

The Engines behind Supernovae and Gamma-Ray Bursts

The Role of Convection and Rotation

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Abstract

We review the different engines behind supernova (SNe) and gamma-ray bursts (GRBs), focusing on those engines driving explosions in massive stars: core-collapse SNe and long-duration GRBs. Convection and rotation play important roles in the engines of both these explosions. We outline the basic physics behind convection in core-collapse and discuss the wide variety of ways scientists have proposed that this physics can affect the supernova explosion mechanism, concluding with a review of the current status in these fields.

1 Introduction

Supernovae and Gamma-Ray Bursts are the two most energetic explosions in the universe since the big bang. For a brief time, they outshine the galaxies in which they reside. Observations have led to the classification of these outbursts based on their observational properties. For supernovae, the classification is based primarily on their spectra: type II SNe have strong hydrogen lines, type I SNe exhibit hydrogen lines at late times if at all. Type I supernovae are further distinguished by the presence of strong silicon lines (Ia), absent in Ib/c supernovae; Ib supernovae have helium lines, absent in Ic supernovae. GRBs are differentiated by the duration of the burst and “hardness” of its spectra: long-hard bursts with durations above a few seconds and the short-soft bursts. A third intermediary category may exist. An additional feature of these observations is that long-duration GRBs have type Ic supernovae associated with them.

Theorists have proposed a wide variety of mechanisms for these explosions, from electrical storms in the earth’s stratosphere to colliding strings in the early universe. Indeed, just 25 years after the discovery of GRBs, over 100 models existed for GRBs[50]. However, only a handful of models have emerged as “favorite” engines behind all the types of GRB and SN explosions. Figure 1 reviews the 3 main types of SN and 2 types of GRB and the energy source/mechanism behind the explosions. Type Ia SNe are produced by the thermonuclear explosion of a white dwarf. In the standard picture, the ignition of the white dwarf occurs when its mass exceeds the Chandrasekhar mass (generally through accretion from a binary companion). This is the only explosion powered by nuclear energy. All other explosions are produced by the release of gravitational energy. Short-Duration GRBs are believed to be produced by gravitational energy released during the merger of two neutron stars and the subsequent accretion onto the black hole formed during this merger.

Type Ib/c SNe, Type II SNe and long-duration GRBs are ALL believed to be produced in the collapse of a massive star down to a compact object. For most SNe, the outburst is produced when a massive star ($\gtrsim 9M_{\odot}$) collapses down to a neutron star. Long-duration GRBs are produced in the collapse of a very massive star ($\gtrsim 20M_{\odot}$) down to a black hole. Although the source of energy for both these explosions is the same potential energy released during the collapse, it is believed that the mechanism that converts this energy into an explosion is very different. The mechanism behind long-duration GRBs also produces associated type Ic SNe, so the line dividing these different explosions is not concrete, but we shall argue below that these “type Ic” SNe have different characteristics from “normal” type Ic SNe.

This paper reviews the role of convection in the explosion mechanisms of core-collapse SNe and GRBs. But before we focus on this particular piece of physics, let’s review the basic engines involved.

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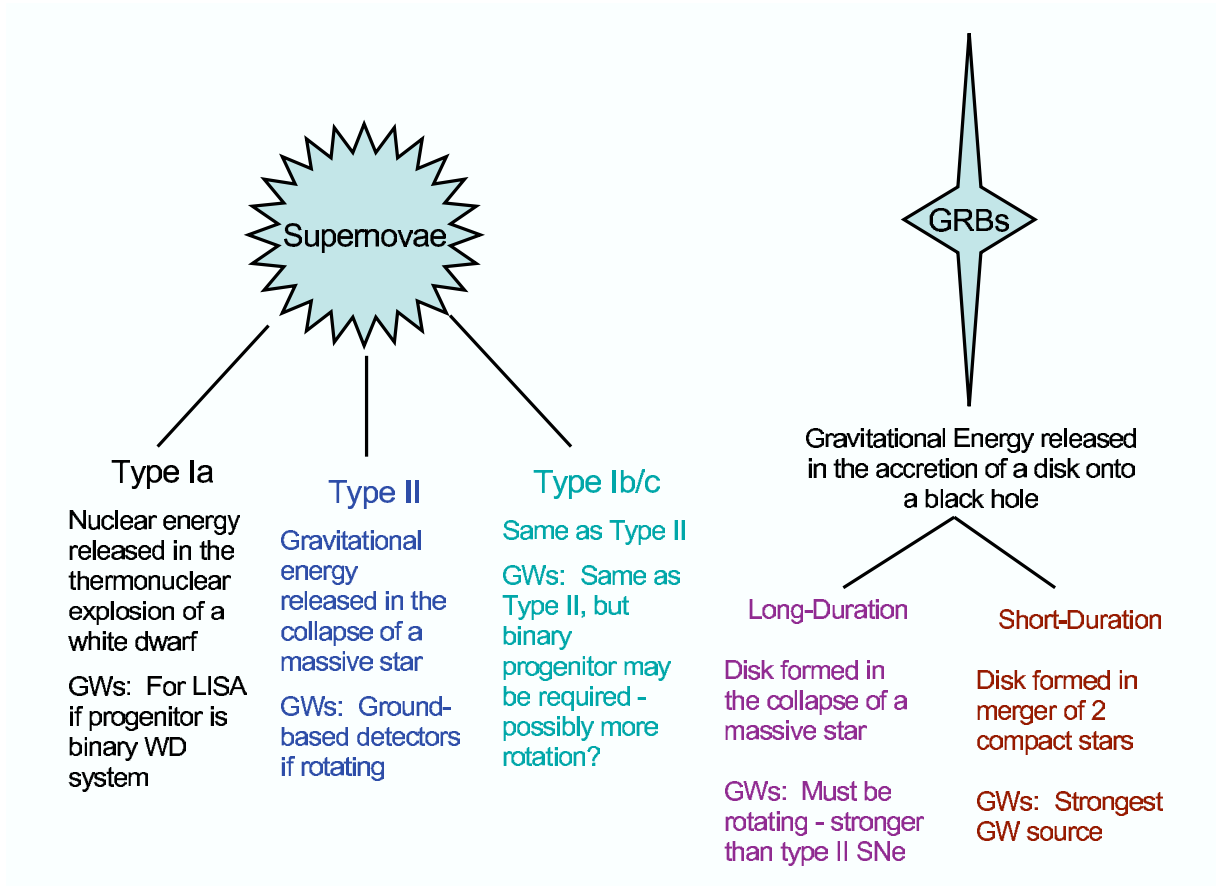


Figure 1: Three basic SN types and two primary GRB types. The basic SN types are distinguished based on the appearance of different elements (H, He, Si) in the spectra of the explosions. The GRB types are distinguished by the duration of the gamma-ray emission. Two SNe types (II,Ib/c) and one GRB class (long-duration) are believed to be produced by the collapse of massive stars. This paper focuses on the role of convection on the explosions behind these core-collapse events.

1.1 The Basic Engine Powering Core-Collapse Supernovae

Stars are powered by the nuclear fusion of material in their cores. For stars like the sun, this fusion stops after the core has “burned” its hydrogen into helium. But for more massive stars, the core is able to become sufficiently hot to fuse the helium ashes into carbon. In stars above $\sim 9M_{\odot}$, this burning process continues through the fusion of silicon into iron. The fusion of iron to heavier elements does not release any further energy and the build up of an iron core marks the end of the life of even the most massive stars. When this core becomes so large that pressures and densities dissociate the iron to lighter atoms (removing thermal support) and induce the capture of electrons onto protons (removing electron degeneracy support), the core collapses.

The core collapses until it reaches nuclear densities where nuclear forces and neutron degeneracy pressure halt the collapse. This marks the initial formation of the proto-neutron star (roughly $0.9M_{\odot}$) and sends a “bounce”-shock back through the star. This shock is extremely hot and most of its energy is stored in thermal energy. Initially, photons, and even neutrinos, are trapped in this shock. But as the shock moves to lower densities, the neutrinos can leak out of the shock, removing most of its energy and causing it to stall. This occurs roughly between 100-200km. The stall leaves behind a region between the proto-neutron star and the stalled shock with a negative entropy gradient (high entropies below lower entropies) that initiates convection in this region. But this convection continues to be driven by a number of instabilities (to be discussed below) that Herant et al.[27] argued would allow the convective region to more-efficiently transfer the potential energy released (stored in the thermal energy of the proto-neutron star) to be converted to kinetic energy and ultimately drive a supernova explosion. After over a decade of debate, there is now a general consensus that this convective region is indeed critical to the supernova explosion engine. It allows the transfer of energy from the proto-neutron star to the stalled shock, pushing back against the infalling star and ultimately driving a supernova explosion.

