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To cite this article: G. A. González-Sprinberg and J. Vidal 2017 *J. Phys.: Conf. Ser.* **912** 012001

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# Tau magnetic moment

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**Abstract.** The  $\tau$  lepton magnetic moment theoretical predictions and measurements are reviewed. While it is believed that such a high mass particle is a good candidate to show up new physics, this is not the case up to now. The magnetic moment of elementary fermions, and in particular the anomalous magnetic moment of the electron, had an historical impact both in relativistic quantum mechanics and in quantum field theories. Besides, many new physics models were discarded when confronted with these magnitudes. More recently, the discrepancy of the experiments and the theoretical predictions for the muon anomalous magnetic moment is still an open issue. For the  $\tau$  lepton, instead, while the theoretical prediction is well known for the standard model and some new physics models, the data are very far of determining even its sign or the first figure. We will discuss the most important theoretical aspects of the  $\tau$  magnetic moment, and also the current accepted measurements and future perspectives, in particular related to B-factories.

## 1. Introduction

The magnetic moment of elementary fermions allows to write a first order interaction of a fermion with a magnetic field. This electromagnetic interaction, for stable fermions in the presence of a magnetic field, provides a way to measure some of these magnetic moments with a unusual precision. For the electron this precision is more than ten figures and the relative precision is 0.7 ppb [1]. This method and a similar precision are not possible in the case of an unstable particle, such as the  $\tau$  lepton, with a mean life  $(290.3 \pm 0.3) \times 10^{-15} \text{s}$ . On the other side, theoretical predictions for the magnetic moments, and in particular for the quantum corrections that give origin to the anomalous magnetic moment (AMM), can be computed with extraordinary precision in QED, and also in electroweak theory. More recently higher order hadronic corrections where also computed.

The magnetic moment anomaly for a spin one-half fermion  $f$  was first computed to leading order in QED by J.Schwinger in 1948 [3], and this is the famous result:

$$a_f \equiv \frac{g_f - 2}{2} = \frac{\alpha}{2\pi} \simeq 0.00116 \quad (1)$$

This expression is flavour independent and in the case of the electron only higher QED corrections are necessary in order to approach the experimental result. For other fermions, and in particular for the muon, corrections proportional to mass enter in the computation of the electroweak and hadronic contributions.



The AMM allows a stringent test for new physics and it was deeply investigated by many authors (see, for example, [4] and cites therein). The discrepancy of several sigma between the theoretical and experimental value of the muon AMM gave origin to a series of impressive experiments of the Muon ( $g-2$ ) Collaboration that resulted in a 0.7 ppm measurement [5, 6, 7]; besides a large number of theoretical work was done in order to understand this discrepancy [8, 9, 10].

The case of the tau is very different. While the theoretical prediction in the standard model is well known with many figures, the experimental precision is far from the first figure and even the sign of the AMM is unknown. The short mean life of the tau produces promptly decays and it is through these secondary particles that the AMM should be traced back. The PDG value for the tau AMM is [1]:

$$-0.052 < a_\tau < 0.013 \quad (95\% C.L.) \quad (2)$$

while the theoretical prediction in the standard model is [11]

$$a_\tau = 117721(5) \times 10^{-8} \quad (3)$$

Note that the PDG value is taken from the DELPHI Collaboration[12] in 2003 and has not changed since then. There, and also in other experiments, the bounds are obtained from total cross section data, like  $e^+e^- \rightarrow e^+e^-\tau^+\tau^-$  for DELPHI or  $e^+e^- \rightarrow Z \rightarrow e^+e^-\tau$ , also in LEP. Note that these total cross sections observables provide a rather indirect bound on the AMM and not a direct measurement of this quantity. Besides, in the same papers, bounds on the CP-violating electric dipole moment of the  $\tau$  are also computed, using the total cross section which is a CP-even observable.

On the other side, the theoretical prediction in [11] updated the QED and electroweak corrections and took into account the leading order hadronic ones (based on  $e^+e^-$  data from BaBar, CMD-2, KLOE and SND), and the light by light contributions. But the comparison between experiment and theory is disappointing: the measurements provide bounds that are one order of magnitude above the theory, and not even the sign of the AMM is known. As expressed some years ago by M.Pertl[2] in the section *Dreams and odd ideas in tau research*: "... It would be very nice to measure  $\mu_\tau$  with enough precision to check this (the Schwinger term  $\alpha/2\pi$ ), as it was checked for the  $e$  and the  $\mu$  years ago. At present such precision is a dream."

In the next section we reviewed some of the results in the theoretical side, next we addressed the ideas already investigated in order to measure the AMM, and finally we presented our conclusions.

