Models of Pregeometry

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Abstract

We discuss an approach to the quantization of gravity, known as *pregeometry*, a notion going back to J.A. Wheeler and A.Sakharov. Over the past twenty years many different pregeometrical models have been proposed and we give a (very) short summary of the most important ones. The emphasis, however, is put on a model due to the author, which is based on random graphs. The predictions of that particular model is discussed, as are its relationship with other approaches to quantum gravity.

Introduction

Modern physics consists essentialy of two different theories, which are both very elegant and powerful. The one is *quantum field theory*, dealing with subnuclear length-scales, while the second is *general relativity*, which deals with stellar and galactical scales. Quantum field theory provides a succesful description of three of the four fundamental forces, namely electromagnetism and the strong and weak interactions, it even allows a unification of these into one grand unified theory. General relativity is a *classical* theory of gravitational phenomena, and has sofar defied quantization: we have no consistent theory of quantum gravity. A lot of work is carried out trying to unite general relativity and quantum theory, this truely unified theory would then work for all length scales (at least in principle).
To get an idea of the complexities involved in quantizing gravity, it is worthwile to note the level of mathematical abstraction involved in the

formulation of the theory. General relativity is formulated in terms of differential geometry, so in order to specify the model we have to specify:

1. Euclidean/Minkowskian geometry (flat space), i.e. the vectorspace R^n equipped with a metric of Euclidean or Lorentzian form.
2. A differentiable structure, i.e. a family (U_α, ϕ_α) of regions U_α and differentiable maps $\phi_\alpha : U_\alpha \to \mathsf{R}^n$, giving us a means of defining coordinates on the manifold $M = \cup U_\alpha$
3. An affine structure, i.e. a connection or covariant derivative allowing us to parallel transport vectors on M (this is essential to the equivalence principle).
4. A symplectic structure, i.e. a Poisson bracket, a Hamiltonian or Lagrangean to define dynamics.
5. Fiber bundles to specify the matter content of the theory.

Clearly, general relativity is a very "high level theory". By this I mean to say that it requires a lot of structure (and hence axioms in a rigorous formulation). Using the same terminology, "low level theories" are, mathematically speaking

- Formal logic.
- The theory of categories and sets.
- General topology.
- General algebra.

These are to some extent complementary (one can be expressed in terms of one of the others). It is at this level pregeometry starts.
The basic idea is to derive the differential geometric structure of general relativity from some underlying, more fundamental, theory, *pregeometry*. To avoid confusion, I give the definition of that word in the sense I'm using it.

> PREGEOMETRY: (Lit. "before geometry") A model in which general relativity is derived as a limiting case from a theory where geometrical concepts (some if not all, at least) were absent to begin with. Eventually, one would like a theory based solely on mathematical logic.

This is a very broad definition and a number of different approaches have been proposed. These can be roughly classified according to the following scheme.

- **Logic approach:** This is the approach originally invented by Wheeler [*Whe*]. The underlying structure is a formal logic, i.e. a language of propositions and ways of relating these (logical connectives such as $\wedge, \vee, \Rightarrow, \Leftrightarrow$ and negation $\neg$). It has especially been developed further by D.Finkelstein [*Fin*1]. One of its virtues is its close relationship with the quantum logic approach to quantum theory going back to Birkhoff and von Neumann. A special sub-class is the approach based on *cellular automatons* and similar abstract machines, this has been pursued by Feynman [*Fey*], 't Hooft [*tH*1] and Finkelstein [*Fin*2]. The problem with such models is, of course, their very high level of abstraction: it is often difficult to relate them to the physical reality.
- **Set theoretic approach:** Here the fundamental structure is taken from set theory. Also here has Finkelstein [*Fin*3] made contributions (quantum sets). An approach I am particularly fond of myself is that of the *causal sets* going back to Penrose [*Pen*] and 't Hooft [*tH*2], but developed further by Bombelli et al. [*Bom*] Such causal sets completely determine not only the topological but also the conformal structure of space-time, as was proven by Hawking et al. [*Haw*].
- **Algebraic approach:** The most famous example is probably the Wigner $6j$-symbol approach to 3D-gravity, this is essentialy based on the theory of representations of SU(2).
- **Field theoretical approach:** This is the approach going back to Sakharov: gravity is considered as a result of all the other interactions in much the same way the elastic properties of a solid follows from the electromagnetic interactions of its constituents. This has been pursued especially in Japan by Terazawa [*Ter*], Akama and Oda [*AO*]. In their approach we typically start out with a family of n scalar fields ϕ^a on a n-dimensional topological space (R^n, say),

and one then defines the viel-bein e^a_μ as $\partial_\mu\phi^a$, and hence the metric becomes $g_{\mu\nu} = \eta_{ab}\partial_\mu\phi^a\partial_\nu\phi^b$. One can prove that the correct action is induced. The main problem, as I see it, is the introduction of a finite dimensional space: Where does it come from?

