ISBN: 978-3-95450-231-8 ISSN: 2673-5490 doi: 10.18429/JACoW-IPAC2023-TUPA093

EFFICIENT SIMULATION OF MULTISTAGE PLASMA ACCELERATORS

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Abstract

Plasma accelerators can sustain accelerating gradients of up to ~100 GeV m^{-1}. However, reaching the high energies required for future particle colliders requires the acceleration to be performed in multiple plasma stages. Solving the challenges posed by multistage acceleration, such a beam quality preservation, requires the capability of simulating large chains of accelerating stages, something that is typically limited by the high cost of full 3D particle-in-cell codes. Thus, there is a growing need for the development of more efficient models that allow for inexpensive collider studies with reduced physics or dimensionality. Here, we present the implementation of a novel gridless quasistatic algorithm in the Wake-T code that, coupled with a laser envelope solver, allows for accurate and efficient simulations of multistage laser-plasma accelerators with axial symmetry, a critical step toward their realization.

INTRODUCTION

The recent Accelerator R&D Roadmap of the European Strategy for Particle Physics Update [1] and the Snowmass Accelerator Frontier Report [2] highlight the potential of plasma-based acceleration technology for the realization of a future compact multi-TeV collider. However, despite its ability to sustain ultra-high GeV m^{-1} gradients, many challenges remain regarding the interstage coupling, wall-plug efficiency, beam quality, repetition rate, etc.

In order to develop new solutions to these issues, simulation studies are essential. However, this also presents significant challenges, in particular in terms of computational cost. Performing 3D simulations with electromagnetic particle-in-cell (PIC) codes typically requires several hours or days of computing time, even on high-performance systems. The large number of plasma acceleration stages in a collider, coupled with the need to resolve beams with nm-level emittance, drastically increases the cost of the simulations. Thus, there is a growing interest in the development of reduced physical models that allow for a drastic cost reduction while retaining the most relevant collider physics.

Here, we show that a novel gridless plasma simulation method [3] coupled with a laser envelope solver [4] can allow for efficient and accurate simulations of multistage laser-plasma accelerators that capture the relevant physics in axial symmetry and can be carried out in minutes, even on a conventional laptop or desktop computer. These numerical schemes have been recently implemented in the Wake-T code [5], which is used here to simulate a proof-of-principle collider setup with 20 acceleration stages.

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NUMERICAL METHODS

Wake-T is a particle tracking code specifically designed for plasma accelerators. It allows the simulation of plasma stages together with other beamline components such as conventional magnets (dipoles, quadrupoles and sextupoles) as well as plasma lenses. The 6D evolution of the particle beams is determined either by a Boris [6] or Runge-Kutta integrator, or by transport matrices (in the case of conventional magnets). In the plasma stages, the fields are computed using a gridless quasistatic method with axial symmetry [3]. This method obtains the quasistatic plasma wakefields by calculating a series of radial sums over the plasma particles, instead of relying on a numerical grid. Coupled with a laser envelope solver [4], this method allows for highly efficient simulations of both laser- and beam-driven accelerators that can be carried out in seconds to minutes on a single CPU core.

The reduced cost of the simulations makes Wake-T a useful tool for the design of novel plasma acceleration concepts [7] and for large optimization studies [8] in cases where the plasma response can be assumed axisymmetric. Thus, it is in principle well suited for the study of multistage beamlines for plasma-based particle colliders.

MULTISTAGE SIMULATION STUDY

The suitability of Wake-T for performing cost-effective simulations of collider setups is studied here using the same multistage configuration that has been recently simulated [9] with the 3D electromagnetic PIC code WarpX [10].

The setup consists of 20 laser-plasma acceleration stages where active plasma lenses (APLs) are used for the beam transport. The stages have a length of 28 cm and assume a flat-top longitudinal plasma profile with an on-axis density $n_0 = 1.7 \times 10^{17}\,\mathrm{cm}^{-3}$ and a radial density distribution for laser guiding given by $n(r) = n_0 + r^2/(\pi r_e w_m^4)$, where $w_m = 40\,\mu\mathrm{m}$ is the matched spot size of the laser if relativistic self-focusing effects are neglected, and r_e is the classical electron radius. The stages are separated from each other by 3 cm with an APL in between. The laser driver has a peak normalized vector potential $a_0 = 2.35$, a FWHM duration $\tau_{\mathrm{FWHM}} = 86.4$ fs, a spot size $w_0 = 36\,\mu\mathrm{m}$ and a wavelength

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$\lambda_0 = 0.8\,\mu\text{m}$. The electron beam is externally injected 63 μm behind the center of the driver, and consists of an idealized test distribution with an initial energy of 1 GeV, a normalized emittance $\epsilon_x = \epsilon_y = 1\,\mu\text{m}$, a length $\sigma_z = 0.1\,\mu\text{m}$ and a charge of 1 fC.

