UR-749

K^* - DOMINANCE OF K_{E3} FORM FACTOR AND SIRLIN'S RELATION

BY

K. YAMADA

K^* - DOMINANCE OF K_{e3} FORM FACTOR AND SIRLIN'S RELATION

K. Yamada

Department of Physics and Astronomy
University of Rochester
Rochester, New York 14672

and

Fermi National Accelerator Laboratory
Batavia, Illinois 60510

Abstract

We discuss the precise momentum-dependence of K_{e3} form factor $f_+(t)$ by studying some of recent experimental results on $K_L{}^o \to \pi^{\pm} e^{\mp} \nu$ decays. The parametrization of $f_+(t)$ based on the assumption of K^*-dominance can considerably improve the agreement of Sirlin's relation with existing data.

I. Introduction

The experimental study of semileptonic decays of kaons is quite interesting in that the momentum dependence of the weak form factor for the decay $K \rightarrow \pi e \nu$ can be directly studied in a time-like region. Most experimental results indicate that, for $K_L{}^0 \rightarrow \pi^{\pm} e^{\mp} \nu$, $f_+(t)$ is approximately a linearly-rising function of $t = -(q_e + q_\nu)^2$, where q_e and q_ν are momenta of $e^{\pm}$ and $\nu(\bar{\nu})$ respectively.[1] This fact has been believed to support the K*-dominance of $f_+(t)$ if the conventional linear-expansion parameter λ_+ is nearly equal to 0.030. However, the existing experimental results are not precise enough to tell us whether the actual form factor has the same curvature or derivative at every t-value as some theoretical predictions. This causes a serious problem when one tries to test the Sirlin's relation[2] on form factors of pseudoscalar mesons.

The purpose of this paper is to estimate a precise behavior of $f_+(t)$, in particular its derivative with respect to t near $t = 0$, based on existing data and theoretical models. We will find that the Sirlin's relation is much better satisfied than originally considered, if one assumes the K*-dominance of the form factor $f_+(t)$.

II. Analysis of Data

The parametrizations which are frequently used are the linear fit:

$$f_+(t) = f_+(0)\ (1 + \lambda_+ t/m_\pi{}^2) \quad , \tag{1}$$

and the pole-type fit:

$$f_+(t) = f_+(0)\ (1 - t/m_V{}^2)^{-1} \quad . \tag{2}$$

Theoretically it is quite natural to assume the t-dependence in the form of (2) with $m_V = m_{K^*}$, because $K\pi$ system has only $K^*(J^P = 1^-)$ as a well-established resonance. Some of recent experiments[3,4] definitely show that m_V is a little smaller than $m_{K^*} = 0.892$ GeV . However, this deviation of m_V from m_{K^*} is rather small and there is little doubt in that the picture of K*-dominance is right. It is tempting, therefore, to consider an alternative parametrization following recent remarks by Sehgal:[5]

$$f_+(t) = f_+(0)\,\frac{1}{1-t/m_{K^*}{}^2}\,\exp\left(\frac{\alpha t}{1-t/\Sigma^2}\right) \quad , \tag{3}$$

where $\sum = m_\pi + m_K$ and α is a real parameter to be determined experimentally. The validity of this choice remains to be tested by high-statistics experiments.[6] Here we assume that it is a good approximation for $m_e^2 \le t \le \Delta^2$ $(\Delta = m_K - m_\pi)$, which is relevant for the semileptonic decay. Eq.(3) is then our statement of generalized K*-dominance of $f_+(t)$. Although parameters λ_+ and m_V have been obtained from experiments, corresponding values of $\partial f_+(t)/\partial t$ at $t = 0$ can be significantly different from each other depending on the choice of parametrization.

