SLAC TRANS -99

A LINEAR ION ACCELERATOR WITH SPATIALLY UNIFORM HARD FOCUSING

by

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Translated by T. Watt (August 1969) from *Lineĭniĭ uskoritel' ionov s prostranstvenno-odnorodnoĭ zhestkoĭ fokusirovkoĭ.* Preprint ITEP-673 of the Institute of Theoretical and Experimental Physics, USSR State Committee on Utilization of Atomic Energy, Moscow, March 1969, 16 pp.

TRANSLATED FOR

STANFORD LINEAR ACCELERATOR CENTER

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by

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The authors propose a linear ion accelerator in which the focusing and acceleration are achieved with a high-frequency electric field produced by a long four-conductor line. The accelerator contains no drift tubes. The advantages of this accelerator are its relative simplicity of design, small transverse dimensions, and possibility of substantially reducing the injection energy while maintaining the accelerated beam at a high intensity.

We know that in the building of linear ion accelerators the problem of keeping the particles near the accelerator axis is very troublesome. In all modern low- and medium-energy linear accelerators the problem is solved by placing magnetostatic quadrupole lenses inside the drift tubes. This solution has proved effective and, in a number of machines, has made it

* Translator's Note: Presumably the Russian abbreviation for "State Committee on Atomic Energy."

possible to achieve extremely high accelerated-beam intensities[1,2]. But drift tubes with magnetic lenses and systems for powering them have grave disadvantages: design and engineering complexity, high cost, very exacting mechanical lens-aligning requirements. Effecting the necessary alignment is rendered difficult by the fact that drift tubes are usually fastened to long stems. The need to place the focusing lenses inside the drift tubes necessitates making the tubes so long that any reduction of the injection energy or wavelength of the accelerating field is thereby limited, even in cases in which such a reduction would be permissible from the standpoint of accelerating a beam with a given intensity.

At different times linear accelerators have been proposed having drift tubes in which the hard focusing is achieved not with built-in static lenses but with the high-frequency accelerating field itself, by doing away with the axial symmetry of the accelerating gaps[3,4]. In principle, this should make it simpler and less expensive to build and operate linear accelerators. Yet in all the mentioned designs the drift tubes have been retained, with all their inherent drawbacks (complex accelerator assembly, difficult mechanical alignment, relatively high injection energies). Now, however, consistent use of the high-frequency field for quadrupole ion focusing makes it possible to dispense with drift tubes altogether[5].

In all linear accelerators with drift tubes the hard-focusing system has a distinct spatial periodicity, with either the polarity of the poles of the quadrupole lenses or the geometry of the poles alternating along the axis. But, using a voltage variable in time, we can have a quadrupole system of focusing electrodes that is uniform along the accelerator axis. Such a system is shown in Figure 1. The quantity V is the peak value of the voltage between adjacent electrodes. Since the high-frequency voltage $\pm \frac{V}{2} \cos(\omega t + \phi)$ is applied to the electrodes, the particles, as they travel along the axis, are successively acted upon by fields whose gradient signs alternate. In a spatially uniform quadrupole system this produces a hard-focusing effect.

A longitudinal accelerating component can be created in the high-frequency focusing field if the distance between opposite electrodes of one polarity is periodically varied along the axis. The spatial period of variation of the distance between electrodes must be equal to the path traveled by a particle during an rf cycle, and the phases of the variation of the distances in the vertical and horizontal planes have to be staggered by a half-cycle. Also, the electric-field potential at the axis is modulated with a period $\beta\lambda$, which then produces a resonance accelerating effect. In Figure 2 the electrode cross-sections are indicated schematically by the planes xoz and yoz, z being the longitudinal coordinate, k the wave number: $k = 2\pi/\beta\lambda$.

