# A Study on the logarithm correction of black hole entropy

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**Abstract**. The logarithm correction of black hole entropy is important in understanding the essence of black hole entropy, providing a more accurate entropy calculation. We reviewed the mainstream method of logarithm correction of black hole entropy, including quantum loop gravity correction, conformal field theory correction, and classical thermal correction. Specifically, the correction of quantum loop gravity presents a stable general expression of logarithm correction, which only depends on the surface area of the black hole and solves the problem of meaningless entropy solution under a large length scale. Besides, the correction of the Cardy formula of conformal field theory is limited for the third term in depends on the mass of the black hole, which will finally lead to the unstable coefficient before the correction term. Finally, the correction deduced by the classical thermal method also gives a general expression of black hole entropy. In contrast, the entropy of BTZ black hole has a different coefficient before the logarithm term comparing to other kinds of the black hole. These results shed light for the research in general logarithm correction of black hole entropy, which is suitable for all kinds of black holes.

**Keywords**: Black hole entropy, Logarithm correction, Quantum loop gravity,Cardy formula

## 1. Introduction

Blackhole entropy is defined as a quantity description and measurement of the inner state and structure of the black hole. The black hole entropy expression deduced by Bekenstein and Hawking is proportional to the surface area and only becomes meaningful at quantum scale [1]. However, the scale of the black hole surface area is commonly larger than the quantum scale, which indicates the necessity to correct the expression of entropy. The general correction form is logarithm:

$$S = S_0 - c\ln(S_0) + \cdots \tag{1}$$

where the $S_0$ is the Bekenstein-Hawking entropy, and c is an under-determined coefficient.

Up to now, there are two common attitudes towards these correction forms. One group of scholars admits the validity of B-H entropy and goes to details of the correction. Some studies try to introduce quantum loop gravity [2] and quantum geometry [3] into the system. Two reasons support this: First, the expression of B-H black hole is working only on the quantum scale, which implies the possibility of quantization of black hole entropy. Meanwhile, due to the development of quantum gravity, researchers realized that the computation of black hole entropy could be given by counting the microstates of the black hole [4]. However, the development of logarithm correction using quantum loop gravity has almost paused since the general solution was proposed [5]. Further developments of this theory in black hole entropy are often combined with conformal field theory to gain the advantage of symmetry [6] and investigate black holes in higher dimensions [7]. To some degree, research of quantum loop gravity correction seems to be given by the form of logarithm and the introduction of other forms of correction that can depict the physical picture of black hole entropy more precisely [8, 9]. Nevertheless, the logarithm correction of quantum loop theory is still the mainstream and commonly used method in research, much more common than the conformal field theory and thermal fluctuation. Although these two methods were trying to give a special explanation of black hole entropy from their point of view, none of them gives a stable general solution of black hole entropy like what quantum loop theory did. Another group of people refuses to accept the concept proposed by Bekenstein for the definition of black hole entropy usually relies on two beliefs:

The black hole is a 3-dimensional system in 4-dimensions space.

The common understanding of black hole entropy is based on Boltzmann-Gibbs entropy.

There is no reason that we have to understand the black hole entropy in this way, and this assumption will finally violate the second law of thermal dynamics [10, 11]. They argued that the black hole entropy should be proportional to the logarithm of surface area and plank length [12]. This argument is reasonable and pulls us back to look carefully at logarithm correction of black hole entropy.

This review will list the basic correction ideas from the point of quantum loop theory, conformal field theory, and classical thermal dynamics and then discuss the future development and dilemma of the logarithm correction.

## 2. The correction of the logarithm by quantization

As discussed in the introduction, the attempt to quantization of black hole entropy has the most convincing evidence if we still adapt the Hawking-Bekenstein entropy. There are different kinds of ways of quantization. Still, no one becomes the mainstream of this topic until the quantum loop theory comes out and quickly becomes the mainstream of black hole entropy interpretation. The reason may relate to the high popularity of quantum gravity and the longing for the grand unified theory.