Determination of the Parameter α

From the published data of Buchanan et al[4], we obtain with 95% confidence level that

$$\begin{aligned} \lambda_+ &= 0.043 \pm 0.0008 && (\chi^2/\text{d.f.} = 8.0/11) \\ m_V &= 0.75 \pm 0.05 \text{ GeV} && (7.9/11) \quad , \\ \alpha &= 0.50 \pm 0.24 \,(\text{GeV}/c)^{-2} && (8.0/11) \quad . \end{aligned} \tag{4}$$

It is difficult to say, from this data only, which parametrization is really favored. Several recent experiments report a considerably smaller value of λ_+ and therefore a larger m_V. In the follwing we describe a simple way to estimate the value of α when only λ_+ or m_V is known, which is often the case. Numbers in (4) are then used to check this method. One should keep in mind that the validity of the parametrization (3) is not tested in this way because χ^2/d.f. cannot be obtained. Let us suppose that Eq.(3) correctly describes the K_{e3} form factor $f_+(t)$. Then the corresponding value of $\lambda+$ for the linear fit is found by solving:

$$\int_{m_e{}^2}^{\Delta^2} dt\left[\lambda(t,m_K{}^2,m_\pi{}^2)\right]^{3/2} \left\{\frac{1}{1-t/m_{K^*}{}^2}\exp\left(\frac{\alpha t}{1-t/\sum^2}\right) \right. \tag{5}$$

$$\left. -f_+(0)\ (1+\lambda_+ t/m_\pi{}^2)\right\}^2 = \text{min.}$$

with $f_+(0)$ and λ_+ considered as free parameters. In (5), $\lambda(a,b,c) = a^2 + b^2 + c^2 - 2ab - 2bc - 2ca$ and $\lambda^{3/2}$ expresses a phase space density.[7] The relation between α and λ_+ is shown in Figure 1. In the same way one can also find the relation between α and m_V, which is also found in Figure 1. The values of α estimated from λ_+ and m_V given in (4) through Figure 1 are $0.51 {}^{+0.22}_{-0.25}$ $(\mathrm{GeV/c})^{-2}$ and $0.49 {}^{+0.28}_{-0.21}$ $(\mathrm{GeV/c})^{-2}$ respectively. They are in a good agreement with α of (4), which was directly obtained from data by χ^2-fit. In applying this method, λ_+ or m_V used as inputs must be those which are extracted by fitting the data with an entire t-range, i.e., $m_e^2 \leq t \leq \Delta^2$. Our estimation of the parameter α from several recent experiments[3,8,9] on $K_L{}^0 \to \pi^\pm e^\mp \nu$ is summarized in Table I. From this we estimate $\alpha = 0.1$ to 0.2 $(\mathrm{GeV/c})^{-2}$. Future experimenters are highly encouraged to analyze their data directly by using Eq.(3) too.

III Sirlin's Relation

Sirlin derived a remarkable relation[2] between weak and electromagnetic form factors of pseudoscalar mesons which means for a special case:

$$6 \left.\frac{\partial f_+(t)}{\partial t}\right|_{t=0} = 1/2[\langle r_{\pi^+}^2\rangle + \langle r_{K^+}^2\rangle] + \langle r_{K^0}^2\rangle + O(\lambda 3^2) \quad , \tag{6}$$

where the last term on the right hand side is a second order correction in V-spin symmetry-breaking interactions. The right hand side of (6) becomes 0.306 ± 0.044 fm^2 if available data of electromagnetic charge radii[10] of π^+, K^+, and K^0 is inserted for $\langle r_{\pi^+}^2\rangle$, $\langle r_{K^+}^2\rangle$ and $\langle r_{K^0}^2\rangle$ respectively, neglecting $O(\lambda 3^2)$. The left hand side depends not only on particular experimental data but also on the way of parametrization. This latter situation occurs only because of insufficient accuracy of existing data as was noticed before. By using high statistics data of ref. 3, and estimating the most probable value of α from Figure 1, the left hand side is equal to