1.2 The Basic Engine Powering Long-Duration GRBs

The engine behind both short and long-duration GRBs is powered by the accretion of matter onto a black hole. If this accreting matter has enough angular momentum to form a disk around the black hole, its energy can be extracted by first converting it into magnetic fields or neutrinos and then depositing this energy further out to drive a relativistic explosion[49]. A number of progenitor models exist to produce such black hole accretion disk (BHAD) systems and the relative merits of the two conversion mechanisms (neutrinos or magnetic fields) have been studied in detail assuming a simple disk picture[52, 17].

In 1993, Woosley[61] proposed that the BHAD engine could be produced in the collapse of a rotating massive star. MacFadyen & Woosley[42] modeled the first such collapse of a massive star in 2-dimensions and, using the simple disk models of Popham et al.[52] found that by injecting energy above the disk, they could produce a jet-like explosion that could explain the features of long-duration GRBs.

Let’s outline the basic picture behind this collapsar engine. It begins with the same evolution as the stars we studied in our basic supernova engine. A massive star collapses, forms a proto-neutron star and this engine convects. However, if the star is more massive than roughly $20M_{\odot}$ [16], the supernova engine is unable to produce a strong explosion. If it produces a weak explosion, it is unable to disrupt the entire star and material will eventually fall back onto the proto-neutron star, causing it to collapse to a black hole. For extremely massive stars ($\gtrsim 40M_{\odot}$), the supernova engine may not work at all, producing a black hole without any supernova explosion at all (but as we shall see, this collapse may still produce an explosion).

If the star is rotating, some of the material falling on the newly-formed black hole will hang up in an accretion disk around the black hole. Then it is assumed that either the neutrinos emitted from the hot disk or magnetic fields generated in the disk will convert the potential energy released into an explosion. Popham et al.[52] made a series of advection dominated accretion flow simulations to show that although both the neutrino and magnetic field conversion mechanisms are plausible, the magnetic field conversion mechanisms may well be required for the strong GRB explosions that are observed.

A series of papers have studied this disk picture in more detail. For the neutrino-annihilation transfer mechanism, semi-analytic calculations can provide quantitatively accurate estimates. di Matteo et al.[13] studied this mechanism at the highest accretion rates and found that neutrino trapping severely limited the amount of energy injected by neutrinos in this regime. Fryer & Mészáros[20] focused on progenitors,

finding that for massive stars, the black hole would become too massive by the time the axis above the disk cleared for the simple neutrino-annihilation disk-mechanism to work. All these studies agreed that it is unlikely that neutrino-annihilation will work for GRBs produced in stellar collapse. It is much more difficult to accurately estimate the role of magnetic fields. Although many papers exist predicting what magnetic fields can do for GRBs produced in stellar collapse[49, 35, 43] only a few quantitative calculations of these magnetic fields have been made[53]. Even these calculations are constrained by a number of assumptions and 2-dimensional calculations.

Three-dimensional calculations of these collapse models have shown us that the basic disk picture is far too simple and the behavior of matter near the black hole is much more similar to a supernova engine than that of an accretion disk[54]. Convective instabilities develop and dominate the flow of matter. Rockefeller et al. [54] also found that viscous forces may also be able to extract angular momentum energy driving outflows. How this leads to a GRB remains to be seen, but it is clear that nature is much more complex than we first imagined and a lot of work remains to fleshing out the GRB engine. But the first step is to understand this convection and we will focus on this aspect of the GRB engine here.

2 Convection in Supernovae

2.1 History

In 1979, Epstein[14] argued that lepton gradients in the proto-neutron star would drive convection that would help transport neutrinos deep in the proto-neutron star to its surface, allowing those neutrinos to better heat the material in the shocked region, driving an explosion². Shortly thereafter, a more complete picture based on the first multi-dimensional calculations of convection in supernovae[41, 3, 57] argued that the convection was driven by the interplay between entropy and lepton gradients. These calculations also focused on the role of convection in the proto-neutron star core and the effect this convection would play on the neutrino flux, but they also realized that the role of convection might lead beyond just transporting neutrinos.

The research in the 1980s focused mostly upon the convection in the neutron star. Lattimer and collaborators[39, 7] took advantage of the latest advances in dense equations of state to better estimate the convection and hence the neutrino fluxes from the proto-neutron star. Burrows[8] argued, using semi-analytic calculations, that convection at the neutrinosphere would grow to encompass the entire proto-neutron star. This convection was later confirmed by multi-dimensional simulations[36]. With appropriate tweaking, Wilson & Mayle[59, 60] found that they could get explosions with this proto-neutron star convection, focusing on the doubly diffusive regime where heat diffusion removed the stable entropy gradient faster than lepton diffusion diminished the unstable lepton gradient. The increased neutrino flux from this convection was just enough to produce an explosion.

But all large-scale, multi-dimensional calculations since Herant et al.[27] that include the shocked region only exhibit weak convection in the proto-neutron stars. Whether or not strong convection occurs in the proto-neutron star is still a matter of debate. A very good review of our current understanding of this analysis has been done by Bruenn et al.[4].

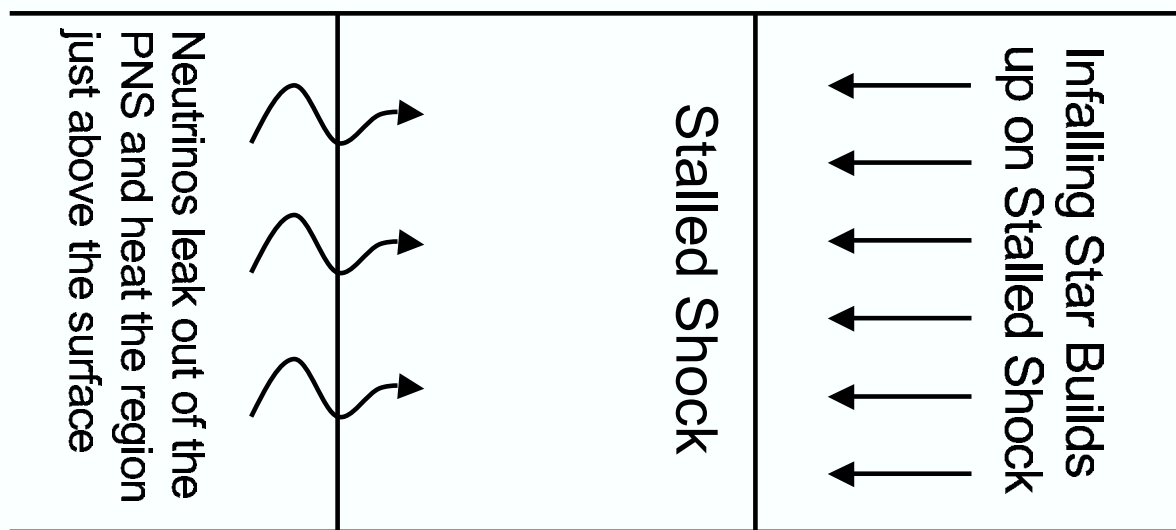
However, in the 1990s, the focus of convection in supernovae turned to the region above the proto-neutron star. In trying to explain the mixing in the ejecta seen in SN 1987A, Herant et al.[26] argued that the convection above the proto-neutron star driven primarily by entropy gradients could be the key to the explosion. Despite discouraging results by Yamada et al.[62] and Miller et al.[45] arguing that the convection above the proto-neutron star was only a minor effect, the simulations of Herant et al.[27] argued that this convection actually played a key role in the explosion. Convection above the proto-neutron star converted thermal energy deposited by neutrinos just above the proto-neutron star to kinetic energy in the explosion.

