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The quantum waves of Minkowski space-time and the minimal acceleration from precanonical quantum gravity

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Abstract. We construct the simplest solutions of the previously obtained precanonical Schrödinger equation for quantum gravity, which correspond to the plane waves on the spin connection bundle and reproduce the Minkowski space-time on average. Quantum fluctuations lead to the emergence of the minimal acceleration a_0 related to the range of the Yukawa modes in the fibers of the spin connection bundle. This minimal acceleration is proportional to the square root of the cosmological constant Λ generated by the operator re-ordering in the precanonical Schrödinger equation. Thus the mysterious connection between the minimal acceleration in the dynamics of galaxies as described by Milgrom's MOND and the cosmological constant emerges as an elementary effect of precanonical quantum gravity. We also argue that the observable values of a_0 and Λ can be obtained when the scale of the parameter $\varkappa$ introduced by precanonical quantization is subnuclear, in agreement with the previously established connection between the scale of $\varkappa$ and the mass gap in quantum SU(2) Yang-Mills theory.

1. Introduction

The approaches to quantum gravity based on applying the standard methods of quantization to different versions of the gravitational Lagrangian [1, 2] almost inevitably lead to fundamental conceptual and technical problems of quantum gravity, such as the mathematical definition of the Wheeler-De Witt equation, the problem of time, the interpretation of (the measurement problem in) quantum cosmology, the problem of the correct classical limit in the loop quantum gravity, and the 122 orders of magnitude discrepancy between the theoretically plausible value of the cosmological constant and the observable one.

The approach called *precanonical quantization* [3–7] was proposed as a response to these problems. It departs from the various forms of canonical quantization, which are based on the canonical Hamiltonian formalism with a necessarily distinguished time variable, and instead uses a space-time symmetric generalization of the Hamiltonian formalism from mechanics to field theory known in the calculus of variations under the name of the De Donder-Weyl (DW) Hamiltonian formulation [8–10]. The precanonical quantization is based on the Dirac quantization of the Heisenberg-like subalgebra of Poisson-Gerstenhaber brackets of differential forms representing dynamical variables, which were found in the DW Hamiltonian formulation in [6, 11–14] and further explored and generalized e.g. in [15–18].

Quantization of brackets defined on differential forms naturally leads to a hypercomplex generalization of quantum theory where operators and wave functions are Clifford-algebra-

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valued [4–7]. The Clifford algebra in question is the complexified Clifford algebra of space-time. The DW Hamiltonian formulation and the quantization of Poisson-Gerstenhaber brackets of differential forms are space-time symmetric by construction. No distinction between space and time variables is required. No notion of field configurations or their initial or boundary data, i. e. the sections of the bundle whose base is space-time and whose fibers are spaces where the fields take values, is required by the procedure of precanonical quantization. The quantum dynamics of fields is described using the sections of the Clifford bundle over the bundle of field variables ϕ^a over the space-time with the coordinates x^μ. These sections are called precanonical wave functions and, in general, have the form (in $n = 1 + 3$ dimensions)

$$\Psi(\phi^a, x^\mu) = \psi + \psi_\mu \gamma^\mu + \frac{1}{2!}\psi_{\mu\nu}\gamma^{\mu\nu} + ... + \frac{1}{4!}\psi_{\mu_1\mu_2...\mu_4}\gamma^{\mu_1\mu_2...\mu_4}. \quad (1)$$

The field variables ϕ^a can be the Yang-Mills field variables A^a_μ [7, 19–21] or metric density variables $h^{\mu\nu}$ [22–25], or tetrad variables e^I_μ [26], or spin connection variables ω^{IJ}_μ [27–31]. The covariant analogue of the Schrödinger equation for the precanonical wave function has the form [22–25, 27–31]

$$i\hbar\varkappa\hat{\nabla\!\!\!/}\Psi - \hat{H}\Psi = 0, \quad (2)$$

where $\hat{H}$ is the operator of the covariant analogue of the Hamiltonian in the DW Hamiltonian-like formulation:

$$H := \partial_\mu\phi^a p^\mu_{\phi^a} - L, \quad p^\mu_{\phi^a} := \frac{\partial L}{\partial\partial_\mu\phi^a}, \quad (3)$$