$$\begin{aligned} &\text{a)} \quad 6\lambda_+/m_\pi{}^2 = 0.372 \pm 0.030 \text{ fm}^2 \ (\lambda_+ = 0.0312 \pm 0.0025) \quad , \\ &\text{b)} \quad 6/m_V{}^2 = 0.331 \pm 0.029 \text{ fm}^2 \ (m_V = 0.840 \pm 0.035 \text{ GeV}) \quad , \\ &\text{c)} \quad 6(m_{K^*}{}^{-2} + \alpha) = 0.324 \pm 0.024 \text{ fm}^2 \ (\alpha = 0.13 \pm 0.10 \text{ GeV}^{-2}) \quad , \end{aligned} \tag{7}$$

respectively, depending on parametrizations (1), (2), and (3). Therefore, if we assume the K*-dominance in the form of (3), then Eq.(6) is well satisfied by α, which was indirectly estimated from data. The value $6\lambda_+/m_\pi^2$ is presumably an overestimation of the left hand side of (6). This point has been also noticed previously by other authors.[6] So, along with the precise data on charge radii of pseudoscalar mesons, the direct measurement of curvature of $f_+(t)$ for $m_e^2 \leq t \leq (m_K - m_\pi)^2$ is highly desirable to test Eq.(6). The positive α means that the total decay rate for $K_L^0 \to \pi^\pm e^\mp \nu$ is enhanced as compared with the simple K*-pole form factor ($\alpha = 0$). This enhancement of decay rate is estimated to be at most a few percent (2% for $\alpha = 0.2$ GeV^{-2}) for $K_L^0 \to \pi^\pm e^\mp \nu$. However, it can have an important consequence if one assumes essentially the same form factor for the decay $D \to Ke\nu$ of charmed mesons. In this case, the decay rate is increased by 35% for $\alpha = 0.2$ GeV^{-2} as compared with the simple F*-dominance form factor.[11] It is due to a large phase-space available for the decay of D mesons. Therefore the precise measurement of the form factor $f_+(t)$ for semileptonic decays of kaons and D mesons will continue to be an important subject on theoretical grounds.

ACKNOWLEDGMENT

We thank Dr. M. K. Gaillard and numerous physicists at both Fermilab and University of Rochester for useful talks.

References

1. We follow the convention of Particle Data Group, Phys. Lett. 75B, (1978).

2. A. Sirlin, Ann. Phys. (N.Y.) 61, 294 (1970), Phys. Rev. Lett. 43, 904 (1979).

3. G. Gjesdal et al., Nucl. Phys. B109 118 (1976).

4. C. D. Buchanan et al., Phys. Rev. D11, 457 (1975).

5. L. M. Sehgal, in Proceedings of the 1979 EPS International Conference on High Energy Physics, Geneva, 1979.

6. For an earlier discussion, see L. M. Chounet, J. M. Gaillard, and M. K. Gaillard, Phys. Rep. 4C, 199 (1972), especially sec. 6.

7. We recall that the total decay rate for $K \to \pi e \nu$ is expressed as:

$$\Gamma(K \to \pi e\nu) = \text{const.} \times \int_0^{\Delta^2} dt[\lambda(m_K^2, m_\pi^2, t)]^{3/2} \times |f_+(t)|^2$$

   by neglecting lepton masses ($\Delta = m_K - m_\pi$).

8. R. Blumenthal et al., Phys. Rev. Lett 34, 164 (1975).

9. D. G. Hill et al., Phys. Lett. 73B, 483 (1978).

10. We used $\langle r_{\pi^+}^2 \rangle$ from A. Quenzer et al., Phys. Lett. 76B, 512 (1978), $\langle r_{K^+}^2 \rangle$ from E. Tsyganov, in Proceedings of the Nineteenth International Conference on High Energy Physics, edited by S. Homma, M. Kawaguchi, and H. Miyazawa (Tokyo, 1979), p. 315, and $\langle r_{K^0}^2 \rangle$ from W. R. Molzon et al., Phys. Rev. Lett. 41, 1213 (1978). See also E. B. Dally et al., Phys. Rev. Lett. 39, 1176 (1977), A. Beretvas et al., Dubna report 12357, 1979.

