The Role of Quark Spin in Hadronisation

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Abstract: The unpolarized and the Collins fragmentation functions (FF) quantitatively describe the hadronization of a polarized quark to unpolarized hadrons. They are needed for mapping the 3D structure of the nucleon from the semi-inclusive deep inelastic scattering experiments. We present our recent results in modeling the polarized quark hadronization in sequential hadron emission picture. Using the spin density matrix formalism, we describe the elementary $q \to q' + h$ process using the eight leading twist quark-to-quark transverse-momentum-dependent (TMD) elementary FFs. The unpolarized and the Collins FFs of light quarks to pions are then calculated using a Monte Carlo (MC) implementation of this model by utilizing the quark-jet framework for the sequential hadronization. We outline the the distinctive features of the resulting Collins FFs that reflect the underlying hadronization mechanism, such as the treatment of the intrinsic transverse momentum of the produced hadrons in the quark-jet framework. These polarized FFs can be precisely measured at FCC-ee, that would allow us to discriminate between different mechanisms for hadronisation, that in turn would provide a detailed description of various deep-inelastic hadron production processes.

Introduction

One of the most challenging topics in high energy physics has been the description of the parton hadronization process because of its non-perturbative nature. The FFs that quantify the hadronization process are universal, in that according to the QCD factorization theorems they enter into the cross sections of various hard scattering processes with observed final state hadrons [1]. The so-called transverse-momentum-dependent (TMD) unpolarized FF can be interpreted as a number density for a quark to produce a hadron that carries a fraction of its light-cone momentum and a transverse momentum with respect to the momentum of the original quark. The modulations of this probability density for unpolarized hadrons produced by a transversely polarized quark is described by the so-called Collins FF [2]. The TMD FFs are needed to extract the TMD parton distribution functions, that encode the 3D structure of the nucleon in the momentum space, from semi-inclusive deep inelastic scattering experiments. One of the most widely used approach for describing hadronization is based on the Lund string model [3] and implemented in the Monte Carlo event generators PYTHIA [4]. Nevertheless, at present the polarized quark hadronization is not implemented in any MC event generator, and it is not possible to simulate the Collins effect.

Recently, we developed a self-consistent description of the polarized quark hadronization and a corresponding MC framework for calculating transversely polarized quark to pion FFs based on the extended quark-jet model [5],[6]. The quark-jet model describes the hadronization of a quark as a sequential emission of hadrons that do not interact with each other or re-interact with the remnant, as schematically depicted in Fig. 1. The quark to hadron fragmentation functions are then calculated as the corresponding number densities, either using integral equations or Monte Carlo techniques [7],[8],[9],[10],[11],[12],[13],[14]. Here we highlight the key findings of Refs. [5] and [6] in the perspective of future precise measurements of polarized FFs in FCC-ee experiment.

Elementary $q \to q' + h$ process

We consider a quark hadronization mechanism where hadrons are produced one at a time in a sequential manner $q \to q_1 + h_1$, $q_1 \to q_2 + h_2$, etc. Thus, to describe this process we need to know both the probability density for the initial quark q to produce a final quark q_1 of a given flavor and momentum, as well as how the polarization is transferred from q to q_1. Let's denote the spin density matrices of q and q_1 as ρ_q and ρ_{q_1} respectively, that are completely determined by the corresponding polarization vectors $\mathbf{s}_q$ and $\mathbf{s}_{q_1}$. The probability density for this transition can be expressed in terms of the respective density matrices ρ_q and ρ_{q_1},

$$f^{q\to q_1} = Tr[\rho_{q_1} A \rho_q \overline{A}], \tag{1}$$

where A is some matrix describing the interaction with the other particles in this process. Then the probability density $f^{q\to q_1}$ should be a linear function in both $\mathbf{s}_q$ and $\mathbf{s}_{q_1}$,

$$f^{q\to q_1}(\mathbf{s}_q, \mathbf{s}_{q_1}) = \alpha_q + \beta_q \cdot \mathbf{s}_{q_1}, \tag{2}$$

where both α_q and β_q are linear functions of $\mathbf{s}_q$ that also depend on the momenta of the quarks. We can express these coefficients in terms of the 8 leading-twist quark-to-quark TMD elementary FFs (see Refs. [5],[6])

