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Pecci-Quinn Extension of the Natural Hybrid Inflation Model

Bumseok Kyae*
Department of Physics, Pusan National University, Busan 609-735

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The "natural hybrid inflation model," which combines the supersymmetric hybrid model and the natural inflation model, was proposed to achieve the spectral index of 0.96, and an axion decay constant of intermediate scale. By introducing both the $U(1)_R$ and a shift symmetry and employing the minimal Kähler potential, we were able to avoid the eta-problem. The two inflaton fields in this model can admit a large non-Gaussianity. For a full realization of the idea of the natural inflation, the shift symmetry in the model should be embedded in a $U(1)_{PQ}$ symmetry.

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I. INTRODUCTION

Supersymmetry (SUSY) has played a key role in progress of particle physics for the last four decades. SUSY introduced in the electroweak scale provides a beautiful resolution to the gauge hierarchy problem [1], and makes the standard model at the electroweak scale possibly embedded in string theory at the fundamental scale [2]. It is possible because SUSY can control quantum corrections to the scalar field's mass so as to be small enough.

SUSY would be important also in cosmology. For instance, the lightest SUSY particle (LSP) present in the minimal superstymmetric standard model (MSSM), *i.e.* the minimal SUSY extension of the standard mdoel might be a strong dark matter candidate. Most of all, SUSY is very helpful in keeping the mass of the inflaton being small against quantum corrections for successful inflation, which is very essential in order to maintain the flatnees of the inflaton potential. Although introduction of SUSY is quite helpful for smallness of inflaton mass, however, it is not enough unlike in particle physics: the positive vacuum energy during inflation, which breaks SUSY, induces the inflaton mass of the Hubble scale in supergravity (SUGRA), which explicitly violates one of the slow-roll conditions. Actually it is a notorious problem in inflation, called the "η(eta)-problem" [3].

The two representatitve resolutions to the η-problem were suggested from the SUSY hybrid and the natural inflationary models. In the SUSY hybrid inflation, the $U(1)_R$ symmetry is introduced, and the minimal form of the Kähler potential is taken. The unwanted Hubble scale induced mass of the inflaton is accidentally cancelled out in this model [4], and inflation is dominantly driven by a log type inflaton potential resulting from the SUSY breaking effect during inflation [5]. In the natural inflaton, a $U(1)_{PQ}$ symmetry is introduced and a pseudo Nambu-Goldstone boson plays the role of the inflaton [6]. The inflaton potential can be generated e.g. by the instanton effect and so it is typically of a sinusoidal type. Thus, only a shift symmetry remains in the inflaton potential. Since the inflaton is the phase part of a scalar field, it can be basically small.

In fact, the $U(1)_R$ symmetry is unique and generically present in $N = 1$ SUSY theories. The $U(1)_{PQ}$ symmetry is necessary to solve the strong CP problem in particle physics [7]. Indeed, the two global U(1) symmetries are quite essential symmetries employed in a large class of SUSY models in particle physics.

In SUSY hyrid inflation models, the observed value of the power spectrum of the cosmic micowave background radiation (CMBR) turns out to be assocated with the breaking scale of the grand unified

*E-mail: bkyae@pusan.ac.kr

theories (GUTs) [5]: CMB temperature anisotropy of $\delta T/T \sim 10^{-5}$ requires a spontaneous symmetry breaking scale of 10^{15-16} GeV. Thus, it provided a good motivation to construct many realistic inflationary models combined with the GUTs in particle physics such as $SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$, $SU(4)_c \times SU(2)_L \times SU(2)_R$, $SU(5) \times U(1)_X$, and SO(10) [8]. However, the SUSY hybrid model has a salient but unrealistic prediction on the spectral index, which is associated with the e-folding number:

$$n_\zeta \approx 1 + 2\eta \approx 1 - \frac{1}{N_e} \approx 0.98 \qquad (1)$$

for $N_e = 50 - 60$ e-folds. It is too large compared to the present bound, $n_\zeta = 0.96^{+0.014}_{-0.013}$. It is quite hard to lower the value, unless the model is seriously modified.

For the slow-roll parameter "η" to be small enough, in the natural inflation the Pecci-Quinn breaking scale or axion decay constant f should be somewhat larger than the Planck scale:

$$f \gtrsim 3M_P, \qquad (2)$$

where M_P implies the reduced Planck mass ($\approx 2.4\times 10^{18}$ GeV). Thus, the $U(1)_{PQ}$ must be valid above the scale of f, which is not naturally acceptable since quantum gravity effects are known to break all global symmetries. To avoid this problem in the framework of the natural inflation, two or (much) more natural inflation sectors are needed [9].

