SHAPES OF β-RAY SPECTRA

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1. FORMULATION OF ALLOWED β DECAY

The shape of β-ray spectra in the allowed approximation is given by[1]

$$N(W)dW = \frac{\xi}{4\pi^3} F(Z,W)pW(W_o-W)^2 \left(1 \pm \frac{b}{W}\right) dW , \qquad (1)$$

where

$$\begin{aligned} \xi &= (|C_S|^2 + |C_V|^2 + |C_S'|^2 + |C_V'|^2)\,|M_F|^2 \\ &\quad + (|C_T|^2 + |C_A|^2 + |C_T'|^2 + |C_A'|^2)\,|M_{GT}|^2 \end{aligned} \qquad (2)$$

and

$$\begin{aligned} b\xi &= \gamma(C_S^*C_V + C_SC_V^* + C_S'^*C_V' + C_S'\,C_V'^*)\,|M_F|^2 \\ &\quad + \gamma(C_T^*C_A + C_A^*C_T + C_T'^*C_A' + C_A'^*C_T')\,|M_{GT}|^2 . \end{aligned} \qquad (3)$$

The plus sign in front of b/W is for β^- decay, and the minus sign for β^+ decay. The Fermi function[2) F will already include the finite size effect. The symbols have their usual meaning: $C_S(C'_S)$ is the parity conserving (parity non-conserving) scalar coupling constant, etc., γ a Coulomb correction term which is almost unity, $M_F(M_{GT})$ the Fermi (Gamow-Teller) matrix element, and b the Fierz interference coefficient. In the case of pure Fermi or pure Gamow-Teller transitions, one has

$$b_F = \gamma \frac{C_S^* C_V + C_S C_V^* + C_S'^* C_V' + C_S' C_V'^*}{|C_S|^2 + |C_V|^2 + |C_S'|^2 + |C_V'|^2} \tag{4}$$

and

$$b_{GT} = \gamma \frac{C_T^* C_A + C_T C_A^* + C_T'^* C_A' + C_T'^* C_A'^*}{|C_T|^2 + |C_A|^2 + |C_T'|^2 + |C_A'|^2} \quad . \tag{5}$$

If the electron polarization is complete ($\mp$ v/c for $\beta^{\mp}$ decay) the Fierz interference terms vanish automatically. This is particularly true of the generally accepted form of the β interaction, the V-A interaction. In fact, the Fierz terms even vanish for any VA interaction. On the other hand, the Fierz terms are the most sensitive tools for checking experimentally whether these assumptions are correct, in the sense that the experimental upper limit for a Fierz coefficient b deduced from the experimental evidence on β interaction type and electron polarization, is larger than that deduced from a direct measurement of the Fierz coefficient. This is at least true of β^+ decay and of Fermi transitions. The best method for the experimental determination of Fierz coefficients consists in measuring the spectral shapes, and not the K/β^+ branching ratios. If $b = 0$ in Eq. (1), the spectral shape is called statistical, because it depends on the statistical sharing of the momentum between the electron and the neutrino in the phase space[3].

Besides the Fierz interference, there are other reasons why the spectrum of an allowed β transition, i.e. a transition with the spin change $\Delta I = 0, \pm 1$ and no parity change, may differ from the statistical shape:

a) The conservation of vector current[4] implies small correction terms which, however, are of second order and therefore vanish in the allowed approximation. They are usually barely observable, but are definitely observed in favourite cases[5] (^{12}B, ^{12}N).

b) Second order effects are always present, but usually observable only if the allowed matrix element is extremely small. This effect has been established[6] for ^{32}P.

c) If the neutrino or antineutrino rest mass does not exactly vanish, there will be a deviation at the upper end of the spectrum. Qualitatively, the same will be true of the neutrino degeneracy, i.e. if there are empty neutrino (antineutrino) states below zero energy, or filled states above zero energy[7].

d) Deviations of the b/W type [Eq. (1)], but with a b differing in its meaning from Eqs. (3), (4) and (5), have been reported as results of experimental work with no satisfactory theoretical explanation[6,8].

2. FORMULATION OF FORBIDDEN β DECAY

The forbidden β decay is a decay which does not take place in the allowed approximation but in a higher order approximation. The first forbidden decay may again be defined by its selection rules $\Delta I = 0$, 1, or 2 with parity change, while the first forbidden approximation contains first order terms in R/λ, where λ is the lepton wavelength and R the nuclear radius, in v_N/c where v_N is the nucleon velocity in the nucleus, and in the Coulomb field. Correspondingly, the higher forbidden transitions are defined by their selection rules, and the higher order approximations by the order of the terms.

It is not the purpose of this review to give all the formulas for the forbidden β decay in full detail, as was done in Eq. (1) for the allowed approximation. Instead of this, the V-A interaction will be assumed unless stated otherwise. The reason is that the basic questions underlying the full description were mostly studied, and are best studied, in the allowed β decay.

It is the general custom to describe the shape of a forbidden β spectrum by a correction factor[3)]

$$S_n = \sum_J S_n^{(J)} \quad , \qquad (6)$$

where the subscript n refers to the degree of approximation (n = 0: allowed; n = 1: first forbidden, etc.), and J refers to the total angular momentum carried away by the two leptons: $J = \Delta I, \Delta I+1, \ldots, I_i+I_f$. The meaning of S_n is as follows:

$$N(W)dW = \frac{\xi}{4\pi^3} F(Z,W)pW(W_o-W)^2 \, S_n(W)dW \; . \qquad (7)$$

In the case of first forbidden β decay, one has in general

$$S_1 = S_1^{(o)} + S_1^{(1)} + S_1^{(2)} \; .$$

In a particular transition, one or two of the $S_1^{(i)}$ may vanish. Table 1 summarizes the selection

TABLE 1

Matrix element		J	ΔI	Δπ
Allowed	$C_V \int 1$	0	0	no
	$C_A \int \vec{\sigma}$	1	0, ± 1 (no 0 → 0)	no
First forbidden	$C_A \int \gamma^5$ $C_A \int \vec{\sigma}\cdot\vec{r}/i$	0	0	yes
	$C_V \int \vec{r}\, i$ $C_V \int \vec{\alpha}$ $C_A \int (\vec{\sigma}\times\vec{r})$	1	0, ± 1 (no 0 → 0)	yes
	$C_A \int i\, B_{ij}$	2	0, ± 1, ± 2 (no 0 → 0, no 0 ⇄ 1, no 1/2 → 1/2)	yes

(J designates the rank of the transition operator, when regarded as a tensor) rules and nuclear matrix elements in the allowed and the first forbidden β decay for the V-A interaction[?].