## 2. Theory

The magnetic moment of a particle  $f$  can be defined by the gyromagnetic ratio as:

$$\vec{\mu}_f = g_f \frac{e}{2m} \vec{s} \quad (4)$$

For a spin 1/2 particle  $f$  the Dirac's equation predicts  $g_f = 2$ , and quantum corrections to this quantity are usually expressed by means of the AMM  $a_f$  defined in Eq.(1), that quantifies the difference with the prediction of the Dirac equation. The quantum corrections, dominated by QED, slightly increases the value of  $a_f$ . Then, the magnetic moment of a spin 1/2 particle of negative charge  $-e$  is just the "Bohr magneton" (but replacing  $m_e$  with the particle mass) times  $(1 + a_f)$ . For the electron, where the mass is well below the electroweak and new physics scales, the AMM provides a strong test of QED and also the best measurement [13] of the fine structure constant  $\alpha$ . The  $\vec{\mu} \cdot \vec{B}$  term naturally arises in the non-relativistic limit of the Dirac's equation. This allows to define the AMM in a precise way in this domain, that is useful for stable particles that can be put in interaction with an external magnetic field. As already stated, the AMM for

the tau is dominated by the Schwinger term, shown in Eq.(1), and all the QED, electroweak and hadronic contributions add up to less than 2%. Due to the high precision experiments for the electron and the muon those contributions played a crucial important role. In general, the other particles in the theory (with mass  $M$ ) contribute with terms proportional to  $m_f^2/M^2$ , and this is true for the standard model and many new physics models. Other dependencies on the new masses and/or scales can also be found in some models. Then, the muon AMM is an scenario where the contributions of all the standard model spectrum can show up. The enhancement factor for the muon with respect to the electron is  $m_\mu^2/m_e^2 \simeq 4 \times 10^4$  while for the tau these factors are  $m_\tau^2/m_e^2 \simeq 1, 1 \times 10^7$  and  $m_\tau^2/m_\mu^2 \simeq 282$ . Then, it is expected that the new electroweak contributions may appear in the AMM of heavier leptons, but this also depends on the precision of the measurements.

The theoretical prediction of the tau AMM was first addressed taking into account higher order QED corrections and electroweak and hadronic corrections as well in ref.[14]. Latter, the computation was updated using more data and new orders and terms in ref.[11], so that they obtained the result quoted in Eq.(3).

In the quantum field theory framework the definition of the magnetic moment and the AMM can be written in several ways. For example, a lagrangian term can be written that reproduces the correct magnetic tensor contributions to the matrix elements. These are originated from loop corrections to the tree level lagrangian of the theory. However, in this approach the particles are implicitly assumed to be on their mass-shell and the coefficients of the lagrangian (i.e., the AMM times some constants) should not depend on the scale of the process. This can be understood as an effective lagrangian approach [15]. Then, necessarily this approach can not have absorptive parts. The magnetic moments and AMM are gauge invariant quantities and for all the particles on-shell are physical quantities. As long as this on-shell condition is relaxed the situation may change, as was investigated for the electroweak theory in ref.[16]. A more precise and general approach in order to define the AMM is just to resort to the on-shell matrix elements, using all the symmetries that are appropriate for the case. Then, one ends with a matrix element with a few terms that are gauge invariant and represent physical properties of the particles. We then define the AMM as the coefficient of a particular term in the Lorentz tensor structure of the matrix element.

The most general Lorentz invariant structure describing the interaction of an on-shell vector boson  $V$  with two on-shell fermions  $f\bar{f}$  can be written in terms of six form factors:

$$\begin{aligned} \langle f(p_-)\bar{f}(p_+)|J^\mu(0)|0\rangle = e\bar{u}(p_-)\Big[(F_1 + F_4\gamma_5)\gamma^\mu \\ + \frac{1}{2m_f}(iF_2 + F_3\gamma_5)\sigma^{\mu\nu}q_\nu + \frac{1}{2m_f}(iF_5 + F_6\gamma_5)q^\mu\Big]v(p_+) \end{aligned} \quad (5)$$

where  $q = p_+ + p_-$ . Since the two fermions are on-shell the form factors  $F_i$  appearing in Eq. (5) are functions of  $q^2$  and  $m_f^2$  only.

In addition, if the current  $J^\mu$  is conserved, we must have

$$i\frac{q^2}{2m_f}F_5 + \left(\frac{q^2}{2m_f}F_6 - 2m_fF_4\right)\gamma_5 = 0 \quad \Rightarrow \quad \begin{cases} F_5 = 0 \\ F_6 = \frac{4m_f^2}{q^2}F_4 \end{cases} \quad (6)$$

so that the final expression for the gauge invariant  $f\bar{f}\gamma$  vertex reduces to:

$$\begin{aligned} \langle f(p_-)\bar{f}(p_+)|J^\mu(0)|0\rangle = \\ e\bar{u}(p_-)\left[\gamma^\mu F_1 + \frac{1}{2m_f}(iF_2 + F_3\gamma_5)\sigma^{\mu\nu}q_\nu + (q^2\gamma^\mu - q^\mu \not{q})\gamma_5 F_A\right]v(p_+) \end{aligned} \quad (7)$$

