THEORETICAL IDEAS ON THE POMERON

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Abstract: We review the mechanisms which can build a "bare" Pomeron and the experimental evidence for and against these mechanisms. Then we discuss the "super-critical" case of Reggeon field theory in order to determine the asymptotic behaviour of the scattering amplitudes when the Pomeron intercept is larger than one.

Résumé : On examine les mécanismes susceptibles d'engendrer un poméron "nu", et les indications expérimentales en faveur et contre ces mécanismes. Ensuite on discute le cas "supercritique" de la théorie des reggéons, afin de déterminer le comportement asymptotique des sections efficaces lorsque l'intercept du poméron est plus grand que 1.

In this talk I will review some theoretical ideas about the Pomeron. Some of these ideas are rather recent, but some date back to fifteen years ago, with the invention of the multiperipheral model[1]. However, if one compares the situation now to that which we discussed four years ago, at the 1973 Moriond meeting on the Pomeron, we see that there has been a lot of clarification from a theoretical point of view, that a rather well-defined theoretical scheme has emerged, and has, to some extent, received experimental verification.

In this scheme, it is useful to distinguish two steps in building the scattering amplitude at high energy. In the first step, one assumes that there exists a basic mechanism for multiparticle production, which gives only short-range correlations in rapidity, or, in other words, which obeys short-range order. A dynamical model which realizes this assumption is of course the multiperipheral model[1], but one may have in mind a dual model[2], or any other model. The only important point is really short-range order, and I need not be too precise about the details of the dynamics.

This production process generates, via unitarity, diffractive amplitudes, for example, the elastic amplitude. Because of the short-range order hypothesis, it will be possible to represent the elastic amplitude by the exchange of a Regge pole[3], namely the Pomeron, more precisely the "bare" Pomeron.

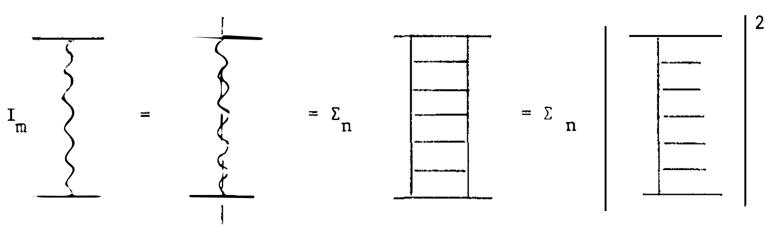

Fig. 1

Now we can go to the second step in building the scattering amplitude, by allowing for multiple exchanges of this Pomeron, Pomeron interactions, etc. The scheme which is derived in this way is Gribov Reggeon calculus[4]

The plan of the talk will follow this idea of a two-step process. In the first part I will discuss how one generates the bare Pomeron, and in the second I will describe some recent results obtained with the Reggeon calculus[5].

1. THE BARE POMERON

I have no time to review in detail the experimental evidence for and against the short-range order picture of multiparticle production. Let me just mention that there is extremely good evidence in favour of an important short-range component in two-particle rapidity correlations, and also good evidence coming from local charge conservation[6].

The important violation of Feynman scaling in the central region goes *a priori* against short-range order: one finds that in the ISR energy range (from $\sqrt{s}$ = 22 GeV to $\sqrt{s}$ = 63 GeV) the inclusive cross-section $d\sigma/dy$ in the central region increases by $\sim$ 40% [7]. However, there may be finite energy effects due to peculiarities of multiperipheral dynamics[8]; also if the Pomeron intercept is larger than one, one expects that $d\sigma/dy$ increases faster than σ_{tot}, owing to the famous AGK cancellation. Nevertheless a 40% increase is a large effect, and one can feel somewhat uneasy with the "explanations" given above.

There is just one result which I would like to examine in some more detail, since it is relatively recent[9]. This result shows for the first time (or at least gives some indication in favour of) the validity of local compensation of transverse momentum[6]. This property, which is a consequence of short-range order, is extremely important, because if transverse momentum were not conserved locally, one would not expect to find a Regge pole out of the unitarity equation[3]. Before this experiment, all the data were in agreement with global conservation[6], thus casting some doubt on the whole theoretical scheme.