A particular class of forbidden transitions are the unique transitions where a unique angular momentum $J = n+1$ contributes only. Neglecting a Coulomb correction factor of about unity, one has[3)]

$$S_n^{(n+1)} = \text{const} \sum_{\nu=0}^{n} p^{2\nu} q^{2(n-\nu)} / [(2\nu+1)!(2n-2\nu+1)!] \quad (8)$$

This reads for first, second, and third forbidden decays, respectively,

$$S_1^{(2)} = \text{const}\ (p^2 + q^2)\ , \quad (9)$$

$$S_2^{(3)} = \text{const}\ [\, p^4 + \frac{10}{3} p^2 q^2 + q^4\,], \quad (10)$$

$$S_3^{(4)} = \text{const}\ [\, p^6 + 7p^2 q^2 (p^2 + q^2) + q^6\,]. \quad (11)$$

Taking Coulomb corrections into account more properly, Eq. (9) is replaced by

$$S_1^{(2)} = \text{const}\ (q^2 + 9L_1)\ . \quad (12)$$

[In connection with this formula, it should be remembered that $F(Z,W)$ is taken to include the normal finite size effect, cf. Section 1.]

In the case of non-unique transitions, the situation is even simpler at first glance: all first forbidden non-unique spectra are expected to show roughly a statistical shape. In practice, however, this statement, which was considered to be valid for most transitions for many years, is no longer true. If the famous RaE spectrum, whose peculiar shape has been known for a long time, is exempted, then the first non-unique first forbidden spectra, where a deviation from the statistical shape was reported, are the ^{186}Re spectra[10]. Nowadays, a large number of such spectra are known. Most of them are of interest in connection with nuclear structure, but not weak interaction. They will be treated briefly in Section 5. Some cases are important for special aspects of weak interaction, such as time reversal invariance, pseudoscalar interaction, or conserved axial vector current. They will be treated in Sections 6, 7 and 8. Particularly with regard to the nuclear structure work, it is often useful to fit the spectral shapes with the formula

$$S_1 = \text{const}\,[1 + aW + (b/W) + cW^2]\,. \qquad (13)$$

For the connection between the parameters a, b, and c on the one hand, and the matrix elements (Table 1) on the other, the paper by Kotani and Ross[11] is recommended.

3. EXPERIMENTAL METHODS

The classical instrument for the study of β-ray spectra is the magnetic spectrometer. If it is iron-free the field strength is strictly proportional to the current, and the field shape does not at all depend on the field strength. However, instruments with iron were also successfully used. One may have a somewhat larger a priori confidence in the data coming from iron-free instruments. In the same way, a lens spectrometer seems to be more suitable than a spectrometer with a transverse field because of a smaller risk of backscattering from the vacuum chamber walls and, in particular, end plates.

Besides the magnetic spectrometer, other types of spectrometers are important, and are becoming even more and more important. These are mainly 4π devices such as the scintillation spectrometer, which seems to be most reliable though also most tedious to operate with the activity distributed homogeneously during the crystal-growing process over the total crystal volume. Semiconductor setups are used preferably in sandwich arramgements. For low energies, proportional counters may be used.

Also in the future, the choice of the right or wrong instrument will furnish material for many discussions. The only general, and generally accepted, conclusion from the experimental results is, however, that there exists no strong correlation between results and types of instruments.

4. EXPERIMENTAL DATA ON ALLOWED SPECTRA

During the last decade, many attempts have been made to measure the shape of allowed spectra with highest possible accuracy. Although conflicting results were reported, the situation may be characterized by the statement that in all cases where the allowed approximation is applicable, the spectra show very accurately statistical shapes. Some transitions have been studied most thoroughly either because they are physically most interesting or because the experimental conditions are most favourable. The following examples will be discussed: ^{22}Na and ^{114}In (Gamow-Teller β^+ and β^- decays, respectively), and ^{13}N (Fermi + Gamow-Teller β^+ decay).

The most accurate shape-factor measurements of ^{22}Na were performed by Leutz and co-workers with an NaI crystal, where the activity was built in during the crystal-growing process. Figure 1 shows a recent result[12]. The shape factor is plotted as a function of the β energy. As the shape factor is a horizontal straight line and the errors are small, the spectrum follows closely that of the statistical shape. There

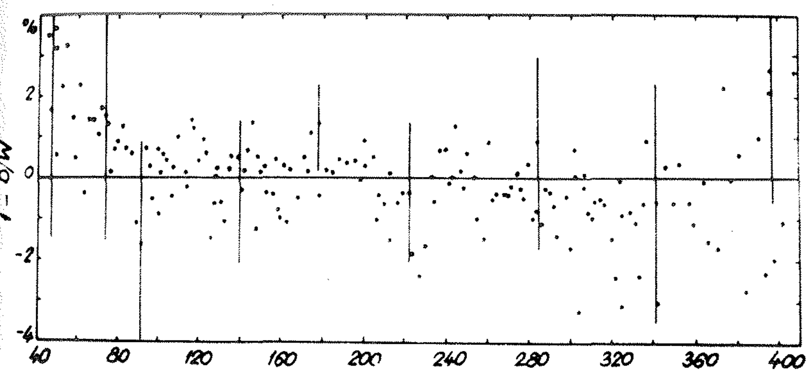

Fig. 1. Shape factor of the ^{22}Na β spectrum as a function of β^+ energy in keV, measured with a 4π scintillation spectrumeter (one of several runs)[12].

is no room for the Fierz interference or any other deviations whatsoever. The value of b is given in Table 2. The work of Leutz et al.[12] confirms the earlier work with a magnetic spectrometer[13].

Although the experimental situation in ^{22}Na is now very clear, the interpretation is not so clear as one may wish with respect to the small experimental errors. The comparative half-life (the ft-value) of this decay is somewhat large for an allowed transition. Therefore, noticeable interference with second order contributions is not a priori excluded. It might be, by mischance, that a small Fierz term present in the allowed approximation just cancels with a second order term. It is therefore very valuable that a completely statistical shape has also been found for the pure Gamow-Teller transition of ^{114}In which has a low ft-value.