$\hat{\nabla\!\!\!/}$ is the operator of the covariant Dirac operator on the space-time, and the parameter $\varkappa$ is an ultraviolet quantity of the dimension of the inverse spatial volume. It appears on purely dimensional grounds given the fact that the physical dimension of classical H is that of the mass density (in $c = 1$ units used throughout the paper). Note that $\varkappa$ is also introduced in the course of the precanonical quantization when the Poisson-Gerstenhaber brackets are replaced by commutators, and the representation of operators corresponding to differential forms is constructed in terms of Clifford-algebra-valued operators. In particular, the 3-dimensional volume element $d\mathbf{x} := dx^1 \wedge dx^2 \wedge dx^3$ is mapped to the Clifford algebra element

$$d\mathbf{x} \mapsto \frac{1}{\varkappa}\underline{\gamma}_0, \quad (4)$$

where $\underline{\gamma}_I$ denote the flat space-time Dirac matrices, $\underline{\gamma}_I\underline{\gamma}_J + \underline{\gamma}_J\underline{\gamma}_I = 2\eta_{IJ}$, $I, J = 0, 1, 2, 3$. This map is very similar to what is known as the "quantization map" or "Chevalley map" in the Clifford algebra literature.

The relation between the description of quantum fields in terms of Clifford-algebra-valued precanonical wave functions $\Psi(\phi^a, x^\mu)$ and the standard QFT can be established if the latter is formulated in the Schrödinger functional picture [32]. In this picture, QFT is described in terms of time-dependent functionals of initial field configurations $\phi^a(\mathbf{x})$, $\mathbf{\Psi}([\phi^a(\mathbf{x})], t)$, which obey the canonical Schrödinger equation

$$i\hbar\partial_t\mathbf{\Psi} - \hat{\mathbf{H}}\mathbf{\Psi} = 0, \quad (5)$$

where $\hat{\mathbf{H}} = \int d\mathbf{x}\hat{T}^0_0$ is the operator of the canonical Hamiltonian.

Taking into account that the precanonical wave function $\Psi(\phi^a, x^\mu)$ gives the probability amplitude of detecting the field value ϕ at the space-time point x and the Schrödinger wave functional $\mathbf{\Psi}([\phi^a(\mathbf{x})], t)$ is the probability amplitude of observing the field configuration $\phi(\mathbf{x})$ on the hypersurface of constant time t, one can anticipate that the Schrödinger wave functional is a continuous product, or product integral, over the spatial points $\mathbf{x}$, of precanonical wave functions restricted to the configuration Σ given by $\phi = \phi(\mathbf{x})$, i.e. $\Psi_\Sigma(\phi = \phi(\mathbf{x}), \mathbf{x}, t)$, and

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transformed from the representation with a diagonal $\hat{H}$ to the representation with a diagonal $\hat{T}^0_0 = \hat{H} - \partial_i\phi^a(\mathbf{x})\hat{p}^i_{\phi_a}$. The resulting expression of the Schrödinger wave functional $\mathbf{\Psi}$ in terms of the precanonical wave functions Ψ is given by the continuous product or the Volterra product integral over $\mathbf{x}$ denoted as $\prod_{\mathbf{x}}$:

$$\mathbf{\Psi} = \mathsf{Tr}\left\{\prod_{\mathbf{x}} e^{-i\phi(\mathbf{x})\gamma^i\partial_i\phi(\mathbf{x})/\varkappa}\Psi_\Sigma(\phi(\mathbf{x}),\mathbf{x},t)\ |_{\frac{1}{\varkappa}\mapsto\underline{\gamma}^0 d\mathbf{x}}\right\}, \tag{6}$$