The effects of convection above the proto-neutron star (PNS) outlined by Herant et al.[27] can be illustrated by comparing the differences between the 1-dimensional (fig. 2) and multi-dimensional (fig. 3) pictures. In the 1-dimensional scenario (fig. 2), material from the infalling star piles up on the stalled shock. For a successful explosion to develop, the engine must be able to push the matter outward. With

²At the PNS surface, electron neutrinos are no longer trapped and, because of this, this surface region becomes extremely electron poor, producing a gradient in the lepton number.

1-Dimensional Supernovae

PNS Surface



The neutrino-heated region becomes hot enough to emit neutrinos itself without pushing out the shock considerably.

Figure 2: Supernovae in the 1-dimensional picture. Matter is not allowed to mix, so the transfer of energy is much more difficult.

Convection Enhanced Supernovae

- **Neutrino Heated Material Rises, Cooling Adiabatically Before Losing Its Energy Via Neutrino Emission**
- **Infalling Material Convects Down to the Proto-Neutron Star. It Does NOT Pile Up at the Accretion Shock.**

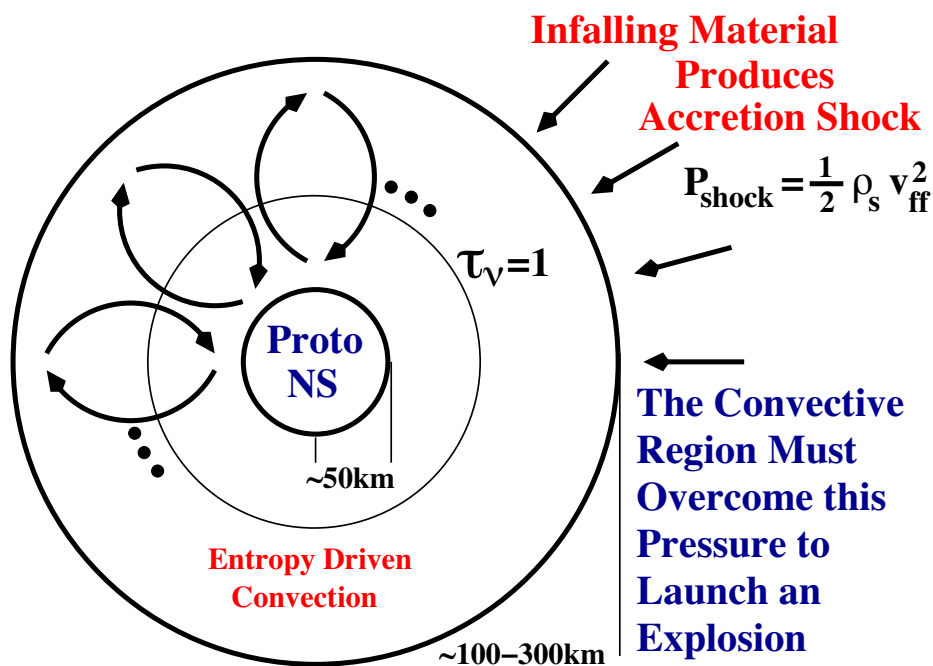


Figure 3: The convection picture from Herant et al.[27].

time, this material piles up on the shock, making the explosion harder and harder to explode. The neutrinos leaking out of the PNS primarily heat the region just above the PNS surface. If this region can not expand, it heats up so high that it too emits neutrinos, losing all of the energy it has gained from the PNS. These two issues make it very difficult for a 1-dimensional calculation to produce a successful explosion.

Convection above the PNS alleviates both of these issues. The infalling material falls onto the shock, but it does not build up on the shock (fig. 3). Instead, convection allows this material to flow down toward the PNS surface. This material does not build up on the shock, so does not contribute to the matter that must be pushed outward to drive a successful explosion. It accretes onto the PNS, depositing additional energy that can help drive a supernova explosion (possibly through neutrino emission). Both of these effects make it easier to produce an explosion. In addition, any heating just above the PNS surface causes the heated material to rise upward, converting its thermal energy into kinetic energy before it gets so hot that it re-emits that energy in the form of neutrinos. So convection also allows the region above the PNS to better convert its thermal energy into kinetic energy to drive an explosion.

What does this look like in a real simulation? Figure 4 shows a snapshot in time of the convective region above the proto-neutron star. The vectors show the direction and the shading denotes entropy. At roughly 300 km one can easily make out the edge of the shocked region where the infalling star hits the top of the convective region. The proto-neutron star ends roughly at 40-50 km. In this region, hot material at the base rises upward and pushes the shock outward. Infalling material pushes back and as it piles up, it flows down onto the proto-neutron star.

Although the Herant et al.[27] result was immediately corroborated by Burrows et al.[9], until recently, the debate about the role of convection met violent arguments. Janka & Müller[33] argued that convection could help the explosion, but only mildly. Mezzacappa et al.[44] argued that convection did not produce explosions. In 2003, Buras et al.[5] argued that explosions could not occur in collapsing stars, even with convection, without the invocation of new microphysics or magnetic fields. On the other extreme, Fryer and collaborators [16, 19, 21, 22, 24] produced explosions for all stars below about $20 M_{\odot}$ unless they artificially damped out convection. Arguments about numerical techniques ensued: e.g. hydrodynamic techniques, transport techniques, errors in the equation of state. Of the 4-5 supernova groups across the world, all but 1 focused on doing better neutrino transport.

In the past few years, this picture has changed. Blondin et al.[1] discovered that a number of instabilities exist in the region above the PNS. Burrows et al.[10] found that these instabilities could drive an explosion independent of neutrino transport scheme. The group led by Janka suddenly started to produce explosions[6]. At this point, over a decade after the Herant et al.[27] work proposed it, there is reasonable consensus in the field that convection above the PNS is critical in the supernova paradigm.

Why does it take so long for scientists to converge on this result? Because it is a difficult problem with a lot of interacting physics, core-collapse explosions must be modeled computationally, and this means scientists must be able to discern between numerical and real effects. And this can be difficult indeed. An example of how hard this is to do can be shown in the range of recent results from the Janka group. In 2003, they argued that convection could not produce explosions without new microphysics or magnetic fields. Two years later, they argued that they could get explosions for stars less massive than $12 M_{\odot}$, but nothing bigger could explode. At the Texas Symposium in Melbourne (Dec. 2006), they admitted that they could get $15 M_{\odot}$ stars to explode without new microphysics or magnetic fields. Issues in their numerics fooled them not once, but twice. This group is very strong in computational astrophysics and is considered by all to be excellent in both computational science and supernovae. But they, as we all do, struggle to disentangle the numerics from the physics. Bear this in mind when we discuss the current state-of-the-art results. What we believe now may be incorrect.

2.2 Convection Basics

The focus of most of the convective instabilities has been on either lepton or entropy gradients. These can be understood fairly easily by linear stability criteria and have been known since the 50s[40, 34]. To determine whether a region is Rayleigh-Taylor unstable to entropy gradients, let's examine a simple case of a homogeneous material with an entropy gradient and a constant gravity vector. Let's take a blob of material in this atmosphere with entropy $[S(r_0) = S_0]$ and slightly raise its entropy ($S_{\text{bub}} = S_0 + \Delta S$).