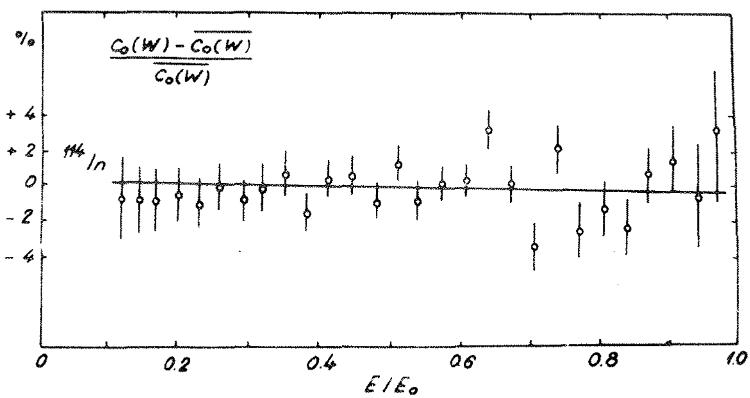

Fig. 2. Shape factor of the ^{114}In β spectrum as measured with a double-lens spectrometer[14].

Figure 2 shows the shape factor [14] of ^{114}In. Again it is a horizontal straight line.

The transitions of ^{22}Na and ^{114}In are pure Gamow-Teller transitions. There are no pure Fermi transitions which offer favourable experimental conditions. However, ^{13}N is known to decay to 70±2% by the Fermi and to 30±2% by the Gamow-Teller interaction. These figures come from the ft-value, the known value of the vector coupling constant, and the model-independent value of the Fermi matrix element.

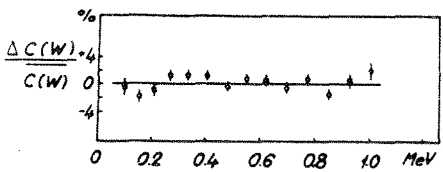

Fig. 3. Shape factor of the ^{13}N β spectrum as measured with a double-lens spectrometer[15].

As the experimental conditions are not bad, ^{13}N is a suitable nucleus for checking the shape of a Fermi spectrum, the Gamow-Teller contribution to b being known from, say, ^{22}Na and ^{114}In. Figure 3 shows the ^{13}N shape factor[15] which is also a straight line.

Figure 4 is a shape factor plot[16] of ^{32}P. There is not much doubt left that this spectrum deviates substantially from the statistical shape. However, there are still arguments regarding the exact form of this deviation - in particular, whether the deviation varies linearly with energy or not - and about its exact size. These questions are not immaterial: if the deviation is linear the polarization is still ex-

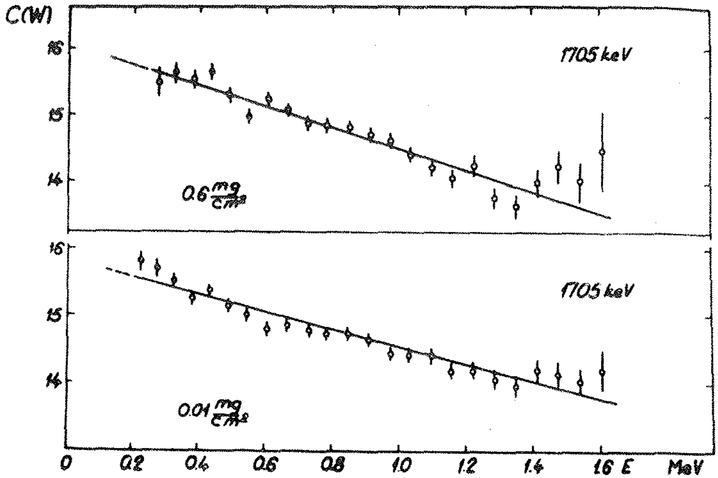

Fig. 4. Shape factor of the ^{32}P β spectrum as measured with a double-lens spectrometer[16). Note the small but not negligible influence of the source thickness on the slope.

pected[17) to be -v/c; if not, no such conclusion can be drawn. Although details of the interpretation are still the subject of discussions, the deviation can in general be understood in terms of nuclear structure effects.

Besides these examples, other spectra have been carefully examined. Table 2 gives a compilation. Although no attempt was made to include every allowed β spectrum ever investigated, this compilation was thought to include at least the majority of recent results.

Inspection of Table 2 shows that, as mentioned at the beginning of this section, β-ray transitions which can be expected to show a statistical shape really do have it. The unexplained b/W deviations were reported by some groups. However, a large number of experimenters[6] did not obtain this effect.

5. EXPERIMENTAL RESULTS ON FORBIDDEN DECAY

This section deals with the general results obtained in forbidden β decay on the basis of V-A interaction.

A lot of work has been done in order to check the shape of unique first forbidden β-ray spectra. They are most reliably studies in transitions where one state (initial or final) has spin zero. In this case J [Eq. (6)] must be 2. These transitions, mostly of the type $2^- \rightarrow 0^+$, are also experimentally very favourable in the sense that many suitable β^- transitions are available. Unfortunately, no suitable β^+ transitions are available.

The most frequently, and perhaps also the most carefully studies decay is ^{90}Y - ^{90}Zr, where ^{90}Y may or may not be separated from its parent ^{90}Sr. Figure 5 shows its shape factor[18] where, other than with allowed decays (Section 4), the expected correction factor $S_1^{(2)}$ [Eq. (12)] is already included in the denominator; hence a horizontal straight line is again expected if $S_1^{(2)}$ [Eq. (12)] describes the shape correctly.

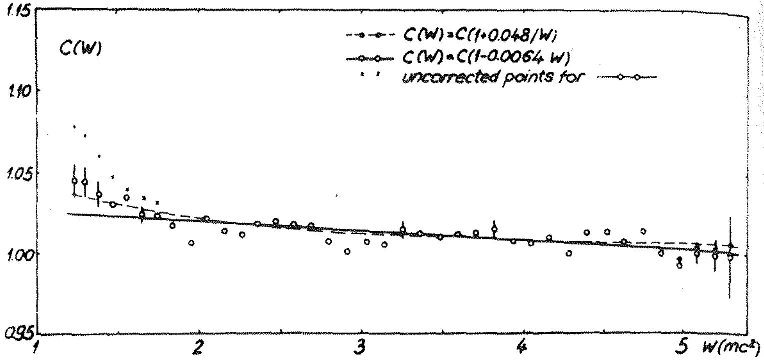

Fig. 5. Shape factor of the ^{90}Y β spectrum as measured with an intermediate image spectrometer[18].

According to Fig. 5 this is not the case. Therefore, the work of Riehs confirms earlier findings[6,14,19] that a slight deviation exists. In fact, a survey of four $2^- - 0^+$ spectra all with normal ft-values (^{42}K, ^{86}Rb, ^{90}Sr, ^{90}Y) showed, within the respective errors, the same percentage-wise decrease of the shape factor between 0 and the maximum kinetic electron energy[14]. An explanation may be found in weak magnetism terms, as Eman et al.[20] have pointed out. It would be very interesting to see whether the sign of deviations is opposite for β^+ decay, but there are experimental difficulties mentioned above. For a report of a large deviation in the β^- decay of ^{166}Ho, cf. Section 8.