where the inverse of the quantization map (4) is used as a natural step of transformation from the Clifford algebraic objects of precanonical quantization to the $\mathbb{C}$-valued functional of the canonical quantum field theory in the Schrödinger representation. In [34, 35] we have shown, for interacting scalar fields in Minkowski space-time, that $\mathbf{\Psi}$ constructed in equation (6) satisfies the standard Schrödinger equation (5) as a consequence of (the flat space-time version of) the precanonical Schrödinger equation (2) restricted to the surface of initial data $\phi^a = \phi^a(\mathbf{x})$ at the moment of time t. A similar relation has been found also for quantum Yang-Mills theory on Minkowski space-time [19, 21] and for scalar fields on curved space-time [36–38]. The existence of this relationship shows that standard QFT based on canonical quantization is the limiting case of QFT based on precanonical quantization corresponding to the inverse quantization map $\frac{1}{\varkappa}\underline{\gamma}_0 \mapsto d\mathbf{x}$ or, loosely speaking, to the limiting case of infinite $\varkappa$ corresponding to the unregularized volume of the momentum space, which equals $\delta^3(\mathbf{x}=0)$.

Let us also note that the existence of the space-time symmetric Hamilton-Jacobi (HJ) theory of fields which is associated with the DW Hamiltonian theory [8–10, 40–42] raises a question about the existence of a formulation of quantum field theories which reproduces the DW HJ theory in the classical limit. The precanonical quantization leads to such a formulation, at least in the case of scalar fields (cf. [4, 43]).

In the previous papers, the precanonical quantization has been applied to general relativity in metric variables [22–25], to the teleparallel equivalent of general relativity [26], and to general relativity in vielbein variables [27–31]. In this paper, we will briefly outline the latter in Section 2 and then, in Section 3, construct the solutions of the precanonical Schrödinger equation for quantum gravity corresponding to the quantum version of Minkowski space-time in Cartesian coordinates. We will find that quantum effects lead to the emergence of minimal acceleration related to the range of the Yukawa modes of the precanonical wave function in the spin-connection space. We will show that this minimal acceleration is related to the square root of the cosmological constant. The latter appears from the reordering of operators in the precanonical Schrödinger equation for gravity. We will also obtain realistic estimations of both quantities, albeit with an error of several orders of magnitude, when the scale of the parameter $\varkappa$ is below approximately 100 MeV, which is consistent with our previous rough estimation of the mass gap in the quantum SU(2) gauge theory [20].

2. Precanonical quantum vielbein gravity

In this section, we mainly collect together the key results from our previous considerations in [27–30]. The construction of precanonical quantum vielbein gravity starts from the Einstein-Palatini Lagrangian density

$$\mathfrak{L} = \frac{1}{8\pi G}\mathfrak{e}e^{[\alpha}_I e^{\beta]}_J(\partial_\alpha\omega^{IJ}_\beta + \omega_\alpha{}^{IK}\omega_{\beta K}{}^J) + \frac{1}{8\pi G}\Lambda\mathfrak{e}, \tag{7}$$

where the vielbein coefficients e^α_I and the spin connection coefficients $\omega_\alpha{}^{IK}$ are the independent field variables, and $\mathfrak{e} := \det(e^I_\mu)$. The DW Hamiltonian formulation leads to the constraints

$$\mathfrak{p}^\alpha_{\omega^{IJ}_\beta} := \frac{\partial\mathfrak{L}}{\partial\,\partial_\alpha\omega^{IJ}_\beta} \approx \frac{1}{8\pi G}\mathfrak{e}e^{[\alpha}_I e^{\beta]}_J, \quad \mathfrak{p}^\alpha_{e^I_\beta} := \frac{\partial\mathfrak{L}}{\partial\,\partial_\alpha e^I_\beta} \approx 0 \tag{8}$$

and the DW Hamiltonian density on the surface of constraints

$$\mathfrak{H} := \mathfrak{p}_\omega \partial\omega + \mathfrak{p}_e \partial e - \mathfrak{L} \approx -\mathfrak{p}^\alpha_{\omega^{IJ}_\beta} \omega_\alpha{}^{IK}\omega_{\beta K}{}^J - \frac{1}{8\pi G}\Lambda\mathfrak{e}. \tag{9}$$