- **Discrete Space-Time Approach**: One way of viewing this approach, is an approximation to the smooth manifold structure of general relativity. One can, however, also view it as an autonoums theory: the smooth space-time then appears as a long wave-length limit. In other words: it is the differentiable manifold that is an approximation, reality is discrete (Kronecker: "God created the natural numbers; the rest is the work of man."). The first break-through was achieved by Regge [*Reg*], and it has later been generalized to *simplical gravity* and *matrix models* (for $d = 2$), see [*Dav*]. Also T.D.Lee has worked on this [*Lee*].

- **Topological approach**: To my mind this is one of the most promising new theories. The starting point is the concept of *quantum topology*, first introduced by C.Isham [*Ish*]. It is closely connected to the logical approach but is closer to physical reality. Important people in this field are, Alvarez, Verdaguer, Céspedes [*ACV1*], [*Ant2*], A.Gribb and R.R. Zapatrin [*GZ*], besides Isham and his coworkers Yu.Kubyshin, and P.Renteln.

- **Random or chaotic approach**: This is the *random dynamics* of H.B.Nielsen [*Nie*]. The known laws of nature (aswell as the structure of space-time) arise as a limiting procedure in which all information of the underlying structure is washed out. Hence, no matter which structure we take as being fundamental at the Planck scale, we will recover the known laws in our low-energy, long wavelength limit (essentially by Taylor expanding to lowest order). Work in this field have also been done by J.Illioupoulos and by Nielsen's collaborators N.Brene, C.Froggat, S.E.Rugh.

- **Synthetic approach**: Here we combine some of the above mentioned approaches. This has been done by Zapatrin [*Zap*] and myself [*Ant*].

We will now continue by setting-up a particular model.

A Particular Model

What is the most basic element of any geometry? Obviously this has to be the notion of a point.[1] The particluar geometrical structure is then given be defining how these points are linked to eachother. This is then what we will take as our basic ingredients: *points* and *links*. In this way the fundamental structure becomes a *graph*. This graph is then going to become *space* in some continuum limit. *Time* will be defined in terms of the dynamics below.

In quantum theory, we introduce operators creating and annihilating the fundamental quantities, allowing us to express all possible processes and all physical quantities in terms of these. In this sense, quantum theory is much more elegant and simple than classical physics where no such structure exists (the closest analogue is the phase-space). Thus we introduce operators $a, a^\dagger, b, b^\dagger$ which annihilates/creates points/links. These basic operators (or perhaps rather *operations*, to avoid the suspicion that we are sneaking in quantum mechanics by the back-door!) are then our key to dynamics. We define an *evolution* as a sequence, $\{G_t\}$, of graphs such that G_{t+1} is obtained from G_t by the application of one of the operators $a, a^\dagger, b, b^\dagger$, e.g. $G_{t+1} = b^\dagger G_t$. Each such application will be called a *time step*, hence we identify the index t with time. The discreteness of the set-up guarantees the uniqueness of this definition.

So each time-step we choose one of the four fundamental operations (this is after all the definition of "time-step"). But how are we to make this choice? The minimal assumption is that this is made at random. We thus introduce four probabilities $p_1, .., p_4$, one for each of the four operations, and we impose the constraint $p_1 + p_2 + p_3 + p_4 = 1$ (at each time-step, *something* happens). The model is now almost completely specified. The only remaining thing is the "interaction" or "mixing" of the point and link operators: What happens if we attempt to delete a point, say, that doesn't exist, or create one, that is already there? What happens to the links going out from a point when we delete it? It seems to me, that the most natural choice is to forbid the deletion (creation) of a point/link that is not there (is already there), and when deleting a point, also to delete all its links.

[1] Actually, one can define *point-less topologies* in terms of categories. I will come back to this point later.