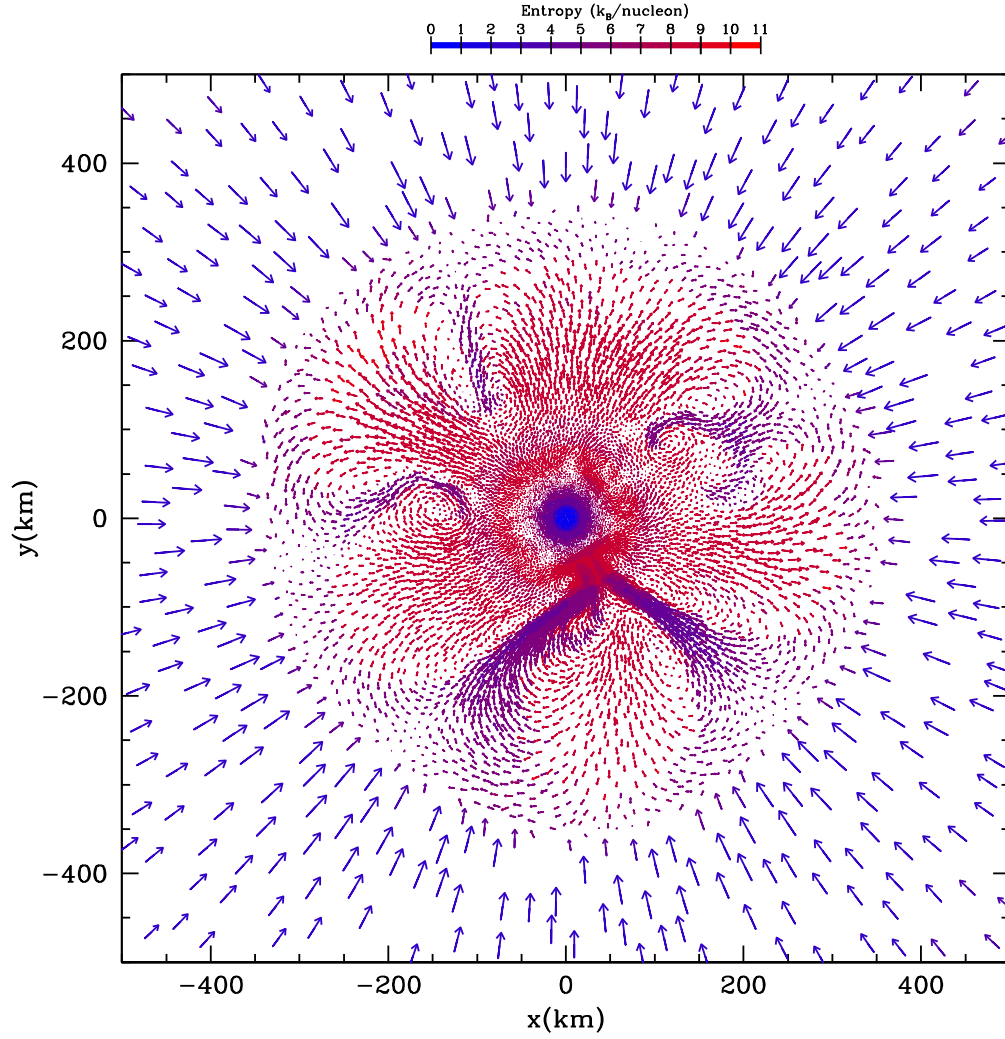


Figure 4: Convection in a core-collapse supernovae[19]. This plot shows a slice of a 3-dimensional simulation at a snapshot in time as the convective region starts to push the star outward[19]. The base of the convective region (surface of the PNS) is at roughly 40-50 km and the top of the convective region (position where the infalling stellar material shocks against the convective region).

As it evolves into pressure equilibrium with the material around it, its density will be lower than its surroundings (for most normal equations of state), and it will rise. If the entropy increases with increasing radius (r), then the bubble will only rise until $S_0 + \Delta S = S(r)$ where the density of the bubble and that of the atmosphere are the same. Such an atmosphere is stable to convection. If the entropy remains constant or decreases at increasing radius, the equilibrated bubble will always have a lower density than its surroundings and will continue to rise. Such an atmosphere is unstable to entropy gradients. Simply stated, the criterion for instability is:

$$dS/dr < 0. \quad (1)$$

If we include lepton (Y_L) gradients, the more general criterion for instability becomes:

$$(\partial P/\partial S)_{\rho, Y_L} dS/dr - (\partial P/\partial Y_L)_{\rho, S} dY_L/dr < 0 \quad (2)$$

where $(\partial P/\partial S)_{\rho, Y_L}$ is the partial derivative of the pressure with respect to the entropy and constant density ρ and lepton number Y_L and $(\partial P/\partial Y_L)_{\rho, S}$ is the corresponding partial derivative of the pressure with respect to the lepton number.

From this general equation, we can understand the bulk of the convective instabilities studied in core-collapse supernovae. The “Lepton-driven” instability is determined by the second term in equation 2. Essentially it says that if there is heavy material on top of light material, the light material will rise while the heavy material descends. The escape of the electron neutrinos at the surface of the PNS produces a region where there is a high neutron/electron ratio ($> 3 - 5$) above a region where this value is below 2.

In the region above the PNS, entropy-driven convection can dominate. As the initial bounce shock moves outward, it weakens and produces a lower shift in the entropy, leaving behind a negative entropy gradient. This starts entropy-driven convection. Heating near the PNS surface continues to maintain this entropy gradient, producing convection.

The electron neutrino emitted during electron capture is less energetic than the energy released during the capture. The neutron-rich region at the surface of the PNS tends to be hotter than the region below it. So although it is unstable to lepton gradients, the entropy gradient stabilizes against this convection. This leads to a doubly-diffusive or “salt-finger” instability³. It is this doubly-diffusive instability that must be understood if we are to solve whether convection within the PNS is important or not.

For the convection above the PNS, the lepton gradient is a negligible effect and we can estimate the convective instability (and timescale) on the entropy gradient alone. One way to estimate the timescale of this convection is to use the Brunt-Väisälä frequency ω [12]:

$$\omega^2 = g/\rho(\partial\rho/\partial S)_P(\partial S/\partial r) \quad (3)$$

where $(\partial\rho/\partial S)_P$ is the partial derivative of the density with respect to entropy at constant pressure, and $g \equiv GM_{\text{enclosed}}/r^2$ is the gravitational acceleration. Here G is the gravitational constant and M_{enclosed} is the enclosed mass at radius r . If $(\partial S/\partial r)$ is negative, ω^2 is negative and the region is unstable. The timescale for this convection (τ_{conv}) is $(|1/\omega^2|)^{1/2}$.

In the limit where radiation pressure dominates the pressure term (reasonably true at the accretion shock), this equation becomes:

$$\omega^2 = g/S(\partial S/\partial r) \approx (1/S)(GM_{\text{enclosed}}/r^2)(\Delta S/\Delta r) \quad (4)$$

where ΔS is the change in entropy over distance Δr . Here we used the following relations: $S \propto T^3/\rho$ and Pressure $\propto T^4$. For the massive $23 M_\odot$ star studied by Fryer & Young[25], where $g \approx 1.5 \times 10^{12} \text{ cm s}^{-2}$, $\Delta S/S \approx 0.2$, and $\Delta r \approx 10^7 \text{ cm}$, the convective timescale is roughly 2 ms. Even on core-collapse timescales, this is extremely rapid and most simulations do not reproduce this result, probably because numerical viscosity (either through SPH artificial viscosity or numerical advection in grid codes) damps out the convection.

Some scientists prefer to estimate the growth time of Rayleigh-Taylor instabilities based on a more simplified equation using the Atwood number A :

$$\omega_{\text{Atwood}}^2 = kgA \quad (5)$$

³So-named because it is like the hot Mediterranean Sea pouring into the Atlantic Ocean. The Sea is saltier but hotter than the Ocean. At first, the two fluids are stable, but as the heat from the Mediterranean diffuses into the water below it, lepton-driven gradients dominate over the stabilizing entropy gradient, allowing convection to develop.

where $A = (\rho_2 - \rho_1)/(\rho_2 + \rho_1)$ and k is the wave number. Such an equation is designed for simplistic examples of a two density fluid chamber. But, if we again assume a radiation pressure dominated gas, this equation becomes:

$$\omega_{\text{Atwood}}^2 = k(GM_{\text{enclosed}}/r^2)(\Delta S/S). \quad (6)$$

If we pick a wave number roughly of the size scale of our convective region, this equation is identical to our equation derived using the Brunt-Väisälä frequency.

2.3 Where Are We Now?

Blondin et al.[1] introduced a new feature into convection above the PNS. Building upon the work of Houck & Chevalier[28], they argued that advective-acoustic (or vortical-acoustic) instabilities would develop low-mode oscillations. This has become the focus of a lot of the multi-dimensional results in the past few years[55, 51, 10, 15] and has brought a new appreciation of the complexity of convection above the PNS.

