Hadronic Interactions from SU(2) Lattice QCD

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Abstract. We study hadronic interactions from Bethe-Salpeter amplitudes in quenched two-color lattice QCD, concentrating on the interactions of two scalar diquarks ($uC\gamma_5 d$) in S-wave scattering states. Between two identical scalar diquarks, we observe repulsive force in short-range region. By defining and evaluating the "quark-exchange part" in the interaction, which is induced by quark-exchange diagrams, or equivalently, by introducing Pauli blocking among some of quarks, we find that the repulsive force in short-distance region arises purely from the "quark-exchange part" and that it disappears when quark-exchange diagrams are omitted. It is qualitatively consistent with the constituent-quark model picture that the origin of short-range repulsion is a color-magnetic interaction among quarks. We also find a novel long-ranged attractive force, which enters in any flavor channels of two scalar diquarks and whose interaction range and strength are quark-mass independent.

1. Introduction

Hadronic interactions are of great importance in nuclear and hadron physics, and baryon-baryon interaction is one of the most significant issues to be clarified in reliable nonperturbative manners. After the successful proposal by Yukawa, the long- and the intermediate-range interactions have been well described by one-boson-exchange potentials (OBEP), in which key players for the hadronic interactions are mesonic modes. On the other hand, the short-distance part of hadronic interactions has not been well clarified so far. Especially, the origin of the strong repulsive core in nucleon-nucleon (NN) channel [2], which is responsible for the high-density nuclear phenomena, has been one of the longstanding problems in nuclear and hadron physics [3, 4, 5, 6, 7, 8, 9, 10].

Whereas it has been long since Quantum ChromoDynamics (QCD), the fundamental theory of strong interactions, was established as the fundamental theory, QCD-based description of hadronic dynamics has not yet been successful. Recently, a nucleon-nucleon (NN) interaction was evaluated from NN Bethe-Salpeter (BS) amplitudes on the lattice [11], and the authors observed the short-ranged repulsive force and the attraction in the intermediate region. In order to clarify the essential structure of hadronic interactions, we study hadronic potential using SU(2) lattice QCD [1], which is simpler than SU(3) QCD and can be a testbed for understanding hadron interactions.

2. Simulation Setup

We follow the strategy proposed by CP-PACS group [1, 12], where asymptotic Bethe-Salpeter wavefunctions on the Euclidean lattice are adopted to evaluate pion scattering length. This method has been further developed and employed for various hadron-hadron channels or interquark potentials [13, 14, 15].

We measure Bethe-Salpeter wavefunctions between two scalar-diquarks ($qC\Gamma_5 q$), and define an interhadron "potential" assuming a nonrelativistic Schrödinger-type equation. We focus on S-wave scattering states of two scalar diquarks by projecting BS-wavefunctions onto A_1^+-wavefunctions, which have overlap with $l = 0$ states. Simulations are performed in SU(2) quenched QCD adopting the standard plaquette gauge action and the Wilson quark action. The lattice extent is $24^3 \times 64$ at $\beta = 2.45$ and the corresponding physical lattice spacing is about 0.1 fm, which is determined to reproduce the string tension $\sqrt{\sigma}$ of 440 MeV. We use four different hopping parameters (quark masses) $\kappa = 0.1350$, 0.1400, 0.1450, 0.1500.

We investigate two-scalar-diquark scattering between $q_i C\Gamma_5 q_j$ and $q_k C\Gamma_5 q_l$, where i, j, k, l denote the flavors of quarks. We consider two different flavor combinations. One combination is $(i, j, k, l) = (1, 2, 1, 2)$, where only two independent flavors exist in four quarks. The other is $(i, j, k, l) = (1, 2, 3, 4)$, where all the quarks have different flavors. We adopt the same hopping parameters (quark masses) for all the quarks in both cases, and therefore all the scalar diquarks degenerate in mass. "Quark-exchange diagrams" in quark-propagator contraction are included only in the $(i, j, k, l) = (1, 2, 1, 2)$ case. The existence of quark-exchange diagrams are considered to be essential for the short-range interactions, since any Pauli-blocking effects among quarks are not included without exchange diagrams. We expect that the origin of short-range hadronic interactions can be accessed by comparing these two flavor combinations.