Intuitively it also seems natural to impose the condition that it is "easier" to delete a point with few (or no) links, than to delete one with many. We can summarize the model in the following rules:

- **Set-Up**: The *fundamental structure* is graph, the *fundamental processes* are deletion and creation of points and links.
- **Simplicity**: We do not allow multiple links between two points, and we do not allow a point to link to itself (i.e. the endpoints have to be different).
- **Time and Evolutions**: An *evolution* is a sequence of graphs, each following from the previous by the application of one of the four fundamental operators. Such an application is known as a *time-step*.
- **Stochasticity**: At each time-step what operator to apply is chosen at random according to a given set of probabilities.
- **Minimal Damage**: When the choice falls upon deletion of a point, we always pick that which has the lowest *degree* (i.e. number of links going out from it). We do not attempt to delete points or links that do not exist, nor to create ones that are already there. When deleting a point we automatically remove all its links aswell.

A graph can be described by its so-called *topological matrix* A, defined by

$$A_{ij} = \begin{cases} 1 & \text{points } i \text{ and } j \text{ linked} \\ 0 & \text{otherwise} \end{cases} \qquad (1)$$

The requirement, that no point links directly to itself implies that A has zeros in the diagonal. As an example, consider the graph shown in figure 1, its topological matrix is

$$A = \begin{pmatrix} 0 & 1 & 0 & 0 \\ & 0 & 1 & 0 \\ & & 0 & 0 \\ & & & 0 \end{pmatrix} \qquad (2)$$

We could introduce an *orientation* of the links by introducing a sign in A: A_{ij} would then be $+1, -1, 0$ depending on whether there was a link

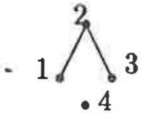

Figure 1: A simple example of a graph.

from i to j, or from j to i, or no links between the two points. Similarly we could let A_{ij} take on any value, the absolute value $|A_{ij}|$ would then give the *length* of the link. But for our purposes we do not need this extra structure.

The set of all graphs will be denoted by Γ and is referred to as *metaspace*.[2] The basic operators act ergodically on this space, i.e. any graph can be obtained from any other by a suitable application of these operators a sufficient number of times. Given two graphs, there is always an evolution going from the one to the other, in fact there is an infinity of such sequences.

The Contents of the Model

Now we have a model, now we start to predict things. As our model is purely topological (in the present formulation), we must look for an interpretation of various important topological concepts, and we must see what the model predicts in these cases. The most important such quantities (in any case, the ones I'm going to discuss) are (1) metric, (2) Euler-Poincaré characteristic and (3) dimensionality.

Note that k'th power of the topological matrix has the property:

$$(A^k)_{ij} = \#\{\text{paths of length } k \text{ between points } i,j\} \qquad (3)$$

This leads to a definition of a metric

$$D_{ij} = \min\{k \mid (A^k)_{ij} \neq 0\} \qquad (4)$$

i.e. the distance between points i, j is equal to the length of the shortest path between these (the length of path is equal to the number of links

[2] I have chosen this name for two reasons: firstly, to emphasize the connection with Wheeler's *superspace*, and, secondly, because I feel we're beginning to trespass the domain of metaphysics with this kind of models.

of which it is made). It is easily checked that this is indeed a metric on each connected component of the graph. In the limit where the number, ν, of points tend to infinity and the length of the links to zero, i.e. in the contiuum limit, this quanitity will tend to a metric function $d(x,y)$ on each connected component of the resulting space, in fact

$$\lim D_{ij} = d(x,y) = \sqrt{2\Omega(x,y)} \tag{5}$$

where $\Omega(x,y)$ is *Synge's world function*

$$\Omega(x,y) = \frac{1}{2}\left(\int_x^y ds\right)^2 \tag{6}$$

where the integral is along a geodesic from x to y. A new formulation of gravity based on this function has recently been proposed by Alvarez and coworkers [*Alv*], [*ACV*2]. One should also note that the same authors also have a pregeometrical model based on the matrix D above, [*ACV*1], [*Ant*2]. Let me just for completeness mention some of the properties of Ω: it can be proven that it contains all the needed information about the manifold, in particular:

$$\lim_{y\to x} \Omega(x,y)_{;\mu\nu} = g_{\mu\nu} \tag{7}$$

More can be found in the abovementioned articles.
In simplicial gravity and Regge calculus one builds-up space-time from some simpler structures, the simplices. This is actually a very general construction (algebraical topology, homology etc., see [*GH*], [*Mas*]). By a *d-cell* we mean a d-dimensional substructure which cannot be separated into other cells of the same dimensionality. Thus a 0-cell is a point, a 1-cell a link, a 2-cell a polygon, a 3-cell a polytope and so on. We're going to restrict our attention to *d-simplices*. A d-cell is a d-simplex if it consists of $d+1$ points and each point is linked to all the others (it thus has $\binom{\nu}{2} = \frac{\nu(\nu-1)}{2}$ links).[3] Hence 0- and 1-cells are simplices, triangles are 2-simplices and tetrahedra are 3-simplices. Let b_d denote the number of d-simplices in a given graph, we define the *Euler-Poincaré characteristic*, χ, to be

$$\chi = \sum_{d=0}^{\infty}(-1)^d b_d \tag{8}$$

[3] In graph theory, a simplex is often known as a *complete graph* or *clique*.

We have already introduced the symbol ν for the number of points, i.e. $b_0 = \nu$. The remaining quantities can be found from the topological matrix in a simple way. Note that $\mathrm{Tr}(A^k)$ is the number of k-gons (polygons with k faces) times $k!$ (due to invariance under permutations of the points), hence

$$b_1 = \frac{1}{2!}\mathrm{Tr}(A^2) \tag{9}$$

$$b_2 = \frac{1}{3!}\mathrm{Tr}(A^3) \tag{10}$$

Restricting ourselves to two-dimensional structures we would get

$$\chi_{2D} = \nu - \frac{1}{2}\mathrm{Tr}(A^2) + \frac{1}{3!}\mathrm{Tr}(A^3) \tag{11}$$

which is essentially the form of the action of the fashionable matrix models, which can thus be considered as following from our model.[4]
Three points i,j,k lie on a triangle if and only if $B_{ijk} \equiv A_{ij}A_{ik}A_{jk} = 1$, and the number of 3-simplices is then

$$b_3 = \frac{1}{4!}\sum_{i,j,k,l} B_{ijk}B_{ijl}B_{ikl}B_{jkl} \tag{12}$$

which we could suggestively write as $b_3 = \frac{1}{4!}\mathrm{Tr}'(B^4)$, thus higher dimensional structures are described by "tensor-models". In this way much of the machinery developed for matrix models could be transferred to $d > 2$, but would only be able to deal with the "static" case (no gravitons, no dynamics).
For a manifold, the dimension is d if and only if $b_{d+1} = b_{d+2} = ... = 0$ and $b_d \neq 0$, it would then be tempting to define the dimensionality of a graph simply as $\max\{d \mid b_d \neq 0\}$. But this is actually quite inappropriate! It could very well happen that, since the dimension is *not* constant all over space in general, that we would have one 100-simplex, say, sitting at the center with thousands of links, i.e. 1-simplices, going out from it. It would be unnatural to say that space then had dimension 100, since for all but a microscopic region the dimensionality would be one! What we need is a

[4] We would in general also have "kinetic terms" due to the possibility of changing the number of points and links, such terms are absent in the much simpler matrix models.

statistically weighted definition, the simplest possible one being, I think, that the dimensionality is equal to d if and only if an arbitrary point has probablity at least 50% for lying on the edge of a d-simplex and less than 50% for being an edge of a $(d+1)$-simplex, i.e. at least half the points sees space as being d-dimensional.

We can now run some simulations on a computer. We choose three probabilities p_1, p_2, p_3 (the last one p_4 is then given by $p_4 = 1 - (p_1 + p_2 + p_3)$), and start with an empty graph. At each time step we then pick at random one of the four fundamental operators with the given probabilities and apply it to the graph. We then calculate the resulting topological matrix and from that the numbers b_d. I have done this for 18 different choices of p_i $(i = 1, 2, 3)$, each time letting the computer go on for 200 time steps. The average values of the dimension d and χ was then calculated for each choice and the result is shown in figures 2 and 3.