The constraints are second class according to the extension of the Dirac classification to the DW theory [39]. The calculation of the generalized Dirac brackets of forms representing the fundamental variables leads to the vanishing brackets of vielbeins and their polymomenta and very simple brackets of spin connection coefficients and their polymomenta, e.g.,

$$\{\![\mathfrak{p}^\alpha_e, e'\varpi_{\alpha'}]\!\}^D = 0,$$
$$\{\![p^\alpha_\omega, \omega'\varpi_\beta]\!\}^D = \{\![p^\alpha_\omega, \omega'\varpi_\beta]\!\} = \delta^\alpha_\beta \delta^\omega_{\omega'},$$
$$\{\![\mathfrak{p}^\alpha_e, \mathfrak{p}_\omega\varpi_{\alpha'}]\!\}^D = \{\![\mathfrak{p}^\alpha_e, \omega\varpi_{\alpha'}]\!\}^D = \{\![\mathfrak{p}^\alpha_\omega, e'\varpi_{\alpha'}]\!\}^D = 0,$$

where $\varpi_\alpha := \partial_\alpha \lrcorner\ dx^0 \wedge dx^1 \wedge ... \wedge dx^3$ is the basis of 3-forms on 4-dimensional spacetime. Quantization of these brackets according to the following generalization of Dirac's quantization rule

$$[\hat{A}, \hat{B}] = -i\hbar\widehat{\mathfrak{e}\{\![A, B]\!\}^D} \tag{10}$$

which leads to the operator representations of the polymomenta of spin connection, the vielbeins, the DW Hamiltonian H, such that $\widehat{\mathfrak{H}} =: \widehat{\mathfrak{e}H}$, and the quantum Dirac operator:

$$\widehat{\mathfrak{p}}^\alpha_{\omega^{IJ}_\beta} = -i\hbar\varkappa\mathfrak{e}\hat{\gamma}^{[\alpha}\frac{\partial}{\partial\omega^{IJ}_{\beta]}}, \quad \text{where} \quad \hat{\gamma}^\alpha := \hat{e}^\alpha_I \underline{\gamma}^I, \tag{11}$$

$$\hat{e}^\beta_I = -8\pi i G\hbar\varkappa\underline{\gamma}^J\frac{\partial}{\partial\omega^{IJ}_\beta}, \tag{12}$$

$$\widehat{H} = 8\pi G\hbar^2\varkappa^2\ \underline{\gamma}^{IJ}\omega_\alpha{}^{KM}\omega_{\beta M}{}^L\frac{\partial}{\partial\omega^{KL}_\beta}\frac{\partial}{\partial\omega^{IJ}_\alpha} - \frac{1}{8\pi G}\Lambda, \tag{13}$$

$$\widehat{\not{\nabla}} = -8\pi i G\hbar\varkappa\underline{\gamma}^{IJ}\frac{\partial}{\partial\omega^{IJ}_\mu}\left(\partial_\mu + \frac{1}{4}\,\omega_{\mu KL}\underline{\gamma}^{KL}\overset{\leftrightarrow}{\vee}\right), \tag{14}$$

where $\overset{\leftrightarrow}{\vee}$ denotes the commutator (antisymmetric) Clifford product $\underline{\gamma}^{IJ}\overset{\leftrightarrow}{\vee}\Psi = \frac{1}{2}\left[\,\underline{\gamma}^{IJ}, \Psi\,\right]$. Hence the precanonical Schrödinger equation for quantum gravity takes the form

