_{\ell}(\tilde{I}_2)_{fk\ell}I^{\rm pc}_{\ell i} &= B^{a[bc]}_f (M_k)^{c'}_c\Big(\sum_{\ell} B^{\ell}_{b[c'a]}B^{r[st]}_{\ell}\Big)B^i_{r[a'b']}H^{[a'b']}_{[st]} \\ &= -\tfrac{1}{3}I_3 + \tfrac{2}{3}I_4 + \tfrac{2}{3}I_5.\end{aligned} \tag{60}$$

The contribution of I_4 cancels in the sum $I^f = 4\tilde{I}_1 + 2\tilde{I}_2$ of the two contributions (60), and one arrives at

$$\hat{A}^{\rm com}_{fki}(s) = 4\sum_{\ell}(\tilde{I}_1)_{fk\ell}I^{\rm pc}_{\ell i} + 2\sum_{\ell}(\tilde{I}_2)_{fk\ell}I^{\rm pc}_{\ell i} = 2I_3 + 4I_5. \tag{61}$$

Doing the same exercise for the u-channel contribution $A^{\rm com}_{fki}(u)$ in Eq. (53) one obtains

$$\begin{aligned}\sum_{\ell'}(I^{\rm pc})_{f\ell'}(\tilde{I}_1)_{\ell' ki} &= B^{a[bc]}_f H^{b'c'}_{bc}\Big(\sum_{\ell'} B^{\ell}_{a[b'c']}B^{r[st]}_{\ell}\Big)B^i_{r[sa']}(M_k)^{a'}_t \\ &= \tfrac{2}{3}\hat{I}_3 - \tfrac{1}{3}\hat{I}_4 + \tfrac{2}{3}I_5, \\ \sum_{\ell'}(I^{\rm pc})_{f\ell'}(\tilde{I}_2)_{\ell' ik} &= B^{a[bc]}_f H^{[a'b']}_{[st]}\Big(\sum_{\ell} B^{\ell}_{b[c'a]}B^{r[st]}_{\ell}\Big)B^i_{s[a'r]}(M_k)^{a'}_t \\ &= -\tfrac{1}{3}\hat{I}_3 + \tfrac{2}{3}\hat{I}_4 + \tfrac{2}{3}I_5,\end{aligned} \tag{62}$$

leading to

$$\hat{A}^{\rm com}_{fki}(u) = 4\sum_{\ell'}(I^{\rm pc})_{f\ell'}(\tilde{I}_1)_{\ell' ki} + 2\sum_{\ell'}(I^{\rm pc})_{f\ell'}(\tilde{I}_2)_{\ell' ki} = 2\hat{I}_3 + 4I_5. \tag{63}$$

Calculating the difference of the s- and u-channel contributions $\hat{A}^{\rm com}_{fki}(s)$ and $\hat{A}^{\rm com}_{fki}(u)$ according to Eq. (53), one obtains the remarkable result that the contribution of the topological tensor invariant I_5 cancels out. The final result is

$$\hat{A}^{\rm com}_{fki} = \hat{A}^{\rm com}_{fki}(s) - \hat{A}^{\rm com}_{fki}(u) = 2(I_3 - \hat{I}_3). \tag{64}$$

We mention that the result $A^{\rm com}_{fki} \sim I_3 - \hat{I}_3$ was already been derived in the early paper [30].

The p.v. pole contributions not dealt with in detail in this paper read

$$\begin{aligned}A^{\rm pole}_{fki} &= -\sum_{\ell}\frac{g_{fk\ell}\, b_{\ell i}}{m_i - m_\ell} - \sum_{\ell'}\frac{b_{f\ell'}\, g_{\ell' ki}}{m_f - m_{\ell'}} \\ &= A^{\rm pole}_{fki}(s) + A^{\rm pole}_{fki}(u),\end{aligned} \tag{65}$$

where $b_{\ell\ell'} = \langle B_\ell|\mathcal{H}^{\rm pv}_{\rm eff}|B_{\ell'}\rangle$ are the p.v. matrix elements which are much smaller than the p.c. matrix elements $a_{\ell\ell'}$ [90,91]. For the same reason, one also skips

$$B^{\rm com}_{fki} = \frac{\sqrt{2}}{f_k}\Big(\sum_{\ell}\langle B_f|M_k|B_\ell\rangle\langle B_\ell|\mathcal{H}^{\rm pv}_{\rm eff}|B_i\rangle - \sum_{\ell'}\langle B_f|\mathcal{H}^{\rm pv}_{\rm eff}|B_{\ell'}\rangle\langle B_{\ell'}|M_k|B_i\rangle\Big). \tag{66}$$

5.2 The parity conserving P-wave amplitude $B^{\rm pole}_{fki}$

Using again the notation of Refs. [54,59–62], the pole contribution to the p.c. P-wave amplitude B is given by

$$\begin{aligned}B^{\rm pole}_{fki} &= \sum_{\ell}\frac{g_{fk\ell}\, a_{\ell i}}{m_i - m_\ell} + \sum_{\ell'}\frac{a_{f\ell'}\, g_{\ell' ki}}{m_f - m_{\ell'}} \\ &= B^{\rm pole}_{fki}(s) + B^{\rm pole}_{fki}(u).\end{aligned} \tag{67}$$

Again, the first and second terms in Eq. (67) represent the s- and u-channel pole contributions, respectively. The sum over the intermediate state labels ℓ and ℓ' extends over all 20 $J^P = 1/2^+$ ground state baryons that have the correct quantum numbers to contribute to the s- and u-channel pole contributions (67). In the present application one has zero, one or maximally two contributions in the sum over intermediate ground state baryons. The latter case occurs when pairs of flavor degenerate states contribute to the sum as e.g. the states (Λ^0, Σ^0) in the $C = 0$ sector or $(\Lambda_c^+, \Sigma_c^+)$, $(\Xi_c^0, \Xi_c^{\prime 0})$ and $(\Xi_c^+, \Xi_c^{\prime +})$ in the $C = 1$ sector. The members of the ground state $J^P = 3/2^+$ 20-plet do not contribute to the sum over intermediate states since the weak transitions $a_{\ell i}$

and $a_{f\ell'}$ with ℓ, ℓ' from the $J^P = 3/2^+$ 20-plet vanish in the SU(3) limit because of the KPW theorem.

The s- and u-channel pole contributions involve the strong coupling coefficients denoted by $g_{fk\ell}$ and the weak parity conserving transition matrix element $a_{\ell i}$ which also appeared in A^{com}_{fki}. One then uses the generalized Goldberger–Treiman relation

$$g_{fk\ell} = \frac{\sqrt{2}}{f_k}(m_f + m_\ell) g^A_{fk\ell} \tag{68}$$

to express the strong coupling coefficients $g_{fk\ell}$ by the appropriate axial vector elements $g^A_{fk\ell}$. One has

$$B^{\text{pole}}_{fki}(s) + B^{\text{pole}}_{fki}(u) = \frac{\sqrt{2}}{f_k}\left(\sum_\ell g^A_{fk\ell}\frac{m_f + m_\ell}{m_i - m_\ell}a_{\ell i} + \sum_{\ell'} a_{f\ell'}\frac{m_i + m_{\ell'}}{m_f - m_{\ell'}}g^A_{\ell' ki}\right). \tag{69}$$

As in the p.v. case we factor out the parameters depending on the dynamic model to remain with a purely flavor dependent contribution $\hat{B}$. In the bag model calculation of Refs. [54,59–62] this is achieved by writing

$$B^{\text{pole}}_{fki}(s) + B^{\text{pole}}_{fki}(u) = \frac{1}{f_k}6\bar{X}_2(4\pi)\frac{2}{3}\bar{Z}(4\pi)\left(\hat{B}^{\text{pole}}_{fki}(s) + \hat{B}^{\text{pole}}_{fki}(u)\right) \tag{70}$$

where

$$\hat{B}^{\text{pole}}_{fki}(s) + \hat{B}^{\text{pole}}_{fki}(u) = \sum_\ell I^{\text{CQM}}_{fk\ell} I^{\text{pc}}_{\ell i} R_{fi}(B_\ell) + \sum_\ell I^{\text{pc}}_{f\ell'} I^{\text{CQM}}_{\ell' ki} R_{if}(B_{\ell'}) \tag{71}$$

with the compact notation for the two mass ratio expressions

$$R_{fi}(B_\ell) = \frac{m_f + m_\ell}{m_i - m_\ell} = R_s(B_\ell),$$
$$R_{if}(B_{\ell'}) = \frac{m_i + m_{\ell'}}{m_f - m_{\ell'}} = R_u(B_{\ell'}), \tag{72}$$

the second ones used as pseudonyms. The flavor invariant I^{CQM}_{fki} denotes the flavor structure of the matrix element $\langle B_{\ell'} M_k | B_i\rangle$ in the constituent quark model. In the bag model the flavor structure of the strong coupling is given by a d/f ratio of $d/f = 3/2$ (cf. Appendix C). In terms of the two building blocks $\tilde{I}_1$ and $\tilde{I}_2$ introduced in Eq. (55) the strong matrix element is given by

$$I^{\text{CQM}}_{fki} = (4\tilde{I}_1 + 5\tilde{I}_2)_{fki} = B^{a[bc]}_f\left(4B^i_{a[bc']} + 5B^i_{b[c'a]}\right)(M_k)^{c'}_c \tag{73}$$

Different from the p.v. S-wave case the sum over intermediate states cannot be taken because of the mass ratio factors which differ for the contributions of the hyperfine doublet partners. However, if one neglects the hyperfine mass splitting, the sum over intermediate states can be performed as in the p.v. S-wave case. To proceed, we define average values of the masses of the set of hyperfine doublet partners $\{B_\ell\}$ denoted by $\bar{m}_\ell$ and correspondingly average values for the mass ratio factors by writing

$$\bar{R}_{fi} = \frac{m_f + \bar{m}_\ell}{m_i - \bar{m}_\ell} = \bar{R}_s(\{B_\ell\}),$$
$$\bar{R}_{if} = \frac{m_i + \bar{m}_{\ell'}}{m_f - \bar{m}_{\ell'}} = \bar{R}_u(\{B_{\ell'}\}). \tag{74}$$

The average mass ratio factors $\bar{R}_{fi}$ and $\bar{R}_{if}$ can then be factored out from the sum over intermediate states and one obtains

$$\hat{B}^{\text{pole}}_{fki}(s) + \hat{B}^{\text{pole}}_{fki}(u) = \bar{R}_{fi}\sum_\ell I^{\text{CQM}}_{fk\ell} I^{\text{pc}}_{\ell i} + \bar{R}_{if}\sum_{\ell'} I^{\text{pc}}_{f\ell'} I^{\text{CQM}}_{\ell' ki}. \tag{75}$$

The sum over the intermediate states can now be performed, using again the completeness relation (12). The task is simplified by the fact that the strong transition $I^{\text{CQM}} = 4\tilde{I}_1 + 5\tilde{I}_2$ is a linear superposition of the same two building blocks $\tilde{I}_1$ and $\tilde{I}_2$ that were used in the evaluation of the corresponding sums in the amplitude A. The result of calculating the sum (75) is then given by

$$\sum_\ell I^{\text{CQM}}_{fk\ell} I^{\text{pc}}_{\ell i} = I_3 + 2I_4 + 6I_5,$$
$$\sum_{\ell'} I^{\text{pc}}_{f\ell'} I^{\text{CQM}}_{\ell' ki} = \hat{I}_3 + 2\hat{I}_4 + 6I_5. \tag{76}$$

The flavor content of the pole model contributions is thus given by

$$\hat{B}^{\text{pole}}_{fki}(s) + \hat{B}^{\text{pole}}_{fki}(u) = (I_3 + 2I_4 + 6I_5)\bar{R}_{fi} + (\hat{I}_3 + 2\hat{I}_4 + 6I_5)\bar{R}_{if}. \tag{77}$$

We have found again the quite remarkable fact that, in the absence of hyperfine mass splittings for the flavor degenerate members of the **20**′ representation, the pole model representation of the p.c. P-wave amplitude B depends only on the initial and final states of the nonleptonic decays and not on the detailed structure of the intermediate states.

Equations (76) can be generalized to the case where the strong d/f ratio takes on general values (see Appendix C). One obtains

$$\sum_\ell I^{\text{gen}}_{fk\ell} I^{\text{pc}}_{\ell i}(s) = \tfrac{5}{3}d(I_3 + 2I_4 + 6I_5) - (\tfrac{5}{3}d - f)(2I_3 + 4I_5),$$
$$\sum_{\ell'} I^{\text{pc}}_{f\ell'} I^{\text{gen}}_{\ell' ki}(u) = \tfrac{2}{3}d(\hat{I}_3 + 2\hat{I}_4 + 6I_5) - (\tfrac{2}{3}d - f)(2\hat{I}_3 + 4I_5). \tag{78}$$

One can check that one recovers Eqs. (76) for $d/f = 3/2$, i.e. for $d = 3/5$ and $f = 2/5$.

6 Some more sample decays

Initiated by a thorough analysis of the methods applied in Refs. [59–62] with details found in Appendix D, in this section we present explicit results for some sample decays on the connection of the current algebra results with the topological tensor invariants. We emphasize again that this connection holds true in the SU(3) limit. The relation between the two approaches derived in Sect. 5 is explicitly verified in these examples. We include at least one decay of each of the classes of decays discussed in Sect. 4. For each decay we include in the header the values of the seven tensor invariants in the same sequence $(I_1^-, I_2^-, I_3, I_4, \hat{I}_3, \hat{I}_4, I_5)$ as in the tables. For a better perception we have underlined the nonvanishing contributions of the topological tensor invariants.

6.1 The CF decay $\Lambda_c^+ \to \Lambda^0 \pi^+$ $12(I_i) = (-2, -2, -2, 4, -2, 4, 1)$

In this case, all seven tensor invariants are non-zero. Together with the SCS decay $\Lambda_c^+ \to \Lambda^0 K^+$, this decay is the only one in which $I_3 = \hat{I}_3 \neq 0$. Since one has $I_3 - \hat{I}_3 = 0$, one concludes that in these decays the W-exchange contribution to the p.v. amplitude A is zero. Nevertheless, there are also tree diagram contributions which will contribute to the amplitude A. Expressed in terms of the topological tensor invariants, the W-exchange contributions to the reduced amplitudes $\hat{A}$ and $\hat{B}$ are given by

$$\begin{aligned}\hat{A}^{\text{com}} &= (\underline{2I_3} + \underline{4I_5}) - (\underline{2\hat{I}_3} + \underline{4I_5}) = 0 - 0,\\ \hat{B}^{\text{pole}} &= (\underline{I_3} + \underline{2I_4} + \underline{6I_5})\, R_s(\Sigma^+) + (\underline{\hat{I}_3} + \underline{2\hat{I}_4} + \underline{6I_5})\, R_u(\Sigma_c^0)\\ &= 1 \cdot R_s(\Sigma^0) + 1 \cdot R_u(\Sigma_c^0).\end{aligned} \tag{79}$$

The same result is obtained by explicit summation over the intermediate states. The intermediate states are Σ^+ in the s-channel and Σ_c^0 in the u-channel. The intermediate states do not contribute to the p.v. amplitude A since F_{π^+} is a conserved charge operator which implies $I^f_{\Lambda^0\pi^+\Sigma^+} = 0$ and $I^f_{\Sigma_c^0\pi^+\Lambda_c^+} = 0$. The contribution of the intermediate states to the p.c. amplitude B is given in terms of the flavor factors multiplying the relevant mass ratio factors $R_s(\Sigma^0)$ and $R_u(\Sigma_c^0)$. One has

$$\begin{aligned}\hat{B}_s^{\text{pole}}(\Sigma^0) &: I^{\text{CQM}}_{\Lambda^0\pi^+\Sigma^+} I^{\text{pc}}_{\Sigma^+\Lambda_c^+} = 1,\\ \hat{B}^{\text{pole}}(u;\, \Sigma_c^0) &: I^{\text{pc}}_{\Lambda^0\Sigma_c^0} I^{\text{CQM}}_{\Sigma_c^0\pi^+\Lambda_c^+} = 1,\end{aligned} \tag{80}$$

where $I^{\text{pc}}_{\Sigma^+\Lambda_c^+} = -4/2\sqrt{6}$, $I^{\text{pc}}_{\Lambda^0\Sigma_c^0} = 4/2\sqrt{6}$, $I^{\text{CQM}}_{\Lambda^0\pi^+\Sigma^+} = -6/2\sqrt{6}$, and $I^{\text{CQM}}_{\Sigma_c^0\pi^+\Lambda_c^+} = 6/2\sqrt{6}$ (see Tables 16 and 18 in Appendix C). The mass ratios read

$$\begin{aligned}R_s(\Sigma^0) &= \frac{m_{\Lambda^0} + m_{\Sigma^0}}{m_{\Lambda_c^0} - m_{\Sigma^0}} = 2.11\\ R_u(\Sigma_c^0) &= \frac{m_{\Lambda_c^0} + m_{\Sigma_c^0}}{m_{\Lambda^0} - m_{\Sigma_c^0}} = -3.54,\end{aligned} \tag{81}$$

Our results are in agreement with the results in Ref. [60] up to an overall sign difference. Note that there is a contribution to $\hat{B}^{\text{pole}}$ in Eq. (79) from the topology III given by the term proportional to I_5. We conclude that this term was in fact included in the analysis of Ref. [60] despite the claim of the authors that the contributions from the type III diagram were omitted.