In this expression,  $F_1$  parametrizes the vector part of the electromagnetic current ( $F_1(0) = 1$ ), and it is identified with the charge,  $F_A = -F_4/q^2$  is the so-called anapole moment,  $F_3$  parametrize the electric dipole moment

$$d_f = \frac{e}{2m_f} F_3(0), \quad (8)$$

while  $F_2(0)$  defines the AMM:

$$F_2(0) = a_f; \quad \mu_f = (1 + a_f) \frac{e}{2m_f} \quad (9)$$

Note that this is true for particles on-shell, while for off-shell particles usually the matrix elements (or the effective vertex) contains many form factors, depending on  $q^2$  and all other possible scalar invariants, and besides can be not gauge-invariant, as it is well known [17]. Similar expression can be written for other vector bosons, such as the  $Z$ , where the weak magnetic moments are now well defined at the  $Z$ -peak, i.e. at  $q^2 = M_Z^2$ . In this case the magnetic moment develops also an absorptive part that can also be measured using suitable observables.

The AMM that is measured in the experiments contains all the contributions of the well established standard model and possible new physics. This last contributions are in general suppressed by the scale of the new physics, typically  $m_f^2/\Lambda^2$ . Then, these contributions are enhanced for high mass particles and promotes the AMM as a probe for new physics.

The tensor structure of the magnetic terms flips chirality, and in the standard model fermion masses are the only source of chirality flips, so one should expect that the AMM appears proportional to mass. Besides, observables that are exactly zero when chirality is conserved are the best candidates in order to measure magnetic moments. These observables only be sensitive to fermion masses and magnetic moments. Furthermore, they will depend linearly on magnetic moments. A measurement using observables of this kind is what one should call a direct observable related to the AMM.

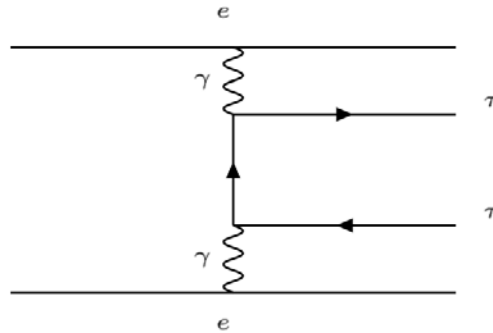
### 3. Experiments

The main new ingredient for the tau AMM is its short mean life, that prevents to perform similar experiments and ideas that were successful for other leptons. Then, the AMM should be searched in the tau decay products, using the several channels that are open for this lepton. In particular, this can be done by means of the energy distribution of some of the these particles [26]. The properties of the weak interaction decays allow to study the AMM using both linear and correlations polarizations of the taus as spin analyzers. Then, the AMM propagates in the linear polarizations and correlations modifying the angular distribution of these decaying particles. This can be useful for both the electromagnetic AMM and also for the weak AMM [27, 28, 29, 30]. In this way some direct observables can be identified and investigated, that are closely related to the anomalous magnetic moments. Besides, other indirect observables, such as the ones coming from total cross sections can also give partial information and bounds on the AMM.

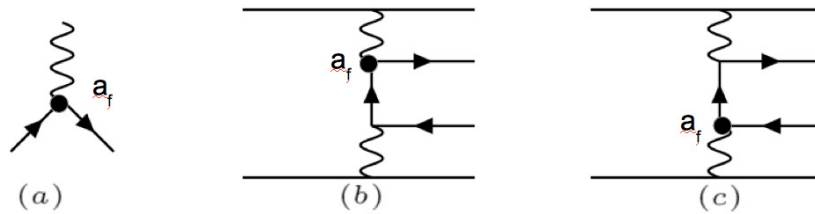
The current PDG limit was obtained by DELPHI [12] using data from the total cross section  $e^+e^- \rightarrow e^+e^-\tau^+\tau^-$  at LEP2, where they selected 2390 events at energies in the range  $183 \text{ GeV} < \sqrt{s} < 208 \text{ GeV}$ . The resulting bound Eq.(2) can also be expressed as:

$$a_\tau = 0.018 \pm 0.017 \quad (10)$$

to be compared with the theoretical prediction in the standard model Eq.(3). Clearly the experiment is far from determining the anomaly for the tau in the standard model. The leading Feynman diagram for this process is shown in Fig.1. The interference of this diagram with the



**Figure 1.** Feynman diagram for the process in DELPHI at LEP2.



**Figure 2.** (a) AMM effective vertex insertion; (b) and (c) Feynman diagram with and AMM insertion that interferes with the diagram in Fig.1 for the process in DELPHI at LEP2.