The important fact is that $d = 3$ is predicted. For a proper three di-

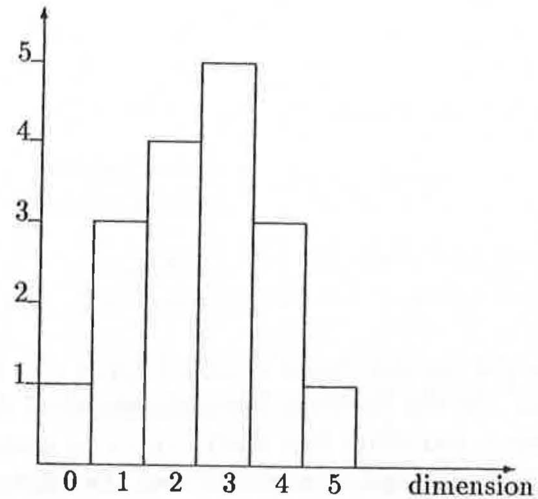

Figure 2: The number of times the different values of dimension occured in the simulations.

mensional manifold χ would vanish identically, the fact that it doesn't just implies that we do not have a proper manifold, i.e. the dimension can

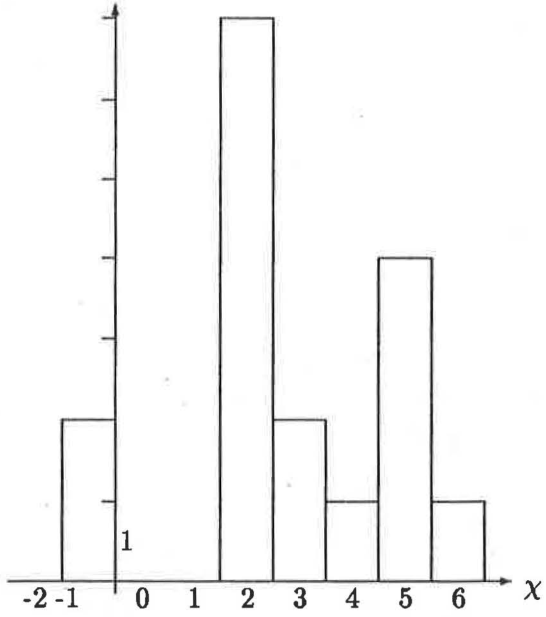

Figure 3: The frequency of the average values of the Euler-Poincaré characteristic χ in the simulations. The averages have been rounded off to the nearest integer value.

change as we move around on it, it can have non-smooth edges or jumps and so on.

A Master-Equation

In the introduction I defined pregeometry as a model which gave general relativity in a certain limit, and, furthermore, I claimed that the proposed model did just that. Time has come now to comment more on this.

The fact that the possible choices of fundamental operations at a given time step only depends on the structure of the graph at that "instant" and not on its previous history implies that the stochastic process is *Markovian*. It can be proven, see [*Gar*], that Markov processes of continous arguments

x, t satisfy a differential equation, the *Chapmann-Kolmogorov equation*, of the form

$$\frac{\partial}{\partial t}P(x,t|x_0,t_0) = -\sum_i \left\{ \frac{\partial}{\partial x_i}(a_i(x)P) + \frac{1}{2}\sum_j \frac{\partial^2}{\partial x_i \partial x_j}(b_{ij}(x)P) \right\} + + \int w(x|y,t)P(y,t|x_0,t_0) - w(y|x,t)P(x,t|x_0,t_0)dy \quad (13)$$

where $P(x,t|x_0,t_0)$ is the probability that the system will evolve from state x_0 at time t_0 to state x at time t. The functions a_i, b_{ij} are essentially the moments of the probability distribution

$$a_i(x) \equiv \lim_{\Delta t \to 0} \frac{1}{\Delta t} \int_{|x-y|>\varepsilon} (x_i - y_i)P(y, t+\Delta t|x,t)dy + O(\varepsilon)$$

$$b_{ij}(x) \equiv \lim_{\Delta t \to 0} \frac{1}{\Delta t} \int_{|x-y|>\varepsilon} (x_i - y_i)(x_j - y_j)P(y, t+\Delta t|x,t)dy + O(\varepsilon)$$

and

$$w(x|y,t) \equiv \lim_{\Delta t \to 0} P(x, t+\Delta t|y,t)$$

The Fokker-Planck, Boltzmann and Liouville equations are all of this type. If we assume that we can carry this equation over into our discrete set-up (in any case, it is supposed to hold in the continuum limit), we get an equation of the form (invoking the summation convention and summing over repeated indices)