There are some issues to this new convective instability that are still being realized by the supernova community. First and foremost, the existence of this instability does not preclude the existence of the Rayleigh-Taylor instabilities scientists have focused on in the past. Indeed, because the growth time of the accretion shock instabilities tend to be > 100 ms, Rayleigh-Taylor instabilities will dominate at early times (Yamada, presented at this meeting[64, 65]). Figure 5 shows the explosion roughly 100 ms after bounce. At this time, the only strong instabilities are Rayleigh-Taylor. As the edge of the shock front moves outward, the Rayleigh-Taylor instabilities will naturally develop lower order modes (Rayleigh-Taylor modes will have instabilities roughly the size of the convective region). Roughly 350 ms after the explosion, the Rayleigh-Taylor instabilities still dominate (Fig. 6), but an $l = 1$ mode convection has developed that almost assuredly is an accretion instability. So this new instability will only play a role in extremely delayed explosions.

The long delay in the development of accretion shock instabilities places limits on the effectiveness of this instability in driving supernova explosions. The explosion energy is derived from the energy stored in the PNS or the convection region above the PNS. Once the explosion is launched, it is very difficult to inject further energy in the shock (unless we invoke magnetic fields). Fryer[23] made some simple estimates of the energy making the following assumptions: 1) the explosion energy is limited to the energy in the convective region, 2) the convective region can be mimicked by a radiation dominated gas ($\gamma = 4/3$), 3) the structure total energy in the convective region is not too different from the equilibrium solution. In pressure equilibrium, the pressure profile $[P(r)]$ of a radiation dominated gas is[11]:

$$P(r) = [1/4M_{\text{NS}}G(S_{\text{rad}}/S_0)^{-1}(1/r - 1/r_{\text{shock}}) + P_{\text{shock}}^{1/4}]^4 \text{erg cm}^{-3}, \quad (7)$$

where M_{NS} is the mass of the proto-neutron star, G is the gravitational constant, S_{rad} is the entropy in Boltzmann's constant per nucleon, $S_0 = 1.5 \times 10^{-11}$ and $r_{\text{shock}}, P_{\text{shock}}$ are the radius and pressure of the accretion shock produced where the infalling stellar material hits the convective region. P_{shock} is set to the ram pressure of the infalling material $\equiv 1/2\rho v_{\text{ff}}^2$ where ρ is the density of the material and v_{ff} is the free-fall velocity at the shock. In our derivation, we will set our free parameter to the accretion rate (\dot{M}_{acc}) of the infalling material. Using mass continuity, $\rho = \dot{M}_{\text{acc}}/(4\pi r_{\text{shock}}^2 v_{\text{ff}})$, the pressure at the shock is:

$$P_{\text{shock}} = (2GM_{\text{NS}})^{0.5} \dot{M}_{\text{acc}} / (8\pi r_{\text{shock}}^{2.5}). \quad (8)$$

The energy density for a radiation dominated gas is just $3 \times P(r)$:

$$\begin{aligned} u(r) = & 3 \left[4.7 \times 10^8 \frac{M_{\text{NS}}}{M_{\odot}} \frac{10 k_{\text{Bnuc}}^{-1}}{S_{\text{rad}}} \left(\frac{10^6 \text{cm}}{r} - \frac{10^6 \text{cm}}{r_{\text{shock}}} \right) \right. \\ & \left. + 1.2 \times 10^6 \left(\frac{M_{\text{NS}}}{M_{\odot}} \frac{\dot{M}_{\text{acc}}}{M_{\odot} s^{-1}} \right)^{0.25} \left(\frac{2 \times 10^7 \text{cm}}{r_{\text{shock}}} \right)^{5/8} \right]^4 \text{erg cm}^{-3}. \end{aligned} \quad (9)$$

Note that the near the neutron star surface, the ram pressure is small compared to the pressure component compensating for the gravity of the neutron star. This means that the energy density in this region is

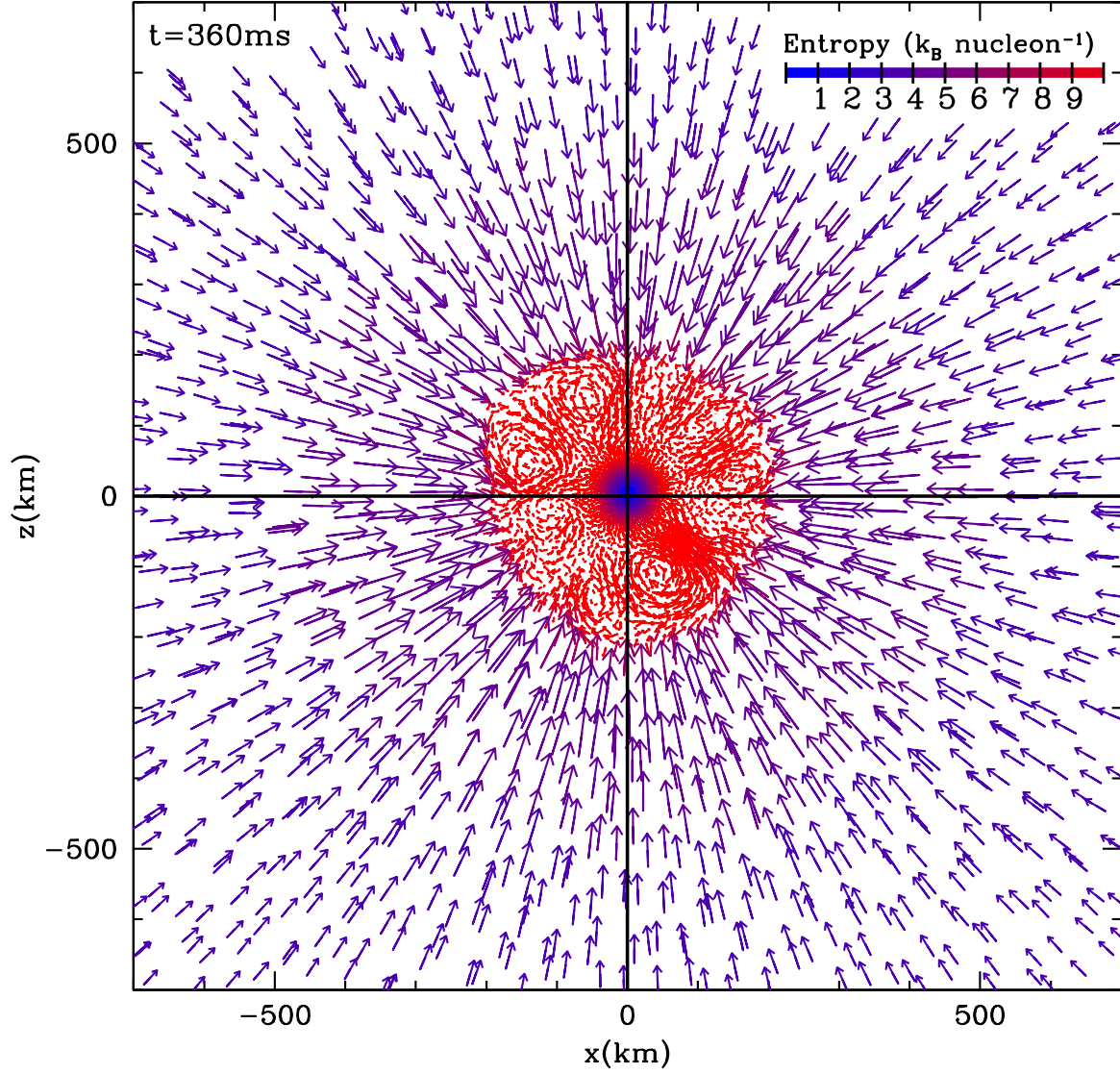


Figure 5: Slice of a 3-dimensional collapse of a $23 M_{\odot}$ roughly 100 ms after bounce[25]. This quickly into the explosion, we do not expect any shock instabilities to have developed. Indeed, the the convective cycles, with sizes comparable to the size-scale of the convection region.