Let us consider a four-conductor quadrupole line. In the general case, in a quasi-stationary approximation, the distribution of the electric-field potential has the form

$$u(r,\psi,z,t) = -\frac{V}{2}\left[F_0(r,\psi) + \sum_{n=1}^{\infty} F_n(r,\psi)\sin knz\right]\cos(\omega t+\varphi), \qquad (1)$$

where $\omega = 2\pi c/\lambda$ is the field's cyclic frequency. The function

$$F_0(r,\psi) = \sum_{s=0}^{\infty} A_{0s}\, r^{2(2s+1)} \cos 2(2s+1)\psi$$

defines the law of the potential distribution in the cross-sections with the coordinate $kz = l\pi$ ($l = 0, 1, 2, \ldots$), where the distribution has exact quadrupole symmetry. This function is determined fairly accurately by the shape of the electrodes in the sections in question. The coefficients F_n for the harmonics of the space modulation of the distances between electrodes are $F_n(r,\psi) = \sum_{s=0}^{\infty} A_{ns} I_{2s}(knr)\cos 2s\psi$.

The particle-energy increment over a path equal to the period of the space modulation $\Delta z = \beta\lambda$ is defined by the expression

$$\Delta W = \frac{ekV}{2}\sum_{n=1}^{\infty} n \int_0^{\beta\lambda} F_n(r,\psi)\cos knz\cos(kz+\varphi)\,dz.$$

For a particle traveling along the accelerator axis, we have $\Delta W = 2eV\Theta\cos\phi$, where ϕ is the field's phase (reckoned from the maximum) at the instant at which the particle is found at the point corresponding to the cross-section with exact quadrupole symmetry; Θ is the acceleration efficiency:

$$\Theta = \frac{\pi}{4} A_{10} \quad . \tag{2}$$

We see that in the first approximation only the first harmonic of the electrodes' space modulation contributes to the acceleration.

We introduce the dimensionless longitudinal coordinate $\tau = \frac{1}{2\pi}(\omega t + \phi)$. The transverse oscillations of the ions are described in the nonrelativistic case by the equation

$$\frac{d^2x}{d\tau^2} = \frac{2\pi^2 eV}{m\omega^2}\left[\frac{\partial F_0}{\partial x}\cos 2\pi\tau + \sum_{n=1}^{\infty}\frac{\partial F_n}{\partial x}\sin n(2\pi\tau - \varphi)\cos 2\pi\tau\right] \tag{3}$$

We confine ourselves to a linear approximation to the quadrupole component of the field and first harmonic of the electrodes' space modulation. Keeping in mind that the principal contribution to the acceleration is made only by the term with the coefficient A_{10}, we set

$$\left.\begin{aligned} F_0(z,\psi) &= A_{00}\, z^2 \cos 2\psi \\ F_1(z,\psi) &= A_{10}\, I_0(kz) \\ F_n(z,\psi) &\equiv 0; \quad n \geqslant 2 \end{aligned}\right\} \tag{4}$$

We then let a be the minimum distance from an electrode to the axis that determines the channel's throughput capacity (acceptance). If the electrodes are not modulated, at any point of the axis the channel radius is equal to a, and with ideal hyperbolic poles the gradient of the electric field is $G_o = V/a^2$. When there is modulation, the mean distance from an electrode to the axis is greater than a, which results in a reduction of the effective focusing gradient. With modulated electrodes the electric-field gradient at the channel axis in the cross-sections with exact quadrupole symmetry is $G = \frac{\partial Ex}{\partial x} = VA_{oo}$. The ratio of the gradient at the channel axis with modulated electrodes in the cross-sections with quadrupole symmetry, to the gradient of a field with ideal unmodulated poles, we shall call the focusing efficiency ($\varkappa$). As readily seen,

$$\varkappa = A_{oo}\, a^2. \tag{5}$$

The equation for the transverse oscillations (3) reduces to the form

$$\frac{d^2x}{d\tau^2} = \left[\frac{\varkappa V}{\varepsilon_o}\left(\frac{\lambda}{a}\right)^2 \cos 2\pi\tau - \frac{2\pi V}{\beta^2 \varepsilon_o}\,\theta \sin\varphi\right] x \,, \tag{6}$$

where ε_o is the ion rest energy in electron-volts. In a smooth approximation the advance of the phase of the transverse oscillations in the period $\beta\lambda$ (or $\Delta\tau = 1$) is defined by the equality

$$\mu^2 = \frac{1}{8}\left(\frac{\varkappa V \lambda^2}{\pi \mathcal{E}_0 a^2}\right)^2 + \frac{2\pi V}{\beta^2 \mathcal{E}_0}\,\theta \sin\varphi. \qquad (7)$$