Higher forbidden unique spectra show the expected behaviour: ^{10}Be follows Eq. (10)[21], and ^{40}K Eq. (11)[6]. The experiments, at least on ^{10}Be, are not accurate enough to prove or disprove the existance of small Coulomb corrections or deviations such as those shown in Fig. 4.

Table 3 is a compilation of unique forbidden shape factors.

Many non-unique first forbidden spectra were found to show substantial deviations from the statistical shape. They are summarized in Table 4, as well as the spectra showing a statistical shape, and the non-unique higher forbidden spectra. The spectra of ^{144}Ce, ^{144}Pr, ^{166}Ho, and ^{210}Bi (RaE) will be treated in special sections.

6. BETA DECAY OF RaE AND TIME REVERSAL INVARIANCE

The time reversal in nuclear β decay can be checked in the most straightforward manner by correlation experiments on the decaying free neutron. This was performed and yielded a value of $\theta_{V-A} = 5 \pm 9^{0}$ for the deviation from V-A (the stated error is the standard deviation)[22]. In the RaE decay, owing to a very peculiar destructive interference, there is also a chance to test this very important symmetry principle. The experiments can be performed with a much better statistical accuracy than in the free neutron case. On the other hand, the interpretation of the experimental data is not so straightforward.

The most thorough evaluation of all experimental

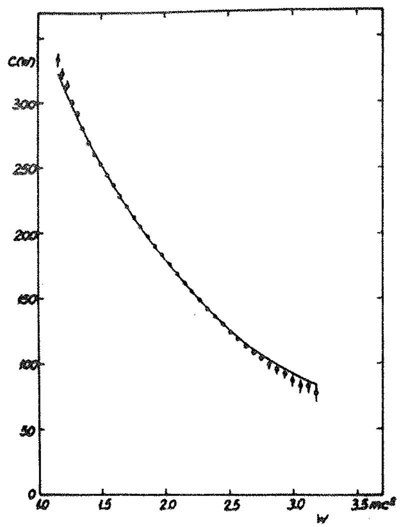

Fig. 6. Shape factor of the RaE β spectrum as measured with a double - lens spectrometer[23].

information which is available includes both the spectral shape and the electron polarization as a function of energy, or more adequately, of c/v. Figure 6 shows the RaE spectral shape factor[23]. As this is not a straight line at all, the spectrum deviates very strongly from the statistical shape. This abnormal behaviour makes the analysis very unpleasant. With the help of the very elaborate theory of Fujita et al.[24], the present author analyzed his shape factor measurement and treated both the matrix elements and θ_{V-A} as completely free parameters. The result was $\theta_{V-A} = 4.5 \pm 1.0^{0}$. This is, however, no real indication of a violation of time reversal, as θ_{V-A} vanishes within three times the experimental standard deviation . The reason why this is so is the large curvature of θ_{V-A} as a function of the experimentally determined quantity α_{V-A}. The usually applied linear error computation, therefore, fails completely in this special case. As stated in the original paper of the RaE spectrum, the experimental result is in agreement with the time reversal invariance. Recent analyses of the RaE decay by Sodemann and Winther, and Vinduška and Šott[25] came to the same conclusion. We can deduce that $\theta_{V-A} < 6^0$ with a confidence level of 95%, in comparison with $\theta_{V-A} < 23^0$ with the same confidence level from the free neutron decay. As the nuclear β decay is leptonic and the decays of the long-lived K meson into two π mesons ($K_L^0 \to \pi^+\pi^-$ and $K^0 \to \pi^0\pi^0$) are non-leptonic, there is no direct connection whatsoever between T (or CP) violation in the latter case and θ_{V-A} for nuclear β decay.

7. PSEUDOSCALAR INTERACTION

By means of a large number of experiments, the weak interaction was shown to be of the V-A type. All evidence is also in favour of the interaction being universal. If one believes in this universality a priori, then the most crucial test for a pseudoscalar part in the weak interaction is the branching ratio of π decay $R = (\pi \to e+\nu_e)/(\pi \to \mu+\nu_\mu)$, where the theoretical estimate for the V-A interaction and the experimental value[26] $R = (1.24 \pm 0.03)10^{-4}$ coincide very nicely. R is small, as the π-e decay is greatly hindered by the helicity requirement for e. If the interaction were pseudoscalar, i.e. with opposite helicity $+v/c$ for the negative electron, then R would be about 5 due to phase space effects.

If one does not believe in the universality of the Fermi interaction a priori, one has to look for direct experimental evidence in favour of or against the pseudoscalar interaction in nuclear β decay. Because of the selection rules, this cannot be done in the allowed β decay, but it can be done in the first forbidden β decay, particularly 0-0 transitions. Besides this primary pseudoscalar interaction, there may also be an induced pseudoscalar interaction which arises from the strong interaction. Both imply the same experimental consequences. The induced pseudoscalar interaction apparently takes place in μ capture.

When searching for the pseudoscalar interaction in nuclear β decay, the $0^- \rightarrow 0^+$ transition $^{144}Pr \rightarrow$ ^{144}Nd is particularly suitable - if it is allowed to call any β transition suitable for this purpose. Bhalla and Rose[27)] first outlined an elaborate theory and compared it with experiments. They concluded $|C_P|/|C_A| < 90$ - which is a large number for an upper limit. Later experimental work done at Heidelberg[28)] also yielded no evidence for a pseudoscalar contribution. An upper limit of $|C_P|/|C_A| < 5$ was reported, but this was due to a numerical error caused by a computer with a too low capacity, as kindly pointed out to the author by F.T. Porter[29)]. A re-evaluation with a more suitable computer is therefore planned.

8. CONSERVATION OF VECTOR AND AXIAL VECTOR CURRENT AND G PARITY

The concept of a conserved vector current[4)] (CVC) is recommended by electrodynamics, as the vector current part of the weak interaction is analogous to electrodynamics, and there the electric charge is conserved. It is also experimentally recommended in order to explain the fact that muon and nuclear vector decay coupling constants are equal, with no renormalization due to strong interaction [leaving a difference of about 2% to be explained by the Cabibbo angle[30]]. Experimental evidence, amongst other, came also from the spectral shape measurements of suitable β transitions (^{12}B, ^{12}N) where, owing to a large energy release and a large M1 matrix element, this twice-forbidden correction to the allowed approximation becomes measurable[5].