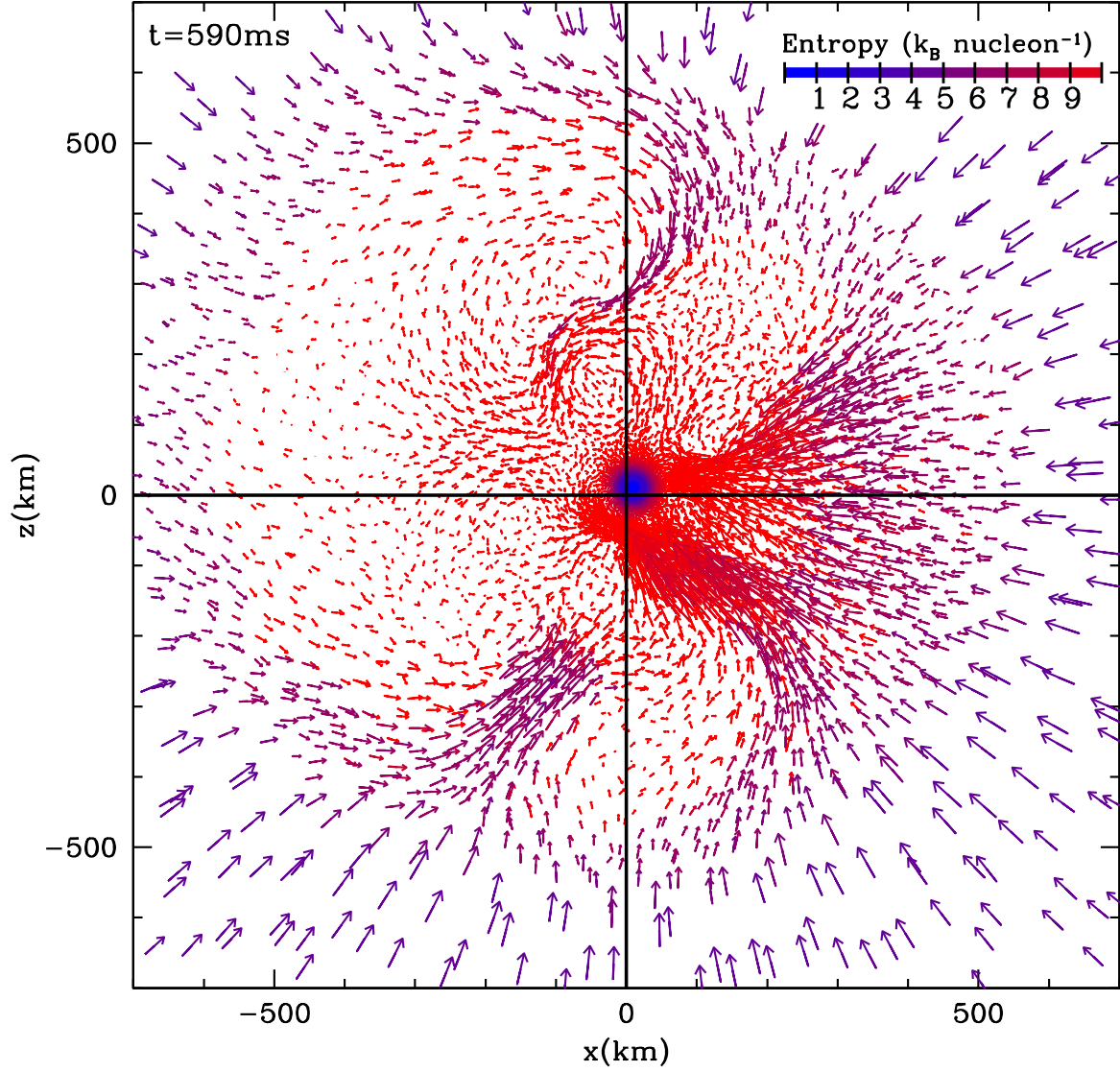


Figure 6: Slice of a 3-dimensional collapse of a $23 M_{\odot}$ roughly 350 ms after bounce[25]. By this time, we expect shock instabilities to have developed, and an $l=1$ mode has developed that could well be caused by this instability. But Rayleigh-Taylor instabilities also exist.

dominated by the neutron star’s mass. At higher radii, the energy density is set by the ram pressure. The ram pressure, in turn, is determined by the density structure of the star, but for stars less massive than $15\text{--}20\text{ M}_\odot$, this infall rate drops considerably after the first 50–200 ms [16].

If the explosion has a long delay, there won’t be much energy stored in the convection region (see Fryer 2006 for more details). It is possible that the explosion energy can be stored in the PNS and have it inject this energy even after the launch of the shock. Burrows et al. [10] suggested that accretion shocks could excite oscillations in the PNS. These oscillations could store energy to drive an explosion, but Yamasaki & Yamada [64, 65] found that this energy is less than 10^{50} erg (and is inefficient at imparting this energy into the envelope), a full order of magnitude less than what we need to produce a “normal” supernova explosion.

The long growth-time of the acoustic instabilities means both that Rayleigh-Taylor instabilities dominate at early times when we need to make the explosion occur if we want a strong explosion. For the accretion shock instabilities to be important for supernovae, we must find a different source for the explosion energy. One possible source is rotational energy in the PNS. With strong magnetic fields, a spinning neutron star can inject considerable energy into the expanding supernova ejecta. To understand that, we must understand the role of convection when rotation is added to the picture.

3 Convection with Rotation

3.1 History

Much of the early work studying rotation in stellar collapse focused on how rotation could drive an explosion [47, 2, 58, 46, 31, 32]. Most of these papers required extremely high rotation rates and strong magnetic fields. But these mechanisms generally predicted an explosion well before the core of the star reached nuclear densities and the neutrino fluxes from these explosion mechanisms fell far short of what was observed in SN 1987A. In addition, the angular momentum required far exceeded what current stellar evolution models could predict.

In the 90s, scientists started focusing on the role rotation played in modifying the convection above the PNS. Yamada and collaborators [56, 63] argued that neutrinos would heat along the rotation axis much more effectively than along the equator, driving stronger convection along this axis. Fryer and collaborators [18, 21] discussed a different effect, the fact that the angular momentum would stabilize against convection along the equator. Both of these effects lead to stronger convection along the rotation axis and a larger asymmetry in the explosion. This asymmetry has been used to explain many observational features of core-collapse supernovae [48, 37, 29, 30].

3.2 Basics of Convection with Rotation

To understand the effect of rotational gradients on convection, we can use a similar linear stability analysis to the one we used for lepton and entropy gradients 7. Let’s consider the net force on the blob in a rotating atmosphere with an entropy gradient:

$$\Delta a = g \left(1 - \frac{\rho + \Delta\rho}{\rho_{\text{blob}}} \right) + \frac{j_{\text{blob}}^2 - (j + \Delta j)^2}{(r + \Delta r)^3} \quad (10)$$

where $j_{\text{blob}} = j$. For acceleration to remain positive (so the blob will continue to rise) any entropy gradient must overcome an angular momentum gradient. The corresponding Solberg-Høiland instability criterion is (Endal & Sofia 1978):

$$\frac{g}{\rho} \left[\left(\frac{d\rho}{dr} \right)_{\text{adiabat}} - \frac{d\rho}{dr} \right] > \frac{1}{r^3} \frac{dj^2}{dr} \quad (11)$$

The left half of this equation corresponds to the entropy gradient condition we’ve already studied. But the condition for instability set to zero is altered by the presence of an angular momentum gradient. If the angular momentum increases with increasing radius as it does for our core collapse models, then the entropy gradient must overcome the angular momentum gradient to drive convection. In our simulations,

Rotation

Models of Rotating Stars Now Exist!

Rotation Important for:

**Neutron Star Spins and the Nature of
Young Pulsars**

**Gravitational Wave Emission:
Bar-Modes and R-modes**

Asymmetric Supernova Explosions!