two similar diagrams but with an insertion of the AMM in the  $\tau\tau\gamma$  vertex, shown in Fig.2, produces contributions in the total cross section that are proportional to two chirality flipping terms, such as AMM times a mass or the squared value of the AMM. These ideas were studied by the authors of ref.[18]. Note that two particles in the effective vertex in diagrams (b) and (c) in Fig.2 are off-shell: the photon and also one tau. This means that the question of gauge invariance should be addressed and, besides, this effective vertex depends on the two scalar quantities that the vertex depends on. The bound was obtained in the range of energies quoted above, and in each of the events the scalar variables in the vertex are different. Furthermore, the effective AMM vertex is not the only one that can be written for this vertex, in which two of the three lines are off-shell, and several form factor can be associated with the vertex (see for example ref.[17]). Related to these topics, it is appropriate to ask oneself if just a bound or a measurement of the (on-shell) AMM predicted in the standard model would ever be found with this kind of observables. Even in the hypothesis of a very high statistics, will the observables allow to determine any of the figures in Eq.(3)? The answer is no, no matter how high the statistics would be. In any case, this result is just a bound, under the hypothesis that all the other form factors do not contribute. This bound is still sensible as long as it is not close to the one loop standard model contributions to the form factors of the vertex; in particular, for the AMM. Then, this bound may be interpreted as a bound on new physics contributions to the AMM, under the hypothesis mentioned before.

Other ideas related to the tau AMM were also investigated, but they were not taken into account by the PDG. Some of them share the same kind of problems in order to interpret their meaning. The authors of ref.[20] explored the total cross section  $e^+e^- \rightarrow \tau^+\tau^-$  at PETRA using data with  $q^2 < (37 \text{ GeV})^2$  looking for bounds on the AMM and also for a scale of compositeness. They found a limit  $|a_\tau| < 0.02$ ; in ref.[34, 35] bounds were investigated from the radiative decay

$Z \rightarrow \tau^+ \tau^- \gamma$  at LEP1, where only the anomalous coupling  $a_\gamma$  was taken into account and the contributions coming from the tau  $Z$ -magnetic coupling  $a_Z$  was neglected. Using this approach, with the inclusion of only linear terms in  $a_\gamma$  in the cross section (were the  $\tau$  which emits the photon is off-shell) the analysis of the L3 Collaboration [23] obtained

$$-0.056 < a_\tau < 0.044 \quad (95\% C.L.) \quad (11)$$

while the OPAL Collaboration [24] obtained the bound:

$$-0.068 < a_\tau < 0.065 \quad (95\% C.L.) \quad (12)$$

In ref.[21, 22] they explored the bounds one can obtain from the process  $e^+e^- \rightarrow \tau^+\tau^-\gamma$  that was latter measured at L3 [23]; in ref.[18, 19] they studied in  $\gamma\gamma \rightarrow \tau^+\tau^-$  the limit that can be found in  $\gamma\gamma$  colliders and in hadron colliders such as the LHC; in ref.[32, 33] they investigated the bounds at the  $Z$ -peak in the channel  $Z \rightarrow \tau^+\tau^-$  using an effective lagrangian approach and without taking into account the weak magnetic moments. In [25] they recently explored the radiative decay of the Higgs boson  $H \rightarrow \tau^+\tau^-\gamma$ . Some of these authors and experiments also investigated the bounds one could get from the same (CP-even) observables on the tau electric dipole moment. Recently, the possibility of obtaining bounds for the AMM in B-factories was also addressed in ref.[37]. Here, they studied the possibility of obtaining bounds for the tau AMM and also for the electric dipole moment using a leptonic radiative decay of the tau lepton, with a photon in the final state. The vertex in which this photon participates may have information associated to the AMM. They concluded that a sensitivity of the order of 0.012 could be obtained with the planned full set of Belle II data. This bound is below (but close) to the DELPHI result, 0.017, already mentioned.

We have also investigated an approach to the problem that took the effective lagrangian approach fully into account in ref.[31]. There, a direct bound on the electromagnetic and weak AMM were obtained on general grounds and in a model independent way. These bounds came from the observation that in general extensions of the standard model it is very difficult to generate a magnetic moment for a lepton without originating a coupling of the  $Z$  boson to the lepton, of the same order of magnitude. Then, as the last is strongly bounded by LEP, one may obtain a rather strong bound on the  $\gamma$ -magnetic moment. A complete analysis of the magnetic moment couplings of the tau to the photon, the  $Z$  and the  $W$  bosons, using the complete amount of data coming from tau-lepton production at LEP1, SLD and LEP2, and data on  $W$  decays into tau leptons from LEP2 and pp collider, was done. LEP1 and SLD are more sensitive the weak magnetic coupling, while LEP2 is sensitive to both the electromagnetic and weak AMM, and data from  $W$  decays into tau in colliders provide some bounds on the magnetic  $W\tau\nu_\tau$  coupling.

