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The nature of the quark-hadron phase transition in hybrid stars and the mass-radius diagram

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Abstract.

In this work, we use a hybrid equation of state that allows us to choose the smoothness of the quark-hadron phase transition, by choosing the value of a continuous parameter μ_c. To describe the hadron phase, we use an equation of state (EoS) based on a chiral effective field theory (cEFT), and for the quark phase we use the equation of state of the MFTQCD (Mean Field Theory of QCD). We solve simultaneously the TOV equations and the tidal deformability equations and contruct the mass-radius and deformability-mass diagrams for several values of the parameter μ_c. We find that the curves in these two diagrams are almost insensitive to the smoothness of the phase transition.

1. Introduction

Neutron stars reproduce the conditions of a still poorly understood region of the QCD phase diagram, where the temperature is low and the chemical potential is high. In this region, matter may deconfine. In this case, the inner core of the star is made of quark matter, the crust is made of hadronic matter and there is a phase transition inside the star. To obtain more information about these compact objects, we use models for the equation of state. With them, we can solve simultaneously the TOV and the tidal deformability equations and thus obtain the measurable properties of these compact stars. Comparing the results with experimental data, we can decide if the chosen model gives a good description of dense and cold matter. A good introduction to the subject can be found in [1].

Until recently, the existing data were compatible with stars made of quark matter, of hadron matter and of a combination of both. During the last years, new data started to appear more rapidly, introducing new and stringent constraints on the mass-radius (M-R) diagram and reducing the freedom in the choice of the equation of state. In this context an interesting question is: can we learn something about the quark-hadron phase transition (if there is any) from the new astrophysical data? In particular, can we know how sharp it is (first order? second order?) and if there is a mixed phase?

A first step along this direction was taken in Ref. [2], where several neutron star observables were calculated with a hybrid equation of state which could be tuned to include a sharper or smoother phase transition, depending on the choice of a contiuously varying parameter.

In this work, we follow the strategy of Ref. [2] using different equations of state for the two phases and using different tools to control the sharpness of the transition.

2. Formalism

To obtain hybrid equations of state with phase transitions of different smoothness, we used the equation proposed in [3]:

$$P = \lambda P_H + (1-\lambda) P_Q + \frac{2\delta(\mu_B)}{\sqrt{(P_Q - P_H)^2 + 4\delta(\mu_B)}}, \tag{1}$$

$$\epsilon = \lambda \epsilon_H + (1-\lambda)\epsilon_Q - \frac{2\left[1 + (\mu_B/\mu_c)^2\right]\delta(\mu_B)}{\sqrt{(P_Q - P_H)^2 + 4\delta(\mu_B)}}, \tag{2}$$

$$\rho_B = \lambda \rho_{B,H} + (1-\lambda)\rho_{B,Q} - \frac{2(\mu_B/\mu_c^2)\delta(\mu_B)}{\sqrt{(P_Q - P_H)^2 + 4\delta(\mu_B)}}, \tag{3}$$

where

$$\delta(\mu_B) = \delta_0 \exp\left[-(\mu_B/\mu_c)^2\right], \quad \lambda = \frac{1}{2}\left[1 - \frac{(P_Q - P_H)}{\sqrt{(P_Q - P_H)^2 + 4\delta(\mu_B)}}\right]. \tag{4}$$

The free parameters δ_0 and μ_c allow us to choose the intensity of the phase transition. The subscript indices H and Q refer to the hadron and quark phase quantities, respectively. Therefore, choosing the values of δ_0 and μ_c and the equations of state of the quark and hadron phases, we get the pressure, energy density, and baryonic density of the hybrid equation of state. Originally, Eqs. (1), (2) and (3) were formulated to describe the phase transition in high-energy nuclear collisions. Applying them to neutron stars, we noticed that the pressure values do not vanish when $\rho_B = 0$. Then, to adapt these equations to the stellar medium, we consider that the hybrid equation of state is given by

$$\text{hybrid EoS} = \begin{cases} \text{hadronic EoS, for } \mu_B < \mu_{B,\text{limit}}, \\ \text{Eqs. (1), (2) and (3) for } \mu_B \geq \mu_{B,\text{limit}}. \end{cases}$$

In this work, we choose the model presented in [4] (called here HLPS) and recently used in [5] for the hadronic phase and the MFTQCD [6, 7, 8] for the quark phase. The HLPS model was designed to be consistent with a chiral effective field theory and is one of the most well accepted models to describe hadronic matter. In this work, we choose the Soft equation of [4]. The MFTQCD equations were obtained assuming that the gluon field can be decomposed into low (soft) and high (hard) momentum components and that the sources of the latter are so intense that the hard gluon field can be treated as a classical mean field. Using this assumption and some other approximations, we can derive the equation of state from the QCD Lagrangian [8].