6.2 The CF decay $\Lambda_c^+ \to \Sigma^0\pi^+$ $4\sqrt{3}(I_i) = (0, 2, 2, 0, -2, 4, 1)$

The second decay we discuss is the CF decay $\Lambda_c^+ \to \Sigma^0\pi^+$. Again, this decay proceeds only by W-exchange. The asymmetry parameter has been measured and is given by $\alpha(\Lambda_c \to \Sigma^0\pi^+) = -0.73 \pm 0.17 \pm 0.07$. This implies that the S-wave amplitude must be nonvanishing. And in fact, a glance at Table 3 shows that one has a s-channel contribution while the u-channel contribution vanishes since $2\hat{I}_3 + 4I_5 = 0$. Let us check this in more detail. The W-exchange contributions to the reduced amplitudes $\hat{A}$ and $\hat{B}$ are given by

$$\begin{aligned}\hat{A}^{\text{com}} &= (\underline{2I_3} + \underline{4I_5}) - (\underline{2\hat{I}_3} + \underline{4I_5}) = 2/\sqrt{3} - 0(u),\\ \hat{B}^{\text{pole}} &= (\underline{I_3} + 2I_4 + \underline{6I_5})\, R_s(\Sigma^+) + (\underline{\hat{I}_3} + \underline{2\hat{I}_4} + \underline{6I_5})\, R_u(\Sigma_c^0)\\ &= 2/\sqrt{3}\, R_s(\Sigma^+) + 3/\sqrt{3}\, R_u(\Sigma_c^0)\end{aligned} \tag{82}$$

A direct summation over the intermediate state results in

$$\begin{aligned}\hat{A}^{\text{com}}(s;\, \Sigma^+) &: I^f_{\Sigma^0\pi^+\Sigma^+} I^{\text{pc}}_{\Sigma^+\Lambda_c^+} = 2/\sqrt{3},\\ \hat{A}^{\text{com}}(u;\, \Sigma_c^0) &: I^{\text{pc}}_{\Sigma^0\Sigma_c^0} I^f_{\Sigma_c^0\pi^+\Lambda_c^+} = 0,\\ \hat{B}^{\text{pole}}(s;\, \Sigma^+) &: I^{\text{CQM}}_{\Sigma^0\pi^+\Sigma^+} I^{\text{pc}}_{\Sigma^+\Lambda_c^+} = 2/\sqrt{3},\\ \hat{B}^{\text{pole}}(u;\, \Sigma_c^0) &: I^{\text{pc}}_{\Sigma^0\Sigma_c^0} I^{\text{CQM}}_{\Sigma_c^0\pi^+\Lambda_c^+} = 3/\sqrt{3}\end{aligned} \tag{83}$$

where we have used $I^f_{\Sigma^0\pi^+\Sigma^+} = -4/2\sqrt{2}$, $I^{\text{pc}}_{\Sigma^+\Lambda_c^+} = -4/2\sqrt{6}$, $I^f_{\Sigma_c^0\pi^+\Lambda_c^+} = 0$, $I^{\text{CQM}}_{\Sigma^0\pi^+\Sigma^+} = -4/2\sqrt{2}$ and $I^{\text{CQM}}_{\Sigma_c^0\pi^+\Lambda_c^+} = 6/2\sqrt{6}$ from Tables 16 and 18 in Appendix C.

6.3 The SCS decay $\Xi_c^0 \to pK^-$ $2\sqrt{6}(I_i) = (0, 0, 0, -2_a, 0, 0, -1_b)$

This decay is interesting from the point of view that there are altogether four states that contribute as intermediate states.

These are (Λ^0, Σ^0) in the s-channel and $(\Lambda_c^+, \Sigma_c^+)$ in the u-channel. The structure of the tensor invariants can be seen to be identical for this decay and the decay $\Lambda_c^+ \to \Xi^0 K^+$ treated in the previous section. We anticipate that the treatment in terms of tensor invariants will be a much simpler undertaking than the explicit summation of the four contributing intermediate states. In our analysis we shall retain the identification of the weak transitions in terms of the (a) and (b) type contributions.

The reduced commutator term $\hat{A}^{\rm com}$ and pole term $\hat{B}^{\rm pole}$ are given by

$$\begin{aligned}\hat{A}^{\rm com} &= (2I_3+4I_5)-(2\hat{I}_3+4I_5)\\ &= (-4_b/2\sqrt{6})-(-4_b/2\sqrt{6})=0,\\ \hat{B}^{\rm pole} &= (I_3+\underline{2I_4}+\underline{6I_5})\bar{R}_s(\Lambda^0,\Sigma^0)\\ &\quad +(\hat{I}_3+2\hat{I}_4+\underline{6I_5})R_u(\Sigma_c^0)\\ &= -(4_a+6_b)/2\sqrt{6}\,\bar{R}_s(\Lambda^0,\Sigma^0)-6_b/2\sqrt{6}\,R_u(\Sigma_c^0).\end{aligned} \tag{84}$$

When taking the intermediate state route, one has to take into account that $I^f_{\Sigma_c^+ K^- \Xi_c^0}=0$ and $I^{\rm CQM}_{\Lambda_c^+ K^- \Xi_c^0}=0$ (see Appendix C). For the relevant products of flavor factors one obtains

$$\begin{aligned}&\hat{A}^{\rm com}(s;\Lambda^0,\Sigma^0): I^f_{pK^-\Lambda^0}I^{\rm pc}_{\Lambda^0\,\Xi_c^0}+I^f_{pK^-\Sigma^0}I^{\rm pc}_{\Sigma^0\,\Xi_c^0}\\ &\quad =\big(-(2_a+4_b)+2_a\big)/2\sqrt{6},\\ &\hat{A}^{\rm com}(u;\Lambda_c^+): I^{\rm pc}_{p\,\Lambda_c^+}I^f_{\Lambda_c^+K^-\Xi_c^0}=(-4_b)/2\sqrt{6},\\ &\hat{B}^{\rm pole}(s;\Lambda^0,\Sigma^0): I^{\rm CQM}_{pK^-\Lambda^0}I^{\rm pc}_{\Lambda^0\Xi_c^0}+I^{\rm CQM}_{pK^-\Sigma^0}I^{\rm pc}_{\Sigma^0\Xi_c^0}\\ &\quad =\big(-(3_a+6_b)-1_a\big)/2\sqrt{6},\\ &\hat{B}^{\rm pole}(u;\Sigma_c^+): I^{\rm pc}_{p\,\Sigma_c^+}I^{\rm CQM}_{\Sigma_c^+K^-\Xi_c^0}=-6_b/2\sqrt{6}.\end{aligned} \tag{85}$$

Keeping in mind that the two S-wave contributions have to be subtracted while the two P-wave contributions have to be added, one finds agreement of the two calculational routes. The net result is that the commutator contribution to the decay $\Xi_c^0 \to pK^-$ vanishes.

6.4 The CF decay $\Omega_c^0 \to \Xi^0 \bar{K}^0$
$2(I_i)=(-1,-1,0,0,-2,0,0)$

The only W-exchange contribution to this decay is from the tensor invariant $\hat{I}_3=-1$ (see Table 8). Therefore, the decay proceeds only via the u-channel, which becomes obvious from

$$\begin{aligned}\hat{A}^{\rm com} &= (2I_3+4I_5)-(\underline{2\hat{I}_3}+4I_5)=0(s)-(-2),\\ \hat{B}^{\rm pole} &= 0(s)+(\underline{\hat{I}_3}+2\hat{I}_4+6I_5)\,R_u(\Xi_c^0,\Xi_c^{\prime 0})\\ &= 0(s)-\bar{R}_u(\Xi_c^0,\Xi_c^{\prime 0}).\end{aligned} \tag{86}$$

where $0(s)$ stands for the absence of an s-channel contribution. The interest in this decay is caused by the fact that both intermediate states Ξ_c^0 and $\Xi_c^{\prime 0}$ contribute to the P-wave amplitude $B^{\rm pole}(u)$. In terms of the flavor invariants I^f, $I^{\rm pc}$ and $I^{\rm CQM}$ one obtains

$$\begin{aligned}&\hat{A}^{\rm com}(u;\Xi_c^{\prime 0}): I^{\rm pc}_{\Xi^0\Xi_c^{\prime 0}}I^f_{\Xi_c^{\prime 0}\bar{K}^0\Omega_c^0}=-2,\\ &\hat{B}^{\rm pole}(u;\Xi_c^0,\Xi_c^{\prime 0}): I^{\rm pc}_{\Xi^0\Xi_c^0}I^{\rm CQM}_{\Xi_c^0\bar{K}^0\Omega_c^0}+I^{\rm pc}_{\Xi^0\Xi_c^{\prime 0}}I^{\rm CQM}_{\Xi_c^{\prime 0}\bar{K}^0\Omega_c^0}=-1.\end{aligned} \tag{87}$$

The results are in agreement with Ref. [62]. There is no need to differentiate between the current algebra approach and the modified current algebra result in this case since $I_5=0$.

In the limit of vanishing hyperfine splitting $m_{\Xi_c^0}=m_{\Xi_c^{\prime 0}}$ one can sum the two P-wave u-channel contributions to obtain

$$\begin{aligned}\hat{B}^{\rm pole} &= 2(\underline{\hat{I}_3}+2\hat{I}_4+6I_5)\frac{m_{\Xi_0}-\bar{m}_{\Xi_c^0}}{m_{\Omega_c^0}-\bar{m}_{\Xi_c^0}}\\ &= -2\frac{m_{\Xi_0}-\bar{m}_{\Xi_c^0}}{m_{\Omega_c^0}-\bar{m}_{\Xi_c^0}},\end{aligned} \tag{88}$$

where $\bar{m}_{\Xi_c^0}$ denotes a suitable mass average of the two flavor degenerate states Ξ_c^0 and $\Xi_c^{\prime 0}$. The decay was analyzed in Ref. [50], again in the context of the current algebra approach. In the first version of Ref. [50] there was the claim that the contributions of the intermediate states Ξ_c^0 and $\Xi_c^{\prime 0}$ cancel each other for $m_{\Xi_c^0}=m_{\Xi_c^{\prime 0}}$. This statement was corrected later on in an Erratum to Ref. [50] with a result in agreement with Eq. (88). That the two contributions do not, in fact, cancel can be easily checked by a mere visual inspection of the tensor invariants in Table 8. We have provided this example to illustrate the power of the current algebra approach when expressed in terms of topological tensor invariants because the compactness of the tensor invariant expressions allows one to easily trace mistakes in a current algebra result.

6.5 The SCS decay $\Omega_c^0 \to \Sigma^0 \bar{K}^0$
$2\sqrt{2}(I_i)=(0,0,0,-2_a,0,0,-1_a)$

The decay proceeds only through W exchange and via the (a)-type weak interaction. The interest in this decay is based on the fact that up to normalization factors the topological invariant structure is identical to the one of the previous decay $\Lambda_c^+ \to \Xi^0 K^+$. The same statement holds true for the decays

$$\begin{aligned}&\text{CF}: \Lambda_c^+\to\Xi^0K^+;\quad \Xi_c^0\to\Sigma^+K^-\\ &\text{SCS}: \Omega_c^0\to\Sigma^+K^-,\ \Sigma^0\bar{K}^0;\quad \Xi_c^0\to\Sigma^+\pi^-,\ pK^-\\ &\text{DCS}: \Xi_c^+\to p\pi^0,\ n\pi^+;\quad \Xi_c^0\to p\pi^-,\ n\pi^0.\end{aligned} \tag{89}$$

One obtains

$$\begin{aligned}\hat{A}^{\rm com} &= (2I_3+4I_5)-(2\hat{I}_3+4I_5)\\ &= (-4_b/2\sqrt{2})-(-4_b/2\sqrt{2})=0,\\ \hat{B}^{\rm pole} &= (I_3+\underline{2I_4}+\underline{6I_5})\bar{R}_s(\Xi^0)\end{aligned}$$

$$+(\hat{I}_3 + 2\hat{I}_4 + \underline{6I_5})\bar{R}_u(\Xi_c^0, \Xi_c'^0) = \\ = -(10_a)/2\sqrt{2}\,\bar{R}_s(\Xi^0) - 6_a/2\sqrt{2}\,\bar{R}_u(\Xi_c^0, \Xi_c'^0) \tag{90}$$

or

$$\begin{aligned}
&\hat{A}^{\text{com}}(s;\,\Xi^0) : I^f_{\Sigma^0\bar{K}^0\Xi^0} I^H_{\Xi^0\Omega_c^0} = -4_a/2\sqrt{2} \\
&\hat{A}^{\text{com}}(u;\,\Xi_c^0) : I^H_{\Sigma^0\Xi_c^0} I^f_{\Xi_c^0\bar{K}^0\Omega_c^0} = -4_a/2\sqrt{2} \\
&\hat{B}^{\text{pole}}(s;\,\Xi^0) : I^{\text{CQM}}_{\Sigma^0 K^0\Xi^0} I^H_{\Xi^0\Omega_c^0} = -10_a/2\sqrt{2} \\
&\hat{B}^{\text{pole}}(u;\,\Xi_c^0, \Xi_c'^0) : I^H_{\Sigma^0\Xi_c^0} I^{\text{CQM}}_{\Xi_c^0\bar{K}^0\Omega_c^0} + I^H_{\Sigma^0\Xi_c'^0} I^{\text{CQM}}_{\Xi_c'^0\bar{K}^0\Omega_c^0} \\
&= (-2_a - 4_a)/2\sqrt{2},
\end{aligned} \tag{91}$$

where

$$\begin{aligned}
R_u(\Xi_c^0) &= \frac{m_{\Omega_c^0} + m_{\Xi_c^0}}{m_{\Sigma^0} - m_{\Xi_c^0}} = 4.83, \\
R_u(\Xi_c'^0) &= \frac{m_{\Omega_c^0} + m_{\Xi_c'^0}}{m_{\Sigma^0} - m_{\Xi_c'^0}} = 4.53.
\end{aligned} \tag{92}$$

6.6 The $\Delta C = 0$ SCS decays $\Xi_c^+ \to \Lambda_c^+\pi^0$ and $\Xi_c^0 \to \Lambda_c^+\pi^-$ $12\sqrt{2}(I_i(\Xi_c^+ \to \Lambda_c^+\pi^0)) = 12(I_i(\Xi_c^0 \to \Lambda_c^+\pi^-)) = (-5_{a'}, 4_{a'}, -2_{b'}, 4_{b'}, -8_{a'}, 4_{a'}, 1_{b'})$

The decays are contributed to by both the factorizing tree diagram and the nonfactorizing W-exchange contributions. The tree diagram contributions induced by the transition (a') $s \to u;\ u \to d$ are purely p.v. S-wave contributions, as follows from the light diquark transition $0^+ \to 0^+ + 0^-$ in the background field of the heavy charm quark. It is therefore interesting to have a closer look at the structure of the W-exchange contributions.

Let us first state that the two decays are related by the $\Delta I = 1/2$ rule as follows (see Table 9)

$$\sqrt{2}\,\mathcal{M}(\Xi_c^+ \to \Lambda_c^+\pi^0) = \mathcal{M}(\Xi_c^0 \to \Lambda_c^+\pi^-). \tag{93}$$

For the tree level contribution this comes about since we are using only the $\mathcal{H}_{\text{eff}}(\mathcal{O}_-)$ contribution. For the W-exchange contributions the $\Delta I = 1/2$ rule is a consequence of the KPW theorem.

