第 35 卷 第 4 期
2018 年 12 月

原 子 核 物 理 评 论
Nuclear Physics Review

Vol. 35, No. 4
Dec., 2018

Article ID: 1007-4627(2018) 04-0356-06

Nuclear Shapes Made Up by Nucleons and Nuclear Forces

T. Otsuka[1,2,3,4], Y. Tsunoda[3], T. Togashi[3], N. Shimizu[3], T. Abe[2]

(*1. RIKEN Nishina Center, 2-1 Hirosawa, Wako, Saitama 351-0198, Japan;*
2. Department of Physics, University of Tokyo, 7-3-1 Hongo, Bunkyo, Tokyo 113-0033, Japan;
3. Center for Nuclear Study, University of Tokyo, 7-3-1 Hongo, Bunkyo, Tokyo 113-0033, Japan;
4. Instituut voor Kern-en Stralingsfysica, K. U. Leuven, Celestijnenlaan 200D, B-3001 Leuven, Belgium)

Abstract: We discuss the quantum self-organization introduced recently as one of the major underlying mechanisms of the quantum many-body systems. Atomic nuclei are actually a good example, because two types of the motion of nucleons, single-particle states and collective modes, interplay in determining their structure. The collective mode appears as a consequence of the balance between the effect of the mode-driving force (*e.g.*, quadrupole force for the ellipsoidal deformation) and the resistance power against it. The single-particle energies are one of the sources to bring about such resistance power: a coherent collective motion is more hindered by larger spacings between relevant single particle states. Thus, the single-particle state and the collective mode are "enemies" against each other in the usual understanding. However, the nuclear forces are rich and complicated enough so as to enhance relevant collective mode by reducing the resistance power by changing single-particle energies for each eigenstate through monopole interactions. This will be demonstrated with the concrete example taken from Zr isotopes. In this way, the quantum self-organization occurs: single-particle energies can be self-organized by (i) two quantum liquids, *e.g.*, protons and neutrons, (ii) monopole interaction (to control resistance). Thus, atomic nuclei are not necessarily like simple rigid vases containing almost free nucleons, in contrast to the naïve Fermi liquid picture *a la* Landau. Type II shell evolution is considered to be a simple visible case involving excitations across a (sub)magic gap. The quantum self-organization becomes more important in heavier nuclei where the number of active orbits and the number of active nucleons are larger.

Key words: nuclear shape; quantum self-organization; single-particle and collective modes

CLC number: O571.6 **Document code:** A **DOI:** 10.11804/NuclPhysRev.35.04.356

1 Motivation

The underlying mechanisms of the multi-nucleon structure of atomic nuclei have been studied for nearly a century as one of the most important objectives of nuclear physics. It has then been understood usually that there are two types of dominant motion of nucleons in the atomic nucleus in low-excitation energy: single-particle states and collective modes. Regarding the single-particle states, Mayer and Jensen introduced the shell structure and associated magic numbers[1–3]. The nuclear shell model developed on these concepts has been extremely successful in the description of the structure of many nuclei (see for example[4–6]). The collective modes with low-excitation energies include various cases. Among them, the ellipsoidal deformation of the nuclear shape has been studied since Rainwater[7], and Bohr and Mottelson[8–9]. The nuclear shapes have been one of the primary focuses of the nuclear structure physics, including spherical, vibrational and rotational ones[10]. The relation between the single-particle states and the collective modes has become of keen and long-standing interest, as mentioned by Bohr and Mottelson in Ref. [10] as "the problem of reconciling the simultaneous occurrence of single-particle and collective degrees of freedom and exploring the variety of phenomena that arise from their interplay".

The atomic nucleus is a many-body quantum system consisting of protons and neutrons, which is often considered to be described in terms of Landau's Fermi Liquid picture. In a rather simplified expression of

Received date: 10 Oct. 2018

Foundation item: HPCI Strategic Program (The origin of matter and the universe) and "Priority Issue on Post-K computer" (Elucidation of the Fundamental Laws and Evolution of the Universe) from MEXT and JICFuS.