This method is reported by Krzysztof A.Meissmer in 2004 [5] and has been improved in details of computation by J Fernando Barbero G and Eduardo J S Villasenor in 2009 [13]. This review shows only the original derivation since the basic physical ideas are the same.

Assuming:

$$N(a) = Ce^{2\pi\gamma_M a} \tag{2}$$

$$\sum_i \sqrt{|m_i|(|m_i| + 1)} < a, \tag{3}$$

the black hole entropy of such quantum loop gravity is given by:

$$S = \ln N(a) = \frac{\gamma_M}{4\gamma}\frac{A}{l_p^2} + O(\ln A) \tag{4}$$

where $\gamma_M$ is a constant which can be determined numerically, A is the surface area of the black hole, $\gamma$ is a parameter introduced by [14], $l_p$ is the plank length given by $l_p = \sqrt{\frac{hG}{c^3}}$, $a = \frac{A}{8\pi\gamma l_p^2}$, and we require $m_i \in \frac{Z}{2}$ and $m_i \neq 0$.

Comparing this result with the result of the Bekenstein-Hawking formula, we know that:

$$\gamma = \gamma_M \tag{5}$$

which implies the so-called "quantum area" has been fixed in this problem. Subsequently, we need to define two functions P(s) and G(s), where P(s) is defined as the Laplace transform of N(a):

$$P(s) = \frac{2\sum_{k=1}^{\infty} e^{-s\sqrt{k(k+2)/4}}}{s(1-2\sum_{k=1}^{\infty} e^{-s\sqrt{k(k+2)/4}})} \tag{6}$$

$$G(s) := \sum_{k=1}^{\infty} e^{-s\sqrt{k(k+1)/4}} \tag{7}$$

The growth of the N(a) is determined by the poles of P(s), i.e.,:

$$N(a) = \sum_{s_i, Re(s_i)>0} res_{s_i} e^{s_i a} + O(a^n) \tag{8}$$

On this basis, $s_i$ needs to be found in order to derive the expression of N(a). Thus, our problem is to solve the equation and get the value of s:

$$G(s) - \frac{1}{2} = 0 \tag{9}$$

By defining a new continuous function in $R(s) \geq 0$:

$$G_r(s) := G(s) - \sum_{k=1}^{\infty} e^{-s(k+1)/2} - \frac{s}{4}\sum_{k=1}^{\infty} \frac{e^{-s(k+1)/2}}{k} \tag{10}$$

Analyzing the distribution around zeroes of Eq. (9), it is easily found that the only real zero is given by:

$$s = \bar{\gamma}_M \tag{11}$$

Then, with the help of Table. 1, P(s) and N(a) can be found, for large a:

$$P(s) \sim \frac{C_M}{s-\bar{\gamma}_M},\ N(a) = C_M e^{\bar{\gamma}_M a} \tag{12}$$

where $C_M = -\frac{1}{2\bar{\gamma}_M G'(\bar{\gamma}_M)} = 0.509202564$.

**Table 1.** The complex zeroes of $G(s) - \frac{1}{2} = 0$ when Re(s)>1 and |Im(s)|<100. This table may help to find the solution and research the behavior of the equation (9) around zeroes [5].

| Re(s) | Im(s) |
|---|---|
| **1.49246359** ... | 0 |
| **1.22393017** ... | ±22.1530069 ... |
| **1.41016352** ... | ±35.9362749 ... |
| **1.30363023** ... | ±58.0472739 ... |
| **1.18587535** ... | ±71.8281531 ... |
| **1.17654302** ... | ±79.9673974 ... |
| **1.07191106** ... | ±87.4120423 ... |
| **1.21660746** ... | ±93.9713059 ... |