Let us concentrate on the W-exchange contribution to the decay $\Xi_c^0 \to \Lambda_c^+\pi^-$. We begin with the tensor invariant representation. Since $2I_3 + 4I_5 = 0$ (see Table 9), the S-wave s-channel contribution vanishes, i.e. one has $A^{\text{com}}(s) = 0$. The nonvanishing S-wave u-channel and P-wave (s, u)-channel contributions read

$$\begin{aligned}
\hat{A}^{\text{com}} &= 0(s) - \big(\underline{2\hat{I}_3} + \underline{4I_5}\big) = 0(s) - (-16_{a'} + 4_{b'})/12, \\
\hat{B}^{\text{pole}} &= \big(\underline{I_3} + \underline{2I_4} + \underline{6I_5}\big)R_s(\Sigma_c^0) + \big(\underline{\hat{I}_3} + \underline{2\hat{I}_4} + \underline{6I_5}\big)R_u(\Xi_c'^+) \\
&= 12_{b'}/12\,R_s(\Sigma_c^0) + 6_{b'}/12\,R_u(\Xi_c'^+),
\end{aligned} \tag{94}$$

where $2\hat{I}_3 + 4I_5 = (-16_a + 4_b)/12$, $I_3 + 2I_4 + 6I_5 = 12_b/12$ and $\hat{I}_3 + 2\hat{I}_4 + 6I_5 = 6_b/12$ in agreement with the result of the intermediate state route where

$$\begin{aligned}
&\hat{A}^{\text{com}}(u;\,\Xi_c^+) : I^H_{\Lambda_c^+\Xi_c^+} I^f_{\Xi_c^+\pi^-\Xi_c^0} = (-16_{a'} + 4_{b'})/12, \\
&\hat{B}^{\text{pole}}(s;\,\Sigma_c^0) : I^{\text{CQM}}_{\Lambda_c^+\pi^-\Sigma_c^0} I^H_{\Sigma_c^0\Xi_c^0} = 12_{b'}/12, \\
&\hat{B}^{\text{pole}}(u;\,\Xi_c'^+) : I^H_{\Lambda_c^+\Xi_c'^+} I^{pc\,(\pi^-)}_{\Xi_c'^+\Xi_c^0} = 6_{b'}/12.
\end{aligned} \tag{95}$$

Note that P-wave transitions are solely induced by the quark level transition (b') $c \to d;\ s \to c$. The transition (a') $s \to u;\ u \to d$ does not contribute because of a quantum number mismatch for the P-wave s-channel amplitude as exemplified by the fact that the W-exchange topologies IIa and III do not admit a contribution (a'). The KPW theorem is responsible for the absence of a contribution (a') to the P-wave u-channel amplitude. Contrary to what is stated in the literature [74,75,92], one obtains a nonvanishing P-wave contribution from the transition (b') which, in addition, is considerably enhanced by the mass ratio factors $R_s(\Sigma_c^0) = 278.82$ and $R_u(\Xi_c'^+) = -17.29$. Such a large P-wave contribution is quite welcome since the present model calculations, which are based on an assumed S-wave transition, are below or far below the experimental rate measurement [17].

6.7 The $\Delta C = 0$ SCS decays $\Omega_c^0 \to \Xi_c^+\pi^-$ and $\Omega_c^0 \to \Xi_c^0\pi^0$ $2\sqrt{6}(I_i(\Omega_c^0 \to \Xi_c^+\pi^-)) = 4\sqrt{3}(I_i(\Omega_c^0 \to \Xi_c^0\pi^0)) = (1_{a'}, -2_{a'}, 2_{b'}, -4_{b'}, 0, 0, 0)$

The decays are contributed to by both tree diagram and W-exchange contributions. Since one now has a $D(1^+) \to D(0^+) + 0^-$ diquark transition in the background field of the heavy charm quark, the tree contribution is purely P-wave.

The W-exchange contributions are induced by the quark level transition (b') $c \to d;\ s \to c$ which is a pure $\Delta I = 1/2$ transition for both $\mathcal{H}_{\text{eff}}(\mathcal{O}_-)$ transitions. Therefore, for the W-exchange contributions one has the $\Delta I = 1/2$ relation

$$\sqrt{2}\,\mathcal{M}(\Omega_c^0 \to \Xi_c^0\pi^0) = -\mathcal{M}(\Omega_c^0 \to \Xi_c^+\pi^-), \tag{96}$$

regardless of whether one invokes the KPW theorem.

As Table 9 shows, the topologies IIb and III do not contribute as can be surmise from the fact that $\hat{I}_3 = \hat{I}_4 = I_5 = 0$, i.e. there are no u-channel W-exchange contributions. One obtains

$$\begin{aligned}
\hat{A}^{\text{com}} &= \big(\underline{2I_3} + 4I_5\big) - \big(2\hat{I}_3 + 4I_5\big) = 4_{b'}/2\sqrt{6} - 0(u), \\
\hat{B}^{\text{pole}} &= \big(\underline{I_3} + \underline{2I_4} + 6I_5\big)\,R_s(\Xi_c'^0) + 0(u) \\
&= -6_{b'}/2\sqrt{6}\,R_s(\Xi_c'^0) + 0(u),
\end{aligned} \tag{97}$$

where $2I_3 = 4_b/2\sqrt{6}$ and $I_3 + 2I_4 = -6_b/2\sqrt{6}$. It is apparent that the W-exchange contributions generate an S-wave contribution through the topology IIa.

Let us confirm that we obtain the same result for the intermediate state representation. The s-channel intermediate states are Ξ_c^0 and $\Xi_c^{\prime 0}$, resp., since $I^f_{\Xi_c^+\pi^-\Xi_c^{\prime 0}} = 0$ and $I^{\rm CQM}_{\Xi_c^+\pi^-\Xi_c^0} = 0$. One therefore has

$$\hat{A}^{\rm com}(s;\, \Xi_c^0) : I^f_{\Xi_c^+\pi^-\Xi_c^0}\, I^{\rm pc}_{\Xi_c^0\Omega_c^0} = 4_{b'}/2\sqrt{6}$$
$$\hat{B}^{\rm pole}(s;\, \Xi_c^{\prime 0}) : I^{\rm CQM}_{\Xi_c^+\pi^-\Xi_c^{\prime 0}}\, I^{\rm pc}_{\Xi_c^{\prime 0}\Omega_c^0} = -6_{b'}/2\sqrt{6}. \tag{98}$$

Again, the P-wave contribution is enhanced by the mass factor $R_s(\Xi_c^{\prime 0}) = 43.51$, but not as much as in the previous case $\Xi_c^0 \to \Lambda_c^+\pi^-$.

6.8 The CF decay $\Xi_{cc}^{++} \to \Xi_c^+\pi^+$ $2\sqrt{6}(I_i) = (-2, 1, 0, 0, -4, 4, 0)$

The decay belongs to the class of decays $B_{cc}(\mathbf{3}) \to B_c(\bar{\mathbf{3}}) + M(\mathbf{8})$. One has

$$\hat{A}^{\rm com} = 0(s) - (\underline{2\hat{I}_3} + 4I_5) = -8/2\sqrt{6},$$
$$\hat{B}^{\rm pole} = 0(s) + \left(\underline{\hat{I}_3} + \underline{2\hat{I}_4} + 6I_5\right) R_u(\Xi_{cc}^+)$$
$$= 4/2\sqrt{6}\, R_u(\Xi_{cc}^+) \tag{99}$$

or

$$\hat{A}^{\rm com}(u;\, \Xi_{cc}^+) : I^{\rm pc}_{\Xi_c^+\Xi_{cc}^+}\, I^f_{\Xi_{cc}^+\pi^+\Xi_{cc}^{++}} = (\tfrac{8}{2\sqrt{6}})(-1) = -8/2\sqrt{6},$$
$$\hat{B}^{\rm pole}(u;\, \Xi_{cc}^+) : I^{\rm pc}_{\Xi_c^+\Xi_{cc}^+}\, I^{\rm CQM}_{\Xi_{cc}^+\pi^+\Xi_{cc}^{++}} = (\tfrac{8}{2\sqrt{6}})(\tfrac{1}{2}) = 4/2\sqrt{6}. \tag{100}$$

6.9 The CF decay $\Xi_{cc}^+ \to \Xi_c^+\pi^0$ $4\sqrt{2}(I_i) = (0, 0, 4, -2, -4, 4, 0)$

This decay also belongs to the class $B_{cc}(\mathbf{3}) \to B_c(\bar{\mathbf{3}}) + M(\mathbf{8})$. The decay is not related to the previous decay $\Xi_{cc}^{++} \to \Xi_c^+\pi^+$ by isospin symmetry, as a comparison of the tensor invariants shows. There are, in fact, two reduced isospin amplitudes describing the decays $\Xi_{cc} \to \Xi_c\pi$. Analyzing the flavor flow in diagram III one concludes that the topological invariant I_5 vanishes. From the general analysis in Sect. 4 we know that $I_3 + 2I_4 = 0$ is a general result for this class of decays. In the language of the current algebra approach this implies that the P-wave s-channel contribution vanishes. Candidates for the intermediate s-channel states are Ξ_c^+ and $\Xi_c^{\prime +}$. Still, $\Xi_c^{\prime +}$ does not contribute since the weak transition $\langle \Xi_c^{\prime +}|H^{\rm pc}|\Xi_{cc}^+\rangle$ vanishes in the SU(3) limit due to the KPW theorem. In terms of the topological tensor invariants one has

$$\hat{A}^{\rm com} = (\underline{2I_3} + 4I_5) - (\underline{2\hat{I}_3} + 4I_5) = 8/4\sqrt{3} - (-8/4\sqrt{3}),$$
$$\hat{B}^{\rm pole} = (\underline{I_3} + \underline{2I_4} + 6I_5)\, R_s(\Xi_c^+) + (\underline{\hat{I}_3} + \underline{2\hat{I}_4} + 6I_5)$$
$$R_u(\Xi_{cc}^+) = 4/4\sqrt{3} R_u(\Xi_{cc}^+) \tag{101}$$

with $2(\underline{I_3} - \widehat{\underline{I_3}}) = 4/\sqrt{3}$ and $\underline{\hat{I}_3} + \underline{2\hat{I}_4} + 6I_5 = 1/\sqrt{3}$. The same result is obtained by the intermediate state analysis where one has

$$\hat{A}^{\rm com}(s;\, \Xi_c^+) : I^f_{\Xi_c^+\pi^0\Xi_c^+}\, I^{\rm pc}_{\Xi_c^+\Xi_{cc}^+} = 8/4\sqrt{3},$$
$$\hat{A}^{\rm com}(u;\, \Xi_c^{++}) : I^{\rm pc}_{\Xi_c^+\Xi_{cc}^+}\, I^f_{\Xi_{cc}^+\pi^0\Xi_{cc}^+} = -8/4\sqrt{3},$$
$$\hat{B}^{\rm CQM}(u;\, \Xi_{cc}^+) : I^{\rm pc}_{\Xi_c^+\Xi_{cc}^+}\, I^{\rm CQM}_{\Xi_{cc}^+\pi^0\Xi_{cc}^+} = 4/4\sqrt{3}. \tag{102}$$

6.10 The CF decay $\Xi_{cc}^+ \to \Sigma_c^{++}K^-$ $2(I_i) = (0, 0, 0, 2, 0, 0, 0)$

This decay belongs to the class $B_{cc}(\mathbf{3}) \to B_c(\mathbf{6}) + P(\mathbf{8})$. From the discussion in Sect. 4 we know that $I_3 = \hat{I}_3 = \hat{I}_4 = I_5 = 0$ for this class of decays. The only nonvanishing contribution is $I_4 = 1$, i.e. there is no factorizing contribution to this decay. When stated in terms of the current algebra approach, this implies that the only nonvanishing contribution to this decay is the P-wave s-channel contribution.

We verify this in explicit form using the current algebra representation. First note that there is no candidate for the u-channel intermediate state. As concerns the s-channel, due to the KPW theorem $\Xi_c^{\prime +}$ does not contribute as intermediate state. On the other hand Ξ_c^+ does not contribute to the S-wave s-channel since $I^f_{\Sigma_c^{++}K^-\Xi_c^+} = 0$. One thus remains with the P-wave s-channel contribution $B^{\rm pole}(s)$. One has

$$\hat{B}^{\rm pole} = (I_3 + \underline{2I_4} + 6I_5)\, R_s(\Xi_c^+) + 0(u)$$
$$= 2R_s(\Xi_c^+) + 0(u), \tag{103}$$

where $I_3 + \underline{2I_4} + 6I_5 = 2$. In terms of the intermediate state path one has

$$\hat{B}^{\rm pole}(s,\ \Xi_c^+) : I^{\rm CQM}_{\Sigma_c^{++}K^-\Xi_c^+}\, I^{\rm pc}_{\Xi_c^+\Xi_{cc}^+} = 2 \tag{104}$$

in accord with the result in terms of tensor invariants. This result is in agreement with the result in Ref. [59]. Note that there is no need for modify the current algebra approach in this case since $I_5 = 0$.

6.11 The CF decay $\Xi_{cc}^{++} \to \Sigma^+ D^+$ $2(I_i) = (0, 0, 0, 0, 0, -2, 0)$

This decay belong to the class $B_{cc}(\mathbf{3}) \to B(\mathbf{8}) + M(\bar{\mathbf{3}})$. The current algebra approach applies to the P-waves only where we can check the structure of the $1/2^+$ pole contributions. As Table 12 shows, there are only u-channel pole contributions since the only nonvanishing tensor invariant is $\hat{I}_4$. This agrees with the observation that, from the flavor flow, there can be no intermediate s-channel contributions. One has

$$\hat{B}^{\rm pole}(u;\, \Sigma_c^+, \Lambda_c^+) = (\hat{I}_3 + \underline{2\hat{I}_4} + I_5)\, \bar{R}_u(\Sigma_c^+, \Lambda_c^+)$$
$$= -2\, \bar{R}_u(\Sigma_c^+, \Lambda_c^+) \tag{105}$$

or

$$\hat{B}^{\text{pole}}(u;\,\Sigma_c^+,\Lambda_c^+) : I^{\text{pc}}_{\Sigma^+\Sigma_c^+} I^{\text{CQM}}_{\Sigma_c^+ D^+ \Xi_{cc}^{++}} + I^{H}_{\Sigma^+\Lambda_c^+} I^{\text{CQM}}_{\Lambda_c^+ D^+ \Xi_{cc}^{++}} = 1/2 - 5/2 = -2. \tag{106}$$

6.12 The CF decay $\Xi_{cc}^+ \to \Sigma^+ D^0$
$2(I_i) = (0, 0, 0, 0, 0, 0, 1)$

This decay belongs to the same class as the previous one, $B_{cc}(\mathbf{3}) \to B(\mathbf{8}) + M(\bar{\mathbf{3}})$. Since $I^{\pm}_{1,2} = 0$, there are no tree diagram contributions. The only nonvanishing tensor invariant is I_5. This implies that one has both s- and u-channel pole contributions. The current algebra approach only applies to the P-wave pole contribution where we can check on the structure of the $1/2^+$ pole contributions. The s-channel intermediate state is Ξ_c^+. In the u-channel both Λ_c^+ and Σ_c^+ contribute as intermediate states. The structure of the P-wave contributions is given by

$$\begin{aligned}\hat{B}^{\text{pole}}(s;\,\Xi_c^+) &= (I_3 + 2I_4 + \underline{6I_5})\, R_s(\Xi_c^+) = 3R_s(\Xi_c^+),\\ \hat{B}^{\text{pole}}(u;\,\Lambda_c^+,\Sigma_c^+) &= (\hat{I}_3 + 2\hat{I}_4 + \underline{6I_5})\, R_u(\Lambda_c^+,\Sigma_c^+)\\ &= 3R_u(\Lambda_c^+,\Sigma_c^+).\end{aligned} \tag{107}$$

Since $I^{\text{CQM}}_{\Sigma^+ D^0 \Xi_c^+} I^{H}_{\Xi_c^+ \Xi_{cc}^+} = 3$ and $I^{\text{CQM}}_{\Lambda_c^+ D^0 \Xi_{cc}^+} + I^{H}_{\Sigma^+\Sigma_c^+} I^{\text{CQM}}_{\Sigma_c^+ D^0 \Xi_{cc}^+} = 1/2 + 5/2 = 3$, the result obtained via intermediate states agrees with this result. Note that the modified current algebra approach predicts that the decay $\Xi_{cc}^+ \to \Sigma^+ D^0$ vanishes.

7 Some general features of the topological tensor and current algebra approaches

The tables in Sect. 4 contain a wealth of physics information on the nonleptonic decays of charm baryons. Just by visual inspection one can identify many interesting patterns of physical relevance concerning the nonleptonic charm baryon decays. We divide the general remarks into two parts. In the first part we list general features of the various charm baryon decays without using any dynamical input. In the second part we identify some general features in the tables using elements of the current algebra approach. We begin with by listing decays that have all zero entries in the tables, i.e. their decay rates are predicted to be zero. They are all DCS decays and read

$$\begin{aligned}&\Gamma(\Omega_c^0 \to \Lambda^0\pi^0,\, \Sigma^0\eta(\eta')) = 0,\\ &\Gamma(\Omega_{cc}^+ \to \Lambda_c^+\pi^0,\, \Sigma_c^+\eta(\eta')) = 0,\\ &\Gamma(\Omega_{cc}^+ \to \Sigma^0 D_s^+) = 0.\end{aligned} \tag{108}$$

A corresponding prediction is given in the following. Note that there are no tree contributions to any of these decays which implies that one only has transitions induced by the effective Hamiltonian $\mathcal{H}_{\text{eff}}(\mathcal{O}_-)$. According to Table 2 the DCS Hamiltonian $\mathcal{H}_{\text{eff}}(\mathcal{O}_-)$ from Eq. (5) transforms as an isosinglet ($\Delta I = 0$). From the isospin assignments of the charm baryons in Table 1 and the fact that $I(D_s^{(*)+}) = 0$ one can see that all the listed decays are isospin forbidden. A related derivation of the vanishing of the decay $\Omega_{cc}^+ \to \Sigma^0 D_s^{*+}$ can also be obtained from a topological point of view and the KPW theorem. Both the isospin forbidden decay $\Omega_{cc}^+ \to \Sigma^0 D_s^{*+}$ and the isospin allowed decay $\Omega_{cc}^+ \to \Lambda^0 D_s^{*+}$ proceed via the topological diagrams IIb and III in which the final state quark pair $[du]$ emerging from the weak vertex is antisymmetric. It then follows that the decay $\Omega_{cc}^+ \to \Sigma^0 D_s^{*+}$ is forbidden while the decay $\Omega_{cc}^+ \to \Lambda^0 D_s^{*+}$ is allowed. In view of this observation the result $\Gamma(\Omega_{cc}^+ \to \Sigma^0 D_s^{*+})/\Gamma(\Omega_{cc}^+ \to \Lambda^0 D_s^{*+}) = 16.71$ obtained in Ref. [78] is rather puzzling.