Biography: Takaharu Otsuka (1952–), male, born in Yokohama, Japan, Professor in The University of Tokyo, working on theory of nuclear structure; E-mail: otsuka@phys.s.u-tokyo.ac.jp.

this picture, protons and neutrons of a nucleus are in a mean potential which is like a rigid "vase", and those nucleons are like free particles moving in this vase, interacting weakly among themselves through a "residual interaction". The single-particle energies (SPE) of such a system exhibit the shell structure, and are split with different SPEs in general. If such splittings are large enough, the many-body structure is dominated by the SPEs: nucleons occupy the lowest single particle states in the ground state, the next lowest configuration gives us the first excited state, and so forth. In such cases, the correlations due to the interaction between nucleons may contribute, but their effects are suppressed, more or less, compared to the effects of SPE splittings. However, if relevant SPE splittings are rather small, the energy gain from such correlations overcomes, and a collective mode dominates the structure of the ground and low-lying states. Although the understanding of the relation between the single-particle states and the collective modes has been pursued in many ways, it seems to remain an open problem to date. As an example, G. E. Brown had kept, throughout his life, the question, how single particle states can coexist with collective modes as quoted from "Fermi liquid theory: A brief survey in memory of G. E. Brown" in Ref. [11]. We shall present a novel mechanism which may be closely related to this subject.

2 Nuclear shapes and quantum phase transition

We shall focus on the quadrupole shape transition in this paper, while similar discussions can be made in other cases. Fig. 1 displays the excitation energy of the 2_1^+ state, or the 2_1^+ level, for Sm and Zr isotopes as functions of the neutron number, N. In the Sm chain, the 2^+ level is lowered rather gradually as N increases, similarly to many other isotopic chains. As shown in Fig. 1, a higher 2^+ level corresponds to a spherical shape and its surface oscillation, while a lower 2^+ level implies an ellipsoidal deformed shape and the rotation of the ellipsoid. In the Zr chain, in contrast, the 2^+ level drops down abruptly as N is changed from 58 to 60. Due to this abrupt change, this phenomenon can fit well to the quantum phase transition[12]. Likewise, the ground-state structure of the Zr isotopes is changed drastically from N=58 to 60, as the sudden transition from the the sphere to the strongly prolate deformed ellipsoid. The Monte Carlo Shell Model (MCSM) describeed both situations including the abrupt change within the same Hamiltonian[12–13].

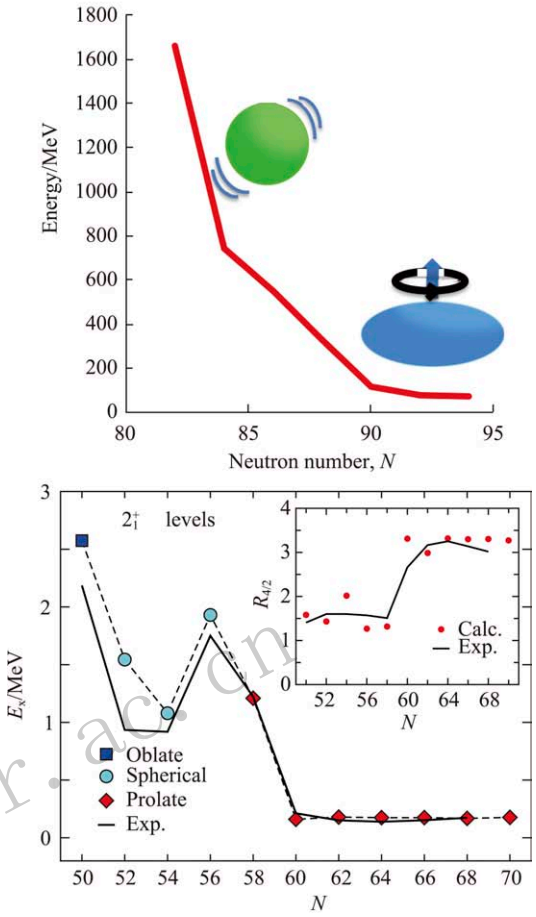

Fig. 1 (color online) Systemtic changes of the 2_1^+ level in (upper) Sm and (lower) Zr isotopes, as functions of N. Data taken from Ref. [14] for Sm. The lower panel is based on Ref. [12].