There is another way of verifying the CVC theory which is even more tightly connected with electrodynamics and is an extension of the Siegert theorem in electrodynamics. In fact, the first experimental evidence for the CVC theory was obtained by Fujita from the RaE spectrum[31]. Many attempts were undertaken later to prove or disprove this theorem on a large number of β transitions, particularly by J. Deutsch and co-workers. As the CVC theory predicts a ratio of matrix elements to have a given value, and matrix elements cannot usually be determined from spectral shapes only, this work is not a subject of a detailed treatment in this paper; in the case of RaE, however, the spectral shape alone is sufficient.

The axial vector current part of the weak interaction has no analogue in electrodynamics. It is not surprising that it is not completely conserved. This is manifested by the ratio[32] $|C_A|/|C_V| = 1.19\pm0.02$ instead of unity.

There may, however, be a partially conserved axial vector current (PCAC). Krmpotić and Tadić[33] made attempts to draw conclusions from the spectral shapes of $0^- - 0^+$ transitions.

Unfortunately, the experimental situation which is the basis of theoretical analysis is not at all clear for the three transitions which are suitable: $^{144}Ce \rightarrow {}^{144}Pr$, $^{144}Pr \rightarrow {}^{144}Nd$, and $^{166}Ho \rightarrow {}^{166}Er$. The most favourable experimental conditions are those found in the decay of ^{144}Pr. Here, a recent result from Heidelberg[28] is in fair agreement with earlier studies[6,34], but there are small differences which may lead to conflicting conclusions[33]. No experimental differences exist for ^{144}Ce, as there is only one measurement[28], but the spectrum was obtained by subtracting the ^{144}Pr spectrum extrapolated down to low energies from the measured ($^{144}Ce + {}^{144}Pr$) spectrum. This extrapolation procedure is, of course, doubtful in the case of complicated and not yet understood spectrum, such as that of ^{144}Pr. The Heidelberg[28] result for ^{166}Ho is also obtained by subtraction of a computed component from the measured sum spectrum. Here the component is of the first forbidden unique type. Its shape should be known, except for an uncertainty coming from the presence or non-presence of a small correction term, treated in Section 5. However, a recent direct measurement[35] of this unique component yielded a large deviation. If this is really true then the new shape factor must be taken for the subtraction, and a differing shape for the $0^- \to 0^+$ component will result from the measurement of the sum spectrum.

The ^{144}Pr shape as measured at Argonne[34] can be explained with a PCAC and with no G-parity violation, while the ^{144}Pr and ^{166}Ho shapes as measured at Heidelberg[28] can be explained with the G-parity violation only[33].

10. NEUTRINO REST MASS AND NEUTRINO DEGENERACY

The question of the neutrino rest mass has always been interesting since the first postulation of this particle by Pauli[36]. As a particle with the vanishing rest mass always moves with the velocity of light, but a particle with the non-vanishing rest mass at small kinetic energy behaves non-relativistically, one expects the largest deviation to show up near the β end point where the neutrino energy is very small. As the absolute resolving width E or p of any of the suitable spectrometers increases with increasing energy E, it is advantageous to investigate transitions with low maximum energy E_o. For β^- decay, tritium (^3H) is best suited. This gives the antineutrino mass. If one does not believe in the particle-antiparticle concept a priori, one has to measure also a β^+ emitter. Unfortunately, no β^+ emitter with high intensity and low maximum energy is provided by nature, because of competing electron capture. ^{22}Na seems to be a suitable compromise.

Neutrino degeneracy, i.e. the availability of empty neutrino or antineutrino states below zero energy or the non-availability of states above zero energy, also gives deviations at the end of the β spectrum, either below the end point extrapolated from the Fermi plot (states of positive energy are filled) or above the end point (states of negative energy are empty). If the particle-antiparticle concept for neutrino and antineutrino (here defined as the chargeless particle emitted in β^+ and β^- decay, respectively) is correct, then empty neutrino states below zero are connected with filled antineutrino states above zero, and vice versa. Detailed, also with regard to cosmology, were given by Weinberg[7].

Experimentally no indications of finite (anti-) neutrino rest mass or of neutrino degeneracy were found. The presently known upper limit for the antineutrino rest mass comes from two experiments on tritium, both already performed a long time ago[37,38]. Although somewhat lower upper limits are given in the respective papers, the present author is inclined to state only an upper limit of 1 keV, at a confidence level of 90%. However, there are experiments in progress which one can hope will yield a lower upper limit.

Figure 7 shows the ^3H Fermi plot as measured by Langer and Moffat[37].

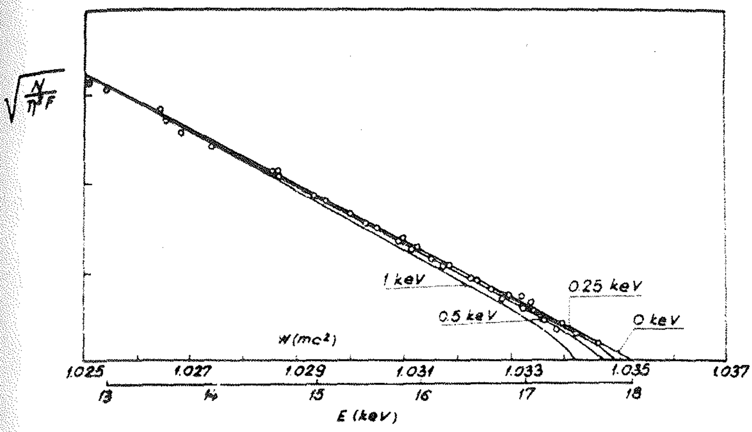

Fig. 7. ^3H Fermi plot as measured with a 180° shaped field spectrometer[37], for the determination of the antineutrino rest mass.

For the neutrino (from β^+ decay) no direct value was available until very recently. Now a preliminary result has been obtained at Heidelberg[39], $m_\nu < 6$ keV at 90% confidence.