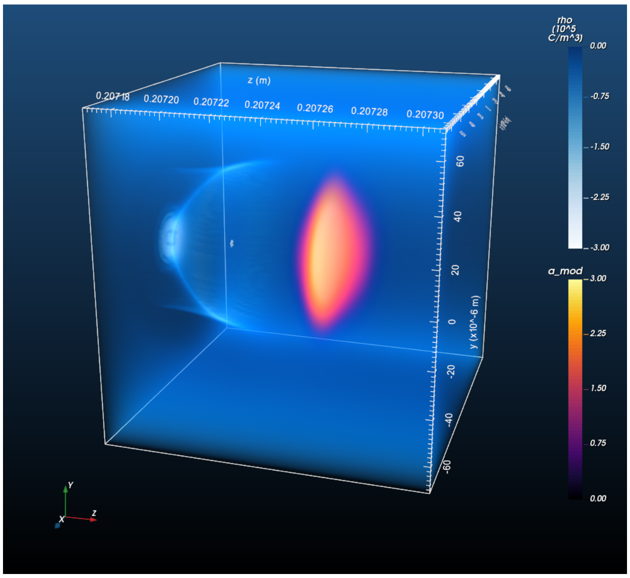

Figure 1: View of the plasma wake (rho), the laser pulse (a_mod) and the electron beam in the first plasma stage as obtained from Wake-T. Visualized with VisualPIC [11].

The simulations consider a plasma that extends up to 100 μm radially and that is discretized into 786 particles. The grid of the plasma (only used to interpolate the field into the particle beam) has a resolution of $\Delta_r = 1\,\mu\text{m}$ radially and $\Delta_z = c\tau_{\text{FWHM}}/40$ longitudinally, where c is the speed of light. This results in a grid with $N_r = 393$ and $N_z = 185$ elements. The plasma wakefields are updated every time the simulation box advances by $\Delta_{z,\text{fields}} = 140\,\mu\text{m}$, which corresponds to the longitudinal size of the box. The laser envelope is initialized in a box with the same physical dimensions but with a finer longitudinal resolution of $\Delta_z/4$ and a time step $\Delta_t = \Delta_{z,\text{fields}}/(4c)$. A view of the resulting plasma wake in the first stage as obtained with these simulation parameters can be seen in Fig. 1. The simulation of each stage takes only about 90 s using a single CPU core on a conventional laptop.

One of the challenges of this multistage setup is to preserve the beam emittance. This requires precise matching of the Courant-Snyder parameters of the beam [12], α_x, β_x and γ_x, which are assumed to be the same in both transverse planes. Two different approaches to realize this have been tested: one by using an analytical formula to determine the length L_{APL} and focusing strength k_{APL} of the APL, and another by performing Bayesian optimization to obtain the best combination of k_{APL} and the length of the drift L_d between the APL and the following plasma stage.

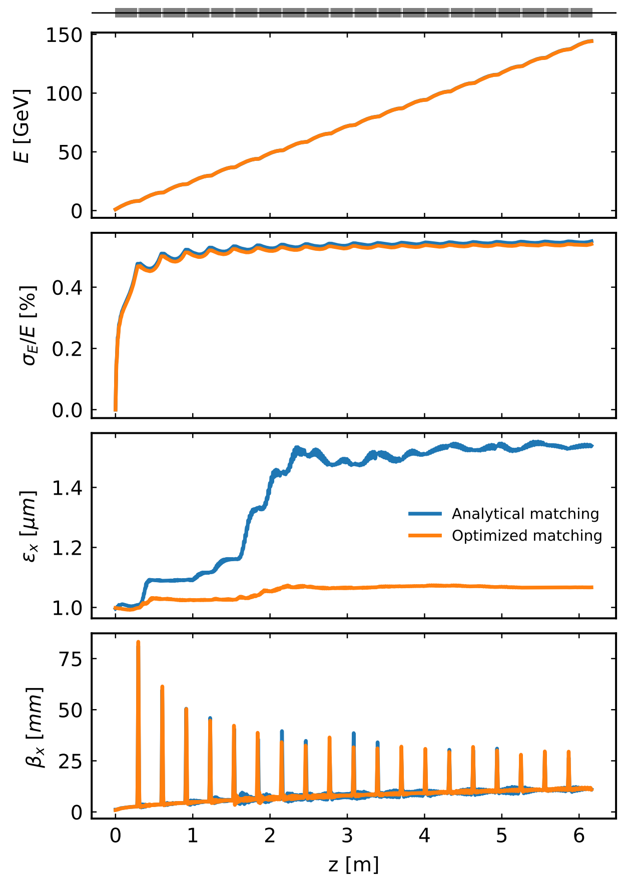

Figure 2: Evolution of the beam parameters along the 20 acceleration stages, comparing the case where the beam transport is determined analytically with the case where it is optimized live during the simulation. (a) Beam energy, (b) energy spread, (c) emittance and (d) beta function.