3 Monte carlo shell model

We here give a brief sketch of the MCSM. For more detailed account, one can refer to Refs. [15–16]. The MCSM is a relatively recent method to obtain eigensolutions of the shell-model calculation in nuclear structure. The shell-model calculation is similar to the configuration interaction (CI) calculation in other fields of science. The major differences can be pointed out as (i) the two ingredients, protons and neutrons, are taken instead of electrons, and (ii) nuclear forces are considered instead of Coulomb or other forces. In conventional shell-model calculations, the matrix of the Hamiltonian with respect to many Slater determinants is obtained first and is then diagonalized. Because a huge number of configurations are needed for the description of large systems, the dimension of the matrix can be prohibitively large, making the calculation infeasible. On the other hand, many interesting and important problems are expected beyond this limit. The MCSM can provide us with a breakthrough in this regard. The MCSM is completely different from the con-

ventional shell-model calculation. A set of Slater determinants, called MCSM basis vectors, is introduced as an importance truncation, and the diagonalization of the Hamiltonian is performed in the Hilbert space spanned by the MCSM basis vectors. Each MCSM basis vector is a Slater determinant comprised of single-particle states that are superpositions of the original usual single-particle states, and the amplitudes of these superpositions are determined through a combination of stochastic and variational procedures. Even when the dimensions are in the order of 10^{23} for the conventional shell model, the problem can be solved, to a good approximation, with up to approximately 50∼200 MCSM basis vectors.

4 Quantum phase transition in Zr isotopes

We shall discuss structure changes in Zr isotopes based on the MCSM calculation. The upper panel of Fig. 2 shows the occupation numbers of relevant proton orbits for some states. The $g_{9/2}$ orbit is not occupied much in the 0_1^+ state of ^{98}Zr, whereas it is occupied by about 3.5 protons in its 0_2^+ state. Note that this 0_1^+ (0_2^+) state has a spherical (deformed) shape. These changes, including the numbers of proton holes in the pf orbits, result in substantial changes of the neutron (effective) SPEs as shown schematically in the middle panel of Fig. 2. The proton-neutron monopole interaction (indicated by the wavy line in the figure), combined with the changes of the occupation numbers, is the origin of those shifts. The bottom panel depicts the actual neutron effective SPEs. One notices substantial changes in the effective SPEs for the different states considered. For instance, one finds that the spacing between the $d_{5/2}$ and $g_{7/2}$ orbits is nearly 5 MeV for the 0_1^+ state of ^{98}Zr, but it is reduced to about 2 MeV in 0_2^+ state. A similar reduced splitting is seen also in the 0_1^+ state of ^{100}Zr which is again strongly deformed.

We next discuss why the SPEs are so different between spherical and deformed states. It is reminded that the nuclear deformation at low excitation energy is a Jahn-Teller effect[17], meaning that the collective motion causing the deformation occurs as a consequence of coherent contributions from some relevant orbits near the Fermi energy. For such coherent effects, in general, larger splittings of SPEs weaken the coherence, leading to less collectivity.

On the other hand, the monopole interaction can change the effective SPEs depending on the occupancy of the other nucleons. If the monopole interaction were uniform with respect to the single-particle orbits,

