On electromagnetic wave tails in curved spacetimes

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Abstract

We calculate the tail term of the electromagnetic potential of a pulsed source in arbitrary bounded motion in a weak gravitational field and demonstrate that generally the received radiation tail arrives after a time delay. We apply the results to a compact binary system and conclude that the tail energy can be a noticeable fraction of the direct pulse energy.

1 Introduction

At the present meeting, one of the keywords is certainly "dimension". So a talk about wave tails is appropriate as the wave tail effect is very sensitive to the spacetime dimension. In fact, the spacetime dimension $n = 4$ is the lowest dimension where the Huygens principle can be valid, i.e., the spacetime can be curved but the wave tails still do not occur. Physically speaking, the wave tails arise because the radiation is backscattered by the spacetime curvature. In certain cases backscattering can influence observations as it weakens and disperses sharp initial pulses. For instance, a single radiation pulse from a pulsed source in the vicinity of a massive body is received by an observer as two distinct pulses: one arriving along the direct route from the source and the other, the tail part, as a reflection from the spacetime curvature.

At present the contribution of gravitational wave tails is recognized as a significant factor and when calculating the wave forms for the detection experiments (LISA, etc.) then the wave tails are taken into account. On the other hand, it seems to us that the electromagnetic wave tails are overlooked. We think it that is easier to detect an electromagnetic wave tail than gravitational radiation.

The aim of this presentation is to provide a compact outline of the results obtained in a recent series of papers [1-6] by the present authors. We are aware of the great number of papers on wave tails (see [7, 6]), however, the limited scope of this paper, unfortunately, does not allow us to refer to them.

2 Exact multipole solution of wave equation

We consider on a pseudo-Riemannian 4-space M a vector wave equation $\mathbf{Lu} = \mathbf{f}$, which in local coordinates reads

$$Lu_c := g^{ab}\nabla_a\nabla_b u_c - R^a_c u_a = f_c, \tag{1}$$

where ∇_a denotes covariant differentation with respect to x^a, and g^{ab} and R_{ac} stand for the the metric and Ricci tensor, respectively. The source term $\mathbf{f}$ in Eq. (1) is in general a distribution, i.e., $\mathbf{f} \in \mathcal{D}'^1(\Omega)$.

Usually the wave equation is solved in the weak-field and slow-motion limit using successive approximations. However, we have recently developed a method [1-4] for calculating the exact solutions of scalar and tensor wave equations whose source terms are arbitrary order multipoles. The method is based on the higher order Green's functions defined by us in Ref. [1]. In one of our papers [3], we obtained an exact multipole solution of a multipole vector wave equation, i.e., we calculated a unique retarded solution $\mathbf{u}^+_\mu$ of the following vector wave equation (with the multipole source term ρ^a_μ)

$$Lu^a_\mu = \rho^a_\mu := (-1)^\mu M^{I(\mu)j}_\mu(t)\nabla_{A(\mu)}(g^{A(\mu)}_{I(\mu)}(x, y(t))g^a_j(x, y(t))\bar{\delta}(x, \xi)). \tag{2}$$

Here $g^a_i(x, y)$ stands for the transport bitensor; $A(\mu) := a_1 \dots a_\mu$ and $I(\mu) := i_1 \dots i_\mu$ denote multiindices with respect to x and y, respectively; the multipole moment $\mathbf{M}_\mu(t)$ is a tensor field of order $\mu + 1$ on the worldline ξ of the source of electromagnetic radiation. The line distribution $\bar{\delta}(x, \xi)$ in Eq. (2) is defined by the relation $(\rho^a_\mu, \phi_a) := \int_\xi M^{I(\mu)j}_\mu(t)\phi_{j;I(\mu)}(y(t))dt$ with $y(t) \in \xi$, where the parameter t is the proper time along the worldline ξ.

Let us suppose that there is a t_0 such that $\mathbf{M}_\mu(t) = 0$ for $t < t_0$. The exact multipole solution u^{+a}_μ can be written as a sum $u^{+a}_\mu = d^{+a}_\mu + \mathcal{V}^a_\mu$, $\forall x \in J^+(\xi)\backslash\{\xi\}$ of the direct wave d^{+a}_μ and the wave tail $\mathcal{V}^a_\mu$. The expression for

d_μ^{+a} can be found in Ref. [3]. Here we are interested in the tail part

$$\mathcal{V}_\mu^a(x) = \frac{1}{2\pi} \int_{t_0}^{\tau(x)} V^a_{j;I(\mu)}(x, y(t)) M_\mu^{I(\mu)j}(t) dt \ , \tag{3}$$

where $\tau(x)$ denotes the retarded time of the observer at x and V_j^a is the tail term of the classical Green's function. The exact form of the classical Green's function is known only for a few particular cases, e.g., for de Sitter, Bianchi I type, and Robertson-Walker metrics. However, approximate Green's functions can be calculated.