We discuss a sample of linear relations between pairs of partner amplitudes that follow from the tables which are based on SU(3) invariance. These relations can also be derived by making use of the SU(2) isospin (I-spin), U-spin and V-spin subgroups of SU(3). The identification of such bilinear relations in terms of their I-spin, U-spin or V-spin origin may sometimes be helpful from the mnemonic point of view. We discuss a few examples for each of the I-spin, U-spin and V-spin subgroup relations where we restrict ourselves to bilinear relations between pairs of particle decay amplitudes, leaving out triangular relations between three decay amplitudes. We thus concentrate on bilinear amplitude relations which are governed by a single I-spin, U-spin or V-spin reduced matrix element.

7.1 I-spin sum rules

As an example we take the CF relation

$$\mathcal{A}(\Lambda_c^+ \to \Sigma^+\pi^0) = -\mathcal{A}(\Lambda_c^+ \to \Sigma^0\pi^+) \tag{109}$$

following from Table 3. The relation (109) can also be derived from the I-spin subgroup using the isospin decomposition $0 \to (\Delta I = 1)\otimes 1 \otimes 1$.[1] We have used a calligraphic notation $\mathcal{A}$ in Eq. (109) where $\mathcal{A}$ stands either for the p.v. amplitude A or for the p.c. amplitude B. If one neglects isospin mass breaking effects, this leads to the equality of the two rates

$$\Gamma(\Lambda_c^+ \to \Sigma^+\pi^0) = \Gamma(\Lambda_c^+ \to \Sigma^0\pi^+). \tag{110}$$

There are many such isospin relations which can be read off from the tables. We do not list them all except for an interesting prediction on the vanishing of amplitudes following from I-spin symmetry. This prediction comes about by noticing

[1] Note that in this section we prefer to use the angular momentum notation instead of the notation in irreducible representations of SU(3) which are characterized by their multiplicity (in boldface). In terms of these irreducible representations, the decomposition at hand reads $\mathbf{1} \to \mathbf{3} \otimes \mathbf{3} \otimes \mathbf{3}$.

that the DCS transition $c \to d;\ s \to u$ is an isospin scalar with $\Delta I = 0$. As a result one has three DCS decays that are altogether isospin forbidden. These are

$$\Gamma(\Omega_c^0 \to \Lambda^0 \pi^0) = 0, \quad \Gamma(\Omega_{cc}^+ \to \Lambda_c^+ \pi^0) = 0, \tag{111}$$

with an isospin decomposition $0 \to (\Delta I = 0) \otimes 0 \otimes 1$ or $0 \to (\Delta I = 0) \otimes 1 \otimes 0$. Of physical relevance are also the related six forbidden decays involving the vector mesons ω, ϕ and D_s^{*+}. First of all, these are the five $\Delta C = 1$ DCS decays

$$\begin{aligned} &\Gamma(\Omega_c^0 \to \Sigma^0 \eta_8,\ \Sigma^0 \eta_1) = 0, \\ &\Gamma(\Omega_{cc}^+ \to \Sigma_c^+ \eta_8,\ \Sigma_c^+ \eta_1) = 0. \end{aligned} \tag{112}$$

Finally, the $\Delta C = 2$ DCS decay $\Omega_{cc}^+ \to \Sigma^0 D_s^+$ is allowed from the topological diagram point of view IIb and III but forbidden by the KPW theorem.

7.2 U-spin sum rules

First of all, note that the three effective CF, SCS and DCS Hamiltonians belong to the same $\Delta U = 1$ multiplet with ΔU_3 quantum numbers $+1$, 0, and -1, respectively (cf. Table 2 and Fig. 2). Again we shall only discuss bilinear U-spin sum rule relations. As an example we consider the four decays $\Xi_c^0 \to \Sigma^+ K^+$ (CF), $\Xi_c^0 \to \Sigma^+ \pi^-$ (SCS), $\Xi_c^0 \to p K^-$ (SCS) and $\Xi_c^0 \to p \pi^-$ (DCS). There is only one U-spin reduced matrix element for all four decays, as can be seen from the decomposition of the U-spin product $0 \to (\Delta U = 1) \otimes 1/2 \otimes 1/2$. Accordingly, one has the amplitude relations

$$\begin{aligned} &\frac{\mathcal{A}(\Xi_c^0 \to \Sigma^+ K^-)}{c^2} = -\frac{\mathcal{A}(\Xi_c^0 \to \Sigma^+ \pi^-)}{cs} \\ &= -\frac{\mathcal{A}(\Xi_c^0 \to p K^-)}{cs} = -\frac{\mathcal{A}(\Xi_c^0 \to p \pi^-)}{s^2}, \end{aligned} \tag{113}$$

where we have divided the amplitudes by the relevant Cabibbo angle factors (one has $V_{us} = \lambda = s = \sin\theta_C \approx 0.224$ and $c = \cos\theta_C \approx 0.975$). The relations (113) are in agreement with the SU(3) analysis in the respective tables. More U-spin relations involving also more than two decay amplitudes can be found in Refs. [29,30].

7.3 V-spin sum rules

Again we shall only discuss bilinear V-spin sum rule relations. For example, using Table 3 for the CF decays $B_c(\bar{\mathbf{3}}) \to B(\mathbf{8}) + M(\mathbf{8})$ one reads off

$$\begin{aligned} &\mathcal{A}(\Lambda_c^+ \to p \bar{K}^0) = -\mathcal{A}(\Xi_c^0 \to \Xi^- \pi^+), \\ &\mathcal{A}(\Xi_c^+ \to \Xi^0 \pi^+) = -\mathcal{A}(\Xi_c^+ \to \Sigma^+ \bar{K}^0) \end{aligned} \tag{114}$$

which also follow from V-spin symmetry via the decomposition of the V-spin products $1/2 \to (\Delta V = 0) \otimes 1 \otimes 1/2$ and $0 \to (\Delta V = 0) \otimes 1/2 \otimes 1/2$. In the same way one finds the V-spin CF amplitude relation

$$\mathcal{A}(\Lambda_c^+ \to \Xi^0 K^+) = -\mathcal{A}(\Xi_c^0 \to \Sigma^+ K^-). \tag{115}$$

The V-spin sum rules (114) and (115) have been listed before in Ref. [30] but have been missed in Ref. [29].

7.4 Combined sum rules

One can then combine the U-spin sum rule (113) and the V-spin sum rule (115) into one sum rule involving the five decays in Eqs. (113) and (115), as will be discussed further on.

The U-spin and V-spin amplitude relations do not directly translate into the corresponding rate relations since the rate formula involves different kinematical factors for the p.v. $|A|^2$ and p.c. $|B|^2$ contributions which can differ considerably from one another, as can be seen by the rate expression (cf. Appendix E)

$$\begin{aligned} \Gamma &= \frac{p}{4\pi m_i^2}\Big(Q_+|A|^2 + Q_-|B|^2\Big) \\ &= \frac{p}{32\pi m_i^2}\Big(|H^{\text{pv}}_{1/2\,0}|^2 + |H^{\text{pc}}_{1/2\,0}|^2\Big) \end{aligned} \tag{116}$$

($Q_\pm = (m_f \pm m_i)^2 - m_k^2$, $p = \sqrt{Q_+ Q_-}/(2m_i)$). In the last expression of Eq. (116) we have written the rate in terms of the two helicity amplitudes of the process. As discussed in the following, it can make a big difference for the SU(3) rate predictions depending on whether one postulates SU(3) invariance for the invariant amplitudes A and B or for the helicity amplitudes $H^{\text{pv}}_{1/2\,0}$ and $H^{\text{pc}}_{1/2\,0}$. The SU(3) analysis in Refs. [26,27,29] has been done using the helicity amplitude option while the SU(3) analysis in Refs. [23,24,28] is based on the invariant amplitude option. The authors of Ref. [25] take an extreme view and postulate SU(3) invariance for the rates or equivalently branching fractions, thereby disregarding all kinematic factors. This may be justified for isospin related decays but not for decays where the final states are composed of differing hypercharges. Savage and Springer advocate the use of an average of the kinematical S-wave and P-wave factors Q_+ and Q_- when comparing rates to SU(3) predictions [19].

7.5 More sum rules

Returning to the sum rules, more sum rules can be obtained by visually scanning the tables for decays with repeating patterns of the values of the tensor invariants. For example, for the pattern (0, 0, −2, 0, 2, −4, −1) one has the amplitude relation

$$\begin{aligned} &\sqrt{2}\mathcal{A}(\Lambda_c^+ \to \Sigma^0 \pi^+) = -\sqrt{2}\mathcal{A}(\Lambda_c^+ \to \Sigma^+ \pi^0) \\ &= -\mathcal{A}(\Xi_c^0 \to \Xi^0 K^0) = \mathcal{A}(\Xi_c^0 \to n \bar{K}^0). \end{aligned} \tag{117}$$

The four listed decays proceed via W-exchange alone.

The four decays $\Lambda_c^+ \to n\pi^+$, $\Lambda_c^+ \to \Lambda^0\pi^+$, $\Lambda_c^+ \to \Sigma^0\pi^+$ and $\Lambda_c^+ \to pK^-$ follow from the pattern $(x, x, x, x, x, -4, -1)$. The tensor invariant relation $\hat{I}_4 = 4I_5$ holds in addition to the four inherent relations listed in Eq. (23). This implies that there are only two topological invariants each for the amplitudes A and B describing the four decays. For definiteness we take the topological invariants $\mathcal{T}_3$ and $\mathcal{T}_5$. In this way one can then derive the sum rule

$$|\mathcal{A}|^2(\Lambda_c^+ \to n\pi^+)/s^2 = 3\,|\mathcal{A}|^2(\Lambda_c^+ \to \Lambda^0\pi^+) + |\mathcal{A}|^2(\Lambda_c^+ \to \Sigma^0\pi^+) - |\mathcal{A}|^2(\Lambda_c^+ \to p\bar{K}^0) \quad (118)$$

involving only squared magnitudes of the amplitudes $\mathcal{A} = A, B$. A corresponding sum rule was listed in Eq. (27) of Ref. [25]. However, the authors of Ref. [25] have written their sum rule in terms of rates, or equivalently, in terms of branching fractions assuming SU(3) to hold for the rates. In view of the widely diverging kinematical factors in the rate expression Eq. (116) the validity of their sum rule may be called into question.

Further sum rules are obtained by visually scanning the tables for decays with repeating patterns of the values of tensor invariants. One of the patterns is $(0, 0, 0, -2, 0, 0, -1)/N_j$, where N_j is the product of normalization factors of the flavor wave functions in the decay process $(B_c \to B + M)_j$. Collecting the decays with this pattern, we leave out decays related to isospin. Left are the four decays $\Xi_c^0 \to \Sigma^+K^-$, $\Xi_c^0 \to \Sigma^+\pi^-$, $\Xi_c^0 \to pK^-$ and $\Xi_c^0 \to p\pi^-$ listed in Eq. (113), and the decay $\Lambda_c^+ \to \Xi^0K^+$ (CF) listed in Eq. (115). The decay $\Omega_c^0 \to \Sigma^+K^-$ (SCS) also possesses the aforementioned flavor pattern and will be included in our analysis by appealing to SU(6) symmetry (as discussed in Sect. 3), even though the decay belongs to the class $B_c(\mathbf{6}) \to B(\mathbf{8}) + M(\mathbf{8})$ and not to the class $B_c(\bar{\mathbf{3}}) \to B(\mathbf{8}) + M(\mathbf{8})$ as the other five decays. The SU(3) analysis can then be done by relating the amplitudes of these six decays by the inverse of the normalization factors $1/N_j$. There are no tree-graph contributions to these decays and, adding a bit of dynamics from the current algebra approach, one expects that the p.v. amplitude A vanishes since $I_3 = \hat{I}_3 = 0$ in these cases. This observation will be the basis of our following analysis where we assume that the decays proceed via p.c. P-wave W-exchange contributions and where we assume that SU(3) invariance holds for the dimensionless amplitude B.

7.6 Spin kinematics

When comparing rates, one has to take into account the spin kinematical P-wave rate factor which, according to Eq. (116) reads $p\,Q_-/m_i^2 = Q_-\sqrt{Q_+Q_-}/2m_i^3$. One can then write down predictions for the six relative rates which read

$$\Gamma_{\Lambda_c^+\to\Xi^0K^+} : \Gamma_{\Xi_c^0\to\Sigma^+K^-} : \Gamma_{\Xi_c^0\to\Sigma^+\pi^-} : \Gamma_{\Xi_c^0\to pK^-} : \Gamma_{\Omega_c^0\to\Sigma^+K^-} : \Gamma_{\Xi_c^0\to p\pi^-} = 1\,c^4 : 2.25\,c^4 : 2.83\,c^2s^2 : 3.86\,c^2s^2 : 19.22\,c^2s^2 : 4.54\,s^4. \quad (119)$$

If instead one assumes that SU(3) holds for the dimensionful helicity amplitudes (see Appendix E) the corresponding kinematical factor is $\Gamma \sim p/m_i^2|H^{\mathrm{pc}}|^2$, and one arrives at the relative rate prediction

$$\Gamma_{\Lambda_c^+\to\Xi^0K^+} : \Gamma_{\Xi_c^0\to\Sigma^+K^-} : \Gamma_{\Xi_c^0\to\Sigma^+\pi^-} : \Gamma_{\Xi_c^0\to pK^-} : \Gamma_{\Omega_c^0\to\Sigma^+K^-} : \Gamma_{\Xi_c^0\to p\pi^-} = 1\,c^4 : 1.13\,c^4 : 1.23\,c^2s^2 : 1.29\,c^2s^2 : 6.70\,c^2s^2 : 1.38\,s^4. \quad (120)$$

It is apparent that the SU(3) symmetry predictions diverge widely depending on whether one postulates SU(3) symmetry for the invariant amplitudes or for the helicity amplitudes. As the numbers in Eqs. (119) and (120) show, the difference in the predicted rate ratios can amount up to a factor of 3.3 for the rate ratio $\Gamma_{\Xi_c^0\to p\pi^-}/\Gamma_{\Lambda_c^+\to\Xi^0K^+}$. The issue of whether to assume SU(3) invariance for the invariant or helicity amplitudes cannot be decided on a purely theoretical basis except for an aesthetic argument favoring the invariant amplitude option because the invariant amplitudes carry no mass dimension. In the long run only experiment can decide which of the two options has to be favored.

7.7 Exclusive W-exchange contributions

The hope is that from these decays one can glimpse a hint of the dynamics governing the W-exchange contributions without interference from the tree diagram contributions. Apart from the six decays listed in Eq. (119) there are a large number of decays that proceed via W-exchange contributions alone. Some examples are the CF decays $\Lambda_c^+ \to \Sigma^0\pi^+$ and $\Xi_c^0 \to \Xi^0\pi^0$ in Table 3. All of the decays $B_{cc} \to B + D$ in Table 12 have solely W-exchange contributions.

7.8 Tree diagrams alone

While theoretical progress on the dynamics of the W-exchange contributions has been rather slow, there exist a large body of literature on the treatment of the tree diagram contributions to charm baryon decays. The tree diagram contributions are related to the current induced charm baryon to light baryon transitions which have been studied in a large number of papers using a variety of theoretical models, including also lattice calculations. Among the models are

(i) the relativistic quark model [93–95]
(ii) the MIT bag model [96]

(iii) the covariant confined quark model [42,97–99]
(iv) the relativistic quark–diquark model [100,101]
(v) the semirelativistic quark model [102,103]
(vi) SU(3) based analysis [25,26]
(vii) light-cone sum rules [104–107]
(viii) the light front quark model [108]
(ix) QCD sum rules [109]
(x) lattice calculations [110].

For the single charm baryon decays one finds only one decay which proceeds via the tree diagram alone: the well-measured SCS decay $\Lambda_c \to p\,\phi$ [6–8] listed in Tables 4 and 5. This decay has gained a certain prominence since it allows one to fix the effective number of colors $N_c(\text{eff})$ coming into play when calculating the nonleptonic decay amplitude from the current induced transition amplitudes.