In Ref. [10], the authors show that such shortcomings in the SUSY hybrid and the natural inflationary models can be overcome by combining the two models ("natural hybrid inflation model"); the spectral index becomes the desired value 0.96, and the decay constant f can be lowered to the intermediate scale. However, the authors considered only a shift symmetry rather than a full $U(1)_{PQ}$, since the shift symmetry is enough to protect the smallness of the inflaton mass from the SUGRA correction. In this paper we attempt to extend the model of Ref. [10] to accommodate a full $U(1)_{PQ}$.

This paper is organized as follows. In section II, we will review the natural hybrid inflation model, in section III we will propose the extended model with $U(1)_{PQ}$. Section IV will be devoted to conclusion.

II. NATURAL HIBRID MODEL

Under the $U(1)_R$ symmetry, the superpotential W and a superfield S are supposed to transform in the same way: $W \to e^{2i\gamma}W$ and $S \to e^{2i\gamma}S$. Under the shift symmetry, a superfield T is supposed to transform as $T \to T + 2\pi i f$. Let us consider the following superpotential and the Kähler potential:

$$\begin{aligned} W &= \kappa S\left[M^2 - m^2 e^{-T/f} - \psi\overline{\psi}(1+\rho e^{-T/f})\right] , \\ K &= |S|^2 + |\psi|^2 + |\overline{\psi}|^2 + \frac{1}{4}(T+T^*)^2 , \end{aligned} \qquad (3)$$

which are consistent with the $U(1)_R$ and the shift symmetry. The superfields of a conjugate pair, ψ and $\overline{\psi}$ are assumed to carry opposite gauge charges. M^2, m^2, f, κ, and ρ are parameters. For simplicity, they all are assumed to be real parameters. The real part of the scalar comonent of S and the imaginary part of T are regarded as the inflaton fields. Assuming a hierarchy among the dimensionful parameters, $m \ll f \ll M$ ($\ll M_P$), slow-roll of the inflatons and large non-Gaussianity turn out to be possible [10].

The $U(1)_R$ forbids S^2, S^3, etc. in the superpotential, which destroy slow-roll inflation. While the mass of the inflaton S is not generated at tree level from the superpotential Eq. (3), it can be done radiatively: a logarithmic inflaton potential is generated by the SUSY breaking effect or positive vacuum energy during inflation, which makes a small slope for S, leading it to the SUSY minimum [5]. Although the SUGRA corrections included, the inflaton mass is still small, if the Kähler potential is given by the minimal form.

The scalar component of T is composed of the two real scalar fields, $T(x) = \phi(x) + ia(x)$. (In this paper, we use the same notation for a superfield and its scalar component.) Thus, the transformation of the shift symmetry, $T \to T + 2\pi i f$ implies just $a(x) \to a(x) + 2\pi f$. Because of the shift symmetry, the Kähelr potential, which is a functional of $T+T^*$, does not contain $a(x)$. Accordingly, the F-term scalar potential in SUGRA is expected not to induce the Hubble scale mass term for $a(x)$ during inflation [11]. However, the Kähler potential in Eq. (3) provides the cononical form of the kinetic terms for $a(x)$ as well as $S(x)$ and $\phi(x)$.

At the minimum of the scalar potential obtained from Eq. (3), ψ and $\overline{\psi}$ develop VEVs of order M, whereas

$S = a = 0$ and $\phi/f \sim \mathcal{O}(m^2/f^2) \ll 1$. Inflation can be initiated by the conditions, $S \gtrsim M$ and $a \neq 0$. Actually it is possible, since extremely high temperature of the early universe would make the scalar fields deviated from their minima. Then, the vacuum energy $\kappa^2 M^4$ dominates the universe, which exponentially inflates the space. Due to the large VEV of S, ψ and $\overline{\psi}$ get heavy masses, and so $\psi = \overline{\psi} = 0$ during inflation. $\phi(x)$ is still stuck near to the origin: $\phi/f \sim \mathcal{O}(M_P^2 m^2/f^2 M^2) \ll 1$. Thus, the scalar potential during inflation is effectively given by

$$V_{\rm inf} = \mu^4 \left(1 + \alpha \log\frac{S}{\Lambda} - \lambda \cos\frac{a}{f}\right), \qquad (4)$$

where the first term corresponds to the positive vacuum energy ($\mu^4 \equiv \kappa^2 M^4$) leading to exponential expansion of the space, and the second term indicates the radiative correction ($\alpha \equiv \kappa^2/8\pi^2$) to the inflaton potential by the vacuum energy, and the third term is the tree level potential of $a(x)$ by the shift symmetry ($\lambda \approx 2m^2/M^2$).