$$\underline{\gamma}^{IJ}\frac{\partial}{\partial\omega^{IJ}_\mu}\left(\partial_\mu + \frac{1}{4}\omega_{\mu KL}\underline{\gamma}^{KL}\overset{\leftrightarrow}{\vee} - \frac{\partial}{\partial\omega^{KL}_\beta}\omega_\mu{}^{KM}\omega_{\beta M}{}^L\right)\Psi(\omega, x) + \lambda\Psi(\omega, x) = 0, \tag{15}$$

where $\lambda := \frac{\Lambda}{(8\pi G\hbar\varkappa)^2}$ is a dimensionless combination of the fundamental constants of the theory, which depends on the operator ordering of ω and ∂_ω.

Note that equation (15) was first obtained in [27] without explicitly specifying the action of the spin connection term on the wave function. The need for the commutator product was understood later in [36], and the coefficient $\frac{1}{2}$ in front of the commutator comes from the consideration of the Ehrenfest theorem similar to that in [7], which is still unpublished.

The scalar product of precanonical wave functions is given by

$$\langle\Phi|\Psi\rangle := \mathrm{Tr}\int \overline{\Phi}\,\widehat{[d\omega]}\,\Psi, \tag{16}$$

where $\overline{\Psi} := \underline{\gamma}^0\Psi^\dagger\underline{\gamma}^0$ and the operator-valued invariant integration measure on the 24-dimensional space of spin connection coefficients

$$\widehat{[d\omega]} \sim \hat{\mathfrak{e}}^{-6}\prod_{\mu IJ} d\omega^{IJ}_\mu. \tag{17}$$

The operator $\hat{\mathfrak{e}}^{-1}$ is constructed from the operators $\hat{e}^{\beta}_{I}$ in (12).

Thus, we arrive at the "spin connection foam" formulation of the geometry of quantum gravity in terms of the Clifford-algebra-valued wave function on the bundle of spin connection coefficients over spacetime, $\Psi(\omega, x)$, and the transition amplitudes on the total space of this bundle, $\langle \omega, x|\omega', x'\rangle$, which are the Green functions of (15). The wave function corresponds to the "quantum fuzziness" of points of the total space and the Green functions correspond to the quantum correlations between the points, i.e. a quantum analogue of the classical connection.

The normalizability of precanonical wave functions: $\langle\Psi|\Psi\rangle < \infty$, leads to the vanishing contribution of the large curvatures $R = d\omega + \omega \wedge \omega$ to the probabilistic measure defined by the norm, and that ensures the quantum-gravitational avoidance of a curvature singularity by the precanonical wave function.

In the context of quantum cosmology, $\Psi(\omega, x)$ defines the spatially homogeneous statistics of local fluctuations of the spin-connection, which is classically given by the Hubble parameter $\dot{a}/a$, not the "distribution of quantum universes according to the Hubble parameter" as suggested by the picture of the superspace of 3-geometries emerging from the canonical quantization. Hence the problem of the "external observer" of the "quantum ensemble of universes" disappears.

The evolution of matter and radiation on the background of quantum gravitational fluctuations whose statistics and correlations are predicted by (15) may lead to predictable consequences for the distribution of matter and radiation at large cosmological scales, which may be observable.

In general, analysis of solutions of precanonical Schrödinger equation for quantum gravity, equation (15), is a formidable task. In the following section, we will construct the simplest solutions which can be interpreted as a quantum wave counterpart of the Minkowski space-time.