It reads:

$$\begin{aligned}
P = &\frac{27\xi^2}{2}\rho_B^2 - B \\
&+ \frac{1}{4\pi^2}\sum_{q=u,d,s}\left[k_{F,q}\mu_{F,q}\left(\mu_{F,q}^2 - \frac{5}{2}m_q^2\right) + \frac{3}{2}m_q^4 \ln\left(\frac{k_{F,q}+\mu_{F,q}}{m_q}\right)\right] \\
&+ \frac{1}{12\pi^2}\left[k_{F,e}\mu_{F,e}\left(\mu_{F,e}^2 - \frac{5}{2}m_e^2\right) + \frac{3}{2}m_e^4 \ln\left(\frac{k_{F,e}+\mu_{F,e}}{m_e}\right)\right], \qquad (5)\\
\epsilon = &\frac{27\xi^2}{2}\rho_B^2 + B \\
&+ 3\frac{1}{4\pi^2}\sum_{q=u,d,s}\left[k_{F,q}\mu_{F,q}\left(\mu_{F,q}^2 - \frac{1}{2}m_q^2\right) - \frac{1}{2}m_q^4 \ln\left(\frac{k_{F,q}+\mu_{F,q}}{m_q}\right)\right] \\
&+ \frac{1}{4\pi^2}\left[k_{F,e}\mu_{F,e}\left(\mu_{F,e}^2 - \frac{1}{2}m_e^2\right) - \frac{1}{2}m_e^4 \ln\left(\frac{k_{F,e}+\mu_{F,e}}{m_e}\right)\right], \qquad (6)\\
\rho_B = &\frac{1}{3\pi^2}\sum_{q=u,d,s}k_{F,q}^3, \qquad (7)
\end{aligned}$$

where $\mu_{F,i} = \sqrt{k_{F,i}^2 + m_i^2}$ is the Fermi energy of particle i and $\xi \equiv g/m_G$. g is the strong coupling constant and m_G, the dynamical gluon mass. To apply these equations to neutron stars, we assumed that quarks and electrons are in chemical equilibrium. We also imposed charge neutrality and baryon number conservation [6, 7]. The first term of Eqs. (5) and (6), proportional to ρ_B^2, arises from the interaction between hard gluons and quarks. This term results in a stiffer equation of state. The second term of these equations originates from soft gluons and is defined by [8]

$$B = \frac{1}{4}\left\langle F^{a\mu\nu}F^a_{\mu\nu}\right\rangle, \qquad (8)$$

where $F^{a\mu\nu}$ is the soft gluon field tensor. We can notice in Eqs. (5) and (6) that this term has the same behavior as the MIT bag constant, which is why we denote it by B. The free parameters of the equation of state of MFTQCD are B and ξ, and in this work, we choose $B = 200$ MeV/fm^3 and $\xi = 0.0016$ MeV^{-1}.

3. Results

Having defined the EoS of the quark and hadron phases, we fixed the values of the free parameters to be $\delta_0 = 50$ (MeV/fm^3)^2 and $\mu_{B,\text{limit}} = 1000$ MeV. By fixing δ_0, μ_c becomes the only parameter that determines the intensity of the phase transition. To consider four phase transitions with different smoothness, we have chosen $\mu_c = 0,\ 700,\ 1000,\ 2000$ MeV. We have checked that these values represent an extremely sharp, two intermediate and a very smooth phase transition, respectively. When $\mu_c > 2000$ the smooth transition curves suffer only sligth changes. The obtained equations of state are shown in Fig. 1. In the panels of this figure, we can see that the higher the value of μ_c, the smoother the phase transition is. Thus, the red curve ($\mu_c = 0$ MeV) represents a sharp phase transition. The yellow, green and blue curves ($\mu_c = 700,\ 1000,\ \ 2000$ MeV, respectively) represent smoother phase transitions.