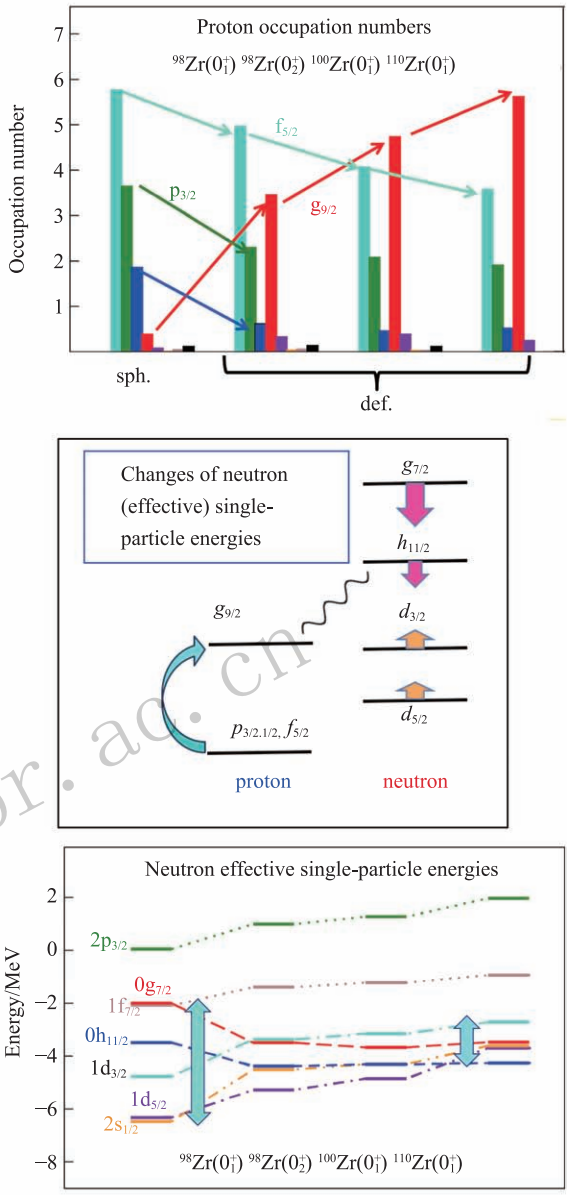

Fig. 2 (color online) (upper) Occupation numbers of proton orbits of Zr isotopes. (middle) Schematic illustration of the changes of neutron (effective) single-particle energies in Zr isotopes. (lower) Actual values of neutron (effective) single-particle energies obtained in the calculation of Ref. [12]. Upper and lower panels are based on Ref. [12].

no configuration dependence would appear, and this change should be absent. From the tensor-force component of the nuclear force, one obtains its monopole interaction attractive or repulsive, depending on the combination of the orbits[18–19]. This is clearly against the uniformity, and its effect can be crucial. The central-force component changes its magnitude significantly depending on the combination of the single-particle orbits mainly due to varying overlaps of radial wave functions of single-particle states involved[19]. Thus, the monopole interaction is indeed far from be-

ing uniform, and the selection of favored configurations can shift the effective SPEs of relevant single-particle orbits substantially. If relevant effective SPEs can be made closer to being degenerate, it helps the deformation. We shall formulate this novel mechanism in the next section.

5 Quantum self-organization

The nuclear deformation is determined by the balance between the effect of the collective-mode driving force and the resistance power against this collective mode. This property is expressed schematically as,

$$\text{deformation} = \frac{\text{quadrupole force}}{\text{resistance power}}. \qquad (1)$$

In the case of the ellipsoidal shape, the collective-mode driving force is the quadrupole (or quadrupole-quadrupole) interaction. This interaction is one of the major components of the proton-neutron realistic nuclear force. A typical example of the resistance power is the pairing interaction, which tends to make the shape more spherical, as all time-reversal pairs are equally favored. Keeping the pairing interaction aside, we shall consider another source of the resistance power. That is nothing but the (effective) SPEs.

We here introduce a novel mechanism called, **Quantum Self-Organization**. This mechanism means the following property: Atomic nuclei can "organize" their single-particle energies by taking particular configurations of protons and neutrons, optimized for each eigenstate, thanks to orbit-dependences of monopole components of nuclear forces (*e.g.*, tensor and central forces). This results in an enhancement of Jahn-Teller effect, *i.e.*, an enhancement of the collective mode. The deformation and the quantum self-organization can be linked in a non-linear way with a positive feedback : once some nucleons are excited to particular orbits, the effective SPEs are shifted in favor of a larger deformation. A larger deformation can promote such excitations with more nucleons. This cycle continues until a self-consistency is achieved, whereas intermediate situations are skipped (or remain at higher excitation energies even with fragmentation). In many cases, massive excitations are involved, and the particle-hole hierarchy is broken, for instance, a 6p-6h deformed state comes right after a 2p-2h near-spherical state, skipping 4p-4h state[20].