The effective Lagrangian considered is,

$$\mathcal{L}_{eff} = \alpha_B \mathcal{O}_B + \alpha_W \mathcal{O}_W + \text{h.c.} , \quad (13)$$

where the couplings  $\alpha_B$  and  $\alpha_W$  real (note that complex couplings will break  $CP$  conservation and would lead to the electric and weak-electric dipole moments). The two operators in Eq.(13) are:

$$\mathcal{O}_B = \frac{g'}{2\Lambda^2} \overline{L}_L \varphi \sigma_{\mu\nu} \tau_R B^{\mu\nu} , \quad (14)$$

and

$$\mathcal{O}_W = \frac{g}{2\Lambda^2} \overline{L}_L \vec{\tau} \varphi \sigma_{\mu\nu} \tau_R \vec{W}^{\mu\nu} . \quad (15)$$

Here  $L_L = (\nu_L, \tau_L)$  is the tau leptonic doublet,  $\varphi$  is the Higgs doublet,  $B^{\mu\nu}$  and  $\vec{W}^{\mu\nu}$  are the  $U(1)_Y$  and  $SU(2)_L$  field strength tensors, and  $g'$  and  $g$  are the gauge couplings. After spontaneous



symmetry breaking, the Higgs gets a vacuum expectation value and the interactions in Eq.(13) can be written in terms of the gauge boson mass eigenstates,  $A^\mu$  and  $Z^\mu$ , as

$$\begin{aligned} \mathcal{L}_{eff} = & \epsilon_\gamma \frac{e}{2m_Z} \bar{\tau} \sigma_{\mu\nu} \tau F^{\mu\nu} + \epsilon_Z \frac{e}{2m_Z s_W c_W} \bar{\tau} \sigma_{\mu\nu} \tau Z^{\mu\nu} \\ & + \left( \epsilon_W \frac{e}{2m_Z s_W} \bar{\nu}_{\tau L} \sigma_{\mu\nu} \tau_R W_+^{\mu\nu} + \text{h.c.} \right), \end{aligned} \quad (16)$$

where  $F_{\mu\nu}$  is the electromagnetic field strength tensor,  $Z_{\mu\nu} = \partial_\mu Z_\nu - \partial_\nu Z_\mu$  and  $W_+^{\mu\nu} = \partial_\mu W_\nu^+ - \partial_\nu W_\mu^+$  are the corresponding strength tensors for the  $Z$  and  $W$  gauge bosons. We have not written the non-abelian couplings involving more than one gauge boson because they are not relevant to our purposes.

The Lagrangian (16) gives additional contributions to the magnetic moments of the tau, which are usually expressed in terms of  $a_\tau$  and similar parameters for the corresponding weak magnetic moments for the  $Z$ -boson,  $a_Z$  [27, 28, 29] and the  $W$ -boson,  $\kappa^W$  [26]. They can be expressed as follows,

$$a_\tau = \frac{2m_\tau}{m_Z} \epsilon_\gamma, \quad (17)$$

$$a_Z = \frac{2m_\tau}{m_Z} \frac{1}{s_W c_W} \epsilon_Z, \quad (18)$$

$$\kappa^W = \sqrt{2} \frac{2m_\tau}{m_Z} \epsilon_W. \quad (19)$$

where the dimensionless couplings are

$$\epsilon_\gamma = (\alpha_B - \alpha_W) \frac{um_Z}{\sqrt{2}\Lambda^2}, \quad (20)$$

$$\epsilon_Z = -(\alpha_W c_W^2 + \alpha_B s_W^2) \frac{um_Z}{\sqrt{2}\Lambda^2}, \quad (21)$$

$$\epsilon_W = \alpha_W \frac{um_Z}{\Lambda^2} = -\sqrt{2} (\epsilon_Z + s_W^2 \epsilon_\gamma). \quad (22)$$

One remarkable result from this effective lagrangian approach is that the three magnetic moments, namely the electromagnetic one  $a_\tau$ , the weak magnetic  $a_Z$  and the non-diagonal weak  $W$ -boson coupling,  $\kappa^W$ , appear as a result of just the two effective operators in Eqs.(14,15), and only depend on the two coefficients  $\alpha_B$  and  $\alpha_W$ . Note that the three magnetic moments are not independent of each other. Besides, in the effective Lagrangian approach, exactly the same couplings that contribute to processes at high energies also contribute to the magnetic moment form factors,  $F^{new}(q^2)$ , at  $q^2 = 0$ . The difference  $F^{new}(q^2) - F^{new}(0)$  only comes from higher dimension operators whose effect is suppressed by powers of  $q^2/\Lambda^2$ , as long as  $q^2 \ll \Lambda^2$ , as needed for the consistence of the effective Lagrangian approach.

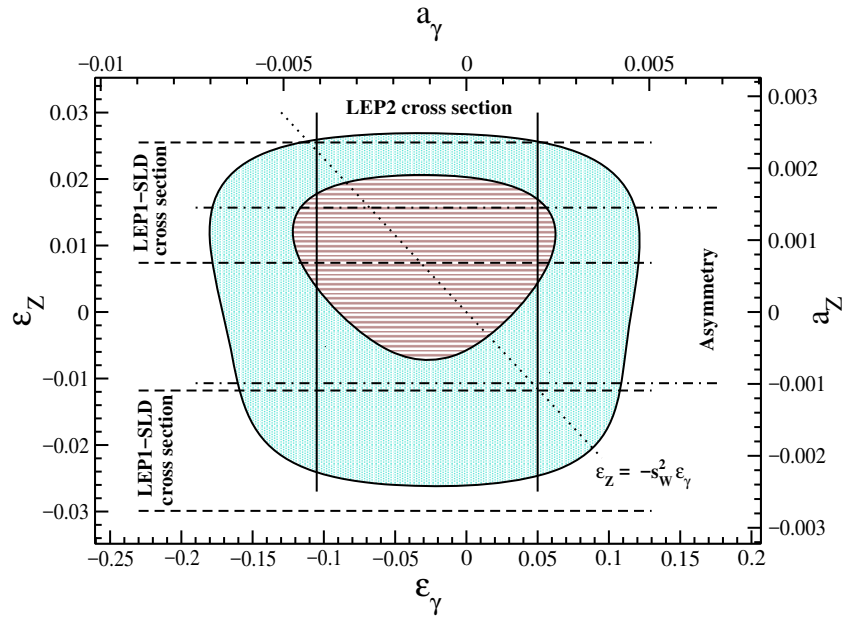
Combining the data cited above (see details in [31]) we obtained the contour plots of Fig.3 and Fig.4.