The quantity which is measured is the fluctuation of transverse momentum transfer[3,6]:

$$\vec{Q}_T\ (y) = \Sigma_i\ \vec{p}_{Ti}\ \theta\ (y - y_i) \qquad (1)$$

where $\vec{p}_{Ti}$ is the transverse momentum of particle i, y_i its rapidity, and $\theta(x)$ the step function. The fluctuation is

$$D_T^2\ (y_1,y_2) = <\vec{Q}_T\ (y_1) .\ \vec{Q}_T\ (y_2)> \qquad (2)$$

Just to illustrate this formula, let me take both arguments equal to zero:

$$D_T^2\ (0,\ 0) = <(\Sigma_{\substack{i \\ y_{cm}<o}}\ \vec{p}_{Ti}\ )^2\ > \qquad (3)$$

It is easy to realize that if transverse momenta are distributed at random (global conservation of $\vec{p}_T$), this fluctuation will be proportional to the average number of particles, $\langle n\rangle$, and to $\langle\vec{p}_T^{\,2}\rangle$. In fact one finds

$$D^2_T\ (0,\ 0) = \frac{1}{6} <n_{ch}> <\vec{p}^{\,2}_T\ (0)> \tag{4}$$

where the formula is written for charged particles only. For example, at 200 GeV/c, $\langle n \rangle_{ch} = 7.8$, $\langle \vec{p}^{\,2}_T \rangle \simeq 0.20\ (\mathrm{GeV})^2$, and one gets from (4) a theoretical prediction of ≃ 0.26. A more refined (but yet not completely correct) model of global $\vec{p}_T$ conservation has been built by the authors of Ref. 9, and one sees from Table 1 that global conservation is ruled out experimentally.

Table 1

$D^2_T(0,0)\ (\mathrm{GeV/c})^2$

	Experiment	Random model
200 GeV/c	0.202 ± 0.009	0.289 ± 0.006
300 GeV/c	0.208 ± 0.008	0.331 ± 0.009

Another important point is that $D^2_T(y_1,y_2)$ should be energy independent. This is beautifully satisfied by the data at 200 and 300 GeV/c. However, even with local $\vec{p}_T$ conservation, one would still expect some energy dependence, reflecting the increase with energy of the particle density ρ, and of $\langle \vec{p}^{\,2}_T \rangle$. Indeed, if one tries an exponential parametrization

$$D^2_T\ (0,\ y) = A\ e^{-|y|\ /\ L} \tag{5}$$

the coefficient A is determined to be

$$A = \frac{1}{2} < \vec{p}^{\,2}_T\ (0) > \rho_{ch}\ (0)\ L$$

Since both $\langle \vec{p}^{\,2}_T(0) \rangle$ and $\rho_{ch}(0)$ increase by ∿ 10% between 200 GeV/c and 300 GeV/c, and since L is approximately constant, one would expect a ∿ 20% increase of D^2_T, as is observed in the case of local conservation of charge[6].

Although there are certainly problems with the details of the experiment, let me nevertheless assume that we can use its result in the Krzywicki-Weingarten bound[3] for the Pomeron slope $\alpha'_{\mathbb{P}}$

$$\alpha'_{\mathbb{P}} \geqslant \frac{1}{8 \int_0^\infty D^2_T\ (0,\ y)\ dy} \simeq 0.16\ (\mathrm{GeV/c})^{-2} \tag{6}$$

From this value, we can assert that we understand at least half of the Pomeron slope, since $\alpha'_{\mathbb{P}} \simeq 0.25\ (\mathrm{GeV/c})^{-2}$.

2. POMERON ITERATIONS

Once the Pomeron has been built, one can forget about its origin and remember only a few parameters:

intercept: $\alpha_{\mathbb{P}}$ or $\mu = \alpha_{\mathbb{P}} - 1$

slope : $\alpha'_{\mathbb{P}}$ or simply α'

coupling to inelastic states: λ

(in order to simplify the discussion, we keep only the triple Pomeron coupling λ). Gribov has shown that these parameters can be inserted into the Lagrangian of a two-dimensional non-relativistic field theory, and that the rules for computing multi-Pomeron exchanges are exactly the Feynman rules of this field theory[4]. The Lagrangian is written in terms of a Pomeron field $\psi(y,\vec{b})$, which is a function of the rapidity y (equivalent to an imaginary time) and of the two-dimensional impact parameter $\vec{b}$. Owing to Pomeron interactions, the Pomeron intercept gets renormalized, $\mu \to \mu_R$. If $\mu_R < 0$, the amplitudes are dominated asymptotically by a pole and the situation is very simple. If $\mu_R = 0$, one has the so-called "critical Pomeron" where for example the total cross-section behaves as $(\log s)^{\eta}$ [10]. Let us call μ_c the value for which μ_R is exactly zero. If $\mu > \mu_c$, we have the so-called "super-critical Pomeron". Various estimates give $\mu \simeq 1.1$ [11] while $\mu_c \simeq 1.01-1.02$, and it thus seems that the super-critical case is realized in nature.