From Fig. 7 one may conclude that the antineutrino Fermi energy is

$$E_F^{(\bar{\nu})} < 1 \text{ keV}.$$

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TABLE 2

Allowed β decay

[References are quoted in Ref. 6)]

Nuclide States log ft	E_0 (MeV)	Coefficients a and b $[a,(mc^2)^{-1};\ b, mc^2]$	Author and year	
^{6}He $0^+ \rightarrow 1^+$ 2.9	3.50	$-0.20 < b < 0.08$	Scwarzschild	57
^{12}B $1^+ \rightarrow 0^+$ 4.1	13.38	a = 0.0093±0.0004	Mayer-Kuckuk et al.	62
		a = 0.0028±0.0005	Lee et al.	63
^{12}N $1^+ \rightarrow 0^+$	16.36	a = 0.0031±0.0004	Mayer-Kuckuk et al.	62
	16.43	a = -0.0027±0.0003	Lee et al.	63
^{13}N $1/2^- \rightarrow 1/2^-$ 3.7	1.190	b = 0.001 ±0.024	Daniel et al.	58
^{18}F $1^+ \rightarrow 0^+$ 3.6	0.635	a = 0.0034±0.0091	Hofmann	64
^{22}Na	0.543	$0.25 < b < 0.35$	Hamilton et al.	58
$3^+ \rightarrow 2^+$	0.545	b = -0.016 ±0.020	Daniel	58
7.4		b = 0.0008±0.0020	Leutz	61
	0.544	$0.1 < b < 0.3$	Brantley et al.	64
		b = 0.001 ±0.003	Leutz et al.	67a
^{24}Na	1.394	$-0.026 < b < 0.020$	Porter et al.	57
$4^+ \rightarrow 4^+$ 6.1	1.389	a = -0.0150±0.0045 or b = 0.072 ±0.023	Daniel	58
	1.388	a = 0.000 ±0.005	Depommier et al.	61
	1.392	a = -0.012 ±0.006	Paul et al.	63
	1.393	a = -0.005 ±0.007	Lehmann	64
	1.394	a = 0.002 ±0.010	Beekhuis et al.	65

a: see Ref. 12)

TABLE 2 (contd.)

Nuclide States log ft	E_0 (MeV)	Coefficients a and b $[a,(mc^2)^{-1};\ b,mc^2]$	Author and year
^{32}P	1.712	$b = 0.03 \pm 0.04$	Pohm et al. 56
$1^+ \to 0^+$	1.712	$b = -0.032 \pm 0.045$	Pohm et al. 56
7.9	1.711	$0.05 < b < 0.093$	Porter et al. 57
	1.705	$a = -0.041 \pm 0.013$	Daniel 58
		$a \approx -0.02$	Graham et al. 58
	1.711	$0.2 < b < 0.4$	Johnson et al. 58
		$a = 0.35$ and $b = 4.9$	Brabec et al. 58
	1.708	$a = -0.0133 \pm 0.0011$	Nichols et al. 61
	1.705	$a = -0.042 \pm 0.010$	Fehrentz et al. 61
	1.706	$a = -0.03$	Depommier et al. 61
		$a = -0.025 \pm 0.007$ and $b = 0.12 \pm 0.05$; $b = 0.195 \pm 0.020$	Ch´ing-Ch´eng Jui et al. 62
	1.700	$a = -0.025$	Sharma et al. 63
		$a \approx 0$	Quivy 64
		$a = 0 \pm 0.01$	Persson et al. 65
	1.71	$a = -0.09$ (300–600 keV) ; $a = -0.01$ (600–1650 keV)	Lehmann 65
^{41}A $7/2^- \to 7/2^-$ 5.0	1.198	$a = -0.017 \pm 0.005$	Paul 64
^{47}Ca $7/2^- \to 7/2^-$ 8.5	1.979	$0 < b < 0.3$	Langer et al. 63
^{56}Mn $3^+ \to 2^+$ 7.2	2.838	$0 < b < 0.3$	Howe et al. 62
^{56}Co $4^+ \to 4^+$ 8.7	1.46	$a = -0.24$ and $b = 0.04$ or $0.2 < b < 0.3$	Hamilton et al. 61
^{58}Co $2^+ \to 2^+$ 6.6	0.474	$b = 0.3$	Rhode et al. 63

TABLE 2 (contd.)

Nuclide State log ft	E_0 (MeV)	Coefficients a and b $[a,(mc^2)^{-1}; b, mc^2]$	Author and year	
^{60}Co $5^+ \to 4^+$ 7.5	0.32	$a \approx 0$	Bonhoeffer	59
^{66}Ga $0^+ \to 0^+$ 7.8	4.153	complicated	Camp et al.	63
^{89}Zr $9/2^+ \to 9/2^+$ 6.1	0.90	$a = -0.39$ <u>and</u> $b = 0.09$ $0.25 < b < 0.45$	Hamilton et al.	60
^{110m}Ag $6^+ \to 6^+$ 8.2	0.529	$a = -0.01 \pm 0.03$	Daniel et al.	63
^{114}In $1^+ \to 0^+$ 4.4	1.996	$0.2 < b < 0.3$	Johnson et al.	58
	1.989	$a = 0.0036 \pm 0.0021$	Nichols et al.	61
	1.987	$b = 0.05 \pm 0.02$	Daniel et al.	61
	1.988	$a = -0.0015 \pm 0.0030$ or $b = 0.005 \pm 0.022$	Daniel et al.	64
	1.980	$a = -0.0005 \pm 0.0020$	André et al.	64
^{131}I $7/2^+ \to 5/2^+$ 6.6	0.606	$a = 0.02 \pm 0.02$	Daniel et al.	64

TABLE 3

UNIQUE FORBIDDEN β SPECTRA

[References are quoted in Ref. 6)]

Nucleus States log ft	E_0 (MeV)	Coefficients a and b [a, $(mc^2)^{-1}$; b, mc^2]	Author and year
^{41}A $7/2^- \rightarrow 3/2^+$ 8.6	2.48	a = 0.00 ± 0.01	Kartashov 61
^{42}K $2^- \rightarrow 0^+$ 8.4	3.52	a = −0.010 ± 0.004	Daniel et al. 64
^{84}Rb $2^- \rightarrow 0^+$ 8.5	1.657	b = 0.2	Langer et al. 64
^{86}Rb $2^- \rightarrow 0^+$ 8.4	1.774	a = −0.017 ± 0.002	Daniel et al. 64
^{90}Sr $0^+ \rightarrow 2^-$ 8.3	0.546	a = −0.054 ± 0.019	Daniel et al. 64
^{88}Y $4^- \rightarrow 2^+$ 8.6	0.76	a = 0.0 ± 0.1	Rhode 63
^{90}Y $2^- \rightarrow 0^+$ 8.3	2.261	0.2 < b < 0.3	Johnson et al. 53
	2.265	b = 0.025	Yuasa 57
	2.271	a = −0.0047±0.0008	Nichols et al. 61
	2.268	b = 0.26 ± 0.03	André et al. 64
	2.273	0.30 < b < 0.40	Langer et al. 64
	2.284	a = −0.0072±0.0032	Daniel et al. 64
	2.280	a = −0.0064±0.0016	Riehs 64
^{91}Y $1/2^- \rightarrow 5/2^+$ 8.5	1.545	0.3 < b < 0.4	Langer et al. 64
^{142}Pr $2^- \rightarrow 0^+$ 7.8	2.164	a = −0.025 ± 0.003	Beekhuis 66

Deviations from unique shape.