3. Lattice QCD results

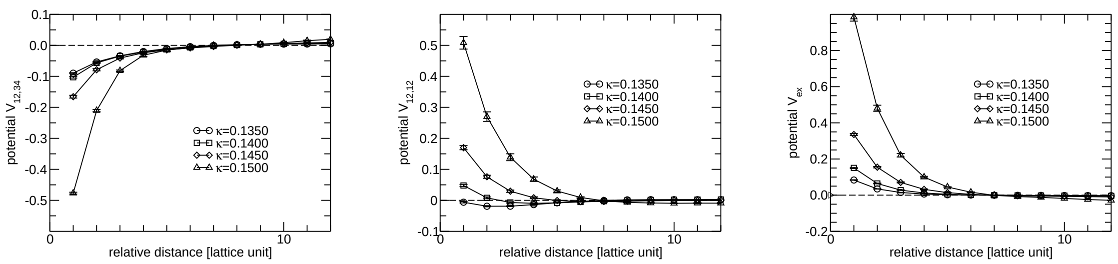

Figure 1. *Left*: Potentials $V_{12,34}(R)$ computed with the flavor combination, $(i, j, k, l) = (1, 2, 3, 4)$, are plotted as functions of relative distance R. *Middle*: Potentials $V_{12,12}(R)$ computed with the flavor combination, $(i, j, k, l) = (1, 2, 1, 2)$, are plotted as functions of relative distance R. *Right*: "Quark-exchange parts" of potentials, which are defined as $V_{\rm ex}(R) \equiv V_{12,12}(R) - V_{12,34}(R)$, are plotted as functions of relative distance R.

3.1. Potentials : $V_{12,34}(R)$ and $V_{12,12}(R)$

Fig. 1 [left] shows the reconstructed potentials $V_{12,34}(R, m_q)$ as functions of the interhadron distance R, which are extracted with the flavor combination $(i, j, k, l) = (1, 2, 3, 4)$. Here, we include no quark-exchange diagram and hence no Pauli-blocking effect among quarks is activated. One finds that the interaction in this channel is always attractive, and that the strength of this attraction depends on quark masses. $V_{12,34}(R, m_q)$ rapidly reduces with decreasing κ (increasing m_q), and finally the potentials at κ=0.1350 and 0.1400 coincide with each other, which implies the existence of a long-range *quark-mass independent* attraction.

The interhadron potentials $V_{12,12}(R, m_q)$ obtained with $(i, j, k, l) = (1, 2, 1, 2)$ can be seen in Fig. 1 [middle]. The important difference from the previous $(i, j, k, l) = (1, 2, 3, 4)$ case is quark-exchange diagrams newly included, which gives rise to Pauli-blocking effect among quarks. We now find *strong repulsions in the short-distance region.*

3.2. Potentials : Quark-exchange part $V_{\rm ex}(R)$

In the following, we define a "quark-exchange part" $V_{\rm ex}(R, m_q)$ in $V_{12,12}(R, m_q)$ as

$$V_{\rm ex}(R, m_q) \equiv V_{12,12}(R, m_q) - V_{12,34}(R, m_q), \tag{1}$$

which is nothing but the difference between $V_{12,12}(R, m_q)$ and $V_{12,34}(R, m_q)$. Taking into account that quark-exchange diagrams are included only in $V_{12,12}(R, m_q)$ and that the diagrams needed for $V_{12,34}(R, m_q)$ are identical with the direct diagrams in $V_{12,12}(R, m_q)$, it can measure "quark-exchange effect" or Pauli-blocking effect among quarks in $V_{12,12}(R, m_q)$. In practice, it is equivalent to assuming

$$V_{12,34}(R, m_q) = V_{\rm dir}(R, m_q), \quad V_{12,12}(R, m_q) = V_{\rm dir}(R, m_q) + V_{\rm ex}(R, m_q), \tag{2}$$

with a "direct part" $V_{\rm dir}(R, m_q)$ measured only with direct diagrams and a "quark-exchange part" $V_{\rm ex}(R, m_q)$ induced by adding exchange diagrams. Such decomposition of a diquark-diquark potential is conceptually similar to the usual decomposition procedure in RGM (Resonating Group Method) calculations in quark cluster models [7, 16]. Thus extracted potential $V_{\rm ex}(R, m_q) \equiv V_{12,12}(R, m_q) - V_{12,34}(R, m_q)$ shows a monotonous behavior as is seen in Fig. 1 [right], and therefore the decomposition seems beneficial for our purpose to clarify strengths or ranges of potentials. One can now find that *the short-range repulsion arises only from* $V_{\rm ex}(R, m_q)$, which implies Pauli-blocking effect among quarks is essential for the short-range repulsion.