Contrary to the single charm decays, there are a large number of double charm decays from the class $B_{cc}(3) \to B_c(6) + M(8)$ in Table 11 which decay via the tree diagram alone. From the CF decays of this class these are the four decays $\Xi_{cc}^{++} \to \Sigma_c^{++}\bar{K}^0$, $\Xi_{cc}^{++} \to \Xi_c^{\prime +}\pi^+$, $\Omega_{cc}^+ \to \Omega_c^0\pi^+$ and $\Omega_{cc}^+ \to \Xi_c^{\prime +}\bar{K}^0$. From the flavor flow of diagram IIa one can check that these four decays cannot proceed via diagram IIa. As concerns the SCS and DCS decays, one finds a number of decays of this class in Table 11 which we do not list separably.

7.9 Weak $1 \to 3$ quark decays

The topological diagrams in Fig. 1 can be divided up into two classes. Diagrams Ia, Ib and IIb are characterized by weak $1 \to 3$ short distance quark transitions while diagrams IIa and III are governed by $2 \to 2$ short distance quark transitions. Diagrams IIa and III both involve the creation of a quark pair from the vacuum. Quantum mechanically such a situation can be handled by the $^3P_0$ model which, however, brings in a considerable amount of uncertainties in terms of model parameters and an unknown energy dependence of the quark pair creation process. The covariant confined quark model (CCQM) is a quantum field theoretical model where the topological diagrams are interpreted as Feynman loop integrals with nonlocal quark particle interactions. In principle, a field theoretical model as the CCQM should also be able to describe transitions involving quark pair creation. However, as it turns out, the CCQM calculation of diagrams IIa and III becomes unreliable because the energy release in charm baryon decays is quite large and the energetic light quark corresponding to one of the light quarks in the created pair probes the nonlocal Gaussian wave functions in a problematic region. The CCQM does, however, provide reliable results for diagrams Ia, Ib and IIb as exemplified by the recent calculation of the decay $\Xi_{cc}^{++} \to \Xi_c^+\pi^+$ which obtains contributions only from diagrams Ia, Ib and IIb. Therefore, it is interesting to provide a list of decays that decay via the rearrangement diagrams Ia, Ib and IIb. It would be interesting to calculate this class of decays either in the quantum mechanical model or in the quantum field theoretical CCQM model.

As an inspection of the tables shows, there are many decays that belong to this class. Listing again only CF decays, we find the quark rearrangement transitions ($I_3, I_4, I_5 = 0$)

$$\begin{aligned} &\Xi_c^+ \to \Sigma^+\bar{K}^0,\, \Xi^0\pi^+, \quad \Omega_c \to \Xi^0\bar{K}^0, \\ &\Xi_{cc}^+ \to \Xi_c^+\pi^+,\, \Sigma^+D^+, \quad \Omega_{cc}^+ \to \Xi_c^+\bar{K}^0. \end{aligned} \tag{121}$$

7.10 No S-wave contributions from W-exchange

According to the current algebra analysis, the W-exchange contribution to the p.v. amplitude A is proportional to $I_3 - \hat{I}_3$, i.e. $A(W\text{-exchange}) \sim I_3 - \hat{I}_3$. The vanishing of $A(W\text{-exchange})$ can be realized in two ways. One can have (i) $I_3 = \hat{I}_3$, $I_3 \neq 0$ or (ii) $I_3 = \hat{I}_3 = 0$. The tables show that the first case $I_3 = \hat{I}_3$, $I_3 \neq 0$ is rather rare. We found only two decays with this signature, namely the CF decay $\Lambda_c^+ \to \Lambda^0\pi^+$ and the SCS decay $\Lambda_c^+ \to \Lambda^0 K^+$. Both of these decays also have tree graph contributions which will also contribute to the S-wave amplitude. Therefore, concerning the vanishing of the p.v. amplitude A and thus about the vanishing of the asymmetry parameter, one cannot say anything general about these two decays. The second case $I_3 = \hat{I}_3 = 0$ occurs rather frequently. Among them are the six decays listed in Eq. (119) including their isospin partners. There are five DCS decays of the Ω_c^0 charm baryon with the signature $I_3 = \hat{I}_3 = 0$. These are

$$\text{DCS}: \Omega_c^0 \to \Sigma^+\pi^-,\ \Sigma^0\pi^0,\ \Sigma^-\pi^+,\ pK^-,\ n\bar{K}^0. \tag{122}$$

Since there are no tree graph contributions to these decays, within the current algebra approach the five decays are thus predicted to have zero asymmetry.

All of the double charm baryon decays $B_{cc}(\mathbf{3}) \to B_c(\mathbf{6}) + M(\mathbf{8})$ listed in Table 11 have the signature $I_3 = \hat{I}_3 = 0$. Of particular interest are those decays that have no tree contributions. These are

$$\begin{aligned} &\text{CF}: \Xi_{cc}^+ \to \Sigma_c^{++}K^-,\, \Omega_c^0K^+, \\ &\text{SCS}: \Omega_{cc} \to \Xi_c^0\pi^+, \\ &\text{DCS}: \Omega_{cc} \to \Sigma_c^+\pi^0. \end{aligned} \tag{123}$$

The asymmetry parameters of these decays are predicted in the current algebra approach to be zero.

As concerns the p.c. amplitude B, one cannot find a single example in the tables for which $B(W\text{-exchange}) = 0$ holds fully nontrivially. The signature would be $I_3 + 2I_4 + 6I_5 = 0$ *and* $\hat{I}_3 + 2\hat{I}_4 + 6I_5 = 0$. Independently, each of these signatures is found in many decays where the other signature is not given. Still, there are a few examples in which both conditions are satisfied, though the second only in the trivial way $\hat{I}^3 = \hat{I}^4 = I^5 = 0$. Except for a single DCS example $\Omega_c^0 \to \Lambda^0 \eta_\omega$ from Table 8, all the examples are from Table 10. We distinguish between cases with and without tree contributions.

(i) The pattern $(0, 0, +4, -2, 0, 0, 0)$ or multiples of it, indicating a transition without tree contributions, is found in the CF decay $\Xi_{cc}^+ \to \Xi_c^+ \eta_\phi$, and in the DCS decays $\Omega_c^0 \to \Lambda^0 \eta_\omega$ and $\Omega_{cc}^+ \to \Lambda_c^+ \eta_\omega$, with the common property that the meson is an η.

(ii) The pattern $(-2, +1, +4, -2, 0, 0, 0)$ is found in the SCS decay $\Omega_{cc}^+ \to \Xi_c^+ \eta_\omega$.

(iii) The pattern $(+2, -1, +4, -2, 0, 0, 0)$ or multiples of it, indicating a transition with tree contributions, is found in the CF decays $\Xi_{cc}^+ \to \Xi_c^0 \pi^+$ and $\Xi_{cc}^+ \to \Lambda_c^+ \bar{K}^0$, in the SCS decays $\Xi_{cc}^+ \to \Xi_c^0 K^+$, $\Omega_{cc}^+ \to \Xi_c^+ \pi^0$ and $\Omega_{cc}^+ \to \Xi_c^0 \pi^+$, and in the DCS decays $\Omega_{cc}^+ \to \Xi_c^0 K^+$ and $\Omega_{cc}^+ \to \Xi_c^+ K^0$, i.e. for decays into pions and kaons.

8 Summary and outlook

We have collected and organized a wealth of material on 196 nonleptonic $B_i(1/2^+) \to B_f(1/2^+) + M_k(0^-)$ decays of single and double charm baryons which else is scattered among many papers in the literature. We have collected some group theoretical material on the symmetry groups $SU(2)_I$, $SU(2)_U$, $SU(2)_V$, SU(3) and SU(6) needed in the analysis of the decays. We have presented the results of calculating the values of the seven topological tensor invariants for each of the 196 decays in a number of tables. Without having to do any explicit numerical calculation, the information contained in the tables leads to a number of important observations on the structure of the nonleptonic decays which we explicate. In the second part of the paper we have discussed a dynamical approach based on current algebra and the pole model, the results of which can be represented very compactly in terms of the topological tensor invariants introduced in the first part of the paper. The compact result was achieved by performing the sum over the intermediate baryon ground states inherent to the current algebra approach, using a completeness relation. We have critically examined a modified version of the current algebra approach introduced recently in the literature. From the experimental point of view, the key experiment to discard or to keep this new modified approach is to measure the asymmetry parameter in the decay $\Lambda_c^+ \to \Xi^0 K^+$ and, for that matter, to measure the asymmetry parameter in all the decays listed in Eq. (119) plus their isospin partners. While the asymmetry parameters in these decays are predicted by the modified current algebra approach to be large, in the standard current algebra approach they are predicted to be zero ore close to zero. Our results on the topological tensor invariants are useful also for other dynamical calculations of charm baryon decays such as constituent quark model approaches. In this context we are planning to refine the covariantized constituent quark model calculation of Ref. [37] and extend it to include also the SCS and DCS decays of single charm baryons.

An overall picture is difficult to obtain, as the experiments are on the way. Therefore, no definite decision can be made on whether and to what extend our estimates are compatible with the current data, except for the examples we have exposed. Still, we hope that the material presented in this paper will aid and provide extra stimulus for the experimental search of the many missing decay modes of charm baryon decays including a measurement of their absolute branching ratios – and, what is of utmost importance, to measure the asymmetry parameters in their decays. We are thus looking forward to new experimental results on charm baryon decays from Belle II, BESIII and LHCb which should be forthcoming in the near future.

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Appendix A: Representations of tensors

For the tensor representations of the meson and baryon flavor wave functions we use the phase convention of the second edition of Lichtenberg [68] (differing from the first edition).

A.1 Flavor space wave functions for the mesons

For the meson flavor wave functions we use Table 12.7 in Ref. [68],

$$\begin{aligned}
&\pi^+ : {M^1}_2 = -1,\\
&\pi^- : {M^2}_1 = 1,\\
&\pi^0 : {M^1}_1 = -{M^2}_2 = 1/\sqrt{2},\\
&K^0 : {M^2}_3 = 1,\\
&\bar{K}^0 : {M^3}_2 = -1,\\
&K^+ : {M^1}_3 = 1,\\
&K^- : {M^3}_1 = 1,\\
&\eta_\omega : {M^1}_1 = {M^2}_2 = 1/\sqrt{2},\\
&\eta_\phi : {M^3}_3 = 1,\\
&\eta_8 : {M^1}_1 = {M^2}_2 = -\tfrac{1}{2}{M^3}_3 = 1/\sqrt{6},\\
&\eta_1 : {M^1}_1 = {M^2}_2 = {M^3}_3 = 1/\sqrt{3},\\
&D^+ : {M^4}_2 = -1, \quad D^- : {M^2}_4 = -1,\\
&D^0 : {M^4}_1 = 1, \quad \bar{D}^0 : {M^1}_4 = -1,\\
&D_s^+ : {M^4}_3 = 1, \quad D_s^- : {M^3}_4 = -1.
\end{aligned} \tag{A1}$$

By employing the conjugate representation $M_a^b := {\bar{M}_a}{}^b = {M^a}_b$ of the flavor space wave functions in the main text we take into account that the outgoing mesons are considered as incoming antiparticles to the interaction. The matrix $M = M_k$ is identical to the conserved vector charge operator of the pseudoscalar meson k given in the first column of this list.

A.2 Flavor space wave functions for the baryons

For the baryon flavor wave function use the second set listed in Table 12.4 labelled as "Octet 2" and the second set listed in Table 12.5 labelled as "Second 20_M" in Ref. [68]. Instead of the ordering $B_{[ab]c}$ we use the ordering $B_{c[ab]}$ such that our flavor wave functions are antisymmetric in the last two indices.

- The light baryon octet:

$$\begin{aligned}
&p(uud): \quad B_{112} = -B_{121} = 1/\sqrt{2}\\
&n(udd): \quad B_{212} = -B_{221} = 1/\sqrt{2}\\
&\Sigma^+(uus): \quad B_{113} = -B_{131} = 1/\sqrt{2}\\
&\Sigma^0(uds): \quad B_{123} = -B_{132} = B_{213} = -B_{231} = 1/2\\
&\Sigma^-(dds): \quad B_{223} = -B_{232} = 1/\sqrt{2}\\
&\Lambda(uds): \quad B_{132} = -B_{123} = B_{213} = -B_{231} = \tfrac{1}{2}B_{312}\\
&\quad = -\tfrac{1}{2}B_{321} = 1/\sqrt{12}\\
&\Xi^0(uss): \quad B_{313} = -B_{331} = 1/\sqrt{2}\\
&\Xi^-(dss): \quad B_{323} = -B_{332} = 1/\sqrt{2}.
\end{aligned} \tag{A2}$$

- The $C=1$ charm baryon antitriplet:

$$\begin{aligned}
&\Lambda_c^+(udc): \quad B_{142} = -B_{124} = B_{214} = -B_{241}\\
&\quad = \tfrac{1}{2}B_{412} = -\tfrac{1}{2}B_{421} = 1/\sqrt{12}\\
&\Xi_c^+(usc): \quad B_{143} = -B_{134} = B_{314} = -B_{341}\\
&\quad = \tfrac{1}{2}B_{413} = -\tfrac{1}{2}B_{431} = 1/\sqrt{12}\\
&\Xi_c^0(dsc): \quad B_{243} = -B_{234} = B_{324} = -B_{342}\\
&\quad = \tfrac{1}{2}B_{423} = -\tfrac{1}{2}B_{432} = 1/\sqrt{12}.
\end{aligned} \tag{A3}$$

- The $C=1$ charm baryon sextet:

$$\begin{aligned}
&\Sigma_c^{++}(uuc): \quad B_{114} = -B_{141} = 1/\sqrt{2}\\
&\Sigma_c^{+}(udc): \quad B_{214} = B_{124} = -B_{142} = -B_{241} = 1/2\\
&\Sigma_c^{0}(ddc): \quad B_{224} = -B_{242} = 1/\sqrt{2}\\
&\Xi_c'^{+}(usc): \quad B_{314} = B_{134} = -B_{341} = -B_{143} = 1/2\\
&\Xi_c'^{0}(dsc): \quad B_{324} = B_{234} = -B_{342} = -B_{243} = 1/2\\
&\Omega_c^{0}(ssc): \quad B_{334} = -B_{343} = 1/\sqrt{2}.
\end{aligned} \tag{A4}$$

- The $C=2$ double charm baryon triplet:

$$\begin{aligned}
&\Xi_{cc}^{++}(ucc): \quad B_{414} = -B_{441} = 1/\sqrt{2}\\
&\Xi_{cc}^{+}(dcc): \quad B_{424} = -B_{442} = 1/\sqrt{2}\\
&\Omega_{cc}^{+}(scc): \quad B_{434} = -B_{443} = 1/\sqrt{2}.
\end{aligned} \tag{A5}$$

The flavor space wave functions of the conjugate $\mathbf{20}'$ states are given by $\bar{B}_{ijk} = B^{ijk}$.

A.3 The weak transition tensor $H_{[kl]}^{[ij]}(\mathcal{O}_-)$

Early foundations for the properties in this section and in Table 2 are found in Ref. [63].

- The $\Delta C = 1$ effective Hamiltonian

$$\begin{aligned}
&CF: \quad c\to s, \quad d\to u: H_{[31]}^{[42]} = -H_{[31]}^{[24]} = -H_{[13]}^{[42]}\\
&\quad = H_{[13]}^{[24]} = 1\\
&\text{SCS}: (a) \quad c\to s, \quad s\to u: H_{[31]}^{[43]} = -H_{[31]}^{[34]}\\
&\quad = -H_{[13]}^{[43]} = H_{[13]}^{[34]} = 1\\
&\qquad (b) \quad c\to d, \quad d\to u: H_{[21]}^{[42]} = -H_{[21]}^{[24]}\\
&\quad = -H_{[12]}^{[42]} = H_{[12]}^{[24]} = 1\\
&\text{DCS}: \quad c\to d, \quad s\to u: H_{[21]}^{[43]} = -H_{[21]}^{[34]} = -H_{[12]}^{[43]}\\
&\quad = H_{[12]}^{[34]} = 1.
\end{aligned} \tag{A6}$$

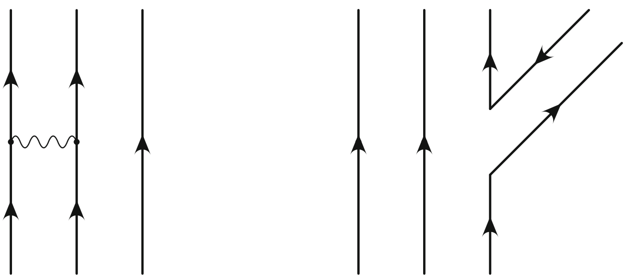

Fig. 4 Weak transition (left) and strong transition (right)

- The $\Delta C = 0$ effective Hamiltonian

$$\begin{aligned} &\text{SCS}: (a') \quad s \to u, \quad u \to d: H^{[31]}_{[12]} = -H^{[13]}_{[12]} \\ &= -H^{[31]}_{[21]} = H^{[13]}_{[21]} = 1 \\ &\quad (b') \quad c \to d, \quad s \to c: H^{[43]}_{[24]} = -H^{[34]}_{[24]} \\ &= -H^{[43]}_{[42]} = H^{[34]}_{[42]} = 1. \end{aligned} \tag{A7}$$

- The $\Delta S = 1$ hyperon decay effective Hamiltonian

$$\begin{aligned} &\text{SCS}: \quad s \to u, \quad u \to d: \quad H^{[31]}_{[12]} = -H^{[13]}_{[12]} \\ &= -H^{[31]}_{[21]} = H^{[13]}_{[21]} = 1. \end{aligned} \tag{A8}$$

Appendix B: Derivation of the completeness relation

The completeness relation (12) can be derived by writing down a sixth rank tensor T^{bcd}_{kmn} built from a linear combination of the products of three δ-functions which is separately antisymmetric under the exchange of the tensor labels $c \leftrightarrow d$ and $m \leftrightarrow n$. One has

$$\begin{aligned} \sum_\ell B^\ell_{k[mn]} B^{b[cd]}_\ell &= a(\delta^b_k \delta^c_m \delta^d_n - \delta^b_k \delta^d_m \delta^c_n) \\ &+ b\Big((\delta^b_m \delta^c_n \delta^d_k - \delta^b_m \delta^d_n \delta^c_k) + (\delta^b_n \delta^c_k \delta^d_m - \delta^b_n \delta^d_k \delta^c_m)\Big). \end{aligned} \tag{B1}$$

By contracting (B1) again with $B^{\ell'}_{b[cd]}$ and using $B^{\ell'}_{m[nk]} + B^{\ell'}_{n[km]} = -B^{\ell'}_{k[mn]}$ one obtains $a - b = 1/2$. The coefficients a and b can be determined by contracting Eq. (B1) with $\delta^k_b \delta^m_c \delta^n_d$ which gives

$$\frac{1}{3} N(N^2-1) = a(N^3 - N^2) + 2b(N - N^2) \tag{B2}$$

due to the normalization and orthogonality relation (11) with the solution $b = -a/2$. One thus has $a = 2/6$ and $b = -1/6$ as in the completeness relation (12).