3. Quantum wave states of Minkowski space-time

The Minkowski metric in Cartesian coordinates:

$$ds^2 = (dx^0)^2 - (dx^1)^2 - (dx^2)^2 - (dx^3)^2,$$

is characterized by the vanishing spin connection coefficients

$$\omega^{IJ}_{\mu} = 0. \tag{18}$$

In this case, the precanonical Schrödinger equation (15) with $\Lambda = 0$ takes the simple form

$$\underline{\gamma}^{IJ}\partial_{\omega^{IJ}_{\mu}}\partial_{\mu}\Psi = 0. \tag{19}$$

In terms of the plane waves on the total space $(x^{\mu}, \omega^{IJ}_{\mu})$

$$\Psi \sim e^{ik_{\mu}x^{\mu} + i\pi^{\mu}_{IJ}\omega^{IJ}_{\mu}}\,\tilde{\Psi}(k_{\mu}, \pi^{\mu}_{IJ}) \tag{20}$$

we obtain

$$\underline{\gamma}^{IJ}k_{\mu}\pi^{\mu}_{IJ} = 0. \tag{21}$$

From (21), we obtain the dispersion relation

$$k_{\mu}\pi^{\mu}_{IJ}k_{\nu}\pi^{\nu IJ} = 0, \tag{22}$$

which reflects a strong anisotropy due to the fibred structure of the (x, ω) space.

Therefore, any solution of (19) has the form

$$\Psi(\omega, x) = \int d^4k \int d^{24}\pi\ \delta(k_{\mu}\pi^{\mu}_{IJ}k_{\nu}\pi^{\nu}_{IJ})e^{ik_{\mu}x^{\mu} + i\pi^{\mu}_{IJ}\omega^{IJ}_{\mu}}\,\tilde{\Psi}(k_{\mu}, \pi^{\mu}_{IJ}). \tag{23}$$

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The solutions of interest should be normalizable. The normalizability on the subspace of vanishing spin connection coefficients $\omega = 0$ takes the form

$$\mathrm{Tr} \int d^{24}\omega\, \delta^{24}(\omega)\overline{\Psi}(\omega, x)\hat{\mathfrak{e}}^{-6}\Psi(\omega, x) = \mathrm{Tr}(\overline{\Psi}(0, x)\hat{\mathfrak{e}}^{-6}\Psi(0, x)) = 1, \tag{24}$$

where the short-hand notations $\overline{\Psi}(0, x)$ and $\hat{\mathfrak{e}}^{-6}\Psi(0, x)$ mean $\overline{\Psi}(\omega, x)$ and $\hat{\mathfrak{e}}^{-6}\Psi(\omega, x)$ taken at $\omega = 0$.

The states which lead to the Minkowski space-time on the classical level have to satisfy the conditions

$$\langle \hat{g}^{\mu\nu} \rangle(x) = \mathrm{Tr} \int d^{24}\omega\delta^{24}(\omega)\overline{\Psi}(\omega, x)\hat{\mathfrak{e}}^{-6}\hat{g}^{\mu\nu}\Psi(\omega, x)) = \eta^{\mu\nu},$$

where the operator of the metric derived from the representation (12) has the form

$$\hat{g}^{\mu\nu} = -(8\pi G)^2\hbar^2\varkappa^2\eta^{IK}\eta^{JL}\partial_{\omega_\mu^{IJ}}\partial_{\omega_\nu^{KL}}. \tag{25}$$

Hence the wave functions which reproduce the Minkowski space-time on average should satisfy

$$\mathrm{Tr}(\overline{\Psi}(0, x)\hat{\mathfrak{e}}^{-6}\hat{g}^{\mu\nu}(\overline{\Psi}(0, x)) = \eta^{\mu\nu}. \tag{26}$$

In terms of the Fourier components (26) implies that

$$\tilde{\Psi}(\pi, k) = \tilde{\Psi}(-\pi, k). \tag{27}$$

By comparison with the normalizability condition, we conclude that

$$-(8\pi G)^2\hbar^2\varkappa^2\eta^{IK}\eta^{JL}\partial_{\omega_\mu^{IJ}}\partial_{\omega_\nu^{KL}}\Psi(0, x) = \eta^{\mu\nu}\Psi(0, x).$$