$$0 = \left(-\frac{\delta}{\delta A_{ij}} G_{ij;kl} \frac{\delta}{\delta A_{kl}} + V(A) + W(A) \right) \Pr(A_0 \to A) \equiv \mathcal{H} \Pr(A_0 \to A) \quad (14)$$

where $V(A), G_{ij;kl}(A)$ are some functions, W is an integral operator

$$W(A)\psi(A) \equiv \int w(A|B)\psi(B)dB \quad (15)$$

and where we have used that our system only depend indirectly on time thus allowing us to replace the time derivative with a variation with respect to the topological matrix. This equation is similar to the Wheeler-DeWitt equation in canonical quantum gravity, the basic difference being (1) the appearance of the probability $\Pr(A_0 \to A)$ and not the wave-functional of the universe, (2) the topological matrix is used instead of the metric, and (3) the evolution described takes place in metaspace and not superspace. We have seen, however, taht we can introduce a metric on each connected components, whereby we could arrive at a Wheeler-DeWitt-like equation on each coordinate patch. Suppose we can make a spectral decomposition of the operator $\Pr(A_0 \to A)$, i.e. suppose we can find eigenstates $Q(A)$ and eigenvalues ξ such that

$$\sum_A \Pr(A_0 \to A)Q(A) = \xi Q(A) \quad (16)$$

It is easily proven that $\Pr(A_0 \to A)$ is a bounded operator (it is in general *not* symmetric: $\Pr(A \to B) \neq \Pr(B \to A)$ - we have irreversibility) and $|\xi| \leq 1$. This eigenvalue equation will in general be a recusrion relation, giving $Q(A)$ as a polynomial in ξ. For instance if the Markov process was random walks on the set of integers, the relation would read $\frac{1}{2}Q_{n-1} + \frac{1}{2}Q_{n+1} = \xi Q_n$, where we have written $Q(n) = Q_n$, the solution is just the Tchebyscheff polynomials. One can actually find the polynomials under some (rather drastic) simplifying assumptions, the details can be found in my thesis [*Ant*]. The probability can now be written

$$\Pr(A_0 \to A) = \frac{\int Q(A_0)^* \xi Q(A) d\sigma(\xi)}{\int |Q(A_0)|^2 d\sigma(\xi)} \quad (17)$$

where σ is a measure such that $\int Q(A)^* Q(B) d\sigma \propto \delta(A-B)$ (such a measure always exist, see again Gardiner's book [*Gar*]). It is impossible to resist the temptation of viewing $Q(A)$ as the analogue of the wavefunctional of the universe, and one sees that it satisfies $\mathcal{H}Q(A) = 0$. In this way, we have a "derivation" (in the Random Dynamics sense) of the Wheeler-DeWitt equation. Hence, our claim that this is a genuine model of pregeometry is justified: it even gives quantum gravity. For the reader who might be interested in the stochastic formulation of quantum theory I refer to [*PC*], [*AZ*] for further details.

It is natural to expand $Q(A)$ on the eigenfunctions of the integral operator W. Denote these by $\Psi_\Lambda(A)$ and let Λ be the eigenvalues, we then have an equation of the form

$$\mathcal{H}\Psi_\Lambda = (-\Delta + V(A) + \Lambda)\Psi_\Lambda(A) = 0 \quad (18)$$

i.e. Ψ_Λ satisfies a Wheeler-DeWitt-like equation with a "cosmological constant" Λ. In the long wavelength limit we would expect to see only fluctuations around an equilibrium solution, which would then have a fixed value for Λ.

Goodbye Points, Hello Logic

Let me just finish off with some (even more) speculative comments. The first concerns the very formulation of the model. While it is perhaps the most pedagogical way of presenting it, the concept of points is rather superflous. We could just aswell imagine an infinity of "latent" points which are linked together by links. This is also more appealing from another point of view: the links have a fermionic character - either they are there or they aren't - whereas points have more a "Boltzmann-statistical" character and are hence somewhat "classical". The most important virtue of such a pointless (in one of the meanings of that word!) description, is that it takes us into the concept of modern *pointless topology*, which already Isham mentioned should be of importance to quantum gravity. For a delighfull introduction to this fascinating area of modern mathematical research see Vickers [*Vic*].