The property shown in Eq. (1) is somewhat analogous to the well-known relation,

$$\text{electric current} = \frac{\text{voltage}}{\text{resistance}}, \qquad (2)$$

where the electric current, voltage and resistance mean the usual quantities regarding the electricity. The higher voltage produces a higher current, but the current can be increased also by reducing the resistance. The quantum self-organization implies that the atomic nucleus finds particular configurations which decrease the resistance power.

The most favorable configurations and associated (effective) SPEs vary for individual eigenstates even within the same type of the collective mode. For instance, prolate, oblate or triaxial shapes belong to the quadrupole deformation, but can appear with different patterns of the effective SPEs within the same nucleus. The oblate shape is less affected by the quantum self-organization, because smaller numbers of nucleons on unique-parity orbits are the key element of the oblate shape in most cases. In those cases, the organization of many orbits are rather irrelevant, and the quantum self-organization may not occur to a sizable extent. This feature has been verified with concrete cases. On the other hand, many orbits contribute coherently to the prolate deformation, and the quantum self-organization is indeed crucial. This has been confirmed by varying artificially the orbit-dependence of the monopole interaction, for instance, closer to the uniform one.

We present a concrete example by taking the case of the prolate band in ^{68}Ni[20]. The monopole interaction between the neutron $1g_{9/2}$ orbit and the proton $1f_{5/2}$ orbit is more attractive than that between the neutron $1g_{9/2}$ orbit and the proton $1f_{7/2}$ orbit mainly due to the robust property of the monopole interaction of the tensor force, and this difference contributes to the quantum self-organization in this particular case: more neutrons in the $1g_{9/2}$ orbit reduces the $1f_{7/2}$-$1f_{5/2}$ spin-orbit splitting for protons[18–19, 21]. The effect of this difference on the deformation can be visualized quantitatively by replacing the strengths of these monopole interactions with the average of their original values, *i.e.*, the same value. Likewise, we reset the monopole interaction between the neutron $1f_{5/2}$ orbit and the proton $1f_{7/2}$ orbit and the one between the neutron $1f_{5/2}$ orbit and the proton $1f_{5/2}$ orbit. Such modifications correspond basically to the removal of the tensor-force monopole contributions, and lead us to the suppression of the present effects of the quantum self-organization. The resulting Potential Energy Surface is shown in Fig. 3 for the axially symmetric ellipsoidal shape, and is compared with the PES obtained from the original Hamiltonian. Around the spherical minimum the energy curves of the two calculations are similar, however, when going to stronger deformation values, the two approaches differ substantially. In particular, the prolate profound local minimum, seen in the original calculation (red solid line), is pushed up by

about 4 MeV, if the quantum self-organization is suppressed as described above (blue dashed line). Thus, the quantum self-organization is a part of the crucial mechanisms producing the nuclear deformation.

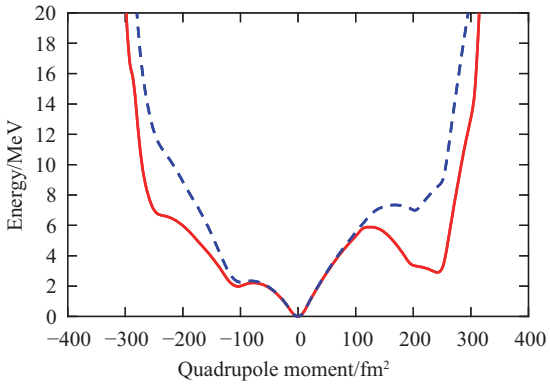

Fig. 3 (color online) Potential Energy Surface with the axially symmetric deformation for ^{68}Ni. The red solid line denotes the energy of the constrained Hartree-Fock calculation with the original Hamiltonian. The blue dashed line implies the same calculation except that the quantum self-organization is suppressed (see the text). Figure taken from Ref. [20].