3 Wave tails in weak gravitational fields

Now we assume that gravitational field is weak, expand the relevant quantities up to first order in a formal small parameter, and calculate the tail term of the classical fundamental solution in the first approximation, obtaining

$$V_a^i(x, y) = \frac{1}{4\pi} \left\{ g_a^i \nabla_b \left[\sigma^{-1}(x, y) P^b(x, y) \right] + \sigma^{-1}(x, y) F_a^i(x, y) \right\} , \tag{4}$$

where

$$P^b(x, y) := \int_{\Sigma(y)} g_p^b(x, z) G^{pq}(z) d\Sigma_q(z), \tag{5}$$

$$F_a^i(x, y) := \int_{S(y)} g_a^p g^{iq} \left(4R^r_{[p} D_{q]r} + R D_{pq} + 2R_{pq} \sigma(x, y) \right) \omega(z). \tag{6}$$

These expressions are essential in the following analysis. Here σ stands for the world function, G^{pq} for Einstein's tensor and $D_{pq} := \sigma_{;[p}(z, x) \sigma_{;q]}(z, y)$. The quantity $P^b(x, y)$ is proportional to 4-momentum of the gravitational field source. But it is more important to pay attention to the domains of integration $\Sigma(y) := C^+(y) \cap J^-(x)$ and $S(y) := C^+(y) \cap C^-(x)$, where $C^-(x)$ is the past light-cone with vertex at x, $D^-(x)$ is the interior of the past light-cone and $J^-(x) = D^-(x) \cup C^-(x)$. $C^+(y)$ denotes the future light-cone with vertex at y. The domains of integration depend on the positions of the wave source and the observer. As in a 4-spacetime the wave source and the observer inevitably move, then for different instants of time the domains of integration are different. In the subdomain $D^*(y) := \{x | x \in D^+(y), C^+(y) \cap \tilde{\Gamma} \subset \Sigma(y), S(y) \cap \tilde{\Gamma} = \emptyset\}$ the wavefront has passed the source of gravitational field $\tilde{\Gamma}$; the intergration region $S(y)$ lies within vaccuum, therefore $R_{pq}(z) = 0$ and $F_a^i(x, y) = 0$. Hence in the subdomain D^* the wave tail has the simple form of (4) with $F_a^i(x, y) = 0$.

For a wave pulse of a finite duration the generation of wave tails $\mathcal{V}_\mu^a$ and the domains of a different wave tail structure are explained in Fig. 1. The

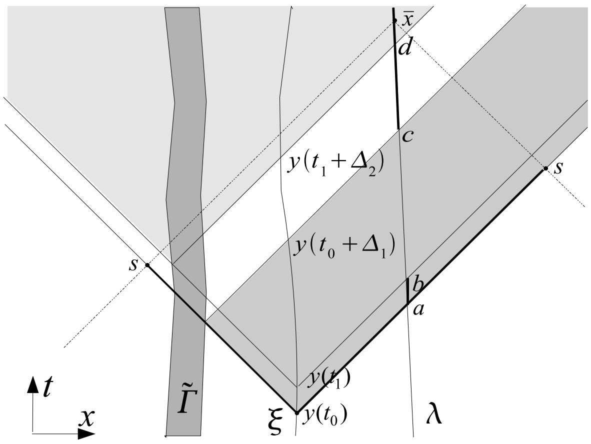

Figure 1: A diagram in Minkowski 2-spacetime: The past light-cone $C^{-}(\bar{x})$ originating from the point $\bar{x}$ is represented by the dotted lines. The bold-faced part of the future light-cone $C^{+}(y(t_0))$ corresponds to the hypersurface $\Sigma(y(t_0))$ with $x = \bar{x}$, and the boundary of the hyperplane $\Sigma(y(t_0))$, i.e., $S(y(t_0))$ is seen as two points s. The remaining components in the figure have been explained in the main text.

worldtube of the source of gravitational field with $R_{ab}(x) \neq 0$ is denoted by $\tilde{\Gamma}$; ξ denotes the worldline of the source of electromagnetic field which radiates only during a finite proper time interval $[t_0, t_1]$, and λ denotes the worldline of observer. It can be seen that the observer represented by the worldline λ receives the principal (direct) pulse within the interval $[a, b]$. During the interval (b, c) on λ a blackout between the direct pulse and the wave tail occurs. During the interval $[c, d]$ the observer receives the wave tail of the general structure and during (d, ∞) the tail of the simple structure.