4. Discussions

We have found that $V_{12,12}(R, m_q)$ and $V_{12,34}(R, m_q)$ can be expressed as shown in Eq.(2). The key property of the R-dependent potentials is twofold; *interaction ranges* and *strengths*. Hadronic interactions are phenomenologically incorporated as one-boson-exchange-potentials (OBEP), which are Yukawa-type and whose interaction ranges are directly connected to exchanged meson masses. Then, such OBE potentials should be sensitive to meson masses, *i.e.* quark masses. On the other hand, the short-range repulsive core is often considered to arise from a color-magnetic interaction induced by one-gluon-exchanges (OGE), whose strength is proportional to the inverse square of (constituent) quark masses, $\sim m_{\rm Q}^{-2}$. Such $m_{\rm Q}$-dependent interactions could be clarified by monitoring the strengths of the potentials.

We evaluate strengths and ranges of these decomposed potentials, $V_{\rm dir}(R, m_q)$ and $V_{\rm ex}(R, m_q)$, by fitting them with an exponential form, $F(x) \equiv A\exp\left(-\frac{x}{B}\right)$, which have two parameters, strength A and range B. (We also tried several other functions. See Ref. [1].) The fit results can be all found in Fig. 2, in which strengths A and ranges B for $V_{\rm dir}(R, m_q)$ and $V_{\rm ex}(R, m_q)$ at each κ are displayed.

4.1. strengths and ranges : $V_{\rm dir}$

In Fig. 2, the fitted parameters, strength $A_{\rm dir}$ and range $B_{\rm dir}$ of the attractive part $V_{\rm dir}(R, m_q)$, are shown as functions of a constituent quark mass $m_{\rm Q}$, which is defined by a half of axialvector-diquark mass in this paper. (Shown as "DIR" in Fig. 2.) Interestingly, both of the strength and the range of $V_{\rm dir}(R, m_q)$ exhibit flattening in the heavy quark-mass region, which indicates that $V_{\rm dir}(R, m_q)$ contains a universal attractive potential $V^U_{\rm att}(R)$. "Universal" here means that neither the strength nor the range of $V^U_{\rm att}(R)$ depends on quark mass and that $V^U_{\rm att}(R)$ always appears in any flavor channels.

We further define a quark-mass dependent part $V_{\rm att}^D(R, m_q)$ as

$$V_{\rm att}^D(R, m_q) \equiv V_{\rm dir}(R, m_q) - V_{\rm att}^U(R). \tag{3}$$

We also fit $V_{\rm att}^D(R, m_q)$ with $F(x)$, and the fitted parameters are shown in Fig. 2. We simply adopt $V_{\rm dir}(R, m_q)$ at $\kappa = 0.1350$ as the universal part $V_{\rm att}^U(R)$, since we observe almost no quark-mass dependence already at this κ. The strength and the range of $V_{\rm att}^D(R, m_q)$ are shown as "ATT(D)". The interaction range $B_{\rm att}^D$ no longer depends on quark mass. That is, $V_{\rm dir}(R, m_q)$ is approximately described by two independent parts;

$$V_{\rm dir}(R, m_q) \sim V_{\rm att}^U(R) + V_{\rm att}^D(R, m_q) = A_{\rm att}^U f_{\rm att}^U(R) + A_{\rm att}^D(m_q) f_{\rm att}^D(R). \tag{4}$$

First part represents a m_q-independent weak long-range force, and the second one does a short-range repulsive interaction that has an m_q-dependent strength and an m_q-insensitive interaction range.