The scalar potential Eq. (4) admits the trajectories of the inflatons, S and a, yielding the center value of WMAP7 for the spectral index, $n_\zeta = 0.96$, an intermediate scale of f, and large non-Gaussianity. Accordingly, the shortcomings of the SUSY hybrid and the natural inflationary models have been overcome. For proper initial values of S and a, TABLE I lists some values of the parameters M, κ, m, f, *etc.* consistent with the data from seven years of WMAP (WMAP7) [12],

$$\mathcal{P}_\zeta = 2.43 \pm 0.115 \times 10^{-9}, \quad n_\zeta = 0.96^{+0.014}_{-0.013}. \qquad (5)$$

As seen in TABLE I, large non-linear parameters $f_{\rm NL}$ resulting in large non-Gaussianity are possible. The WMAP7 data shows that the initial density perturbation is almost scale invariant and Gaussian with $-10 < f_{\rm NL}^{\rm local} < 74$ at the 95% confidence level. In the near future, however, more precise data on $f_{\rm NL}$ are expected from the on-going satellite experiments such as Planck [13].

As pointed above, however, the authors of Ref. [10] considered only a shift symmetry rather than a full $U(1)_{PQ}$. Thus, now we attempt to introduce a $U(1)_{PQ}$ and consider its breaking so that the model can be a realization of the natural inflation. Through the process, a special form of the model in [10] could be obtained.

III. PECCI-QUINN EXTENSION

We consider the following form of the superpotential and the Kähler potential with the $U(1)_R$ and $U(1)_{PQ}$ symmetries:

$$\begin{aligned} W &= \kappa S\left(M^2 - \psi\overline{\psi}\right) + \Sigma\left(\kappa_1 X\overline{X} - \kappa_2 Z\overline{Z}\right), \\ K &= |S|^2 + |\psi|^2 + |\overline{\psi}|^2 + |X|^2 + |\overline{X}|^2 \\ &+ \left(\frac{\kappa_3}{M_P} Z^\dagger X S + \frac{\kappa_4}{M_P} Z^\dagger H_u H_d + {\rm h.c.}\right), \end{aligned} \qquad (6)$$

where $\kappa_{1,2,3}$ are dimensionless couplings. The $U(1)_R$ and $U(1)_{PQ}$ charges of the superfields appearing in Eq. (6) are presented in TABLE II. Z and $\overline{Z}$ are the sources of the $U(1)_{PQ}$ and SUSY breaking. They are spurion fields, whose scalar- and F-components develop VEVs:

$$Z = v + \vartheta^2 F_z, \quad \text{and} \quad \overline{Z} = \overline{v} + \vartheta^2 \overline{F}_z, \qquad (7)$$

where v, $\overline{v}$ are assumed to be of the intermediate scale, 10^{9-12} GeV, and

$$F_z \sim \overline{F}_z \sim m_{3/2} M_P \sim 10^{21}\ {\rm GeV}^2. \qquad (8)$$

As a result, the soft SUSY breaking parameters of TeV scale can be generated in the visible sector through the gravity mediation. From the last term of the Kähler potential in Eq. (6), the MSSM μ term of the desired scale can also be generated via the Giudice-Masiero mechanism [14].

The VEV of $Z\overline{Z}$ ($= v\overline{v}$) triggers the VEVs of X and $\overline{X}$ from the equation of motion by the last two terms of the superpotential in Eq. (6), *e.g.* $\partial W/\partial\Sigma = 0$. On the other hand, $\Sigma = 0$ at the SUSY minimum. Then, the superfields X and $\overline{X}$ are reformulated in terms of the new superfields h and T as [15]

$$X = (v_{PQ} + h)\, e^{-T/v_{PQ}}, \quad \text{and} \quad \overline{X} = (v_{PQ} + h)\, e^{T/v_{PQ}}, \qquad (9)$$

where $v_{PQ} = \sqrt{(\kappa_2/\kappa_1) v\overline{v}}$. Since h is a heavy mode, we will ignore it at low energies. T in Eq. (9) is identified with the "T" discussed above, and so its scalar component is composed of $\phi(x) + ia(x)$. Then the κ_3 term in the Kähler potential can provide the "$\kappa S m^2 e^{-T/f}$" term in the superpotential of Eq. (3):

$$\begin{aligned} W &\supset \kappa_3 \frac{F_z^*}{M_P} v_{PQ} e^{-T/v_{PQ}} S \\ &\approx \kappa_3 (m_{3/2} v_{PQ}) e^{-T/v_{PQ}} S \equiv \kappa\, m^2 e^{-T/f} S. \end{aligned} \qquad (10)$$