According to the results, Krzysztof A. Meissner proposed the P(s) deduced by a different statistical model, including the Boltzmann statistics, the intermediate statistics, and Bose statistics. Now, introducing $N(a,p)$ which satisfies Eq. (3) and $\sum_i m_i = p$ where $p \in \frac{Z}{2}$ and $a \geq \sqrt{|p|(|p|+1)}$ . By solving the recurrence relation of $N(a,p)$ and then imposing the constrain:

$$\sum_i m_i = 0 \tag{13}$$

In other words, let $p = 0$, the expression of $N(a,0)$ can be solved as:

$$N(a,0) = \frac{c_M}{\sqrt{4\pi \quad _M a}} e^{2\pi\gamma_M a} \tag{14}$$

Therefore, the expression of the entropy is given by:

$$S = \ln N(a,0) = \frac{\gamma_M}{4\gamma}\frac{A}{l_p^2} - \frac{1}{2}\ln\left(\frac{A}{l_p^2}\right) + O(1) \tag{15}$$

This expression does not strictly follow the general expression of equation (1), but the correction does contain the essential feature of black hole entropy. More importantly, the entropy of a black hole is usually treated on a small length scale, for the entropy becomes diverging when the surface area of the black hole is relatively large. Based on these results, the black hole entropy can be defined and meaningful even the ratio $\frac{A}{l_p^2}$ is big with the logarithm correction. The problems related to this correction are almost unrelated to the correction itself but more about the definition of black hole entropy [15] and the validity of the quantum loop gravity [16].

In 2007, Alejandro and his team used quantum loop theory to research the loop quantum gravity area spectrum and get an unexpected result in the entropy–area plot, as shown in Figure 1 [17].

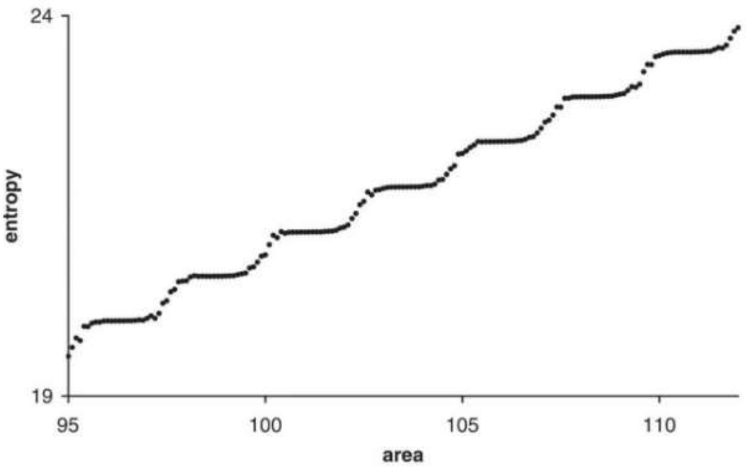


**Figure 1.** The plot of the area of the black hole surface versus the entropy of the black hole

We can see periodical discontinuous is the plot that indicates the black hole surface [17].

They estimated that the step shown in the plot is independent of the parameter $\gamma$ shown in [14], and the expression of the step should consequently be:

$$\Delta S \approx 2\gamma_0 \ln(3) \tag{16}$$

This is break-out progress in the quantization of black hole entropy correction and gives a standard to prove the correction in experiments.

Meanwhile, from the perspective of algebraic, A. A. Bytsenko and A. Tureanu compared the result of partition function computation for quantum gravity to the conformal field theory partition function, which we will discuss in the next chapter. Additionally, they improve understanding of representation theory, providing more possibility of the method of entropy correction from a mathematical point of view [18].

In conclusion, the logarithm correction of the quantum loop gravity is a relatively mature method and can combine other theories. The next step to make further progress is predicted to be the experimental evidence or some new interpretation or definition of black hole entropy.

## 3. Cardy Formula from conformal field theory

### *3.1. The derivation of the Cardy formula*

Conformal field theory is a useful tool for investigating Ads and CFT space since it provides enough symmetry conditions for the physical system. We will start by the derivation of the Cardy formula to review the correction of the density of critical stages, for it has been proved that, in a two-dimensional conformal field theory, to the density of states of the physical system, the computation of black hole entropy normally based on a certain critical stage of Cardy formula [6, 19].