This is precisely the situation which I would like to examine in what follows. As will be seen, the problem is a rather complicated one, and I would like to begin with a simple remark. The Pomeron propagator in $(y,\vec{b})$ space is

$$D(y,\vec{b}) = \frac{1}{4\alpha' y}\, e^{\mu y - \vec{b}^2 / 4\alpha' y} \qquad (7)$$

At fixed y and $\vec{b}$ large enough ($|\vec{b}| \gg \sqrt{4\alpha'\mu}\,y$), the exchange of one pole is going to dominate, since the cuts are multiplicative in impact parameter space. This remark will be extremely important in the following. For the moment let me only use it to make a very naïve guess at the asymptotic behaviour of the amplitude. For $|\vec{b}| \gtrsim \sqrt{4\alpha'\mu}\,y$, perturbation theory, as we have seen, is likely to be valid. However, for $|\vec{b}| \lesssim \sqrt{4\alpha'\mu}\,y$ we may expect that the amplitude, at fixed impact parameter, goes to a constant. If this constant is non-zero, we see at once that the cross-section will be $\sim (4\alpha'\mu)(\log s)^2$, corresponding to a disk expanding in impact parameter space with a velocity $\sqrt{4\alpha'\mu}$.

Let me now try to explain how this picture emerges from a more complete analysis, by considering the two-point Green's function

$$G\,(y, \vec{b}) = \langle 0|\psi(y,\vec{b})\ \psi\ (0,\vec{0})|0\rangle \;=\; \langle 0|\psi(0,\vec{b})\ c^{-Hy}\ \vec{\psi}(0,\vec{0})|\ 0\rangle \tag{8}$$

where H is the Reggeon field theory Hamiltonian. We have chosen to keep the original vacuum of the theory, and not to switch to a new vacuum, although RFT for $\mu > 0$ is superficially similar to a situation in which there is spontaneous symmetry breaking (as in $\lambda\phi^4$ with $\mu^2 < 0$). Indeed we wish to recover perturbation theory at large $\vec{b}$ (at y fixed) and this is automatic if we do not change the vacuum.

Suppose one is able to prove that the spectrum of H is positive and that the first excited state is separated from the ground state (vacuum) by a gap Δ. Then by inserting intermediate states in (8) we obtain (apart from factors of log s)

$$G\,(y) \sim e^{-\Delta y} = s^{-\Delta}$$

This would be the case for $\mu_R < 0$, with $\Delta = -\mu_R$. At the critical point, $\Delta = 0$, and we expect something a ground-state degeneracy, as in a standard second-order phase transition; above the critical point ($\mu > \mu_c$) this ground-state degeneracy would of course persist, and transitions between the two degenerate ground states could give rise to a constant S-matrix (always at a fixed impact parameter).

However, thinking of a normal field theory ($\lambda\phi^4$), one immediately sees a difficulty, because the two degenerate ground states are indeed completely disconnected: the matrix-elements of field operators between the two ground states are strictly zero. This problem is solved in RFT because the Hamiltonian is non-Hermitian. This implies two properties, which are completely at variance with those of the Hermitian theory: i) the perturbative vacuum is one of the ground states; ii) the field-operators have non-zero matrix elements between the two ground states. This allows the second ground state $|\psi_0\rangle$ to contribute as an intermediate state in (8) and to give rise to a constant S-matrix.

To show these properties explicitly, it is convenient to go to a lattice in impact parameter space. This is equivalent to an ultraviolet cut-off (which is in any case present in the theory) and should be harmless in the vicinity of the critical point. Now H has the form

$$H = \Sigma_j\ H_j + K_{in}\ \text{term}$$

where H_j is the Hamiltonian at the lattice point j and the kinetic term connects neighbouring lattice points. Let us begin by determining the spectrum of H_j, neglecting for the moment intersite interactions. In a $\lambda\phi^4$ theory with $\mu^2 < 0$, one discovers that the spectrum of H_j is that of a double well potential.