$$\alpha_q \equiv \hat{D}(z_1, p_{1\perp}^2) + \frac{(\mathbf{p}_{1\perp} \times \mathbf{s}_T)\cdot \hat{\mathbf{z}}}{z_1 \mathcal{M}}\ \hat{H}^{\perp}(z_1, p_{1\perp}^2), \tag{3}$$

$$\beta_{q\parallel} \equiv s_L\ \hat{G}_L(z_1, p_{1\perp}^2) - \frac{(\mathbf{p}_{1\perp}\cdot \mathbf{s}_T)}{z_1 \mathcal{M}} \hat{H}_L^{\perp}(z_1, p_{1\perp}^2), \tag{4}$$

$$\begin{aligned} \beta_{q\perp} \equiv & \frac{\mathbf{p}'_{1\perp}}{z_1 \mathcal{M}} \hat{D}_T^{\perp}(z_1, p_{1\perp}^2) - \frac{\mathbf{p}_{1\perp}}{z_1 \mathcal{M}} s_L \hat{G}_T(z_1, p_{1\perp}^2) \\ & + \mathbf{s}_T\ \hat{H}_T(z_1, p_{1\perp}^2) + \frac{\mathbf{p}_{1\perp}(\mathbf{p}_{1\perp}\cdot \mathbf{s}_T)}{z_1^2 \mathcal{M}^2}\ \hat{H}_T^{\perp}(z_1, p_{1\perp}^2), \end{aligned} \tag{5}$$

where z_1 and $\mathbf{p}_{1\perp}$ are the light-cone momentum fraction and the transverse momentum of q_1 with respect to q, while $\mathcal{M}$ is the mass of q_1. The momentum vector $\mathbf{p}'_{1\perp} \equiv (-p_{1,y}, p_{1,x})$. The unit vector $\hat{\mathbf{z}}$ denotes the direction of the 3-momentum of q, which also helps to define $\mathbf{s}_T$ and s_L as the transverse and longitudinal components of $\mathbf{s}_q = (\mathbf{s}_T, s_L)$. In this work we use hats on TMD elementary FFs to distinguish them from the analogous TMD FFs .

Let us not that the quark q_1 is unobserved, then its polarization is completely determined by $\mathbf{s}_q$, z_1 and $\mathbf{p}_{1\perp}$. It can be expressed as $\mathbf{s}_{q_1} = \beta_q/\alpha_q$. The probability to produce quark q_1 with light-cone momentum fraction z_1 and transverse momentum $\mathbf{p}_{1\perp}$ is determined from Eq. (2), $\hat{f}^{q\to q_1}(z_1, \mathbf{p}_{1\perp}; \mathbf{s}_q) = \alpha_q$. The next fragmentation steps $q_1 \to q_2$, can be treated in a completely analogous manner, where the results are expressed via light-cone momentum fraction η_2 and transverse momentum $\mathbf{p}_{2\perp}$ of quark q_2 relative to q_1. Nevertheless, since $\mathbf{s}_{q_1}$ itself is determined by $\mathbf{s}_q$, we can infer that $\mathbf{s}_{q_2}$ should also be completely determined by $\mathbf{s}_q$, as well as the light-cone momentum fraction z_2 and transverse momentum $\mathbf{P}_{2\perp}$ of quark q_2 with respect to q. Then, in the quark-jet framework, the probability of the $q \to q_2$ transition is given by

$$\hat{f}^{(2)}_{q\to q_2}(z_2, \mathbf{P}_{2\perp}; \mathbf{s}_q) = \hat{f}^{q\to q_1}(z_1, \mathbf{p}_{1\perp}; \mathbf{s}_q) \otimes \hat{f}^{q_1\to q_2}(\eta_2, \mathbf{p}_{2\perp}; \mathbf{s}_{q_1}), \tag{6}$$

where the convolution $\otimes$ relates the corresponding relative momenta. We can then iterate this procedure for the subsequent fragmentation steps in a completely analogous manner.