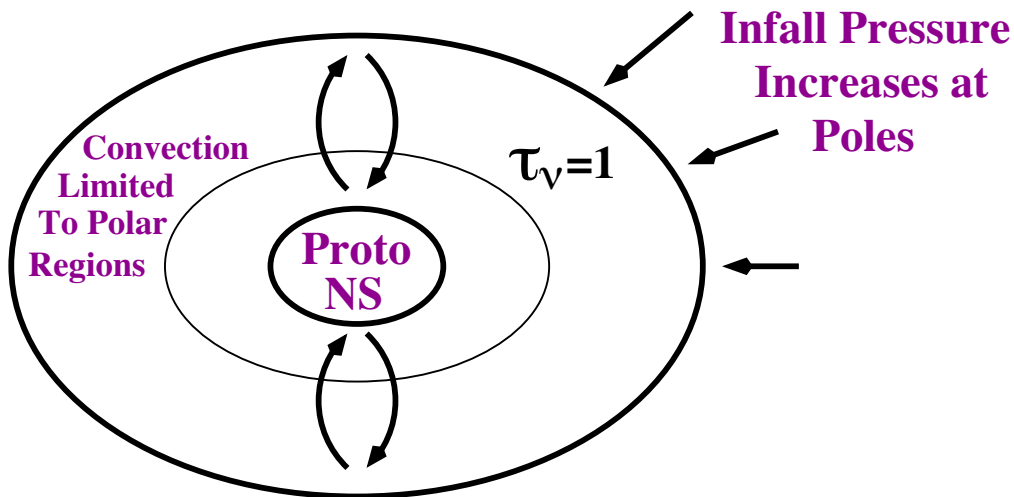


Figure 7: Rotation in Core-Collapse supernovae. Increased neutrino heating along the rotation axis and constraints on the convection in the equator combine to produce asymmetries in the supernova explosion.

the high entropy bubbles are unable to rise through the large angular momentum gradient and the convection is constrained to the polar region. The overwhelming effect of rotation on supernova models is this constraint on the convection and it causes weaker, asymmetric explosions.

But the pressure is also altered by the angular momentum. It produces a more condensed profile along the rotation axis and this will lead to more neutrino energy deposition along these poles[62, 56, 38].

3.3 Where We are Now?

Although rotation has been modeled in collapse extensively, we still do not have accurate progenitor models to definitively determine the role of rotation on convection. In addition, although estimates of the magnetic fields have been made, magnetic fields have not yet been modeled self-consistently. This is the next big step for rotation stellar collapse.

4 Rotation in GRBs

Right now, the history of GRB models is fairly limited. Only a handful of calculations have been made[42, 53, 54]. The simple disk picture was shattered by the first set of 3-dimensional simulations. Angular momentum can help provide support around the black hole and convection cycles can develop. Figure 8 shows a slice of a 3D calculation where the y-axis is the rotation axis. Far from being a simple disk along the equatorial region, vigorous convection has developed. The angular momentum in this matter also contributes to additional instabilities, and this angular momentum can drive gravitational waves.

This problem is in its infancy, and much more work on any number of topics must be done before we know what issues must be studied and what physics is relevant. Among these is the role of magnetic fields and a better instability analysis of the Rockefeller et al.[54] simulations.

5 Gravitational Waves

Rotation clearly plays a key role in the explosions of GRBs, and because of this, we are also assured of producing gravitational waves (GWs). The signal peaks shortly after the collapse of the massive star and quickly drops off (9). The rotation provides an ideal way method to produce a time-varying quadrupole moment in the mass motions. Even so, the GW signal from collapsars is strong enough for advanced LIGO to observe only if the GRB is very near to the earth (within the Galaxy or the local group).

In comparison, there is no guarantee that normal supernovae will produce a strong enough time-varying quadrupole moment to be observed at all. Convection alone produces a GW signal more than 100 times weaker than what is produced in collapsars (10). With rotation or asymmetries (especially in the neutrino emission), this signal could easily be an order of magnitude greater, detectable by advanced LIGO if the supernova occurs in the local group. Because of the much higher rate of supernovae over GRBs, we are more likely to detect supernovae in GWs than GRBs⁴. Despite the difficulty in detecting stellar collapse in GWs, GW observations will tell us much about the rotation and asymmetries in the mechanism itself.

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⁴Note that short duration bursts are believed to be produced by the merger of two neutron stars. Such GRBs will be accompanied by strong GW emission.

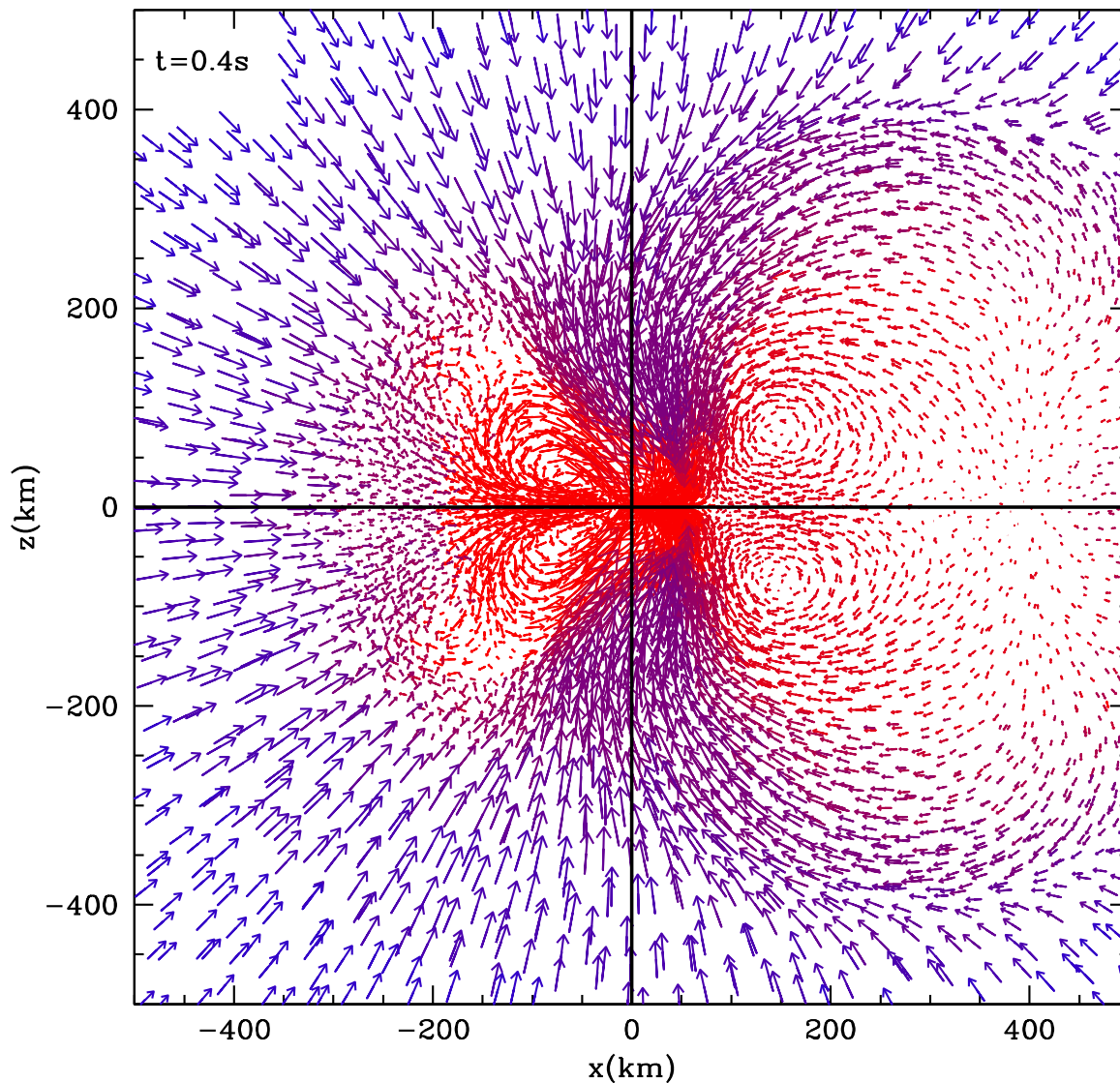


Figure 8: A 3-dimensional slice of a collapsar[54]. Angular momentum prevents direct accretion onto the black hole. Convective instabilities develop and move outward. It is hard to tell that angular momentum is even present in this calculation from this slice of the model. But angular momentum does more than prevent the accretion of material onto the black hole. It contains considerable energy, and viscous forces heat the material near the black hole considerably, driving an explosion.