Generally, it is often required the standard Virasoro algebra contained in a two-dimensional conformal field theory with a central charge c and is given by the formula:

$$\begin{cases} [L_m, L_n] = (m-n)L_{m+n} + \frac{c}{12}m(m^2-1)\delta_{m+n,0} \\ [\bar{L}_m, \bar{L}_n] = (m-n)\bar{L}_{m+n} + \frac{c}{12}m(m^2-1)\delta_{m+n,0} \\ [L_m, \bar{L}_n] = 0 \end{cases} \tag{17}$$

where the generators $L_m$ and $L_n$ are holomorphic and anti-holomorphic diffeomorphisms.

Meanwhile, we can define the partition function on the 2-tours of modulus $\tau = \tau_1 + i\tau_2$ as:

$$Z = \sum \rho(\Delta, \bar{\Delta})\, e^{i2\pi\Delta\tau} e^{-i2\pi\bar{\Delta}\bar{\tau}} \tag{18}$$

where ρ is the number of states which owns eigenvalues $\Delta = L_0, \bar{\Delta} = \bar{L}_0$ in unitary theory.

Subsequently, to calculate the density of states, we need to compute the partition function by contour integration. Assuming that $\tau$ and $\bar{\tau}$ are independent complex variables and $q = e^{i2\pi\tau}$, $\bar{q} = e^{-i2\pi\tau}$ to simplify the computation, we will obtain the expression below:

$$\rho(\Delta, \bar{\Delta}) = \frac{1}{(i2\pi)^2} \int \frac{dq}{q^{\Delta+1}} \frac{d\bar{q}}{\bar{q}^{\bar{\Delta}+1}} Z(q, \bar{q}) \tag{19}$$

Note that, the integration is evaluated along the contours when $q = \bar{q} = 0$. Normally, it is always hard to find the exact expression of the partition function. However, Cardy argued that [6, 19] the partition function can be found by:

$$Z(\tau, \bar{\tau}) = \frac{Tr e^{i2\pi\left(\bar{L}_0 - \frac{c}{24}\right)\tau} e^{-i2\pi\left(\bar{L}_0 - \frac{c}{24}\right)\bar{\tau}}}{e^{\frac{\pi c}{6}\tau_2}} \tag{20}$$

Fortunately, this expression is modular invariant, which implies the transformation $\tau \to -\frac{1}{\tau}$ is allowed, i.e., it is a universal result. Thus, by choosing the lowest eigenvalue $\Delta_0 = L_0$, we can compute the integral by steepest descent and define partition function as:

$$Z = \sum \rho(\Delta)\, e^{i2\pi(\Delta-\Delta_0)\tau} = \rho(\Delta_0) + \rho(\Delta_1) e^{i2\pi(\Delta_1-\Delta_0)\tau} + \cdots \tag{21}$$

For further simplification, one ignores the $\bar{\tau}$ dependence, and derives:

$$\rho(\Delta) = \int d\tau e^{-i2\pi\Delta\tau} e^{i2\pi\Delta_0 \frac{1}{\tau}} e^{\frac{i2\pi c}{24}\tau} e^{\frac{i2\pi c}{24}\frac{1}{\tau}} \tilde{Z}(-1/\tau) \tag{22}$$

If $\tau_2$ is relatively large, $\tilde{Z}(-1/\tau)$ tends to be constant, i.e., can evaluate the integral at the saddle point. Expanding the integral:

$$\mathrm{I[a,b]} = \int d\tau e^{i2\pi a\tau + \frac{i2\pi b}{\tau}} f(\tau) \tag{23}$$

around the extremum $\tau_0 = \sqrt{\frac{b}{q}}$, one acquires:

$$\mathrm{I[a,b]} \approx \int d\tau\, e^{i4\pi\sqrt{ab} + \frac{i2\pi b}{\tau_0^3}(\tau-\tau_0)^2} f(\tau_0) = \left(-\frac{b}{4a^3}\right)^{\frac{1}{4}} e^{i4\pi\sqrt{ab}} f(\tau_0) \tag{24}$$