At this point, we mention that the effective SPE being discussed corresponds somehow to the spherical terms in the Nilsson model[10] which are comprised of the $\ell\ell$ and ℓs terms as well as the harmonic-oscillator-quanta term. As their strengths are independent of the deformation parameter, the present effect is not included in the Nilsson model.

Type-II shell evolution[20] has been discussed, for instance, in Co/Ni region[21–23], where neutrons are excited from the pf shell to $g_{9/2}$ across the N=40 sub-magic gap. The neutrons in $g_{9/2}$ and neutron holes in $f_{5/2}$ provide similar sizable monopole effects, as discussed above. A smaller $1f_{7/2}-1f_{5/2}$ spin-orbit splitting for protons reduces the resistance power against deformation, pulling down the prolate band as seen in Fig. 3. Type II shell evolution was introduced as the particle-hole excitation over a magic or sub-magic gap. Clearly, such kind of mechanism is a very simple and transparent case of the quantum self-organization. On the other hand, the quantum self-organization occurs in more complex ways. Such a complex way can be found in the shape transition of Sm isotopes (see Fig. 1), where nothing like a magic or sub-magic gap is involved. We can see the spherical-vibrational-rotational shape transition in MCSM calculations, as will be reported elsewhere.

Likewise, the shape coexistence, particularly odd-even staggering in Hg/Pb isotopes, have been studied. In those cases, the quantum self-organization provides unique contributions on the pattern of the shape coexistence, as reported also elsewhere.

6 Summary and perspectives

We presented, in this paper, a novel mechanism on the relation between single-particle states and collective modes, as summarized below.

- The atomic nuclei are not like simple rigid vases containing almost free nucleons interacting only weakly among themselves. This is in contrast to the naïve Fermi liquid picture.
- Nuclear forces are rich enough to optimize, to a certain extent, single-particle energies for each eigenstate, and can lead to the quantum self-organization.
- Thus, single-particle energies can be self-organized, being enhanced by the features
  (i) two quantum liquids
  (*e.g.*, protons and neutrons)
  (ii) two major force components
  e.g., quadrupole interaction: to drive collective mode; monopole interaction: to control resistance
- Type II shell evolution is a simple visible case of the quantum self-organization involving excitations across (sub)magic gap.
- Actual cases such as shape coexistence, quantum phase transition, octupole vibration/deformation, super deformation, *etc.* can be studied with this scope with a variety of different appearance patterns.
- The quantum self-organization becomes more important in heavier nuclei where the number of active orbits and the number of active nucleons are larger. With larger numbers of them, the effects of the organization can be more significant. This feature may be linked to fission and superheavy elements. On the other hand, the quantum self-organization may not be so visible in light nuclei except for particular cases like intruder bands or cluster (or multiple particle-hole excited) states.
- Time-dependent version of quantum self-organization may be of another interest for reactions and fission.

Acknowledgements This work was supported in part by the HPCI Strategic Program (The origin of matter and the universe) and "Priority Issue on Post-K computer" (Elucidation of the Fundamental Laws and Evolution of the Universe) from MEXT and JICFuS. The MCSM calculations were performed on the K computer at RIKEN AICS (hp140210, hp150224, hp160211, hp170230).

References:

[1] MAYER M G. Phys Rev, 1949, **75**: 1969.
[2] HAXEL O, JENSEN J H D, SUESS H E. Phys Rev, 1949, **75**: 1766.
[3] MAYER M G, JENSEN J H D. Elementary Theory of Nuclear Shell Structure [M]. New York: Wiley, 1955.
[4] DE SHALIT A, TALMI I. Nuclear Shell Theory[M]. New York: Dover, 2004.
[5] HEYDE K L G. The Nuclear Shell Model[M]. Berlin: Springer, 1990.
[6] CAURIER E, MARTÍNEZ-PINEDO G, NPWACKI F, *et al*. Rev Mod Phys, 2005, **77**: 427.
[7] RAINWATER J. Phys Rev, 1950, **79**: 432.
[8] BOHR A. Mat Fys Medd Dan Vid Selsk, 1952, **26**: 1.
[9] BOHR A, MOTTELSON B R. Mat Fys Medd Dan Vid Selsk, 1953, **27**: 1.
[10] BOHR A, MOTTELSON B R. Nuclear Structure Vol 2 [M]. New York: Benjamin, 1975.
[11] SCHAEFER T. Nucl Phys A, 2014, **928**: 180.
[12] TOGASHI T, TSUNODA Y, OTSUKA T, *et al*. Phys Rev Lett, 2016, **117**: 172502.
[13] KREMER C, ASLANIDOU S, BASSAUER S, *et al*. Phys Rev Lett, 2016, **117**: 172503.
[14] NNDC. Nudat 2, 2016.
[15] OUSUKA T. Prog Part Nucl Phys, 2001, **47**: 319.
[16] SHIMIZU N. Prog Theor Exp Phys, 2012, **2012**: 01A205.
[17] JAHN H A, TELLER E. Proc R Soc A, 1937, **161**: 220.
[18] OTSUKA T, SUZUKI T, FUJIMOTO R, *et al*. Phys Rev Lett, 2005, **95**: 232502.
[19] OTSUKA T, HONMA M, UTSUNO Y, *et al*. Phys Rev Lett, 2010, **104**: 012501.
[20] OTSUKA T, TSUNODA Y. J Phys G: Nucl Part, 2016, **43**: 024009.
[21] TSUNODA Y, OTSUKA T, SHIMIZU N, *et al*. Phys Rev C, 2014, **89**: 031301(R).
[22] MORALES A, BENZONI G, WATANABE H *et al*. Phys Lett B, 2017, **765**: 328.
[23] LEONI S, FORNAL B, MÄRGINEAN N, *et al*. Phys Rev Lett, 2017, **118**: 162502.

核子与核力决定的核形状

T. Otsuka[1,2,3,4,1)], Y. Tsunoda[3], T. Togashi[3], N. Shimizu[3], T. Abe[2]

(1. 日本理化研究所尼许纳中心，和光区广泽 2-1，崎玉　351-0198;
2. 日本东京大学物理系，文京区汉高 7-3-1，东京　113-0033;
3. 日本东京大学原子核研究中心，文京区汉高 7-3-1，东京　113-0033;
4. 比利时鲁汶大学原子核与辐射物理研究所，赛莱思迪贺内拉恩 200D，鲁汶　B-3001)

摘要： 讨论了最近提出的作为量子多体系统重要潜在机制之一的量子自组织，原子核无疑是最好的实例。由于原子核内核子的单粒子和集体运动共存，它们的相互制约决定了核结构。集体模式因其驱动力，如使椭球形变的四极力及其阻力达到平衡形成，而单粒子能量就是产生阻力的一种根源。当存在较大单粒子能隙时，相关的集体运动更易受到阻碍。因此，一般认为，单粒子运动和集体运动是相互对抗的“天敌”。然而，由于核力的多样和复杂性，单极相互作用使单粒子能量改变也能减小其对集体运动的阻碍而加强集体模式，该现象将通过Zr同位素实例加以说明。这就导致了量子自组织的产生：单粒子能量由两种量子液体(质子和中子)和控制阻力的单极相互作用自组织。于是，不同于朗道费米液体理论的结论，原子核不一定像填装了自由核子的刚性瓶。II型壳演化即是包含跨准幻壳能隙激发的直观实例。在重核中，量子自组织因其轨道和核子数更多而更为重要。

关键词： 核形状；量子自组织；单粒子与集体模式

收稿日期： 2018-10-10；　**修改日期：** 2018-6-30

基金项目： 日本MEXT和JICFuS超高性能计算革新研究战略计划(宇宙与物质起源)及“优先问题的K后超算”(宇宙演化和基本定律阐明)项目资助

1) E-mail: otsuka@phys.s.u-tokyo.ac.jp。