The first term on the right-hand side of (7) is determined by the hard-focusing effect. The second term is different from zero only in those cases in which the distance between electrodes is modulated ($\Theta \neq 0$). This term is associated with the field's defocusing effect, which shows up simultaneously with the acceleration effect.

The solutions of equation (6) are Mathieu functions. The transverse oscillations of a particle with the phase ϕ are stable when the conditions

$$\begin{gathered} \frac{\varkappa V}{2\pi^2 \mathcal{E}_0}\left(\frac{\lambda}{a}\right)^2 < 1 - \frac{2V}{\pi \beta^2 \mathcal{E}_0}\,\theta \sin\varphi \\ \frac{\varkappa^2 V}{8\pi^3 \mathcal{E}_0}\left(\frac{\lambda}{a}\right)^4 > -\frac{2\theta \sin\varphi}{\beta^2} \end{gathered} \qquad (8)$$

are satisfied[6]. Analysis of these inequalities shows that the stability of the transverse oscillations is virtually assured---as in accelerators with static lenses---for any values of the synchronous phase (to -90°).

Confining ourselves in the potential-distribution function (1) to only the terms of the first approximation (4), which mainly determine the acceleration and focusing effects, we get

$$\mathcal{U}(r,\psi,z,t) \approx -\frac{V}{2}\left[\varkappa\left(\frac{r}{a}\right)^2 \cos 2\psi + \frac{4\theta}{\pi} I_0(kr) \sin kz\right] \cos(\omega t + \varphi) \qquad (9)$$

The solution (9) is exact if the surfaces of the poles satisfy the equations

$$r^2\cos 2\psi = \frac{a^2}{æ}\left[1 - \frac{4\theta}{\pi} I_0(kr)\sin kz\right]$$
$$r^2\cos 2\psi = -\frac{a^2}{æ}\left[1 + \frac{4\theta}{\pi} I_0(kr)\sin kz\right]. \qquad (10)$$

The depth of the electrodes' space modulation can be defined by the coefficient m , which is equal to the ratio of the maximum distance,between an electrode and the axis,to the minimum distance (Figure 2). From equations (10) we determine the acceleration and focusing efficiencies for ideal poles as a function of the channel aperture a and electrode-modulation depth m :

$$\theta = \frac{\pi}{4} \cdot \frac{m^2 - 1}{m^2 I_0(ka) + I_0(mka)}$$

$$æ = \frac{I_0(ka) + I_0(mka)}{m^2 I_0(ka) + I_0(mka)} .$$

In the general case the acceleration and focusing efficiencies should be determined from relations (2) and (5).

For a given channel aperture, as the modulation depth is increased within broad limits, the acceleration efficiency is enhanced, but the focusing efficiency declines. There exists, therefore, an optimal modulation of the electrodes. As numerical estimates show, the mean proton-energy increment per unit length in the proposed system is of the order of 1 MeV/m, the throughput capacity (normalized acceptance) of the order of 1.5 cm-mrad, up to energies of 20-30 MeV with a high-frequency field strength of up to 150 kV/cm. Fields of that strength are being used in modern accelerators. In view of the spatial continuity of the focusing and accelerating structure, the latter can be effectively used with low injection energies of 100-200 keV, the mean energy increment per unit length substantially increasing in the low-energy region. Acceleration efficiencies of $\Theta = 0.2$ and focusing efficiencies of $\mathfrak{X} = 0.6$ are obtainable in the 0.2-1 MeV energy range, and of $\Theta = 0.5$ and $\mathfrak{X} = 0.4$, respectively, in the 5-20 MeV range.