When $\Delta_0 \ll c$ while $\Delta$ is large, the integral will become:

$$\rho(\Delta) = \left(\frac{c}{96\Delta^3}\right)^{\frac{1}{4}} \exp\left[2\pi\sqrt{\frac{c\Delta}{6}}\right] \tag{25}$$

This Cardy formula could be applied to the correction of black hole entropy, especially BTZ black hole. In this review, two ways will be demonstrated in black hole entropy correction using the Cardy formula.

### *3.2. The correction of black hole entropy*

S.Carlip argued that: By taking the horizon of a black hole as a boundary condition and researching the algebra of diffeomorphisms of the "r-t plane" near the boundary, we can apply a certain universal Virasoro algebra at the horizon [20-22]. Substituting central charge $c = \frac{3A}{2\pi G}\frac{\beta}{\kappa}$, and an eigenvalue $L_0 = \Delta = \frac{A}{16\pi G}\frac{\kappa}{\beta}$ (A is the surface area of the black hole, $\kappa$ is the surface acceleration constant and $\beta$ is an undetermined periodicity) into the Cardy formula and Eq. (25), we can get the logarithmic correction:

$$\rho(\Delta) \sim \frac{c}{12}\left(\frac{A}{8\pi G}\right)^{-\frac{3}{2}} \exp\left(\frac{A}{4G}\right) \tag{26}$$

In general, the constant c should be independent of the surface area A, which requires us to choose an appropriate β. Eventually, we can find entropy and study its behavior, as shown in Figure 2.

$$S \sim \frac{A}{4G} - \frac{3}{2}\ln\left(\frac{A}{4G}\right) + const + \cdots \tag{27}$$

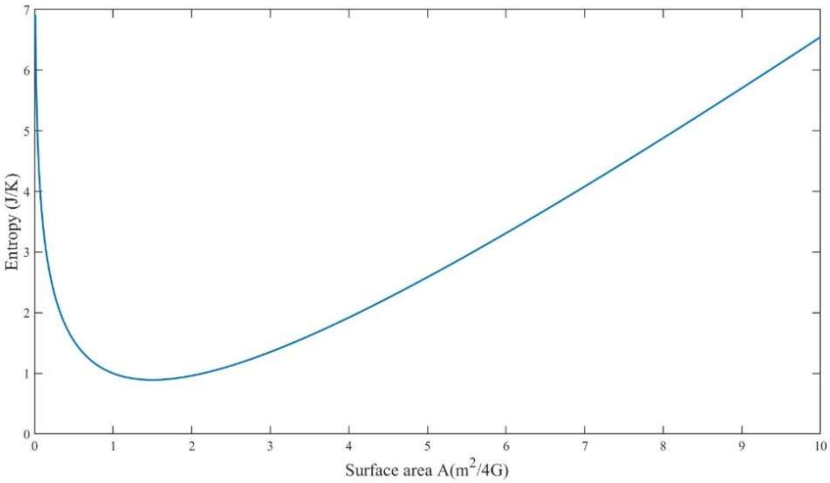


**Figure 2.** The black hole entropy after logarithm correction

The entropy of the black hole firstly decreases with the surface area of the horizon. After critical point A=6G, the entropy increases with area increasing but at a relatively low rate. This implies that, under this correction, there should exist one kind of black hole with the lowest entropy.

This result is agreed with the result deduced by string theory and quantum Geometry [23]. Nevertheless, as S. Carlip argued by himself, based on this computation, the third term, a constant which should be some function of angular momentum and other charges of the black hole, is not independent of other variables, leading to high requirements of fundamental conformal field theoretical explanation of black hole entropy. This will finally lead to the less universality of the constant $\frac{3}{2}$ before logarithm [24].