Table 14 SU(3) properties of the weak transition matrix elements

	Matrix element	Direct product	$N_{\mathrm{SU(3)}}$
$\Delta C = 1$	$\langle B(\mathbf{8})\vert H^{\mathrm{pc}}(\mathbf{6})\vert B_c(\bar{\mathbf{3}})\rangle$	$\bar{\mathbf{3}} \otimes \bar{\mathbf{6}} \otimes \mathbf{8}$	1
	$\langle B(\mathbf{8})\vert H^{\mathrm{pc}}(\mathbf{6})\vert B_c(\mathbf{6})\rangle$	$\mathbf{6} \otimes \bar{\mathbf{6}} \otimes \mathbf{8}$	1
	$\langle B_c(\bar{\mathbf{3}})\vert H^{\mathrm{pc}}(\mathbf{6})\vert B_{cc}(\mathbf{3})\rangle$	$\mathbf{3} \otimes \bar{\mathbf{6}} \otimes \mathbf{3}$	1
	$\langle B_c(\mathbf{6})\vert H^{\mathrm{pc}}(\mathbf{6})\vert B_{cc}(\mathbf{3})\rangle$	$\mathbf{3} \otimes \bar{\mathbf{6}} \otimes \bar{\mathbf{6}}$	0
$\Delta C = 0$	$\langle B_c(\bar{\mathbf{3}})\vert H^{\mathrm{pc}}(\mathbf{8})\vert B_c(\bar{\mathbf{3}})\rangle$	$\bar{\mathbf{3}} \otimes \mathbf{8} \otimes \mathbf{3}$	1 (a', b')
	$\langle B_c(\mathbf{6})\vert H^{\mathrm{pc}}(\mathbf{8})\vert B_c(\bar{\mathbf{3}})\rangle$	$\bar{\mathbf{3}} \otimes \mathbf{8} \otimes \bar{\mathbf{6}}$	1 (b')
	$\langle B_c(\bar{\mathbf{3}})\vert H^{\mathrm{pc}}(\mathbf{8})\vert B_c(\mathbf{6})\rangle$	$\mathbf{6} \otimes \mathbf{8} \otimes \mathbf{3}$	1 (b')
	$\langle B_c(\mathbf{6})\vert H^{\mathrm{pc}}(\mathbf{8})\vert B_c(\mathbf{6})\rangle$	$\mathbf{6} \otimes \mathbf{8} \otimes \bar{\mathbf{6}}$	1 (b')

Appendix C: SU(3) structure of weak and strong transition matrix elements

In this appendix we present explicit values for the weak and strong matrix elements needed in the current algebra analysis of Sect. 6 and Appendix D and depicted in Fig. 4. We begin with a discussion of the weak transition matrix elements.

C.1 Weak matrix elements

In Table 14 we list the SU(3) decomposition of the four classes of decays each for the $\Delta C = 1$ and $\Delta C = 0$ decays. The SU(3) analysis shows that all matrix elements but one are allowed. The forbidden transition is the $\Delta C = 1$ transition $\langle B_c(\mathbf{6})|H^{\mathrm{pc}}(\mathbf{6})|B_{cc}(\mathbf{3})\rangle$ which vanishes in the SU(3) limit since the direct product $\mathbf{3} \otimes \bar{\mathbf{6}} \otimes \bar{\mathbf{6}}$ does not contain the unit representation. All the other transitions are governed by one SU(3) reduced matrix element. While the $\Delta C = 0$ quark transition (b') $c \to d$; $s \to c$ contributes to all four possible matrix elements, the quark transition (a') $s \to u$; $u \to d$ contributes only to the matrix element $\langle B_c(\bar{\mathbf{3}})|H^{\mathrm{pc}}(\mathbf{8})|B_c(\bar{\mathbf{3}})\rangle$ because of the KPW theorem.[2] This pattern is confirmed by the tensor analysis. The only possible connected tensor invariant is given by the contraction

$$I^{\mathrm{pc}} = B^{a[bc]} B_{a[b'c']} H^{[b'c']}_{[b\,c]}. \tag{C1}$$

The notation I^{pc} is a short-hand notation for $I^{\mathrm{pc}}_{B_f B_i} = \langle B_f|H|B_i\rangle$. That the tensor invariant I^{pc} vanishes for the transition $B_{cc}(\mathbf{3}) \to H(\mathbf{6}) + B_c(\mathbf{6})$ can be understood by realizing that the flavor of the noninteracting quark line a must be charmed. But then the quark lines b and c that go into the final baryon must be light and symmetric $\{bc\}$ in the $\mathbf{6}$ representation. This clashes with the antisymmetry of the flavor wave function $B^{a[bc]}$ of the final state.

[2] We do not agree with the $\Delta C = 0$ analysis of Cheng et al. [75,92] who assert that the nondiagonal matrix elements $\langle B_c(\mathbf{6})|H^{\mathrm{pc}}(\mathbf{8})|B_c(\bar{\mathbf{3}})\rangle$ and $\langle B_c(\bar{\mathbf{3}})|H^{\mathrm{pc}}(\mathbf{8})|B_c(\mathbf{6})\rangle$ are zero.

Table 15 $\Delta C = 1$ values of the weak transition matrix element $I^{\rm pc} = \langle B_f | H^{\rm pc}(\mathbf{6}) | B_i \rangle$. The suffix labelling (a) and (b) for the SCS decays is explained in the caption of Table 8

	Matrix element	$I^{\rm pc}$		Matrix element	$I^{\rm pc}$
CF	$2\sqrt{6}\langle \Sigma^+ \vert H^{\rm pc}(\mathbf{6}) \vert \Lambda_c^+ \rangle$	-4	SCS	$2\sqrt{6}\langle p \vert H^{\rm pc}(\mathbf{6}) \vert \Lambda_c^+ \rangle$	$+4_b$
	$2\sqrt{6}\langle \Xi^0 \vert H^{\rm pc}(\mathbf{6}) \vert \Xi_c^0 \rangle$	$+4$		$2\sqrt{6}\langle \Sigma^+ \vert H^{\rm pc}(\mathbf{6}) \vert \Xi_c^+ \rangle$	-4_a
	$2\sqrt{2}\langle \Sigma^+ \vert H^{\rm pc}(\mathbf{6}) \vert \Sigma_c^+ \rangle$	$+4$		$4\sqrt{3}\langle \Sigma^0 \vert H^{\rm pc}(\mathbf{6}) \vert \Xi_c^0 \rangle$	-4_a
	$2\sqrt{2}\langle \Sigma^0 \vert H^{\rm pc}(\mathbf{6}) \vert \Sigma_c^0 \rangle$	$+4$		$12\langle \Lambda^0 \vert H^{\rm pc}(\mathbf{6}) \vert \Xi_c^0 \rangle$	$-4_a - 8_b$
	$2\sqrt{6}\langle \Lambda^0 \vert H^{\rm pc}(\mathbf{6}) \vert \Sigma_c^0 \rangle$	$+4$		$2\sqrt{2}\langle p \vert H^{\rm pc}(\mathbf{6}) \vert \Sigma_c^+ \rangle$	-4_b
	$2\sqrt{2}\langle \Xi^0 \vert H^{\rm pc}(\mathbf{6}) \vert \Xi_c^{\prime 0} \rangle$	$+4$		$4\langle \Sigma^0 \vert H^{\rm pc}(\mathbf{6}) \vert \Xi_c^{\prime 0} \rangle$	$+4_a$
	$2\sqrt{6}\langle \Xi_c^+ \vert H^{\rm pc}(\mathbf{6}) \vert \Xi_{cc}^+ \rangle$	$+8$		$4\sqrt{3}\langle \Lambda^0 \vert H^{\rm pc}(\mathbf{6}) \vert \Xi_c^{\prime 0} \rangle$	$+4_a - 8_b$
	$2\sqrt{2}\langle \Xi_c^{\prime +} \vert H^{\rm pc}(\mathbf{6}) \vert \Xi_{cc}^+ \rangle$	0		$2\langle \Xi^0 \vert H^{\rm pc}(\mathbf{6}) \vert \Omega_c^0 \rangle$	$+4_a$

Table 16 $\Delta C = 0$ values of the weak transition matrix element $I^{\rm pc} = \langle B_f | H^{\rm pc}(\mathbf{8}) | B_i \rangle$. The suffix labelling (a') and (b') is explained in the caption of Table 9

	Matrix element	$I^{\rm pc}$
SCS	$12\langle \Lambda_c^+ \vert H^{\rm pc}(\mathbf{8}) \vert \Xi_c^+ \rangle$	$-16_{a'} + 4_{b'}$
	$2\sqrt{6}\langle \Sigma_c^0 \vert H^{\rm pc}(\mathbf{8}) \vert \Xi_c^0 \rangle$	$-4_{b'}$
	$4\sqrt{3}\langle \Lambda_c^+ \vert H^{\rm pc}(\mathbf{8}) \vert \Xi_c^{\prime +} \rangle$	$-4_{b'}$
	$2\sqrt{6}\langle \Xi_c^0 \vert H^{\rm pc}(\mathbf{8}) \vert \Omega_c^0 \rangle$	$+4_{b'}$
	$2\sqrt{2}\langle \Xi_c^{\prime 0} \vert H^{\rm pc}(\mathbf{8}) \vert \Omega_c^0 \rangle$	$+4_{b'}$

We will not provide an exhaustive list of weak matrix elements. Instead, in Tables 15 and 16 we list all the weak matrix elements that appear in the sample decays discussed in Sect. 6 and Appendix D. In Table 15 we confirm the vanishing of the matrix element $\langle \Xi_c^{\prime +} | H^{\rm pc} | \Xi_{cc}^+ \rangle$ by explicit calculation.

C.2 The $\Delta Y = 1$ hyperon decay

For the $\Delta Y = 1$ hyperon decays there are two reduced SU(3) matrix elements as can be seen by the reduction of the direct product $\mathbf{8}\otimes\mathbf{8}\otimes\mathbf{8} \to 2\cdot\mathbf{1}\oplus\cdots$. It is common practise to characterize the two couplings by their symmetry structure which are labelled by d (symmetric coupling) and f (antisymmetric coupling). As we shall see in a moment, the invariant (C1) has a d/f structure of $d/f = -1$. In addition to the tensor invariant (C1) one has a second tensor invariant given by the contraction $B^{j[bc]} B_{i[bc]} H^{[ir]}_{[rj]}$. This second invariant does not contribute to the $\Delta C = 1$ transitions because i) there are no two same light quarks in the effective Hamiltonian for the CF and DCS transitions and ii) the two contributions from the internal contractions (a) $(s\bar{s})$ and (b) $(d\bar{d})$ cancel in the SCS contributions.

To analyze the d/f structure of the two invariants it is convenient to switch to the second rank tensor representation of the baryon flavor wave function and the effective Hamiltonian. For the baryon flavor wave function this is achieved by the transformation

$$B_{abc} = \tfrac{1}{\sqrt{2}}\, \epsilon_{ibc} B^i_a \quad \Leftrightarrow \quad B^i_a = \tfrac{1}{\sqrt{2}}\, \epsilon^{ibc} B_{abc} \tag{C2}$$

$$B = \begin{pmatrix} -\frac{\Lambda^0}{\sqrt{6}} + \frac{\Sigma^0}{\sqrt{2}} & -\Sigma^+ & p \\ \Sigma^- & -\frac{\Lambda^0}{\sqrt{6}} - \frac{\Sigma^0}{\sqrt{2}} & n \\ \Xi^- & -\Xi^0 & \frac{2\Lambda^0}{\sqrt{6}} \end{pmatrix}. \tag{C3}$$

Compared to the conventional 3×3 representation of the baryon octet we have changed the phases of the Σ^+, Λ^0 ad Ξ^0 components to be in agreement with our phase convention from Lichtenberg [68]. In the same vein one can transform the fourth rank tensor $H^{[ab]}_{[a'b']}$ to a second rank "spurion" tensor S^i_j by writing

$$H^{[ab]}_{[a'b']} = \epsilon_{ia'b'}\, \epsilon^{jab} S^i_j \quad \Leftrightarrow \quad S^i_j = \tfrac{1}{4} \epsilon_{jrs}\, \epsilon^{ir's'} H^{rs}_{r's'}. \tag{C4}$$

The so-called spurion has $S^2_3 = 1$ as only nonvanishing component. One obtains

$$\begin{aligned} & B^{a[bc]}_{\ell} B^{\ell'}_{a[b'c']} H^{[b'c']}_{[bc]} = 2\bar{B}^a_i B^j_a S^i_j = 2\,\mathrm{tr}(\bar{B} B S) \\ & \quad = \mathrm{tr}\left((B\bar{B} + \bar{B}B)S - (B\bar{B} - \bar{B}B)S\right), \\ & B^{j[bc]}_{\ell} B^{\ell'}_{i[bc]} H^{[ir]}_{[rj]} = B^a_i B^j_a S^i_j = \mathrm{tr}(B\bar{B}S) \\ & \quad = \mathrm{tr}\left(\tfrac{1}{2}(B\bar{B} + \bar{B}B)S + \tfrac{1}{2}(B\bar{B} - \bar{B}B)S\right), \end{aligned} \tag{C5}$$

indicating the ratios $d/f = +1$ and $d/f = -1$ for the two couplings $\tilde{C}_F = \sqrt{2}\,\mathrm{tr}([B, \bar{B}]S)$ and $\tilde{C}_D = \sqrt{2}\,\mathrm{tr}(\{B, \bar{B}\}S)$ mentioned by Lichtenberg in Sec. 9.7 of Ref. [68].