TABLE 3 (contd.)

Nucleus states log ft	E_o (MeV)	Coefficients a and b [a, $(mc^2)^{-1}$; b, mc^2]	Author and year	
^{166}Ho $0^- \rightarrow 2^+$ 8.0	1.78	a = -0.105 ± 0.010	Beekhuis	66
^{172}Tm $2^- \rightarrow 0^+$ 8.6	1.87	[shape like ^{90}Y]	Hansen	66
^{172}Tm $2^- \rightarrow 4^+$ 9.8	1.61	[shape like ^{90}Y]	Hansen	66
^{198}Au $2^- \rightarrow 0^+$ 11.3	1.37	[see original paper]	Elliott	54
^{204}Tl $2^- \rightarrow 0^+$ 9.0	0.76	a ≈ -0.02 deviation at E < 90 keV	Egelkraut Leutz	60 62
^{10}Be $0^+ \rightarrow 3^+$ 14.5	0.555	[upper 3/4 2nd unique]	Feldman et al.	50a
^{22}Na $3^+ \rightarrow 0^+$ 13	1.83	[2nd unique]	Wright	53b
^{60}Co $5^+ \rightarrow 2^+$ 12.7	1.48 1.48	[p^2 + 5.79 q^2] 2nd unique shape	Keister Wolfson	54 56
^{40}K $4^- \rightarrow 0^+$ 18.1	1.30	3rd unique shape	Leutz	65

a: see Ref. 21).

b: B.T. Wright, Phys. Rev. 90, 159 (1953).

TABLE 4

Non-unique forbidden β spectra

[References are quoted in Ref. 6)]

Nucleus States log ft	E_0 (MeV)	Coefficients a and b $[a,(mc^2)^{-1};b,mc^2;c,(mc^2)^{-2}]$	Author and year
^{42}K $2^- \to 2^+$ 7.5	1.98	$0 < a < 0.01$	Pohm et al. 56
	2.00	$a = -0.12 \pm 0.04$ $b = 0.67 \pm 0.06$ $c = 0.013 \pm 0.008$	André et al. 64
	2.00	$a = 0.15 \pm 0.26$ $b = 0.81 \pm 0.47$ $c = -0.02 \pm 0.03$	Daniel et al. 65
^{72}Ga $3^- \to 2^+$ 9.2	3.15	$[q^2 + \lambda_2 p^2 + 15 \pm 10]$	Langer et al. 60
^{72}Ga $3^- \to 2^+$ 8.7	2.52	$[q^2 + 0.95p^2 \pm 7]$	Langer et al. 60
^{76}As $2^- \to 2^+$ 8.2	2.42	$a = 0.00 \pm 0.04$	Pohm et al. 56
^{84}Rb $2^- \to 2^+$ 7.1	0.78	$b \approx 0.3$	Langer et al. 64
^{86}Rb $2^- \to 2^+$ 7.7	0.72	$0.4 < b < 0.6$	Robinson et al. 58
	0.722	$a \approx 0.00 \pm 0.05$	Deutsch et al. 61
		$a = -0.7 \pm 0.7$ $b = -0.5 \pm 0.7$ $c = 0.14 \pm 0.15$	Daniel et al. 65
		$a \approx 0$	Spejewski 66

TABLE 4 (contd.)

Nucleus states log ft	E_o (MeV)	Coefficients a and b $[a,(mc^2)^{-1};b,mc^2;c,(mc^2)^{-2}]$	Author and year
^{91}Y $1/2^- \to 1/2^+$ 8.8	0.32	$a \approx 0$	Johnson et al. 60
^{111}Ag $1/2^- \to 3/2^+$ 7.9	0.69	$a \approx -0.17$	Robinson et al. 58
^{115m}Cd $11/2^- \to 9/2^+$ 8.7	1.618	$a = -0.78$ and $b = -17.2$	Sharma et al. 63
^{115m}Cd $11/2^- \to 11/2^+$ 8.3	0.34	$a \approx 0$	Johnson et al. 59
^{124}Sb $3^- \to 2^+$ 10.2	2.317	$[q^2 + 0.874p^2 + 15 \pm 5]$ $[q^2 + 0.874p^2 + 7 \pm 4]$	Langer et al. 60
^{130}I $5^- \to 5^+$ 6.5	1.04	$a \approx 0.00 \pm 0.04$	Daniel et al. 65
^{130}I $5^- \to 6^+$ 5.7	0.62	$a \approx 0.00 \pm 0.04$	Daniel et al. 65
^{140}La $3^- \to 2^+$ 9.2	2.175	$[q^2 + 0.845p^2 + 18 \pm 5]$	Langer et al. 60
^{141}Ce $7/2^- \to 7/2^+$ 6.9	0.432	$a = 0.00 \pm 0.15$	Deutsch et al. 61
^{142}Pr $2^- \to 2^+$ 7.1	0.56	$a \approx 0.0 \pm 0.1$	Hess et al. 64
^{144}Ce $0^+ \to 0^-$ 7.5	0.316	$a = -0.342 \pm 0.008$	Daniel et al. 66

TABLE 4 (contd.)

Nucleus States log ft	E_0 (MeV)	Coefficients a and b $[a,(mc^2)^{-1};b,mc^2;c,(mc^2)^{-2}]$	Author and year
^{143}Pr			
$5/2^+ \to 7/2^-$	0.933	$0.1 < b < 0.35$	Hamilton et al. 58
7.6		$a = -0.018 \pm 0.010$ $b = 0.06 \pm 0.03$	Persson et al. 6[illegible]
		$b = 0.3$	Spejewski 6[illegible]
^{144}Pr			
$0^- \to 0^+$	2.99	[see original paper]	Laubitz 56
6.5	2.984	$[\lambda = i\int\gamma_5/\int\sigma r > 0]$	Graham et al. 58
	2.992	$0 < \lambda < 10$	Freeman 58
	2.996	$\lambda = 5 \pm 2$	Porter et al. 58
	3.000	$a = 0.0376$ $b = -0.118$ $c = -0.0077$	Daniel et al. 6[illegible]
^{147}Nd			
$5/2^- \to 5/2^+$	0.790	$a = -0.23$	Sharma et al. 62
7.4	0.806	$a = -0.07 \pm 0.01$	Beekhuis et al. 66
^{147}Nd			
$5/2^- \to 5/2^+$	0.360	$a \approx 0$	Sharma et al. 62
7.0	0.364	$a = -0.20 \pm 0.15$	Beekhuis et al. 66
^{147}Pm			
$5/2^+ \to 7/2^-$	0.224	$0 < b < \approx 0.3$	Hamilton et al. 58
7.4			
^{148}Pm			
$1^- \to 0^+$		$[\int \sigma \times r / \int ir = -2.2 \pm 0.4]$	Baba et al. 63
9.1			
^{150}Eu			
$1^-,0^- \to 0^+$	1.020	$a \approx 0$	Yoshizawa et al. 63
6.2			
^{152}Eu			
$3^- \to 2^+$	1.48	$[q^2 + 0.79p^2 + 5 \pm 2]$	Langer et al. 60
12.3	1.492	as above	Schneider 60