Solving simultaneously the TOV equations and the tidal deformability equations (for more details see [6]), we obtain Fig. 2. In the left panel of the mass-radius diagram we see that the four hybrid curves show the same behavior as the hadron curve in the region of $R \gtrsim 12$ km. This behavior is expected, as this is the region of the M-R diagram where we find neutron stars

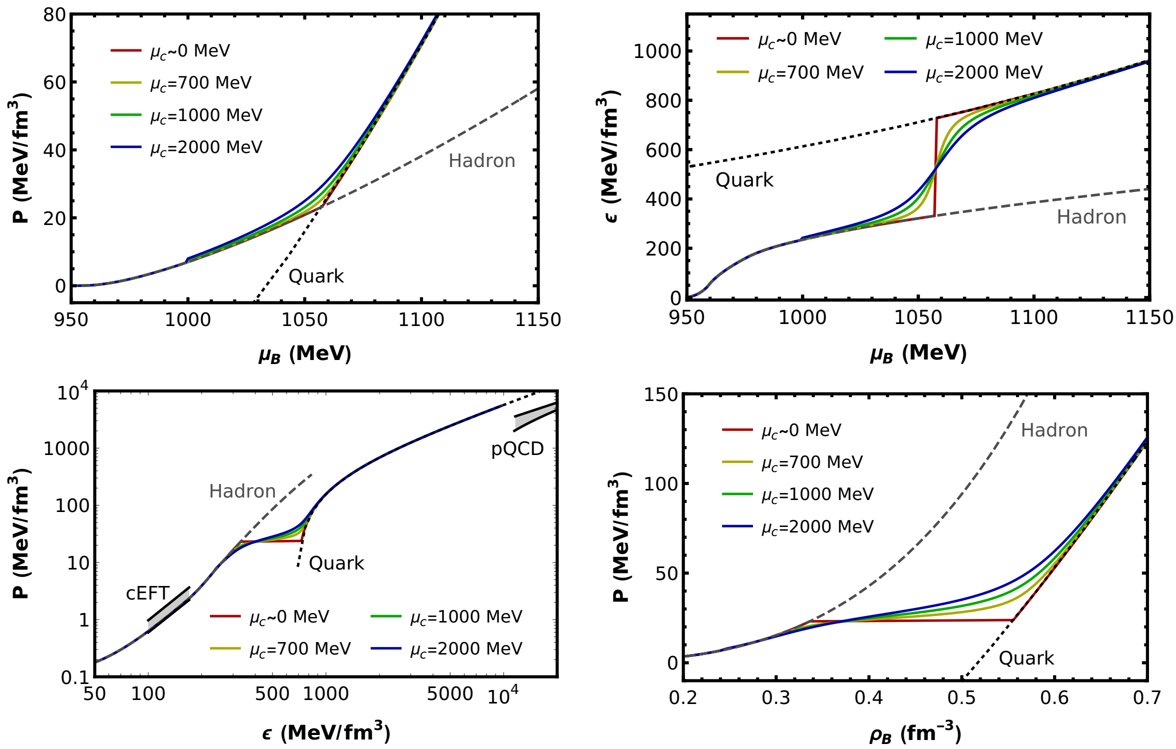

Figure 1. Hybrid equations of state with the MFTQCD for the quark phase and the Soft equation of the HLPS model for the hadron phase. The marks in the $p(\epsilon)$ diagram show the low and high energy constrainsts imposed by chiral perturbation theory and perturbative QCD respectively.

that do not have a central pressure high enough to deconfine matter. In the region of $R \lesssim 8$ km, the curves with different values of μ_c overlap in a behavior close to a quark star. In this region of the M-R diagram neutron stars have very high central pressure. Therefore, these stars are formed mostly of matter in the quark phase.

Changes in the smoothness of the phase transition produce effects only in the intermediate region of the M-R diagram, i.e. for 8 km $\lesssim R \lesssim$ 12 km. Smoother phase transitions lead to slightly higher masses for the same radius value. For these transitions the hybrid star curve goes smoothly from the hadron star curves ($R \gtrsim 12$ km) to the quark star curves ($R \lesssim 8$ km). We notice that in the graph of $P \times \mu_B$ (graph at the top, on the left, of Fig. 1), the curves with higher values of μ_c present higher values of pressure in the region of the phase transition. Therefore, in this region, EOS with smoother phase transitions result in stiffer equations of state. This implies larger values of mass in the mass-radius diagram. The same conclusion is valid for the tidal deformability graph. The region of higher (smaller) values of masses of the hybrid curves presents a behavior of quark (hadron) stars, which is consistent with the high (low) values of central pressure of this region. The importance of the smoothness of the phase transition is shown in the central region of the diagram ($0.7\ \mathrm{M}_\odot < M < 1.3\ \mathrm{M}_\odot$). Curves with higher values of μ_c have smoother changes. We also observe that higher values of μ_c lead to higher values of tidal deformability in this intermediate region.