Therefore, for the plane waves,

$$(8\pi G)^2\hbar^2\varkappa^2\pi^\mu_{IJ}\pi^{\nu IJ} = \eta^{\mu\nu}. \tag{28}$$

Then, from the dispersion relation, it follows

$$\eta^{\mu\nu}k_\mu k_\nu = 0. \tag{29}$$

Thus, the states corresponding to the (1+3)-dimensional Minkowski space-time in the classical limit have:

- the light-like modes (29) along the space-time dimensions (the base of the total space of the bundle of spin connection coefficients over space-time);
- 4 massive (Yukawa) modes (28) in the spin-connection spaces (the fibers of the total space of the bundle of spin connection coefficients over space-time), which propagate in 6-dimensional subspaces with the coordinates ω_μ^{IJ} for each $\mu = 0, 1, 2, 3$;
- the range of those massive modes in ω-space is $8\pi G\hbar\varkappa$, whose value we estimate below;
- the modes corresponding to the spatial $\mu = 1, 2, 3$ are tachyonic. Those tachyonic modes, however, do not violate the causality in space-time as they propagate along the fibers associated to each point of space-time rather than in the space-time itself.

Note that the spin connection has the mass dimension +1, $\varkappa$ has the mass dimension +3, and the square of the Planck length $G\hbar$ has the mass dimension −2. Hence the range of the Yukawa modes in the spin connection space, $8\pi G\hbar\varkappa$, is given in the units of mass dimension +1, which is also the mass dimension of acceleration.

If $\varkappa$ were Planckian, which is a seemingly natural first guess, then the range of Yukawa modes in the spin connection space is also Planckian, and they could be attributed to the quantum foaminess of space-time at the Planck scale, as is usually assumed. However, our study of quantum Yang-Mills theory from the perspective of precanonical quantization (see below) has produced evidence that $\varkappa$ is more likely a sub-nuclear scale quantity, which leads to a drastically different scale of the phenomena in question.

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3.1. An estimation of $\varkappa$ from the mass gap in pure gauge theory

From the Lagrangian of a pure non-abelian Yang-Mills theory we can derive the corresponding DW Hamiltonian function [7, 19] and precanonically quantize it. It leads to the following expression for the DW Hamiltonian operator for the quantum pure YM field with the coupling constant g [7, 19, 20]

$$\widehat{H} = \frac{1}{2}\hbar^2\varkappa^2 \frac{\partial}{\partial A_a^\mu \partial A_\mu^a} - \frac{1}{2} i g \hbar \varkappa C^a{}_{bc} A_\mu^b A_\nu^c \gamma^\nu \frac{\partial}{\partial A_\mu^a} \,. \tag{30}$$

The fact that the eigenvalues of the DW Hamiltonian operator for the pure YM field yield the spectrum of masses of the propagating modes is manifested in the precanonical Schrödinger equation in flat space-time

$$i\hbar\gamma^\mu\partial_\mu\Psi = \frac{1}{\varkappa}\widehat{H}\Psi. \tag{31}$$

In [21], we have shown that equations (30) and (31) for the wave function $\Psi(A, x)$ reproduce the functional Schrödinger equation for the wave functional $\mathbf{\Psi}([A(\mathbf{x})], t)$ after the (3+1) decomposition and the "dequantization map" $\frac{1}{\varkappa}\gamma_0 \mapsto \varpi_0 = d\mathbf{x}$ (cf. (4)).