These two figures represent the same fit, but just expressed in terms of different parameters. Projecting into the axes, one can find bounds to the non-standard contributions to the anomalous electromagnetic and weak magnetic moments  $a_\tau$ ,  $a_Z$ :

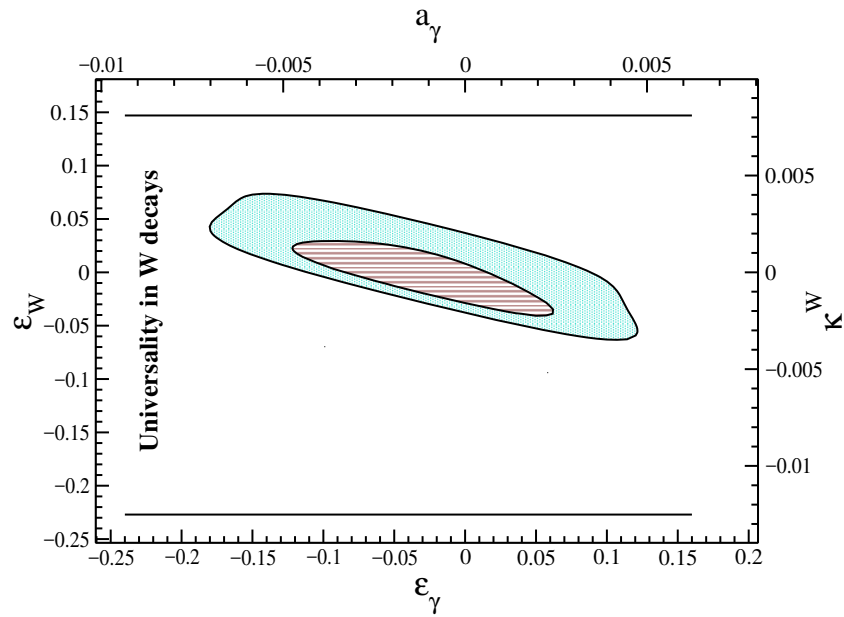
$$(1\sigma) \rightarrow \begin{cases} -0.005 < a_\tau < 0.002, \\ -0.0007 < a_Z < 0.0019, \end{cases} \quad (23)$$

$$(2\sigma) \rightarrow \begin{cases} -0.007 < a_\tau < 0.005, \\ -0.0024 < a_Z < 0.0025. \end{cases} \quad (24)$$





**Figure 3.** Global fit including all constraints discussed in ref.[31]. 95% CL and 68% CL contours are shown. The bands between straight lines show the allowed regions ( $1\sigma$ ) coming from the different experiments: solid (LEP2-189 GeV), dashed (LEP1-SLD cross section), dot-dashed (asymmetry). We also have plotted the line  $\epsilon_Z = -s_W^2 \epsilon_\gamma$  (dotted line) that corresponds to the results of refs.[32, 33]. This relationship appears when only the operator  $\mathcal{O}_B$  contributes.



**Figure 4.** The global fit of Fig. 3 is now plotted in the plane  $\epsilon_\gamma$  and  $\epsilon_W$  to show the combined bounds on  $\epsilon_W$ . As in Fig. 3 95% CL and 68% CL contours are shown. For comparison we also draw as straight lines the direct  $1\sigma$  bounds obtained from universality tests in  $W$  decays.

For  $a_\gamma$  these limits are only about one order of magnitude larger than the standard model contribution in Eq.(3) and well below the PDG bound.

Using the relationship among  $\epsilon_\gamma$ ,  $\epsilon_Z$ ,  $\alpha_B$  and  $\alpha_W$  at a given value of the scale of new physics, one can easily obtain bounds on  $\alpha_B$  and  $\alpha_W$ . Alternatively, by assuming that  $\alpha_B/4\pi$  or  $\alpha_W/4\pi$  are order unity one can obtain bounds on the scale of new physics  $\Lambda$  ( $\Lambda > 9$  TeV).