As can be seen in the left panel of Fig. 2, our curves do not reproduce the experimental data. This is due to the parameter choices made for the equations of state. In a future work, we intend to use the hybrid equations (1), (2), and (3) with other parameters and/or other EoS for the quark and hadron phases. For the purposes of this work, the disagreement with data is not crucial. The most important is the conclusion that the nature of the phase transition has almost no visible effects in the mass-radius diagram or in the deformability-mass diagram. Consequently these observables will not help us in discriminating a strong first order phase transition from other smoother transitions.

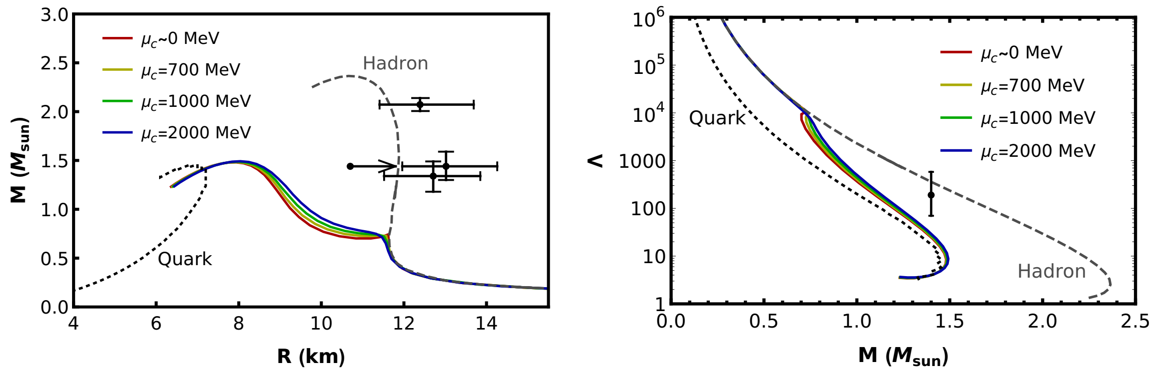

Figure 2. Mass-radius diagram, on the left, and tidal deformability as a function of mass, on the right. The points show the experimental data (for details see [6]).

4. Conclusions

We worked with a hybrid equation of state in which, through the free parameter μ_c, it was possible to choose the smoothness of the quark-hadron phase transition. For the quark phase, we used the EoS of the MFTQCD with $B = 200$ MeV/fm^3 and $\xi = 0.0016$ MeV^{-1}, while for the hadron phase, we used the Soft EoS of the HLPS model. We obtained four EoS just by changing the value of μ_c. We found that higher values of μ_c result in EoS with a smoother phase transition. Solving simultaneously the TOV and the tidal deformability equations, we observed that the equation of state with smoother phase transition results in slightly higher values of the mass and tidal deformability. However, the most striking conclusion is that the mass-radius diagram seems to be very insensitive to the nature of the quark-hadron phase transition. We also conclude that the chosen EoS does not agree with current observational data, although this fact is not essential for this work. This conclusion can be made more robust by considering other EoS for both phases and also by selecting EoS which yield a good description of experimental data.

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References
[1] N. Glendenning, *Compact stars*, (Springer, New York, 2000).
[2] S. Han and A. W. Steiner, Phys. Rev. D **99**, 083014 (2019).
[3] Yogiro Hama et al., Nucl. Phys. A **774**, 169 (2006).
[4] K. Hebeler et at, The Astrophys. Jour. **773:11**, (2013).
[5] A. Parisi, C. V. Flores, C. H. Lenzi, C. S. Chen and G. Lugones, JCAP **06**, 042 (2021).
[6] M. B. Albino, R. Fariello and F. S. Navarra, Phys. Rev. D **104**, 083011 (2021).
[7] B. Franzon, D. A. Fogaca, F. S. Navarra and J. E. Horvath, Phys. Rev. D **86**, 065031 (2012).
[8] D. A. Fogaça, F.S. Navarra, Phys. Lett. B **700**, 236 (2011).