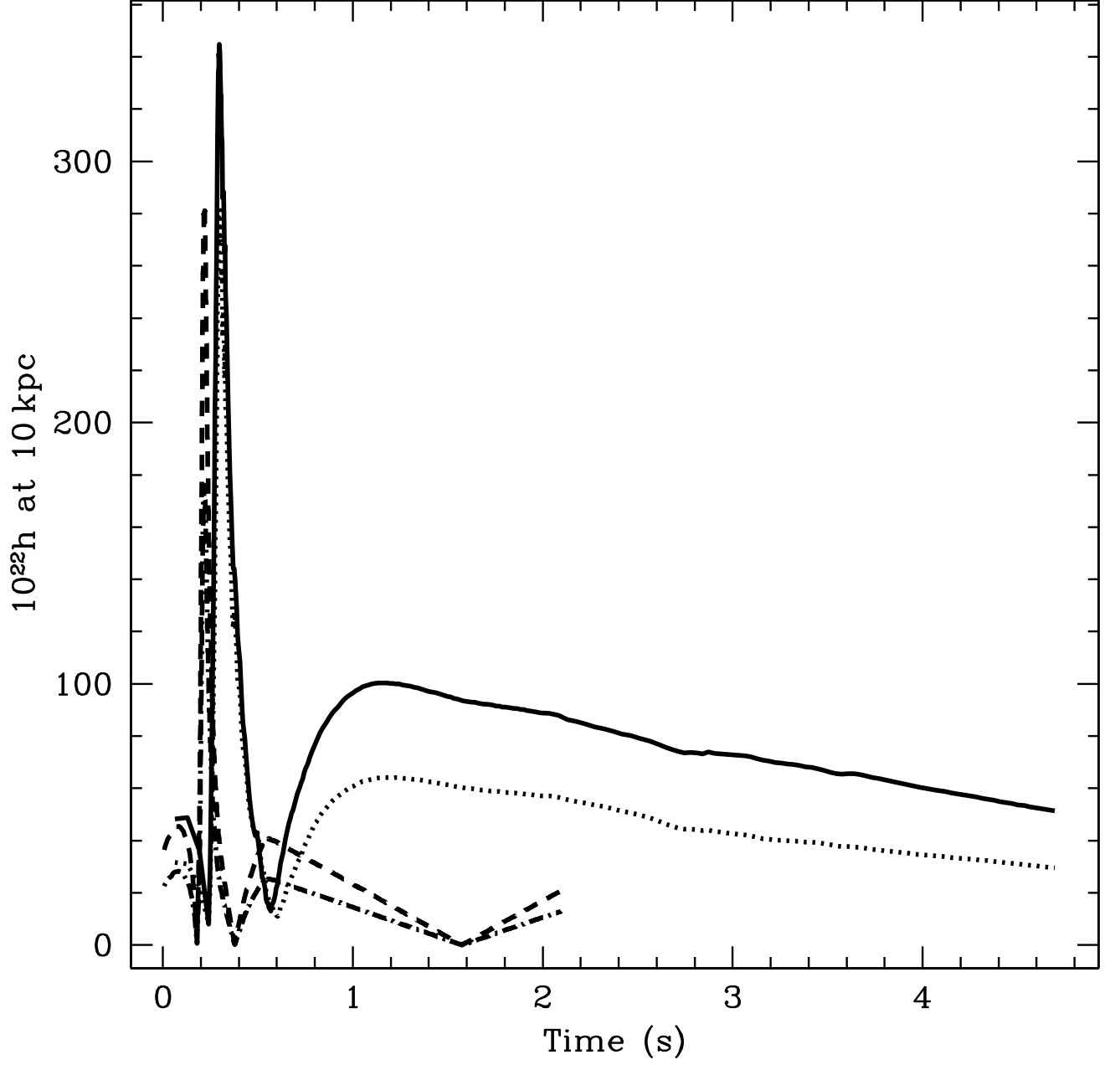


Figure 9: Gravitational wave emission from a $60 M_{\odot}$ star[54]. Angle-averaged wave amplitude of the gravitational wave emission arising from mass motions as a function of time for both a rapidly-rotating (solid/dotted lines for the two GW components) and slowly rotating (dashed/dot-dashed) simulations.

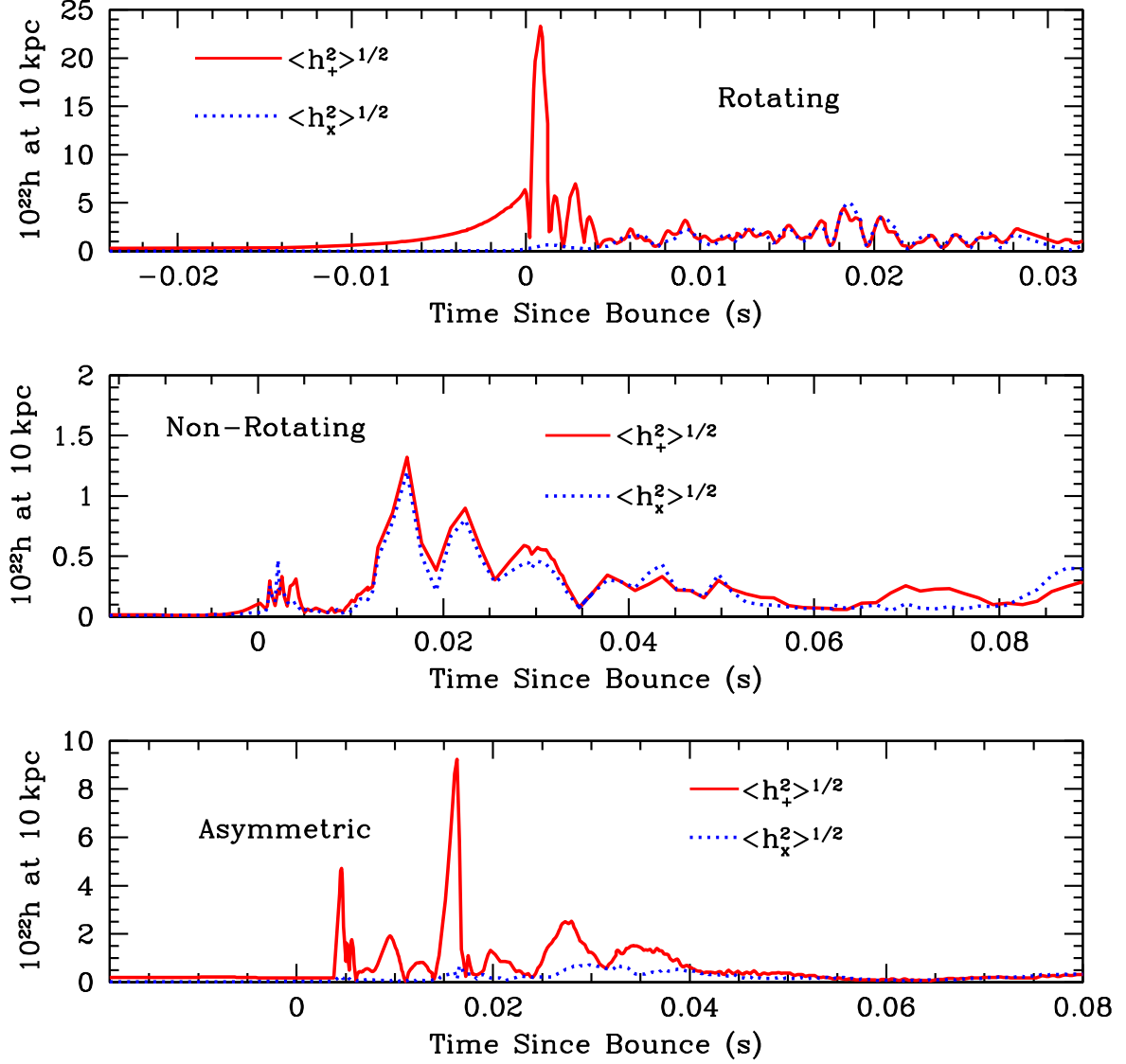


Figure 10: Angle averaged wave amplitudes from mass motions of 3 representative models of rotating, non-rotating, and asymmetric collapse supernovae. The fast-rotator produces the strongest signal, occurring at bounce. The asymmetric collapse simulation produces a reasonably strong signal, but not necessarily at bounce. The asymmetric neutrino signal will actually dominate the GW signal. The non-rotating case does not produce a strong signal.

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