A schematic representation of the geometry of wave tail generation in a spacelike plane x^1x^2 is given in Fig. 2. The plane of the figure is determined by locations of the wave source y, of the observer x and of the center of the gravitational source. The surface $S(y)$, which spreads with time, is the boundary of the ellipsoid of revolution $\Sigma(y)$, with foci at the locations of the observer x on worldline λ and of the wave source y on worldline ξ.

The ellipses S_1 and S_2 are the intersections of the plane of the figure with the surface $S(y)$ that, respectively, correspond to the instants of time at which a delta-like wave pulse emitted by the wave source reaches and passes the source of gravitation. Now we have the following picture. The wave source y emits an instantaneous wave pulse. The direct pulse propagates along the direct route yx to the observer x, after which there occurs a blackout before the arrival of the first tail contribution. The duration of the blackout is $\Delta_1 = l_1 + l_1^* - |\vec{x} - \vec{y}|$.

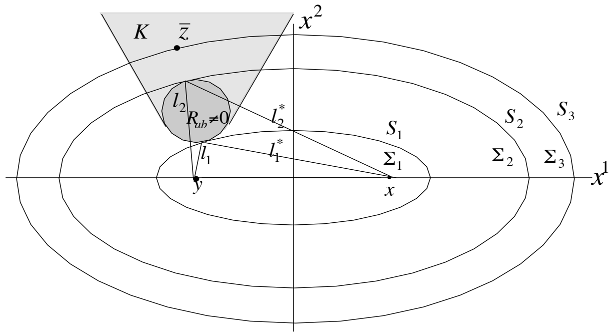

Figure 2: A schematic representation of the geometry of wave tail generation on a spacelike plane x^1x^2. Here y is the wave source, x is the observer, the darker shadowed circular disk is the gravitational source and K is the focusing region. The remaining components in the figure have been explained within the main text.

If the surface $S(y)$ has not yet reached the gravitational source, then $\mathbf{V} = 0$ and there is no wave tail. If the surface $S(y)$ has passed the gravitational source, the source will forever remain inside the domain of integration $\Sigma(y)$, while $\mathbf{P}(x,y)/8\pi = \text{const}$ will be the 4-momentum of the gravitational source and $F^i_a(x,y) = 0$. The area K, which includes the source of gravitational field, corresponds to the region where the wave tail is predominantly generated by gravitational focusing which deforms the direct wave fronts.

It should be mentioned that the occurrence of a time delay between the principal pulse and the wave tail in the case of a weak gravitational field, as a rule, remains unrevealed by expansion of the wave equation into spherical harmonics as in this case it is natural to choose the worldline of the multipole radiation source inside the source of gravitation.

4 Energy carried by wave tails

To get an idea about energy magnitudes, we have constructed an example for which we have obtained an estimate of the ratio of the intensity of the wave tail, I, to the intensity of the direct pulse, I_0, radiated during the time interval $[t_0, t_1]$ (see, [5, 6]), namely,

$$\frac{I(t_1 + \Delta_2(x))}{I_0} = \left(\frac{2M}{\tilde{\Delta}_2(x)}\right)^2 \left(\frac{T}{T + \tilde{\Delta}_2(x)}\right)^2, \tag{7}$$

where $2M$ is the Schwarzschild radius of the source of gravitational field; T denotes the duration of the direct pulse in the observer time; $\tilde{\Delta}_2$ and Δ_2

denote the duration of the blackout between the direct pulse and wave tail of the simplest structure, respectively, in the observer time and in the proper time of the source. To illustrate the above estimate, we have considered a wave source rotating in a circular orbit of radius r_0 around a spherical source of radius r_s of gravitational field. We have found that if $r_0 > r_s^2/4M$, then in the case when the wave pulse is emitted in the region of the geometric shadow of the source of gravitation or in its vicinity it is valid that $\tilde{\Delta}_2(x) \sim 2M$, and according to the estimate the intensity of the tail can be of the same magnitude as the intensity of the primary pulse. For the model under discussion we have found that the ratio of the energy $\mathcal{E}$ transferred by the tail term (beginning from the time $t_1 + \Delta_2(x)$) to the energy of the direct term $\mathcal{E}_0$ is equal to: $\mathcal{E}/\mathcal{E}_0 \approx 0.119(2M)^2/\tilde{\Delta}_2^2(x)$ if $T \approx 1.57\tilde{\Delta}_2(x)$ and can in this case make up nearly 10% of the energy of the direct pulse.

5 Summary

(i) The present results indicate that the electromagnetic wave tails should be carefully considered when calculating the energy radiated by astrophysical systems. The energy carried away by the tail can amount to approximately 10% of the energy of the low-frequency modes of the direct pulse. (ii) The delay effect of the electromagnetic wave tails may be of great importance for their observational detection.

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