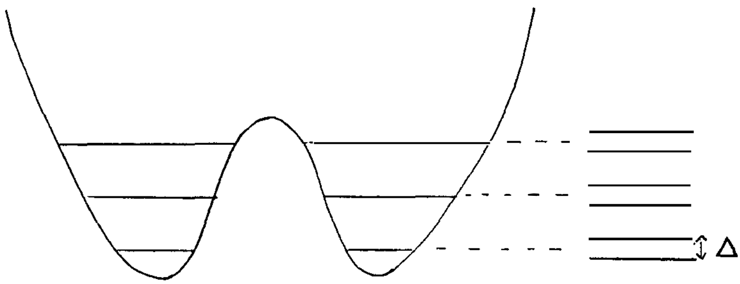

Fig. 2

Because of tunnelling, each energy level is almost doubly degenerate. However, this is true only if one neglects intersite interactions. Indeed, in the ordered situation, the tunnelling disappears and the two ground states are obtained by putting all "particles" in the same well. The wave function in a well is a coherent state*)

$$\chi_{\pm,j} = e^{-\frac{1}{2}\frac{\mu^2}{\lambda^2}} \; e^{\pm \frac{\mu}{i\lambda} a_j^\dagger} \chi_{o,j} \tag{9}$$

where $\chi_{0,j} = |0,j\rangle$ is the perturbative vacuum at each site.

By solving RFT in zero-space dimension, one can show that the spectrum of H_j is quite similar to that of Fig. 2; however, the ground state is the vacuum, $\chi_{0,j}$, while the first excited state (almost degenerate with $\chi_{0,j}$) is approximately again a coherent state

$$\chi_{1,j} = \chi_{0,j} - e^{\frac{\mu}{i\lambda}\bar{\psi}_j} \chi_{0,j} \tag{10}$$

Its energy Δ is proportional to $\sim e^{-\mu^2/2\lambda^2}$, an expression typical of tunnelling, although we do not have a potential well of the form drawn in Fig. 2.

Now comes the crucial point. Suppose one builds a collective state in $\lambda\phi^4$

$$\Phi_\pm = \prod_j \chi_{\pm,j}$$

It is easy to realize that all non-diagonal matrix elements of field operators between Φ_+ and Φ_- (or between $|0\rangle$ and $\Phi_\pm$) will have a factor $\exp\left[-N(\mu^2/\lambda^2)\right]$, where N is the number of states, and all these matrix elements will go to zero in the limit $N \to \infty$.

*) We have slightly redefined the parameters of the $\lambda\phi^4$ theory, so that the formulae look similar to those of RFT.

On the contrary, in RFT, one notices that the state $\chi_{1,j}$, as defined in (10), is correctly normalized to one (in fact -1!) in the limit $\mu/\lambda >> 1$. One has to remember that the "bras" $\bar{\chi}_{0,j}$ are eigenstates of H^+, which means replacing λ by $-\lambda$. Hence

$$(\bar{\chi}_1, \chi_1) = (\bar{\chi}_0, \chi_0) - (\bar{\chi}_0, e^{\frac{\mu}{i\lambda}\bar{\psi}} \chi_0) - (\bar{\chi}_0, e^{\frac{\mu}{i\lambda}\psi} \chi_0) + + (\bar{\chi}_0, e^{\frac{\mu}{i\lambda}\psi} e^{\frac{\mu}{i\lambda}\bar{\psi}} \chi_0) = -1 + e^{-\mu^2/\lambda^2} \tag{11}$$

The collective state ψ_0 is built as

$$\psi_0 = \Pi_j \chi_{0_j} - \Pi_j e^{\frac{\mu}{i\lambda}\bar{\psi}_j} \chi_{0_j} \tag{12}$$

This state is correctly normalized (to -1) and is an approximate eigenstate of H in the limit $\mu/\lambda \to \infty$. When inserted as an intermediate state in G(y), it gives a non-zero contribution [in fact $\sim (\mu/\lambda)^2$].

To be more quantitative, one builds a spin-model by keeping only the two lowest states at each lattice site (owing to the structure of the spectrum, this is a reasonable approximation). The calculations are made in detail in Ref. 5. In the second paper, one shows in particular how the expanding disk is generated, at least in the framework of this spin model.

3. CONCLUSION

I think that there exists at present a theoretical scheme for high-energy scattering which is in reasonable agreement with experiment (although there are some difficulties), and which seems to be internally consistent. Owing to the fact that $\mu > \mu_c$, this scheme predicts that at ultra-high energies, the total cross-sections will grow like $(\log s)^2$ [and the average multiplicity like $(\log s)^3$]. However, I must mention that there are other theoretical approaches to the theory of the super-critical Pomeron, which predict quite a different asymptotic behaviour[12], and there remains a lot of work to be done before the situation is completely clarified.

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