C.3 Strong matrix elements

In Sect. 5 we have introduced the two basic flavor tensor invariants $\tilde{I}_1$ and $\tilde{I}_2$ to describe the strong matrix element $\langle B_f M_k | B_i \rangle$. They read

$$\tilde{I}_1 = B^{a[bc]} B_{a[bc']} M^{c'}_c, \quad \tilde{I}_2 = B^{a[bc]} B_{b[c'a]} M^{c'}_c. \tag{C6}$$

Depending on the class of the transition, the two basic tensor invariants are not always independent. This becomes evi-

Table 17 SU(3) properties of the strong transition matrix elements

Matrix element	Direct product	$N_{\mathrm{SU(3)}}$	$\tilde{I}_1/\tilde{I}_2$
$\langle B(\mathbf{8})M(\mathbf{8})\|B(\mathbf{8})\rangle$	$\mathbf{8}\otimes\mathbf{8}\otimes\mathbf{8}$	2	Indefinite
$\langle B_c(\bar{\mathbf{3}})M(\mathbf{8})\|B_c(\bar{\mathbf{3}})\rangle$	$\bar{\mathbf{3}}\otimes\mathbf{3}\otimes\mathbf{8}$	1	−5/4
$\langle B_c(\mathbf{6})M(\mathbf{8})\|B_c(\bar{\mathbf{3}})\rangle$	$\bar{\mathbf{3}}\otimes\bar{\mathbf{6}}\otimes\mathbf{8}$	1	−1/2
$\langle B_c(\bar{\mathbf{3}})M(\mathbf{8})\|B_c(\mathbf{6})\rangle$	$\mathbf{6}\otimes\mathbf{3}\otimes\mathbf{8}$	1	−1/2
$\langle B_c(\mathbf{6})M(\mathbf{8})\|B_c(\mathbf{6})\rangle$	$\mathbf{6}\otimes\bar{\mathbf{6}}\otimes\mathbf{8}$	1	+1/0
$\langle B(\mathbf{8})M(\bar{\mathbf{3}})\|B_c(\bar{\mathbf{3}})\rangle$	$\bar{\mathbf{3}}\otimes\mathbf{8}\otimes\mathbf{3}$	1	+1
$\langle B_c(\bar{\mathbf{3}})M(\bar{\mathbf{3}})\|B_{cc}(\mathbf{3})\rangle$	$\mathbf{3}\otimes\mathbf{3}\otimes\mathbf{3}$	1	−2
$\langle B_c(\mathbf{6})M(\bar{\mathbf{3}})\|B_{cc}(\mathbf{3})\rangle$	$\mathbf{3}\otimes\bar{\mathbf{6}}\otimes\mathbf{3}$	1	0/ − 1
$\langle B_{cc}(\mathbf{3})M(\mathbf{8})\|B_{cc}(\mathbf{3})\rangle$	$\mathbf{3}\otimes\bar{\mathbf{3}}\otimes\mathbf{8}$	1	−1

dent when one analyzes the strong transitions in terms of the three partaking SU(3) multiplets, as shown in Table 17. In the second column we list the direct products of the participating SU(3) representations and in the third column we register the number $N_{\mathrm{SU(3)}}$ of SU(3) reduced matrix elements needed to describe the transitions. For the cases $N_{SU(3)}=1$ with only one SU(3) reduced matrix element we list the ratio $\tilde{I}_1/\tilde{I}_2$ of the two basic couplings (C6) in column 4. The respective ratios can be obtained by rewriting the tensor invariant $\tilde{I}_2$ in terms of $\tilde{I}_1$ i) following the flavor flow of the transitions, ii) using the Jacobi identity, and iii) making use of the wave function symmetries of the involved baryons. By the same reasoning one finds $\tilde{I}_2=0$ for the class of decays $\langle B_c(\mathbf{6})M(\mathbf{8})|B_c(\mathbf{6})\rangle$ and $\tilde{I}_1=0$ for the class of decays $\langle B_c(\mathbf{6})M(\bar{\mathbf{3}})|B_{cc}(\mathbf{3})\rangle$. This is specified in column 4 of Table 17 by the notation "1/0" and "0/1". Note that the class of light baryon transitions $\langle B(\mathbf{8})M(\mathbf{8})|B(\mathbf{8})\rangle$ is described by two SU(3) invariant amplitudes, i.e. the ratio $\tilde{I}_1/\tilde{I}_2$ is not definite for this class of transitions as annotated in Table 17. One can relate the two invariants $\tilde{I}_1$ and $\tilde{I}_2$ to the usual pair of SU(3) d and f couplings as has been done for the weak transition matrix elements (see Eq. (C5)). One finds

$$\begin{aligned}\tilde{I}_1&=-\tfrac{1}{2}\,\mathrm{tr}(\bar{B}B\bar{M})=\;-\tfrac{1}{4}\,\mathrm{tr}\left((B\bar{B}+\bar{B}B)\bar{M}\right)\\&\quad+\tfrac{1}{4}\,\mathrm{tr}\left((B\bar{B}-\bar{B}B)\bar{M}\right)=\;-\tfrac{1}{4}I^d+\tfrac{1}{4}I^f,\\\tilde{I}_2&=\tfrac{1}{2}\,\mathrm{tr}\left((B\bar{B}+\bar{B}B)\bar{M}\right)\;=\;\tfrac{1}{2}I^d.\end{aligned}\tag{C7}$$

In the usual way of SU(3) labelling, the invariant $\tilde{I}_1$ can be seen to correspond to a coupling ratio $d/f=-1$ while the invariant $\tilde{I}_2$ corresponds to a pure d coupling. One obtains

$$I^d=2\tilde{I}_2,\quad I^f=4\tilde{I}_1+2\tilde{I}_2.\tag{C8}$$

It is common knowledge that the generator of SU(3) is proportional to the antisymmetric coupling of two baryon octets. As Eq. (C7) shows, the antisymmetric coupling is proportional to the linear combination $I^f\sim 2\tilde{I}_1+\tilde{I}_2$. The normalization of the f coupling must be chosen such that the expectation value of the charge operator between proton states is given by the charge of the proton which is 1 in units of the elementary charge e. Let us verify this for the choice $I^f=4\tilde{I}_1+2\tilde{I}_2$. First we express the 3×3 charge operator Q in terms of its π^0 and η_8 components. One has

$$Q=\begin{pmatrix}2/3&0&0\\0&-1/3&0\\0&0&-1/3\end{pmatrix}=\frac{1}{\sqrt{2}}Q(\pi^0)+\frac{1}{\sqrt{6}}Q(\eta_8),\tag{C9}$$

where the charge operator components are given by

$$Q(\pi^0)=\frac{1}{\sqrt{2}}\begin{pmatrix}1&0&0\\0&-1&0\\0&0&0\end{pmatrix},\quad Q(\eta_8)=\frac{1}{\sqrt{6}}\begin{pmatrix}1&0&0\\0&1&0\\0&0&-2\end{pmatrix}.\tag{C10}$$

The expectation value of the charge operator between two proton states can then be calculated to be 1 according to the decomposition of the charge operator (C9). One has

$$\langle p|Q|p\rangle=\frac{1}{\sqrt{2}}I^f_{p\pi^0p}+\frac{1}{\sqrt{6}}I^f_{p\eta_8p}=1.\tag{C11}$$

This is in accordance with Table 18 where we list the values of the strong transition matrix elements for the invariants $\tilde{I}_1$, $\tilde{I}_2$, and for the linear combinations of invariants $I^d=2\tilde{I}_2$ (not shown), $I^f=4\tilde{I}_1+2\tilde{I}_2$ and $I^{\mathrm{CQM}}=4\tilde{I}_1+5\tilde{I}_2$ which appear in the current algebra description of the charm baryon nonleptonic decays. The notation $\langle B_f,M_k|B_i\rangle$ means either $\langle B_fM_k|B_i\rangle$ or $\langle B_f|M_k|B_i\rangle$, depending on whether one deals with I^f or I^{CQM}, respectively. The same result can be obtained by direct calculation of the matrix elements $\tilde{I}_1$ and $\tilde{I}_2$ for the charge operator. The result is $\tilde{I}_1=1/6$ and $\tilde{I}_2=1/6$ which confirms again that the invariant $I^f=4\tilde{I}_1+2\tilde{I}_2$ has the correct normalization. Table 18 shows that the generator matrix element $I^f_{\Sigma^-\pi^+\Lambda^0}=\langle\Sigma^-|Q(\pi^-)|\Lambda^0\rangle$ vanishes which follows from the fact that $Q(\pi^-)$ is a generator of the subgroup SU(2).

We want to ascertain that $I^f=4\tilde{I}_1+2\tilde{I}_2$ is the correctly normalized generator matrix element also from SU(4). As an example we calculate the charge of the charm baryon state $|\Xi_c^+\rangle$. In SU(4) the charge operator is given by $Q=\mathrm{diag}\,(2/3,-1/3,-1/3,\ 2/3)$. For the matrix elements of the charge operator one finds $\tilde{I}_1=1/4$ and $\tilde{I}_2=0$ which gives $I^f=4\tilde{I}_1+2\tilde{I}_2=1$, as required. Table 18 shows that $I^f=0$ for the two classes of transitions $\langle B_c(\bar{\mathbf{3}})M(\mathbf{8})|B_c(\mathbf{6})\rangle$ and $\langle B_c(\mathbf{6})M(\mathbf{8})|B_c(\bar{\mathbf{3}})\rangle$. The vanishing of the f-type matrix elements follows from the fact that the vector charges M_k associated with the members M_k of light meson octet $M(\mathbf{8})$ are generators of SU(3) which do not connect different SU(3) multiplets.

The second linear combination of $\tilde{I}_1$ and $\tilde{I}_2$ that enters the current algebra calculation is the constituent quark model (CQM) tensor invariant I^{CQM} which reads

$$I^{\mathrm{CQM}}=4\tilde{I}_1+5\tilde{I}_2=\tfrac{3}{2}I^d+I^f.\tag{C12}$$

Table 18 Values of tensor invariants for the strong transitions $B_i \to B_f + M_k$. I^f and $I^{\rm CQM}$ are related to $\tilde{I}_1$ and $\tilde{I}_2$ by $I^f = 4\tilde{I}_1 + 2\tilde{I}_2$ and $I^{\rm CQM} = 4\tilde{I}_1 + 5\tilde{I}_2$

	$\tilde{I}_1$	$\tilde{I}_2$	I^f	$I^{\rm CQM}$
$2\sqrt{2}\langle p, \pi^0 \vert p\rangle$	0	+1	+2	+5
$2\sqrt{6}\langle p, \eta_8 \vert p\rangle$	+2	−1	+6	+3
$2\sqrt{2}\langle p, K^- \vert \Sigma^0\rangle$	−1	+1	−2	+1
$2\sqrt{6}\langle p, K^- \vert \Lambda^0\rangle$	+1	+1	+6	+9
$2\sqrt{2}\langle \Sigma^-, \pi^+ \vert \Sigma^0\rangle$	−1	0	−4	−4
$2\sqrt{6}\langle \Sigma^-, \pi^+ \vert \Lambda^0\rangle$	−1	+2	0	+6
$2\sqrt{6}\langle \Lambda^0, \pi^+ \vert \Sigma^+\rangle$	+1	−2	0	−6
$2\sqrt{2}\langle \Sigma^0, \pi^+ \vert \Sigma^+\rangle$	−1	0	−4	−4
$2\langle \Xi^0, K^+ \vert \Sigma^+\rangle$	0	+1	+2	+5
$2\sqrt{2}\langle \Sigma^0, \bar{K}^0 \vert \Xi^0\rangle$	0	−1	−2	−5
$2\langle \Xi^0, K^+ \vert \Sigma^+\rangle$	0	+1	+2	+5
$2\sqrt{6}\langle \Lambda_c^+, \pi^- \vert \Sigma_c^0\rangle$	+1	−2	0	−6
$2\sqrt{6}\langle \Sigma_c^0, \pi^+ \vert \Lambda_c^+\rangle$	−1	+2	0	+6
$12\langle \Xi_c^0, K^+ \vert \Lambda_c^+\rangle$	−5	+4	−12	0
$4\sqrt{3}\langle \Xi_c'^0, K^+ \vert \Lambda_c^+\rangle$	+1	−2	0	−6
$12\sqrt{2}\langle \Xi_c^+, \pi^0 \vert \Xi_c^+\rangle$	+5	−4	+12	0
$2\sqrt{6}\langle \Sigma_c^{++}, K^- \vert \Xi_c^+\rangle$	−1	+2	0	+6
$12\langle \Xi_c^+, \pi^- \vert \Xi_c^0\rangle$	+5	−4	+12	0
$4\sqrt{3}\langle \Xi_c'^+, \pi^- \vert \Xi_c^0\rangle$	+1	−2	0	−6
$12\langle \Xi_c^+, \pi^- \vert \Xi_c^0\rangle$	+5	−4	+12	0
$4\sqrt{3}\langle \Xi_c'^+, \pi^- \vert \Xi_c^0\rangle$	+1	−2	0	−6
$12\langle \Xi_c^+, \pi^- \vert \Xi_c^0\rangle$	+5	−4	+12	0
$4\sqrt{3}\langle \Xi_c'^+, \pi^- \vert \Xi_c^0\rangle$	+1	−2	0	−6
$12\langle \Xi_c^0, \pi^+ \vert \Xi_c^+\rangle$	−5	+4	−12	0
$2\sqrt{12}\langle \Xi_c'^0, \pi^+ \vert \Xi_c^+\rangle$	−1	+2	0	+6
$4\sqrt{3}\langle \Sigma_c^+, K^- \vert \Xi_c^0\rangle$	−1	+2	0	+6
$12\langle \Lambda_c^+, K^- \vert \Xi_c^0\rangle$	−5	+4	−12	0
$2\sqrt{6}\langle \Xi_c^0, \bar{K}^0 \vert \Omega_c^0\rangle$	−1	+2	0	+6
$2\sqrt{2}\langle \Xi_c'^0, \bar{K}^0 \vert \Omega_c^0\rangle$	−1	0	−4	−4
$2\sqrt{2}\langle \Xi_{cc}^{++}, \pi^0 \vert \Xi_{cc}^{++}\rangle$	+1	−1	+2	−1
$2\langle \Xi_{cc}^+, \pi^+ \vert \Xi_{cc}^{++}\rangle$	−1	+1	−2	+1
$2\sqrt{2}\langle \Xi_{cc}^+, \pi^0 \vert \Xi_{cc}^+\rangle$	−1	+1	−2	+1
$2\sqrt{6}\langle \Sigma^+, D^0 \vert \Xi_c^+\rangle$	+1	+1	+6	+9
$2\sqrt{6}\langle \Lambda_c^+, D^+ \vert \Xi_{cc}^{++}\rangle$	−2	+1	−6	−3
$2\sqrt{2}\langle \Sigma_c^+, D^+ \vert \Xi_{cc}^{++}\rangle$	0	−1	−2	−5
$2\sqrt{6}\langle \Lambda_c^+, D^0 \vert \Xi_{cc}^+\rangle$	−2	+1	−6	−3
$2\sqrt{2}\langle \Sigma_c^+, D^0 \vert \Xi_{cc}^+\rangle$	0	+1	+2	+5

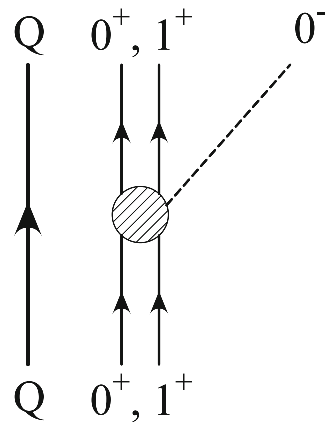

Fig. 5 Light diquark transitions

The constituent quark model coupling I^{CQM} can be seen to correspond to a d/f ratio of 3/2. Table 18 shows that the constituent quark model coupling I^{CQM} vanishes for the class of decays $\langle B_c(\bar{\mathbf{3}})M(\mathbf{8})|B_c(\bar{\mathbf{3}})\rangle$. This can be understood from a LS coupling analysis of the light-side transition $0^+ \to 0^+ + 0^-$. In the constituent quark model (or in the Heavy Quark Effective Theory (HQET)) the light side transition effectively decouples from the heavy side charm quark transition as illustrated in Fig. 5. The light side diquark has the quantum numbers $J^P = (0^+, 1^+)$ depending on whether the two light quark spins are in the spin singlet or the spin triplet state. One can then do a LS analysis of the light side transitions $D(0^+) \to D(0^+) + 0^-$, $D(0^+) \to D(1^+) + 0^-$, $D(1^+) \to D(0^+) + 0^-$ and $D(1^+) \to D(1^+) + 0^-$. From parity the orbital angular momentum of the final state must be odd. One concludes that the transition $D(0^+) \to D(0^+) + 0^-$ must vanish since even for the lowest possible angular momentum $L = 1$ the final state $D(0^+)+0^-$ cannot couple to the initial $D(0^+)$ state because the total spin S of the final state is $S = 0$. A similar LS analysis for the other three diquark transitions shows that these transitions are allowed. The coupling/decoupling arguments presented here do not hinge on the diquark representation of the light quarks but follow also in a single quark picture as long as the two light quarks move independently [66,111].

In Sect. 5 we have also considered the possibility that the strong transition is described by a general mixture of SU(3) d and f couplings. We denote the general coupling by $I^{\text{gen}}(d/f)$ which we choose to parametrize by

$$I^{\text{gen}}(d/f) = d(\tfrac{5}{3}\, I^{\text{CQM}} - \tfrac{2}{3}\, I^f) + f\, I^f \tag{C13}$$

with $d + f = 1$. We have chosen the parametrization (C13) such that i) for $d = 0$ and $f = 1$ one recovers the f-coupling structure $I^f = 4\tilde{I}_1 + 2\tilde{I}_2$ and ii) for $d = 3/5$ and $f = 2/5$ one recovers the coupling structure $I^{\text{CQM}} = 4\tilde{I}_1 + 5\tilde{I}_2$.