The first approach uses the formula

$$L_{\text{APL}} = \frac{1}{k_{\text{APL}}^{1/2}} \arctan\left(\frac{-2\alpha_{x,0}}{\beta_{x,0} k_{\text{APL}}^{1/2} + \gamma_{x,0}/k_{\text{APL}}^{1/2}} \right) \quad (1)$$

to determine the APL length needed to invert the sign of $\alpha_{x,0}$, where $\alpha_{x,0}$, $\beta_{x,0}$ and $\gamma_{x,0}$ are the Courant-Snyder parameters at the entrance of the APL. Eq. (1) is derived from the expression of a thick focusing lens and, by inverting the value of $\alpha_{x,0}$, makes sure that the electron beam arrives to the following stage with the same transverse properties that it had at the exit of the previous stage. Thus, assuming that the focusing fields at the entrance and exit of the plasma stages are the same, and that the beam has no energy spread, this would lead to perfect matching at the entrance of every stage. However, this is not the case in practice due to the strong evolution of the laser, which changes the properties of the wake, and the finite energy spread of the beam. Thus, as seen in Fig. 2, the emittance grows by close to 60 % at the end of the accelerator chain. A positive outcome is that emittance growth appears to saturate at this value, once the transverse profile of the beam reaches an equilibrium with

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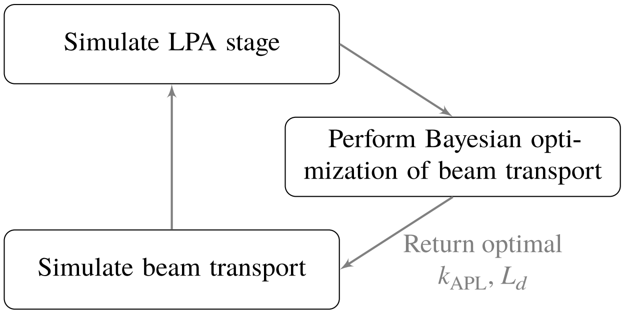

Figure 3: Workflow of the multistage simulation where, after each plasma stage, an optimization is launched to determine the best configuration of the beam transport, thus minimizing emittance growth.

the periodic focusing of the beamline. The final beam energy is 140 GeV.

Another approach that can lead to improved matching is to perform an optimization of the beam transport within the simulation itself after each plasma stage. This would allow us to find the optimal settings of the APL without relying on the assumptions behind Eq. (1). Since Wake-T is fully written in Python, it can be easily coupled with external libraries for additional features. In particular, we have used the OPTIMAS library [8, 13], which leverages the LIBENSEMBLE [14] and Ax [15] packages to perform parallel Bayesian optimization, to determine the optimal beam transport within the main simulation. After every plasma stage, the main Wake-T script launches an optimization consisting of 5 batches of 32 parallel simulations with the objective of minimizing the emittance growth during the following stage. For this, the optimizer can tune both k_{APL} and L_d, assuming that $L_{\mathrm{APL}} = 3$ mm. Once the optimization is done, the best configuration is returned to the main Wake-T script and the simulation continues. This process, which is illustrated in Fig. 3, continues until the last stage is reached. This approach, although more computationally intensive due to the large amount of simulations needed for the optimization, results in an optimal tuning of the accelerator that limits the emittance growth to a mere 6 %.

CONCLUSION

Using the parameters of the 20-stage study presented in Ref. [9], Wake-T has demonstrated the capability of performing efficient collider-relevant simulations where the plasma response can be assumed axisymmetric, requiring only ~90 s per stage on a single CPU core. In addition to multistage studies, the code can be easily coupled with external libraries to perform live optimization within the main simulation. Here, this has allowed us to demonstrate for the first time emittance preservation on the few-percent level over 20 plasma acceleration stages. Beyond this proof-of-principle work, Wake-T could enable the multistage modeling of collider concepts and allow for large optimization and tolerance studies in cases where axial symmetry can be assumed for the plasma response. This is a necessary step before studying tolerance to effects that break the axisymmetry with other models. Future directions in Wake-T include the development of new features for modeling ion motion in the plasma and the capability of efficiently resolving nm-level emittances.

ACKNOWLEDGEMENTS

AFP would like to thank C. Benedetti for fruitful discussions and assistance regarding the implementation of the laser envelope model in Wake-T. This research was supported in part through the Maxwell computational resources operated at DESY, Hamburg, Germany. This work was partially supported by the European Union's Horizon Europe Marie Sklodowska-Curie Staff Exchanges programme under grant agreement no. 101086276. This material is based in part upon work supported by the CAMPA collaboration, a project of the U.S. Department of Energy, Office of Science, Office of Advanced Scientific Computing Research and Office of High Energy Physics, Scientific Discovery through Advanced Computing (SciDAC) program, and by the Director, Office of Science, Office of High Energy Physics, U.S. Dept. of Energy under Contract No. DEAC02-05CH11231.

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