4.2. strengths and ranges : $V_{\rm ex}$

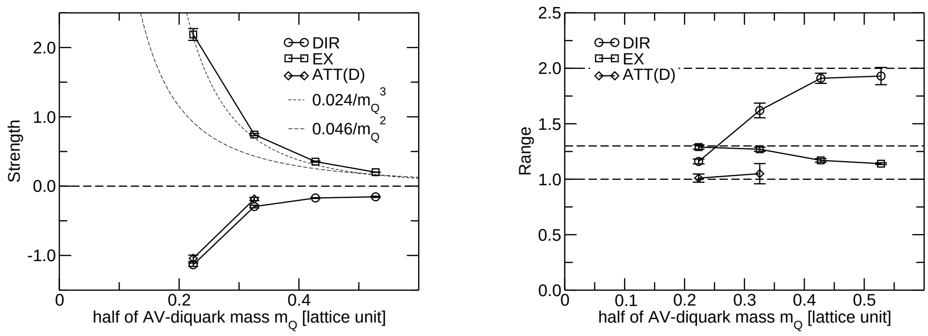

Figure 2. *Left*: The fitted strengths, $A_{\rm dir}$, $A_{\rm ex}$, and $A_{\rm att}^D$, of the potentials, $V_{\rm dir}(R, m_q)$, $V_{\rm ex}(R, m_q)$, and $V_{\rm att}^D(R, m_q)$, are plotted as functions of half of axialvector-diquark mass $m_{\rm Q}$. *Right*: The fitted interaction ranges, $B_{\rm dir}$, $B_{\rm ex}$, and $B_{\rm att}^D$, of the potentials, $V_{\rm dir}(R, m_q)$, $V_{\rm ex}(R, m_q)$, and $V_{\rm att}^D(R, m_q)$. The parameters for $V_{\rm dir}(R, m_q)$, $V_{\rm ex}(R, m_q)$, and $V_{\rm att}^D(R, m_q)$ are respectively shown as "DIR", "EX", and "ATT(D)".

The fitted parameters for the quark-exchange part $V_{\rm ex}(R, m_q)$ are shown in Fig. 2. (Shown as "EX" in Fig. 2.) Let us first have a look at the range parameter $B_{\rm ex}$, which are shown as "EX" in Fig. 2 [right]. One can find that the range $B_{\rm ex}$ in $V_{\rm ex}(R, m_q)$ is almost quark-mass independent. The constituent quark-mass (half of AV-diquark mass) variation from 0.53 to 0.22 gives rise to only 10% change in the range $B_{\rm ex}$, which is much smaller than expected from the meson-mass change. Moreover, at two largest κ's (two lightest quark masses), the range $B_{\rm ex}$ remains unchanged. $V_{\rm ex}(R, m_q)$ can be approximately expressed as

$$V_{\rm ex}(R, m_q) \sim A_{\rm ex}(m_q) f_{\rm ex}(R). \tag{5}$$

The quark-mass dependence appears only in the strength $A_{\rm ex}$. From these observations, we conjecture that the short-range repulsion observed here is not generated by meson poles as OBEP, but by some other origins.

One most possible mechanism for the short-range repulsion is a color-magnetic (CM) interaction among quarks as suggested by constituent-quark models. A CM interaction accompanied by Pauli blocking among quarks raises the energy of closely located two hadrons, which results in strong repulsions. In the first-order perturbation, the contributions from the CM interaction are proportional to its strength, whose m_q-dependence is given by $m_{\rm Q}^{-2}$.

The strengths $A_{\rm ex}$ are also shown in Fig. 2 [left]. $A_{\rm ex}$ increases in the light-quark-mass region and it is qualitatively consistent with the CM interaction as the origin of repulsion. In Fig. 2 [left], two fit functions, $Cm_{\rm Q}^{-2}$ and $Cm_{\rm Q}^{-3}$, are plotted as a long-dashed and a dashed line. The strength $A_{\rm ex}$ is well reproduced by $Cm_{\rm Q}^{-3}$ rather than $Cm_{\rm Q}^{-2}$, which implies that the quark-mass dependence of repulsion seems stronger than that of the strength of the CM interaction itself. However, considering that the Pauli blocking among quarks is essential for the repulsion and that the interaction range of the repulsive force has a very weak quark-mass dependence, the present results are consistent with the quark-model interpretation that short-range repulsion between hadrons arises from CM interaction among quarks.

5. Summary and Outlooks

We have evaluated inter-hadron interactions in two-color lattice QCD. We have considered two-diquark scatterings, whose flavors are (i,j) and (k,l), and extracted its interhadron potential $V_{ij,kl}(R)$. We have found

- *The existence of universal attraction between two hadrons.*
- *The measure of Pauli-blocking effect,* $V_{\rm ex}(R) \equiv V_{12,12}(R) - V_{12,34}(R)$*, always shows repulsive contribution at any* κ*'s.*
- *The quark-mass dependence of the strength of* $V_{\rm ex}$ *is consistent with or stronger than* m_Q^{-2}.
- *The quark-mass dependences of the interaction ranges of direct and exchange parts of potentials (*$V_{\rm dir}$,$V_{\rm ex}$*) are much weaker than expected from meson exchanges.*

Since quark-mass dependence in interaction ranges does not appear, the origin of the interactions we observed so far would be all gluonic interactions and/or flavor-exchange processes, *e.g.* color-magnetic or color-Coulomb interactions, and so on, rather than mesonic contributions. As was found in our analyses, attractive forces are readily masked by the strong repulsive force in $V_{12,12}$ in the light quark-mass region. If the universal attraction in hadronic interaction appears also in SU(3) QCD, they might be observed in a channel with no or less quark-exchange contribution between hadrons, such as $N\phi$ scattering state. In the lighter quark-mass region, meson-exchange contributions could largely emerge and be predominant, which is left for further studies.

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