Appendix D: Scrutinizing the Cheng et al. modification of the current algebra approach

As the authors of Ref. [60] put it, the CF single charm baryon decay $\Lambda_c^+ \to \Xi^0 K^+$ deserves special attention. First, there is no factorizing contribution to the decay as evidenced by Table 3, i.e. the decay is contributed to only by the nonfactorizing W-exchange contributions. Second, as we reconfirm below, the W-exchange contribution to the S-wave amplitude $A^{\text{com}} = A^{\text{com}}_{fki}$ vanishes for this decay in the standard current algebra approach since $I_3 = 0$ and $\hat{I}_3 = 0$ (see Table 3). Thus the current algebra approach predicts the decay to be purely P-wave. This implies that the asymmetry parameter in this decay is predicted to be zero. Cheng et al. point out that it will be difficult to reproduce the rather large experimental branching ratio of this decay ($\mathcal{B} = (5.5 \pm 0.7) \times 10^{-3}$ [112]) with a P-wave contribution alone. They also suggest a way out of what they call a puzzle by reinstalling an S-wave contribution by appealing to the topological structure of the decay. We believe that the construction of Ref. [60] is based on an erroneous assumption about the contribution of the topological diagram IIa to the S-wave amplitude A^{com} as we shall demonstrate in the following.

In order to shed more light on the problem, we list the current algebra W-exchange contributions to the amplitudes A^{com} and $B^{\text{pole}} = B^{\text{pole}}_{fki}$ in terms of the intermediate baryon states for the s- and u-channel contributions. The intermediate state in the s channel is given by Σ^+. In the u channel the intermediate states are given by the flavor degenerate pair of states Ξ_c^0 and $\Xi_c^{\prime 0}$ where Ξ_c^0 contributes to $A^{\text{com}}(u)$ and $\Xi_c^{\prime 0}$ contributes to $B^{\text{pole}}(u)$ since $\langle \Xi_c^{\prime 0}|K^+|\Lambda_c^+\rangle = 0$ and $\langle \Xi_c^0 K^+|\Lambda_c^+\rangle = 0$ (see Appendix C). One therefore has

$$\begin{aligned}
A^{\text{com}} &= \frac{1}{f_K} 6(4\pi \bar{X}_2)\Big(I^f_{\Xi^0 K^+ \Sigma^+}\, I^{\text{pc}}_{\Sigma^+ \Lambda_c^+} - I^{\text{pc}}_{\Xi^0 \Xi_c^0} I^f_{\Xi_c^0 K^+ \Lambda_c^+}\Big),\\
B^{\text{pole}} &= \frac{1}{f_K} 6(4\pi)\bar{X}_2 \tfrac{2}{3}(4\pi)\bar{Z}\Big(I^{\text{CQM}}_{\Xi^0 K^+ \Sigma^+} I^{\text{pc}}_{\Sigma^+ \Lambda_c^+} R_s(\Sigma^+)\\
&\quad + I^{\text{pc}}_{\Xi^0\, \Xi_c^{\prime 0}}\, I^{\text{CQM}}_{\Xi_c^{\prime 0} K^+ \Lambda_c^+} R_u(\Xi_c^{\prime 0})\Big).
\end{aligned} \tag{D1}$$

In line with the fact that we are working in the SU(3) limit we define $\bar{X}_2$ and $\bar{Z}$ as SU(3) averages of the bag model integrals listed in Ref. [60]. Note that the bag model integral X_1 does not appear in Eqs. (D1), since X_1 vanishes in the SU(3) limit. Intrinsic SU(3) breaking effects are accounted for in the bag model calculations but are quite small of $\mathcal{O}(1-2)\%$, as can be estimated from the ratio X_1/X_2 and the relative size of the bag model integrals X_2^d and X_2^d, and Z_1 and Z_2 in Ref. [60]. One can check that Eqs. (D1) agree with Eq. (40) of Ref. [60] when the SU(3) limit is taken.

The flavor coefficients in Eqs. (D1) are given by

$$S\text{-wave } (s;\, \Sigma^+) : I^f_{\Xi^0 K^+ \Sigma^+}\, I^{\text{pc}}_{\Sigma^+ \Lambda_c^+} = -4/2\sqrt{6}$$
$$= 2I_3 + \underline{4I_5}$$

$$S\text{-wave }(u;\ \Xi_c^0,\ \underbrace{\Xi_c^{\prime 0}}_{0}) : I^{\rm pc}_{\Xi^0\Xi_c^0} I^f_{\Xi_c^0 K^+\Lambda_c^+} = -4/2\sqrt{6}$$
$$= 2\hat{I}_3 + \underline{4I_5}$$
$$P\text{-wave }(s;\ \Sigma^+) : I^{{\rm CQM}(K^+)}_{\Xi^0\Sigma^+} I^{\rm pc}_{\Sigma^+\Lambda_c^+} = -10/2\sqrt{6}$$
$$= I_3 + \underline{2I_4} + \underline{6I_5}$$
$$P\text{-wave }(u;\ \underbrace{\Xi_c^0}_{0},\ \Xi_c^{\prime 0}) : I^{\rm pc}_{\Xi^0\,\Xi_c^{\prime 0}} I^{{\rm CQM}(K^+)}_{\Xi_c^{\prime 0}\,\Lambda_c^+} = -6/2\sqrt{6}$$
$$= \hat{I}_3 + 2\hat{I}_4 + \underline{6I_5}, \tag{D2}$$

where we have included the corresponding results in terms of the topological tensor invariants. As proven in Sect. 4, the two results agree with each other. We emphasize that the topological tensor invariants depend only on the initial and final state particles. The summation over intermediate states is automatically accounted for. It is for this reason that the representation of the current algebra results in terms of topological tensor invariants is much compacter than the representation in terms of intermediate states.

The S-wave contribution in the first line of Eqs. (D1) clearly vanishes for the decay $\Lambda_c^+ \to \Xi^0 K^+$ since $A^{\rm com} \sim I_3 - \hat{I}_3$, and both I_3 and $\hat{I}_3$ vanish. As emphasized in Sect. 5, the contribution of the nonvanishing topological invariant I_5 cancels out in the total sum of the s- and u-channel contributions. Therefore, there is no need to banish the contribution of the topological diagram III represented by the single topological invariant I_5 as done in Ref. [60] since I_5 does not contribute to the S-wave amplitude altogether. Instead, Cheng et al. reinstall an S-wave contribution by appealing to the topological structure of the nonleptonic transitions. Their idea is that diagram IIa (see also Fig. 1b in Ref. [60]) allows for an s-channel S-wave contribution to $\Lambda_c^+ \to \Xi^0 K^+$, since $I_4 \neq 0$. However, our analysis shows that the topological tensor invariant I_4 associated with diagram IIa does not contribute to the S-wave s-channel amplitude $A^{\rm com}$. The flaw in the reasoning of Ref. [60] results from an incomplete knowledge of how the topological diagrams are connected with the current algebra contributions which are now available from our analysis.

Nevertheless, let us present the results of the modified current algebra approach of Ref. [60] where we again use the representation in terms of topological tensor invariants. In the modified current algebra approach the u-channel contributions to $\Lambda_c^+ \to \Xi^0 K^+$ are relinquished by observing that i) there are no contributions from the topological diagram IIb and from the postulate that ii) contributions from the topological diagram III must be set to zero. In the modification of the current algebra approach one therefore has

$$A^{\rm com} = \frac{1}{f_K} 6(4\pi\bar{X}_2)\Big(I^f_{\Xi^0 K^+\Sigma^+}\, I^{\rm pc}_{\Sigma^+\Lambda_c^+} - 0(u)\Big),$$
$$B^{\rm pole} = \frac{1}{f_K} 6(4\pi\bar{X}_2)\tfrac{3}{2}(4\pi\bar{Z})\Big(I^{\rm CQM}_{\Xi^0 K^+\Sigma^+} I^{\rm pc}_{\Sigma^+\Lambda_c^+} R_s(\Sigma^+) + 0(u)\Big), \tag{D3}$$

where $0(u)$ stands for the banished u-channel contributions. As a result of their modifications, in their numerical evaluation Cheng et al. obtain a rather large value for the asymmetry parameter $\alpha_{\Lambda_c^+\to\Xi^0 K^+} = 0.90$ due to the fact that $A^{\rm com}$ is no longer zero. This result is in crass contradiction to the current algebra result where $\alpha_{\Lambda_c^+\to\Xi^0 K^+} = 0$. The issue of a vanishing or non-vanishing S-wave contribution to the decay $\Lambda_c^+ \to \Xi^0 K^+$ can be settled by a measurement of the asymmetry parameter in this decay. Unfortunately, such a measurement is not available at present. As concerns the rate, however, Cheng et al. succeed in their original goal to increase the branching ratio to $\mathcal{B} = 7.1 \times 10^{-3}$ close to the experimental branching ratio $\mathcal{B} = (5.5 \pm 0.7) \times 10^{-3}$ [112].

Since the arguments in Ref. [60] are based on an incomplete knowledge of how the topological invariants contribute to the current algebra results, it is difficult to follow their reasoning in the treatment of some of the other charm baryon decays. A case in point is the SCS decay $\Xi_c^+ \to \Sigma^0\pi^+$. The W-exchange contributions proceed via the $(c \to s;\ s \to u)$ transitions (called (a) in Appendix A) with the nonvanishing tensor invariants $I_3 = 2/4\sqrt{3}$ and $I_5 = 1/4\sqrt{3}$ (see Table 4). The intermediate state in the s-channel is Σ^+ for both $A^{\rm com}(s)$ and $B^{\rm pole}(s)$ while the intermediate states in $A^{\rm com}(u)$ and $B^{\rm pole}(u)$ are Ξ_c^0 and $\Xi_c^{\prime 0}$, respectively, as follows from the fact that $\langle \Xi_c^{\prime 0}|\pi^+|\Xi_c^+\rangle = 0$ and $\langle \Xi_c^0\pi^+|\Xi_c^+\rangle = 0$ (see Appendix C). One has

$$A^{\rm com} = \frac{1}{f_K} 6(4\pi\bar{X}_2)\Big(I^f_{\Xi^0\pi^+\Sigma^+}\, I^{\rm pc}_{\Sigma^+\Lambda_c^+} - I^{\rm pc}_{\Xi^0\Xi_c^0} I^f_{\Xi_c^0\pi^+\Lambda_c^+}\Big),$$
$$B^{\rm pole} = \frac{1}{f_K} 6(4\pi)\bar{X}_2\tfrac{2}{3}(4\pi)\bar{Z}\Big(I^{\rm CQM}_{\Xi^0\pi^+\Sigma^+} I^{\rm pc}_{\Sigma^+\Lambda_c^+} R_s(\Sigma^+)$$
$$+ I^{\rm pc}_{\Xi^0\,\Xi_c^{\prime 0}}\, I^{\rm CQM}_{\Xi_c^{\prime 0}\pi^+\Lambda_c^+} R_u(\Xi_c^{\prime 0})\Big) \tag{D4}$$

and

$$S\text{-wave }(s;\ \Sigma^+) : I^f_{\Sigma^0\pi^+\Sigma^+}\, I^{\rm pc}_{\Sigma^+\Xi_c^+} = (-\sqrt{2})(-2\sqrt{6})$$
$$= 2/\sqrt{3} = \underline{2I_3} + \underline{4I_5}$$
$$S\text{-wave }(u;\ \Xi_c^0,\ \underbrace{\Xi_c^{\prime 0}}_{0}) : I^{\rm pc}_{\Sigma^0\Xi_c^0} I^f_{\Xi_c^0\pi^+\Xi_c^+}$$
$$= (-1/\sqrt{3})(-1) = 1/\sqrt{3} = 2\hat{I}_3 + \underline{4I_5}$$
$$P\text{-wave }(s;\ \Sigma^+) : I^{\rm CQM}_{\Sigma^0\pi^+\Sigma^+} I^{\rm pc}_{\Sigma^+\Xi_c^+} = (-\sqrt{2})(-\sqrt{2/3})$$
$$= 2/\sqrt{3} = \underline{I_3} + 2I_4 + \underline{6I_5}$$

$$P\text{-wave } (u;\ \underbrace{\Xi_c^0}_{0},\ \Xi_c'^0) : I^{\rm pc}_{\Sigma^0\,\Xi_c'^0}\, I^{\rm CQM}_{\Xi_c'^0\pi^+\Xi_c^+}$$

$$= (1)(\sqrt{3}/2) = \sqrt{3}/2 = \hat{I}_3 + 2\hat{I}_4 + \underline{6I_5}, \qquad \text{(D5)}$$

leading to

$$\hat{A}^{\rm com} = (\underline{2I_3} + \underline{4I_5}) - (2\hat{I}_3 + \underline{4I_5})$$
$$\hat{B}^{\rm pole} = (\underline{I_3} + 2I_4 + \underline{6I_5})\, R_s(\Sigma^+)$$
$$+(\hat{I}_3 + 2\hat{I}_4 + \underline{6I_5})\, R_u(\Xi'^0). \qquad \text{(D6)}$$

The results can be checked to be in agreement with the results in Ref. [60] up to sign differences due to different sign conventions for the flavor wave functions. The above results for the decay $\Xi_c^+ \to \Sigma^0\pi^+$ in Ref. [60] were calculated in the modified current algebra approach as defined earlier on in the paper. An obvious question is why the u-channel contributions were dropped in Eqs. (D1) and not in the decay $\Xi_c^+ \to \Sigma^0\pi^+$. In both cases the u-channel contributions are proportional to the tensor invariant I_5 which are banished in one case but not in the other case.

Another case of puzzlement are the SCS decays $\Omega_c^0 \to \Sigma^+\bar{K}^-$ and $\Omega_c^0 \to \Sigma^0\bar{K}^0$ treated in a follow-up paper [62], again in the framework of the modified current algebra approach. That the authors of Ref. [62] use the modified current algebra approach is evident from the fact that they do not even list the decays $\Omega_c^0 \to pK^-$ and $\Omega_c^0 \to n\bar{K}^0$ in their list of Ω_c^0 decays since these decays are proportional to I_5 alone (see Table 8). Up to a normalization factor, the above two decays $\Omega_c^0 \to \Sigma^+\bar{K}^-$ and $\Omega_c^0 \to \Sigma^0\bar{K}^0$ can be seen to have a topological invariant structure which is identical to the one in the decay $\Lambda_c^+ \to \Xi^0K^+$ treated above (see Tables 3 and 8). Therefore, one would expect substantial S-wave contributions and large values of the asymmetry parameter in these decays in the modified current algebra approach, contrary to the numerical results listed in Ref. [62].

Apart from the misidentification in the S-wave contribution mentioned above, a second serious objection against the prescription of Cheng et al. is the following. As written down in Eq. (20), the topological tensor invariant I_5 projects onto the reduced topological matrix element $\mathcal{T}_5$. The reduced matrix element $\mathcal{T}_5$ in turn is calculated in terms of bag model integrals which certainly do not vanish as shown in Refs. [59–62]. However, one cannot arbitrarily set $\mathcal{T}_5$ to zero and keep the other reduced topological matrix elements at their nonzero bag model values. As an additional justification of their prescription to banish the contributions proportional to I_5, the authors of Ref. [60] cite the constituent quark model calculation of Ref. [37] where the contributions proportional to I_5 were found to be numerically small. However, in Ref. [37] the reduced matrix element $\mathcal{T}_5$ given in terms of an overlap integral H_3 was found to be numerically small which had no implications for the other two topological contributions IIa and IIb since they are proportional to an unrelated overlap integral H_2.

Appendix E: Amplitudes, rates and asymmetry parameter

As before we shall use the abbreviations $Q_\pm = (m_i \pm m_f) - m_k^2$ such that the magnitude of the rest frame momenta of the daughter particles read $p = \sqrt{Q_+Q_-}/2m_i$. We follow the conventions of Ref. [37] except that we change the sign of the p.c. amplitude B.

$$\text{Invariant amplitudes: } \langle B_f M|\mathcal{H}|B_1\rangle = \bar{u}_f(A - B\gamma_5)u_i$$
$$\text{Helicity amplitudes: } H^{\rm pv}_{1/2\,0} = 2\sqrt{Q_+}A, \quad H^{\rm pc}_{1/2\,0} = 2\sqrt{Q_-}B$$
$$\text{Rate: } \Gamma = \frac{p}{32\pi n_i^2}\left(|H^{\rm pv}_{1/2\,0}|^2 + |H^{\rm pc}_{1/2\,0}|^2\right) = \frac{p}{8\pi m_i^2}\left(Q_+|A|^2 + Q_-|B|^2\right)$$
$$\text{Asymmetry parameter: } \frac{|H_{1/2\,0}|^2 - |H_{-1/2\,0}|^2}{|H_{1/2\,0}|^2 + |H_{-1/2\,0}|^2} = \frac{2\,{\rm Re}(H^{\rm pv}H^{\rm pc\,*})}{|H^{\rm pv}|^2 + |H^{\rm pc}|^2} = \frac{2\sqrt{Q_+Q_-}\,{\rm Re}(AB^*)}{(Q_+|A|^2 + Q_-|B|^2)}. \qquad \text{(E1)}$$

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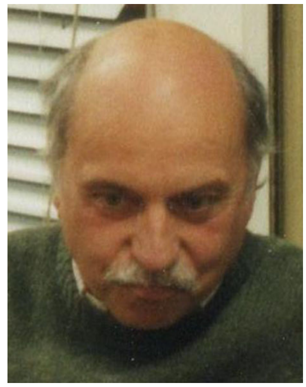

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