Cardy formula is also commonly used in the correction of the product rule of black hole entropy. For the black hole with two horizons, the classical product rule based on Bekenstein-Hawking entropy is given by:

$$S_+S_- = \left(\frac{k_B c^2}{hG}\right)^2 \frac{A_+A_-}{16} \tag{28}$$

where $S_+$ and $S_-$ are the entropy of outer and inner surface, $A_+$ and $A_-$ are the outer and inner surface of the black hole. This indicates that the entropy of the black hole is proportional to the product of the geometric quantity of the black hole. By applying the same method, finally, the density of states for the black hole with charges $c = \bar{c} = \frac{3l}{2G_3}$ is given by:

$$\rho_\pm(\Delta_\pm, \bar{\Delta}_\pm) \approx \frac{8G_3 l^2}{(r_\pm^2 - r_\mp^2)^{\frac{3}{2}}} e^{\frac{2\pi r_\pm}{4G_3}} \tag{29}$$

Subsequently, we obtain the product rule corrected by logarithm [25]:

$$\begin{aligned} S_+S_- = {} & \frac{\pi^2}{4G_3} r_+ r_- - \frac{3\pi}{4G_3}\left[r_+ \ln\left|\frac{2\pi r_+}{G_3}\right| + r_- \ln\left|\frac{2\pi r_-}{G_3}\right|\right] - \frac{3\pi}{4G_3}\left[r_+ \ln|\kappa_- l| + r_- \ln|\kappa_+ l|\right] \\ & + \frac{9}{2}\left[\ln\left|\frac{2\pi r_+}{G_3}\right| \ln|\kappa_- l| + ln\left|\frac{2\pi r_-}{G_3}\right| ln|\kappa_+ l|\right] \\ & + \frac{9}{4}\ln\left|\frac{2\pi r_+}{G_3}\right| ln\left|\frac{2\pi r_-}{G_3}\right| + \frac{9}{4}\ln|\kappa_- l|\, ln|\kappa_+ l| + const. + \cdots \end{aligned} \tag{30}$$

It is also not to be a universal expression since it hasn't been quantized and depends on the mass parameter.

Other efforts mainly focus on combining the Cardy formula and other modern theories, e.g., supersymmetry [7], and the similar correction of different kinds of black holes. Nevertheless, no further improvements were made either in the universal entropy solution or the interpretation of black hole entropy. Different kinds of ways can be used to improve the correction of the Cardy formula, including adapting the Cardy formula in higher dimensions, combining it with quantum loop theory, or just applying it to more kinds of black holes and waiting for a general solution.

## 4. Classical correction method

In this chapter, classical correction methods will be discussed. Those methods are not based on special beliefs, interpretations, or complete physical systems, e.g., the quantum loop gravity and conformal field theory. They are just analysis and computation of black hole entropy, meaning that they are not attempting to provide further understanding or interpretation of black hole entropy. However, these methods are meaningful, especially when we have finished reading the first two chapters, i.e., stopping the abstract mental analysis of the concept and essence of black hole entropy and concentrating on the phenomenological understanding of black hole entropy and its correction. This review will not present the mathematical details of the derivation but mainly focus on the physical analysis.

In this review, we use General Logarithm correction [26]. This method was reported by Saurya Das in 2002, trying to find a general solution to logarithm correction. The basic idea is assuming the black hole is an equilibrium thermal system and computing the density of microstate of the black hole, which is a similar process to what we did in quantum loop gravity and conformal field theory, since:

$$\mathrm{S} := \ln\rho(\mathrm{E}) \tag{31}$$

This requires to start from the inverse transform of partition function:

$$\rho(\mathrm{E}) = \frac{1}{2\pi}\int_{c-i\infty}^{c+i\infty} Z(\beta)e^{\beta E}d\beta = \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} e^{S(\beta E)}d\beta \tag{32}$$

where $\beta = 1/\mathrm{T}$.