Table 1. Some parameter values which give large $f_{\rm NL}$ at the end of inflation with $N_e = 60$. μ, α, and λ in TABLE I of Ref. [10] are converted here into M, κ, and m, respectively. The values of M and f are listed in the unit of GeV. M and f are around the GUT and intermediate scales, respectively. Here we have already imposed the constraints from the data on the power spectrum and the spectral index in WMAP7 presented in Eq. (5)

Case	M $(=\mu/\sqrt{\kappa})$	κ $(=\sqrt{8\pi^2\alpha})$	m $(\approx\sqrt{\lambda/2}M)$	f	$f_{\rm NL}$
1	$4.5\cdot 10^{15}$	$1.4\cdot 10^{-3}$	$2.2\cdot 10^{9}$	$5.4\cdot 10^{12}$	43
2	$1.9\cdot 10^{15}$	$8.2\cdot 10^{-5}$	$6.6\cdot 10^{6}$	$3.8\cdot 10^{10}$	72
3	$5.9\cdot 10^{14}$	$2.5\cdot 10^{-6}$	$5.9\cdot 10^{3}$	$1.1\cdot 10^{8}$	67

Table 2. R and Pecci-Quinn charges of the superfields. The up-type singlet (s)quarks and the Majorana (s)neutrinos carry the unit Pecci-Quinn charges. The SU(2) doublets (singlets) in the MSSM *matter* superfields carry the zero (unit) R charges.

Superfields	S	ψ	$\overline{\psi}$	Σ	X	$\overline{X}$	Z	$\overline{Z}$	H_u	H_d
$\mathrm{U}(1)_R$	2	0	0	-1	0	3	2	1	1	1
$\mathrm{U}(1)_{PQ}$	0	0	0	0	-1	1	-1	1	-1	0

Note that “$e^{-T/v_{PQ}}$” is invariant under the shift of T, $T \to T + 2\pi i v_{PQ}$. Hence, v_{PQ} can be identified with “f” in the previous superpotential in Eq. (3). Thus, f should be given by the $\mathrm{U}(1)_{PQ}$ breaking scale in this model, which is consistent with the values of TABLE I. On the other hand, the “ρ” term in Eq. (3) should be suppressed, $\rho \ll 1$. After $\mathrm{U}(1)_{PQ}$ breaking, the Kähler potential also shows the shift symmetry of $T \to T + 2\pi i v_{PQ}$ [15],

$$K \supset |X|^2 + |\overline{X}|^2 = 2v_{PQ}^2 \cosh\left(\frac{T+T^*}{v_{PQ}}\right) \approx (T+T^*)^2. \tag{11}$$

After canonicalizing the kinetic terms of ϕ and a, the Kähler potential of Eq. (3) can be reproduced.

From Eq. (10), one can estimate the magnitude of κ_3 using the relation,

$$\kappa_3 = \frac{\kappa m^2}{f m_{3/2}}. \tag{12}$$

While κ_3 is of order unity in Case 1 ($\kappa_3 \approx 1.3$ for $m_{3/2} =$ 1 TeV), it is too suppressed in Case 2 ($\sim 10^{-4}$) and 3 ($\sim 10^{-9}$). Hence, Case 1 is the natural case in this model.

IV. CONCLUSION

We have reviewed the “natural hybrid inflation model,” proposed in Ref. [10]. By introducing the $\mathrm{U}(1)_R$ and a shift symmetries, it combines the two inflationary models, which avoids the η-problem. Moreover, it resolves the problems in the SUSY hybrid and the natural inflation models: the spectral index of 0.96 and the decay constant of the intermediate scale can be achieved. In addition, non-Gaussianity can be large, since the two inflaton fields drive inflation. In this paper, we extended the model such that it accommodates a full $\mathrm{U}(1)_{PQ}$ symmetry.

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