This leads us to our second point. Pointless topology is closely related to non-standard logics, especially to what is known under the names of Intuitionistic Logic, Constructivist Logic or Modal S4-Logic, see Goldblatt [*Gol*] or Bell [*Bel*]. This kind of logic contains all of classical logic. Contrary to quantum logics, it *is* distributive, but the connectives have unusual properties, most importantly the failure of the Law of Excluded Middle (the law stating $A \wedge \neg A = 1$). An interesting property of this kind of logics is

$$\neg\neg\neg A = \neg A \text{ but } \neg\neg A \neq A \tag{19}$$

Now, quantum logics comes about through the structure of state-space (it is originally the lattice of closed subspaces of the Hilbert space). In my thesis [*Ant*], it is argued that, identifying graphs which differ only by a number of isolated points, we get a state-space with Intuitionistic Logic as the corresponding lattice structure. It is also argued that *aswell* classical as quantum logics can be considered as subclasses of Intuitionistic Logic, this is also the theme of a forthcomming paper. To go into details would require

the introduction of too many technical terms. Suffice to say that it is based on a *translation* of statements in terms of classical logical connectives into intuitionistic ones $\wedge \mapsto \wedge^\circ, \vee \mapsto \vee^\circ$.... originally introduced by K.Gödel to prove that classical logic was contained within intuitionistic logic. See [*Kle*] for details on this.

Conclusion

Let us stop before this texts gets too long and summarize. We started by noting that general relativity required an incredible amount of structure, and that a more fundamental theory could perhaps be found by restricting oneself to something simpler (mathematically speaking). This approach is known as pregeometry, and we discussed (briefly, admittedly) some contributions to this field which have been made over the last few decades. Not content with that, we set out to invent a model of our own. We took the concepts of points and links to be the most fundamental (we later got rid of the points, though) and thus considered the deletion and creation of these. This gave us a model for pregeometry based on random graphs. We saw that the topological information about this graph (representing "space") was encoded in a matrix A_{ij}, and we saw that the Euler-Poincaré characteristic could be expressed as a polynomial in this matrix, which established a connection with the fashionable matrix models and with the work in quantum topology of Alvarez, Verdaguer and Céspedes.

The model was put on a computer, and to our great surprise it turned out that a dimensionality of space of about 3 was predicted. The resulting space was not, however, a proper manifold ($\chi \neq 0$, whereas, for a proper three dimensional manifold χ would have to vanish).

Encouraged by this success, we turned our attention towards the master equation governing the evolution of such graphs. Our stochastic process was Markovian, and hence the Chapmann-Kolmogorov equation gave us a master equation which looked like the Wheeler-DeWitt equation. In fact, we was able to reexpress this equation in terms of the amplitude $Q(A)$ and not just the probability $\Pr(A_0 \to A)$, thus making it impossible to resist the temptation of viewing $Q(A)$ as a kind of wave-functional of the universe. The resulting equation did, however, show some new phenomenon, most remarkably perhaps the presence of a global term (the integral operator), which we could reinterpret as a cosmological constant term. We should also remember, that the variables in the equation are not the metric tensor g_{ij} as in the proper Wheeler-DeWitt equation, but rather either the topological matrix A_{ij} or the resulting metric on the graph. The latter becoming Synge's world function in the continuum limit.

The review of the model ended (rather suddenly) with some comments on the appropriate logical structure, which was argued to be non-Boolean, and even non-quantum. But quantum logics could be interpreted as being a special case by using some methods invented by K.Gödel who used them to prove that Boolean logics was contained as a special case. This method essentially consists in a reinterpretation of the *logical connectives* $(\wedge, \vee, \Rightarrow, ...)$. The resulting logic is known as *Intuitionistic Logic* and is closely related to topology.

I ought to mention some shortcommings of the model too. The nummerical simulations were only based on a small number of points, a restriction due to limited computer power. Hence the true continuum limit is not exactly known, but rather I had to resolve to handwaving in discussing the probable large scale structure. Also, the validity of the "derivation" of the Wheeler-DeWitt like equation is questionable. The arguments should hold in the continuum limit (by the correspondence principle), but they might not hold without modifications on the microscopic level. However, I would expect the possible modifications to be unimportant for the development of the large scale structure.

Let me finish by thanking the organizers of this meeting for inviting me here to Saint Petersburg, my supervisor Holger Bech Nielsen who found the needed fundings and the people with whom I have had discussions on this subject (Alvarez, Verdaguer, Kubyshin, Zapatrin, Gribb,...).

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