The above bounds are completely model independent and no assumption has been made on the relative size of couplings  $\alpha_B$  and  $\alpha_W$  in the effective Lagrangian (13). For the sake of comparison with published data in ref.[32, 33] we present now the limits that can be found by considering separately only operator  $\mathcal{O}_B$  or only operator  $\mathcal{O}_W$  in the Lagrangian (13). Consider that only  $\mathcal{O}_B$  is present, as in Ref. [32, 33], it is equivalent to impose the relation  $\epsilon_Z = -s_W^2 \epsilon_\gamma$ . Thus, from Fig. 3, it is straightforward to obtain that the bounds on the anomalous magnetic moment (at  $2\sigma$ ) are reduced to  $-0.004 < a_\tau < 0.003$ , while little change is found on the weak-magnetic moment  $-0.0019 < a_Z < 0.0024$ .

In most of the above mentioned experiments some of the particles in the vertex are off-shell. The interpretation of off-shell form factors is then problematic since they can hardly be isolated from other contributions and gauge invariance can be a problem. In the effective Lagrangian approach all those problems are solved because form factors are directly related to couplings, which are gauge invariant. Besides, as already mentioned, the difference  $F^{new}(q^2) - F^{new}(0)$  only comes from higher dimension operators whose effect is suppressed by  $q^2/\Lambda^2$ . In this way a stringent and clear bound on the AMM was obtained.

More recently, some ideas were investigated having in mind the potential of the high statistics (in tau pairs) B-factories, close to  $10^{12}$ . They now have by far the highest statistics of tau pairs, as it is the case for BaBar and Belle, while BELLE-II soon will also produce a huge number of tau pairs. Some time ago, the SuperB [36] project considered the possibility of vertex detectors and polarized electron beams, but the project was finally canceled.

We investigated the potential of the B-factories in order to get bounds on the tau AMM in ref.[29, 30]. There, the taus are produced  $q^2 \simeq (10 \text{ GeV})^2$ , and, then, the problem of gauge invariance also appears. Tau pairs are obtained by both resonant and direct production. Cross section and asymmetries related to the linear and spin-spin correlations were studied on top of the  $\Upsilon$  resonances. In particular, total cross section and the normal  $\tau$  polarization -for the case of unpolarized electron beams- and the transverse and longitudinal ones -for the electron polarized beams- were investigated.

When attempting to extract the value of  $F_2(0)$  from scattering experiments there are additional contributions of other Feynman diagrams, not related to the magnetic form factor. In the case of  $e^+e^- \rightarrow \tau^+\tau^-$  there are contributions not only from the usual  $s$ -channel one-loop vertex corrections but also from box diagrams. The contributions of the latter may interfere in the experimental determination of what we call  $F_2(q^2)$ , i.e. the magnetic part coming *only* from the vertex, and should be somehow “subtracted out”. This may be done either by computing the box contributions and subtracting them from the cross-section, or by performing the measurement in a kinematic region where the boxes happen to be numerically subleading, i.e. on top of the  $\Upsilon$  resonances. In this kinematic regime the non-resonant box diagrams are numerically negligible, and only one loop corrections to the  $\gamma f \bar{f}$  vertex are relevant.

The direct computation of the magnetic part of the standard one-loop QED vertex yields

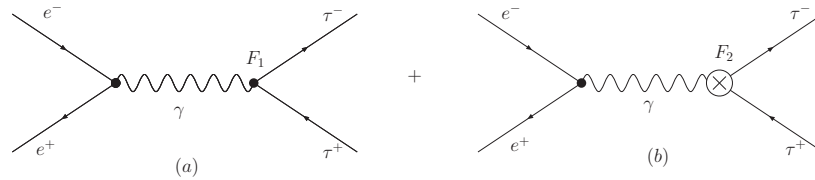
$$F_2(s) = \left(\frac{\alpha}{2\pi}\right) \frac{2m_\tau^2}{s} \frac{1}{\beta} \left(\log \frac{1+\beta}{1-\beta} - i\pi\right), \quad \text{for } q^2 = s > 4m_\tau^2, \quad (25)$$

where  $\alpha$  is the fine structure constant and  $\beta = (1 - 4m_\tau^2/s)^{1/2}$  is the velocity of the  $\tau$ . For  $M_\Upsilon \sim 10 \text{ GeV}$ , we obtained:

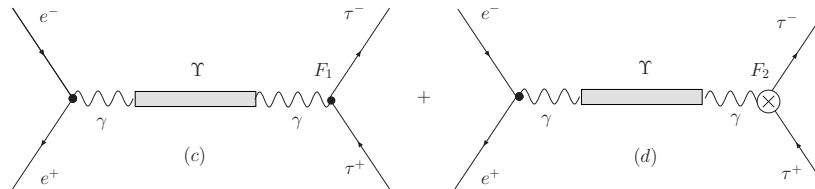
$$F_2(M_\Upsilon^2) = (2.65 - 2.45i) \times 10^{-4}. \quad (26)$$

Note that at this energy the real and imaginary parts are of the same order of magnitude. The above expression for  $F_2(s)$  is a QED gauge-independent quantity, despite being an off-shell amplitude. This fact may be verified through an explicit calculation of the vertex diagram. The gauge-independence of  $F_2(s)$  may also be understood in terms of the way the gauge-cancellations organize themselves in the QED  $S$ -matrix elements: first, the vacuum polarization is gauge-independent by itself; the gauge-dependence of the direct box cancels exactly against that of the crossed box and finally the gauge-dependence of the vertex correction can therefore cancel only against the fermion self-energy graphs renormalizing the external (on-shell) fermions. The latter however are proportional to  $\gamma_\mu$ . Therefore the contribution of the vertex proportional to  $\sigma_{\mu\nu}q^\nu$  must be individually gauge invariant. With these ideas in mind we investigated the QED gauge invariant magnetic tau form factor that can be accessed at B-factories.