MC Implementation and Results

The iterative mechanism for the quark polarization transfer described in the previous section allows us to readily adapt the extended quark-jet framework for MC simulations of the polarized quark hadronization process with a finite number of produced hadrons, similar to our previous work in Refs. [9],[12], and [13]. The basic concept is to adapt the number density implementation of the FFs, which then can be calculated using Monte Carlo techniques as averages of these densities taken over a large number of quark hadronization event simulations. In the instance of polarized quark fragmentation into unpolarized hadrons, the corresponding number density is the following polarized fragmentation function:

$$D_{h/q^{\uparrow}}(z, p_{\perp}^2, \varphi) = D^{h/q}(z, p_{\perp}^2) - H^{\perp h/q}(z, p_{\perp}^2) \frac{p_{\perp} s_T}{z m_h} \sin(\varphi_C), \tag{7}$$

where $D^{h/q}(z, p_{\perp}^2)$ and $H^{\perp h/q}(z, p_{\perp}^2)$ denote the unpolarized and Collins fragmentation function, respectively. The variables z and $p_{\perp}^2$ are the light-cone momentum fraction and the transverse momentum squared of the produced hadron with respect to the momentum of the initial fragmenting quark, and m_h denotes its mass. Here, s_T is the modulus of the transverse component of the quark's polarization. The Collins angle for the hadron $\varphi_C \equiv \varphi - \varphi_s$ is defined as the difference of the azimuthal angles of the produced hadron's transverse momentum φ and the transverse polarization of the initial quark φ_s. We calculate $D_{h/q^{\uparrow}}(z, p_{\perp}^2, \varphi)$ by computing the average number of hadrons h with given momenta produced in the hadronization chain of q. This can be accomplished by sampling the remnant quark's momentum according to the elementary quark-to-quark splitting functions, and calculating the type and the momentum of the produced hadron using flavor and momentum conservation. Then we calculate the polarization of the remnant quark using its momentum from the polarization of the fragmenting quark. We can continue the hadronization chain until we reach some predetermined termination condition, which we choose as a given number of produced hadrons N_L. The hadrons produced at the nth step in the hadronization chain are called rank-n hadrons.

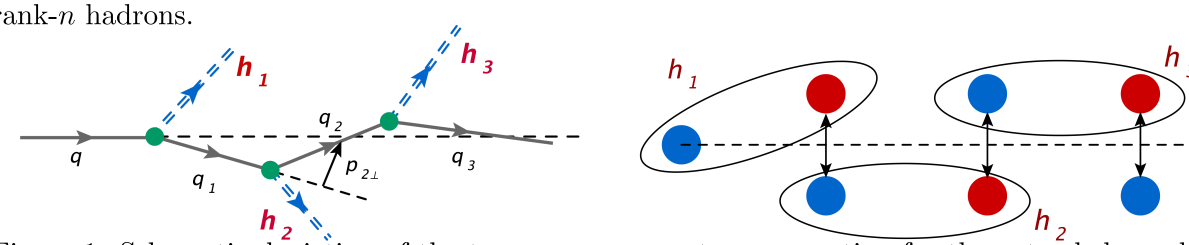

Figure 1: Schematic depiction of the transverse momentum generation for the extended quark-jet framework and the Lund model.

Here we point out a distinctive feature of the quark-jet model in describing the intrinsic transverse momentum of the produced hadrons, which is schematically depicted in the left panel of Fig. 1. In the i-th hadron emission step, we sample the light cone momentum fraction η_i and the transverse momentum $\mathbf{p}_{i\perp}$ of the remnant quark q_i with respect to the fragmenting quark q_{i-1}. The light-cone momentum fraction z_i of the remnant quark with respect to the initial quark q is given by the simple relation $z_i = \eta_1 \cdot ... \cdot \eta_i = z_{i-1} \cdot \eta_i$. To obtain the transverse momentum in the initial quark frame, we need to perform a Lorentz transform that preserves the light-cone momentum fraction. The resulting expression,

$$\mathbf{P}_{i\perp} = \eta_i \; \mathbf{p}_{i-1\perp} + \mathbf{p}_{i\perp}, \tag{8}$$

shows that the transverse momentum of the remnant quark (and the emitted hadron via momentum conservation), gets a contribution from the transverse momenta of the preceding quarks in the hadronization chain. The same is true for the produced hadrons, where the transverse momentum of the hadron emitted at a given step gets a contribution from the recoil of the transverse momenta of previously emitted hadrons. This is different in Lund model, where the different string breaks, that create the $q\overline{q}$ pairs, are causally disconnected. Thus, the direction of the string does not rotate after each quark pair creation, and only the transverse momenta of the hadrons of neighboring ranks can be correlated. For example, the transverse momenta of hadrons h_1 and h_3 in the left panel of Fig. 1 depicting the quark-jet model are correlated, while those in the right panel, schematically depicting the Lund model, are not.