In all cross-sections the poles described by equations (10) are close to hyperbolic in shape, the squares of the distances between the electrodes and axis being modulated according to a sinusoidal law. To permit creation of a sufficiently large difference in potentials between adjacent poles, the latter should be trimmed*, as in ordinary electrostatic

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* Translator's Note: Sic! Russian verb means *cut*, *pare*, *trim*.

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lenses. While this does result in an increase of the field's nonlinear components, it has virtually no effect on the acceleration efficiency. Since poles with limited generatrices are used in all static lenses, the observed nonlinear effects are not specific to the spatially uniform structure.

The beam behavior in a spatially uniform hard-focusing channel has characteristic features. A matched beam possesses dimensions that are constant along the channel axis and pulsate in time. In each of the planes (*xoz* or *yoz*) the maximum cross-sectional size occurs at the instant at which the channel focuses in that plane, the minimum size at the instant at which the channel defocuses in that plane. The conditions of matching the beam to the channel input periodically vary with the frequency of the electric field. Hence, generally speaking, a dynamic matching of the beam to the channel is necessary. Such matching is achieved by means of high-frequency quadrupole lenses. The envelope of the unmatched beam oscillates in time with twice the mean frequency of the particles' transverse oscillations. Superimposed on these are the fast pulsations at the field frequency. The amplitude of the "slow" oscillations runs along the beam at the speed of travel of the particles.

A high-frequency four-conductor line can be produced in the form of a four-chamber cylindrical resonator. All the resonator chambers are excited at the lower, natural frequency so that the vector of the magnetic field's strength is directed along the resonator, it being in the opposite phase in the adjacent chambers. The maximum high-frequency difference in potentials develops in the gap of the interchamber partitions, near the axis. Along the resonator the difference in potentials remains constant at every instant. The distribution of the fields in a resonator with unmodulated electrodes is shown in Figure 3. Figure 4 shows cross-sections of a resonator with modulated electrodes in the planes corresponding to the coordinates $kz = \frac{\pi}{2} + 2\pi l$ and $kz = \frac{3}{2}\pi + 2\pi l$. The closing of the magnetic flux is done at the ends of the resonator, which at the ends is sealed off with flat "bottoms" that do not come in contact with the interchamber partitions.

The natural frequencies of a four-chamber resonator with unmodulated V-shaped electrodes are determined with the equation

$$f\left(\frac{\omega b}{c}\right) = -\frac{2}{\pi}\left(p + \ln\frac{2b}{a}\right),$$

where c is the speed of light, b the resonator radius, a the radius of the interaction region (Figure 3). The parameter p depends on the chamber's angular dimension α ; for the field's quadrupole mode it is $p = \frac{4\sin^2\alpha}{\pi\alpha} - \gamma$; for the dipole mode $p = -\frac{32}{\pi\alpha}\sin\frac{\alpha}{2}\sin\frac{\alpha}{4}\left(\sin\frac{\alpha}{4} + \cos\frac{\alpha}{4}\right) - \gamma$; $\gamma = 0.5772...$ is the Euler constant. The function $f(x)$ is $f(x) = \frac{N_1(x)}{J_1(x)} - \frac{2}{\pi}\ln x$, where J_1, N_1 are first-order Bessel and Neumann functions, respectively. The estimates show that for the field's quadrupole mode the resonator's natural frequency is determined mainly by its radius and is not critically dependent on the parameters a and α . For a wave $\lambda = 2$ m we have $b \gtrsim 25$ cm. The natural frequency of the dipole mode differs appreciably from the frequency of the quadrupole mode.

The figure of merit of a four-chamber H-resonator is much lower than the figure-of-merit values for E_{010}-wave-excited cylindrical resonators. Yet because of the small reserve of high-frequency energy stored in the resonator volume the pulsed losses of high-frequency power in the copper do not exceed values usual for drift-tube-loaded E-resonators. On the other hand, the resonator's low figure of merit permits a considerable shortening of the high-frequency-pulse duration and reduction of the mean power losses. Accelerating intensive beams of long duration requires compensation of the high-frequency-energy losses.

R e f e r e n c e s

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* Translator's Note: Obviously a translation from the English. Correct spelling of author's name uncertain; wording of original English title unknown.