The direct production mechanism is shown in Fig.3 while the resonant one is depicted in Fig.3. The interference of the diagrams shown in each figure gives the leading contribution to terms in  $F_2(q^2)$ . In this way, an expression for the differential and total cross section can be



**Figure 5.** Diagrams: (a) direct  $\gamma$  exchange, (b)  $F_2$  in  $\gamma$  exchange.

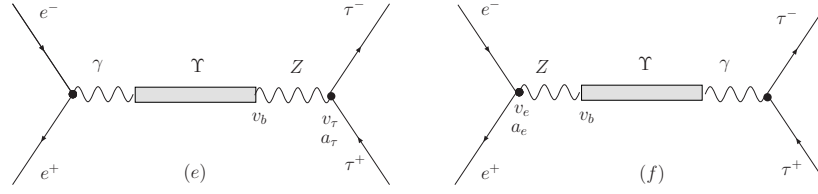


**Figure 6.** Diagrams: (c)  $\Upsilon$  production, (d)  $F_2$  in  $\Upsilon$  production.

computed, where only the real part of  $F_2$  appears. The normal polarization instead, can be obtained through an asymmetry constructed with the angle distribution of the decay products of the tau (see details in ref.[29, 30]), and results proportional to the imaginary part of  $F_2$ . Using polarized electron beams more possibilities are allowed that will not be discussed here.

Also,  $Z$  contamination through the interference with the resonant and direct diagrams shown in Fig.3 can be properly addressed: it is either very suppressed or it does not contribute (for example to the normal polarization asymmetry). Table 1 shows the upper limit to the sensitivity to the real and imaginary parts of  $F_2$  assuming different integrated luminosities for BaBar, SuperB and Belle. No detector efficiencies are taken into account and only statistical errors are considered.

Other observables related to spin-spin correlations give sensitivities similar to the ones shown here, and can be found in ref.[30]. Note that some of these observables depend on the possibility of having polarized electron beams. In ref.[37] the authors discussed the energy resolution of the beams that may limit the possibility of disentangling the direct and resonant productions, and also the fact that the beam energy at Belle II will be far from some of the  $\Upsilon$  resonances. All these facts may have some impact in these ideas.



**Figure 7.**  $\gamma$  and  $Z$  interchange on  $\Upsilon$  production.

**Table 1.** Sensitivity of the  $F_2$  measurement at the  $\Upsilon$  energy ( $ab = \text{attobarn} = 10^{-18}b$ ).  
\* means polarized electron beam required.

EXPERIMENT	O B S E R V A B L E		
	Cross Section	Normal Asymmetry	Transverse and Longitudinal Asymmetry combined*
	$Re(F_2)$	$Im(F_2)$	$Re(F_2)$
$\Downarrow$ Babar+Belle $2ab^{-1}$	$4.6 \times 10^{-6}$	$2.1 \times 10^{-5}$	$1.0 \times 10^{-5}$
Super B/Flavor Factory (1 yr. running) $15ab^{-1}$	$1.7 \times 10^{-6}$	$7.8 \times 10^{-6}$	$3.7 \times 10^{-6}$
Super B/Flavor Factory (5 yrs. running) $15ab^{-1}$	$7.5 \times 10^{-7}$	$3.5 \times 10^{-6}$	$1.7 \times 10^{-6}$

This approach shows that a high sensitivity and several figures in the magnetic properties of the tau lepton can be obtained using these ideas.

#### 4. Conclusions

We have review the main experiments and theoretical studies related to the tau AMM. The interpretation of these results, as well of the appropriate way to express the physics in the different formalisms, were discussed. The tau AMM is still a largely unknown quantity, despite the fact that many of the tau electroweak properties are well known [38]. It appears to be that we are far from the time where the tau AMM could play a role similar to the one that the AMM for the electron and for the muon have played. However, tau physics, being the tau such a high mass lepton, is a wonderful scenario where new physics may show up. More efforts should be addressed to this physics and there is a great opportunity for the new Belle II experiment for discoveries.

##### 4.1. Acknowledgements

This work has been supported, in part, by the Ministerio de Economía y Competitividad (MINECO), Spain, under grants FPA2014-54459-P and SEV-2014-0398; by Generalitat Valenciana, Spain, under grant PROMETEOII2014-087. G.A.G-S. acknowledges the support of CSIC and Pedeciba, Uruguay.

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