The input for the MC calculations of the polarized FFs are the eight TMD elementary quark-to-quark FFs that describe the one step process and the polarization transfer. These elementary FFs have been modeled in Ref. [6]. Here we discuss the results for the calculations of the unpolarized and Collins functions of pions produced by an up quark. The isospin symmetry, assumed to be exact in the model, then can be used to extract the results for the down quark FFs. The plots in Figs. 2 show the unpolarized FFs, and the analyzing powers for an up quark fragmenting to pions. The results for the analyzing power of the Collins effect show the opposite sign for the favored and unfavored channels, and become equal in size at $z \simeq 0.2$. These are in agreement with recent the results by COMPASS, STAR and BELLE experiments that measured significant asymmetries at $z \simeq 0.2$ of opposite signs for the favored and unfavored FFs. In the future work, we can tune our results to best fit experimental data by changing the input elementary quark-to-quark FFs, as we have demonstrated in Ref. [6] a significant dependence of our results on these functions.

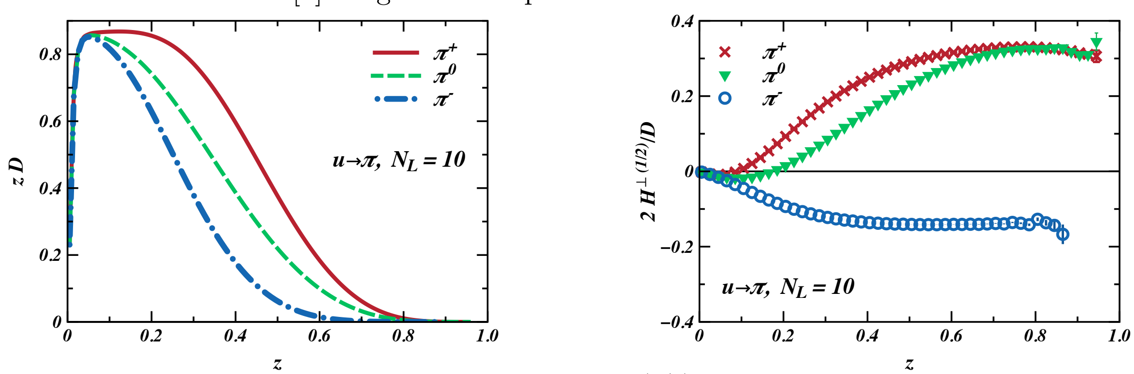

Figure 2: Fitted values of zD (right panel), and $2H^{\perp(1/2)}/D$ (left panel) as a function of z from Monte Carlo simulations for $u \to \pi$, with $N_L = 10$ emitted hadrons.

Conclusions

The accurate description of the polarized quark hadronization process remains one of the most challenging aspects in the phenomenological description of deep inelastic scattering processes. For example, the treatment of the quark polarization and the corresponding correlations are, to date, not included in any of the well-known event generators, such as PYTHIA [4] . Here we presented a self-consistent model for polarized quark hadronization in an iterative setting, and the MC implementation of this model using the extended quark-jet hadronization framework, as first described in Refs. [5] and [6]. The MC approach was used to calculate the TMD polarized FFs of light quarks to pions, namely the unpolarized and Collins FFs. The results for the unpolarized FFs and the ratios of the 1/2 moments of the Collins functions to the unpolarized FFs were presented in Fig. 2. The analyzing powers demonstrated distinctive features: opposite sign for the large z values for favored and unfavored channels. The results for the favored channel then fall off in magnitude more rapidly than the unfavored ones with decreasing z, and they cross the zero at some small z. These features reflect the underlying quark-jet hadronization mechanism, including the treatment of the hadron transverse momentum. It is also interesting to note that the shapes of the analyzing powers and the zero crossover points for the favored ones drastically depend on the forms of the input splitting functions [6]. The inclusion of the strange quarks and kaons, as well as the vector meson production and strong decays, will allow one to precisely describe a large range of phenomena that involve polarized quark hadronization. The computation of various polarized dihadron FFs will provide an improved set of predictions compared to our previous work [14] with a simplistic model. Further work on the model calculations of the input TMD FFs would give more predictive power to the framework which can be tested by precisely measuring the polarized FFs at FCC-ee in the future. At the same time, the polarization transfer mechanism used in this work can be readily adapted into the well-known MC event generators such as PYTHIA [4], with parametric forms for the input functions that can be tuned to best reproduce various experimental data.

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