Based on evaluating the integral through expanding the entropy, it is easy to get the expression of the density of state:

$$\rho(\mathrm{E}) = \frac{e^{S_0}}{\sqrt{2\pi S_0''}} \tag{33}$$

Considering the physical system is in equilibrium, the entropy is given by:

$$\mathrm{S} \coloneqq S_0 - \frac{1}{2}\ln cT^2 + (higher\ other\ term) \tag{34}$$

Then, we can apply this function to different black hole systems to get their entropy expression. All this work can be done by analyzing the small heat fluctuations around the equilibrium. The results are summarized in Table 2.

**Table 2.** The entropy expression of different black hole systems

| Types of Blackhole | The entropy expression |
|---|---|
| BTZ black hole | $S = S_0 - \frac{3}{2}\ln(S_0) - \frac{J^2/M^{\frac{3}{2}}}{2S_0} + \ldots$ |
| Anti-de Sitter Schwarzschild | $S = S_0 - \frac{d}{2(d-2)}\ln(S_0) + \cdots$ |
| Reissner-Nordstrom | $S = S_0 - \frac{(d-4)}{2(d-2)}\ln(S_0) + \ln(F_1^2 F_2) + \cdots$ |

Applying the conformal field theory, these expressions can be further simplified, and we deduced the results listed in Table 3:

**Table 3.** Simplified expressions of the entropy

| Types of Blackhole | The entropy expression |
|---|---|
| BTZ black hole | $S = S_0 - \frac{3}{2}\ln(S_0) - \frac{J^2/M^{\frac{3}{2}}}{2S_0} + \ldots$ |
| Anti-de Sitter Schwarzschild | $S = S_0 - \frac{1}{2}\ln(CT_H^2) + \cdots$ |
| Reissner-Nordstrom | $S = S_0 - \frac{1}{2}\ln(S_0) + \ln(F_2) + \cdots$ |

According to the results, it is consistent with conformal field theory. A similar result has been obtained by Sudipta Mukherji and Shesansu Sekhar Pal, following the same assumption based on a different method [27]. This team obtained the result in a more classical thermal dynamical way, which researched the heat capacity and chemical potential of the system, and expresses the entropy in terms of heat capacity:

$$S = S(\beta_0) - \frac{1}{2}\ln(C_v) + \cdots \tag{35}$$

After applying the formula to Ads-black hole, they derive:

$$S_{BH} = S_{BH} - \frac{1}{2}\ln(S_{BH}) + \cdots \tag{36}$$

where the coefficient before the correction is the same as the result of Saurya Das. By applying the transformation, it is easy to prove that the entropy, in this case, can be transformed into the expression $S_{BH} = cT^2$.

Comparing to the result that we found by the Cardy formula, it is obvious to see that the coefficient before the correction deduced by the Cardy formula is not general but still can be applied to BTZ black. This turns out the general solution of entropy deduced from the Cardy formula is right locally. On the other hand, comparing with the correction of quantum loop gravity, the coefficient agrees with each other except the BTZ black hole. This implies two things:

Quantum loop gravity does capture some essential physics that can be generalized.

There should be some transformation between the BTZ black hole and other kinds of black holes waiting to be discovered.

## 5. Conclusion

In summary, we reviewed the general method of logarithm correction of black hole entropy from the perspective of quantum loop theory, the Cardy formula of conformal field theory, and classical thermal dynamics. The correction of quantum loop theory is relatively mature since it has found the general expression of the black hole entropy. The computation process has been perfected and proved by later research. The correction of the Cardy formula is unsuccessful but does point out some essential physics in BTZ black hole. The correction of thermal fluctuation is universal in all common black hole types except BTZ black hole, which implies a further combination with the correction of the Cardy formula. These results offer a guideline for finding a more general logarithm correction expression for black hole entropy that can include all types and even dimensions of black holes.

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