Submitted for publication
25 March 1969

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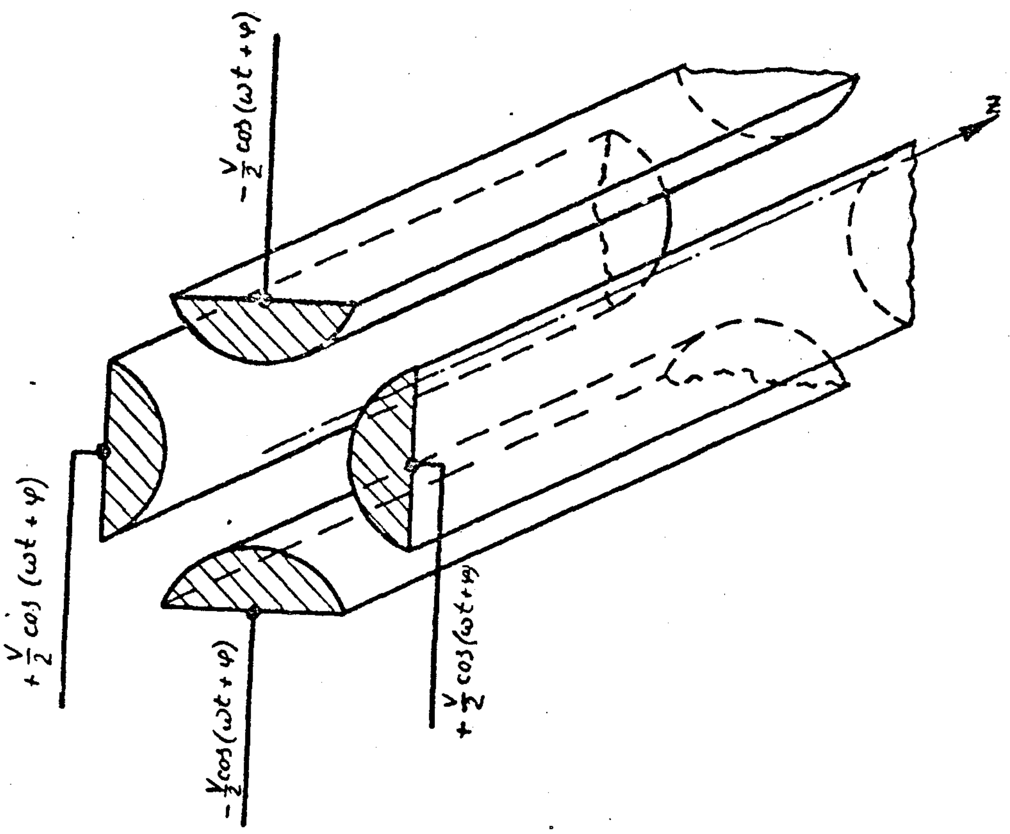