TABLE 4 (contd.)

Nucleus states log ft	E_0 (MeV)	Coefficients a and b [a, $(mc^2)^{-1}$; b, mc^2; c, $(mc^2)^{-2}$]	Author and year
^{154}Eu $3^- \to 2^+$ 11.8	1.855	$[q^2 + 0.807p^2 + 20 \pm 5]$	Langer et al. 60
^{166}Ho $0^- \to 0^+$ 8.0	1.857	$[1 - 0.87W - 1.03/W + 0.225W^2 - 0.021W^3]$	Daniel et al. 66
	1.846	$a = -0.21 \pm 0.03$ $c = 0.038 \pm 0.004$	Beekhuis 67a
^{170}Tm $1^- \to 2^+$ 9.3		$a = 1.11$ $b = -0.85$ $c = -0.24$	Spejewski 66
^{172}Tm $2^- \to 2^+$ 8.4	1.79	[shape like ^{90}Y]	Hansen et al. 66
^{176m}Lu $1^- \to 2^+$ 6.5		$a = -0.005 \pm 0.010$	Deutsch 65
^{185}W $3/2^- \to 5/2^+$ 6.5		$a = 0$	Spejewski 66
^{186}Re $1^- \to 0^+$ 7.7	1.071	$a \approx 0.12$	Porter et al. 56
^{186}Re $1^- \to 2^+$ 8.0	0.934	$a \approx -0.12$	Porter et al. 56
^{194}Ir $1^- \to 2^+$ 8.6		$a = -0.01 \pm 0.02$	Deutsch et al. 65

... see Ref. 35).

TABLE 4 (contd.)

Nucleus States log ft	E_o (MeV)	Coefficients a and b $[a,(mc^2)^{-1};b,mc^2;c,(mc^2)^{-2}]$	Author and year	
^{198}Au	0.966	$a = -0.110 \pm 0.017$	Wapstra et al.	59
$2^- \rightarrow 2^+$	0.968	$a = -0.142 \pm 0.010$	de Vries et al.	60
7.3	0.964	$a = -0.046 \pm 0.010$	Graham	60
	0.962	$a = -0.062 \pm 0.007$	Chabre et al.	61
	0.957	$a \approx -0.02$	Sharma et al.	62
	0.960	$a = -0.33$, $c = 0.074$	Hamilton et al.	62
		$a = -0.30$, $c = 0.07$	Newbolt	64
	0.965	$a = -0.155 \pm 0.015$	Lehmann et al.	64
	0.960	$a = -0.017 \pm 0.006$	Keeler et al.	65
		$a = -0.014 \pm 0.024$	Lewin	65
	0.960	$a = -0.33 \pm 0.09$ $c = 0.068 \pm 0.022$	Parsignault	65
	0.962	$a = -0.34 \pm 0.04$ $c = 0.10 \pm 0.02$	Lachkar et al.	65
	0.961	$a = -0.057 \pm 0.006$	Paul	65
	0.962	$a = -0.050 \pm 0.010$	Beekhuis	65
		$a = -0.073 \pm 0.008$	Spejewski	66
^{199}Au $3/2^+ \rightarrow 1/2^-$ 7.6	0.46	$-0.2 < a < -0.4$	Lehmann	66
^{206}Tl $0^- \rightarrow 0^+$ 5.2	1.57	$a = -0.154$, $b = -0.484$	Howe et al.	61
^{210}Bi $1^- \rightarrow 0^+$ 8.0	1.155	[$\approx$ below]	Plassmann et al.	54
	1.160	$a = 0.578$ $b = 28.466$ $c = -0.658$	Daniel	62
^{36}Cl $2^+ \rightarrow 0^+$ 13.3	0.714	$[p^2 + 0.6q^2]$	Feldman et al.	52
	0.714	$[p^2 + (0.57 \pm 0.03)q^2]$	Johnson et al.	56

TABLE 4 (contd.)

Nucleus states log ft	E_o (MeV)	Coefficients a and b $[a,(mc^2)^{-1};b,mc^2;c,(mc^2)^{-2}]$	Author and year
^{46}Sc $4^+ \to 2^+$ 13	1.48	$[\sim(p^2 + 0.6q^2)]$	Wolfson 56
^{59}Fe $3/2^- \to 7/2^-$ 10.9	1.573	$[p^2 + 3.3\, q^2]$	Wortman et al. 63
^{58}Co $2^+ \to 0^+$ 12.9	1.3	$[\sim p^2]$	Daniel 58b
^{99}Tc $9/2^+ \to 5/2^+$ 12.3	0.290	$[p^2 + (2.0 \pm 0.5)q^2]$	Feldman 52
^{129}I $7/2^+ \to 3/2^+$ 13.5	0.150	$[p^2 + (10 \pm 1)q^2]$	der Mateosian et al. 53
^{135}Cs $7/2^+ \to 3/2^+$ 13.2	0.205	$[p^2 + (10 \pm 1)q^2]$	Lidofsky et al. 53
^{137}Cs $7/2^+ \to 3/2^+$ 12		$[q^2 + 0.0085p^2]$	Langer et al. 51
		$[q^2 + 0.003p^2]$	Yamazaki et al. 58
		$[q^2 + (0.015 \pm 0.004)p^2]$	Daniel et al. 62
		$[q^2 + (0.004 \pm 0.002)p^2]$	Hsue et al. 66
^{87}Rb $3/2^- \to 9/2^+$ 17.6	0.275	[see original paper]	Egelkraut et al. 61

b: H. Daniel, Z. Phys. 150, 144 (1958).