In the temporal gauge $A_0^a = 0$, we can limit ourselves to the operator (30) written only in terms of the spatial components A_i^a. For SU(2) theory with $a, b, c = 1, 2, 3$ and $i, j = 1, 2, 3$, we were able to estimate the gap between the ground state and the first excitation with the vanishing non-abelian charge (a "color" or rather "isospin" in the context of SU(2))

$$\langle \frac{1}{\varkappa}\hat{H}\rangle > \left(\frac{8g^2\hbar^4\varkappa}{32}\right)^{1/3} |\mathsf{ai}'_1|, \tag{32}$$

where ai'_1 is the first root of the derivative of the Airy function. This gap in the spectrum of the DW Hamiltonian operator can be identified with the mass gap

$$\Delta\mu \approx 0.86(g^2\hbar^4\varkappa)^{1/3}. \tag{33}$$

Therefore, the scale of $\varkappa$ is close to the scale of the mass gap. For SU(3) YM theory, this formula will have a different numerical coefficient in front and a different value of the coupling constant g. The SU(3) QCD mass gap lies between the pion masses at 130 MeV and the alleged glueball masses at a few GeV. The numerical factor in (33) for SU(3) may change several times, and the coupling constant g is of the order $10^{-2} - 10^{-1}$ (in the units of $\sqrt{\hbar}$). With those uncertainties, we estimate $\varkappa^{1/3}$ is below 1 GeV with an error of up to 2 orders of magnitude.

3.2. The minimal acceleration

With the GeV-scale $\varkappa$ we obtain

$$8\pi G\hbar\varkappa \sim 10^{-23\pm3\times2} \text{ cm}^{-1}. \tag{34}$$

This quantity is compatible with the scale of the Hubble radius $R_H \sim 10^{28}$ cm and the cosmological constant $|\Lambda| \sim 10^{-56}$ cm^{-2}, if the scale of $\varkappa$ is below 100 MeV, which is on the edge of our margin of error. In this case, $8\pi G\hbar\varkappa$ coincides with the scale of the minimal acceleration $a_0 \sim \sqrt{\Lambda}$ which is known from Milgrom's theory of MOND [44–48]. Note that the Yukawa modes in the spin-connection space, which set the threshold of acceleration $8\pi G\hbar\varkappa$, emerge from quantum fluctuations of spin connection around the vanishing value of the spin connection of Minkowski space-time. They establish the limit below which quantum fluctuations of space-time violate the notion of acceleration-less inertial frames which underlies classical Minkowski space-time. Note also that our value of the minimal acceleration appears here in the context of a quantum analogue of the Minkowski space-time and it may slightly change for the quantum analogues of more realistic cosmological space-times.

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3.3. The cosmological constant

We have already pointed out that the expression of the precanonical Schrödinger equation for gravity (15) is defined up to the operator ordering of ω and ∂_ω. A reordering in the spin connection term will produce a constant of the order $\frac{1}{4}4^3$ added to the dimensionless $\lambda = \frac{\Lambda}{(8\pi G\hbar\varkappa)^2}$ constructed from the bare cosmological constant Λ. If the latter equals zero, then the contribution to the cosmological constant from the reordering of operators, i.e. essentially from the quantum fluctuations of the spin connection, can be estimated as $\Lambda_\omega \sim 4^2(8\pi G\hbar\varkappa)^2$. For $\varkappa \sim 10^{0\pm 3\times 2}$ GeV^3, in agreement with the estimation in Section 3.1, we obtain $\Lambda_\omega \sim 10^{-45\pm 2\times 6}$ cm^{-2}. This estimation is again consistent with the observed value of the cosmological constant if the scale of $\varkappa$ is below approximately 100 MeV. In this case, the minimal acceleration in (34) is related to Λ_ω as follows

$$a_0 \approx \frac{1}{4}\sqrt{\Lambda_\omega}, \tag{35}$$

which is close to the current observed value $a_0 \approx 1.2 \times 10^{-10}$ m·s^{-2} or $a_0 \approx 10^{-29}$ cm^{-1} in the $c = 1$ units. Thus the mysterious connection between the phenomenological minimal acceleration in the dynamics of galaxies as described by MOND (as an alternative to the dark matter) and the cosmological constant (as the simplest dark energy) emerges as an elementary effect of precanonical quantum gravity.

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