Рис. 1

Figure 1

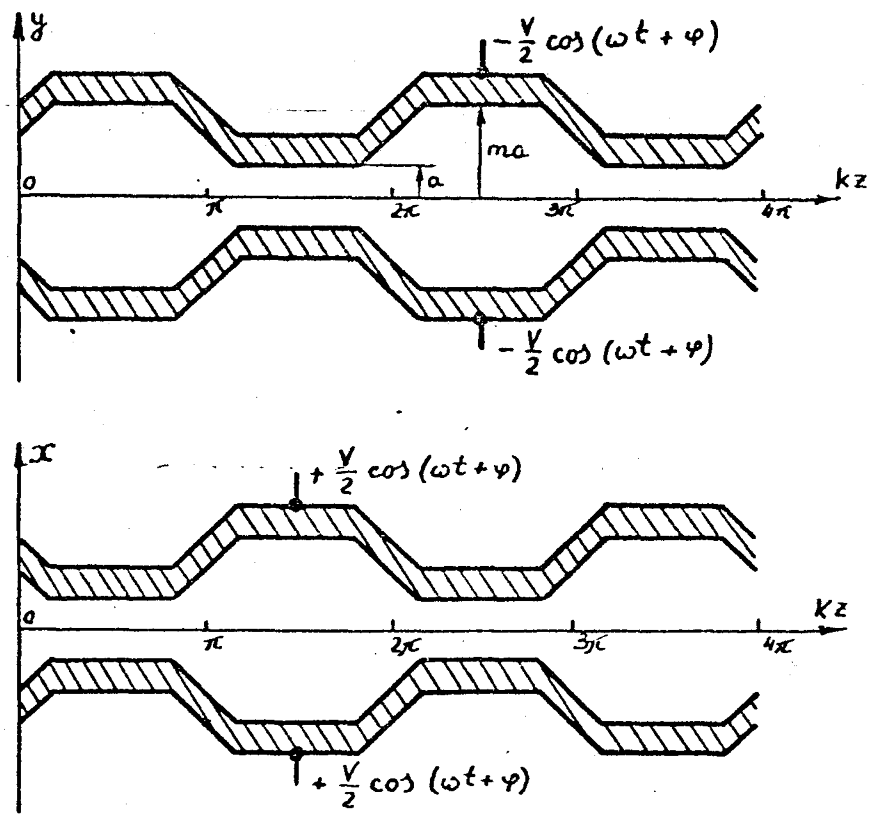

Рис. 2

Figure 2

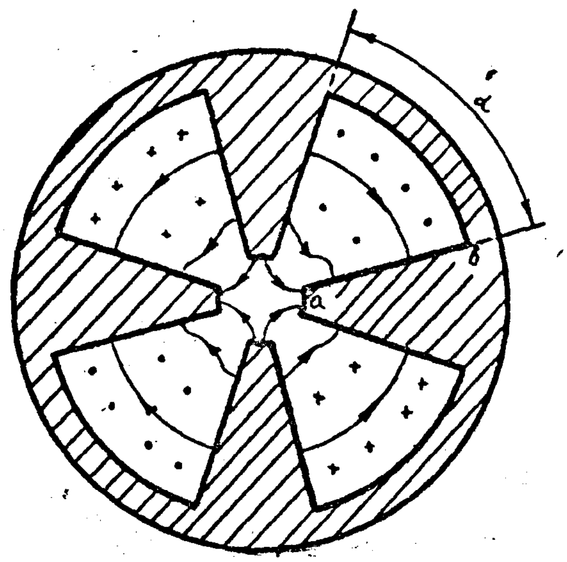

Рис. 3

Figure 3

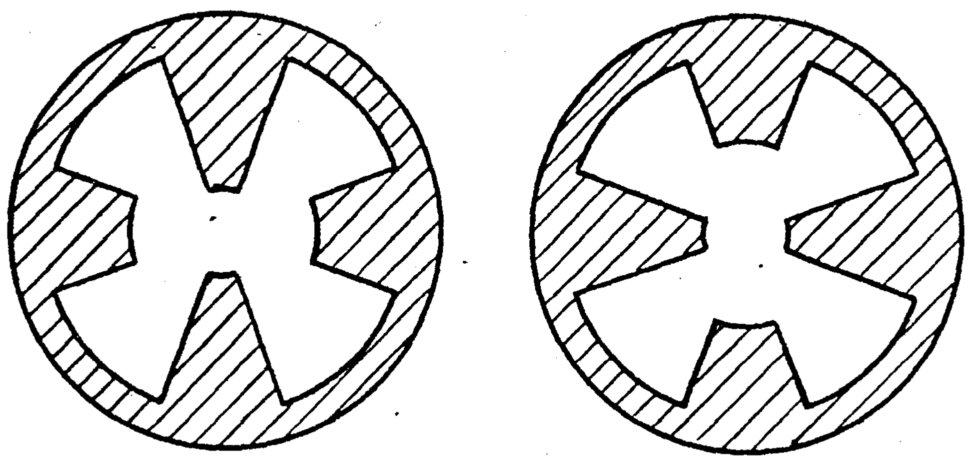

Рис. 4

Figure 4