11. K. Yamada, Phys. Rev. D, to be published.

Table Caption

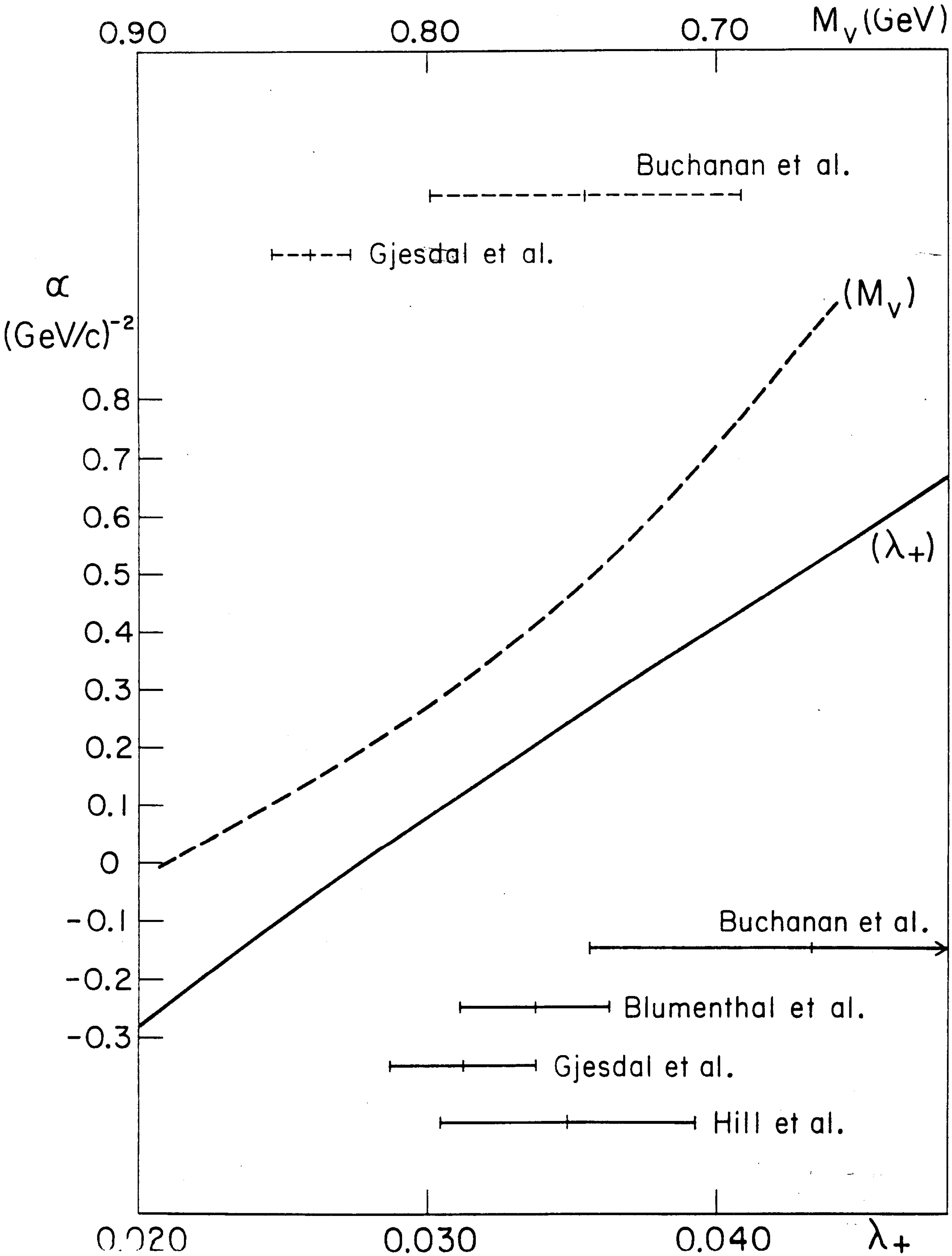
Table 1 The most probable values of α estimated from several recent experiments.

Figure Caption

Figure 1 Relations between various parametrization. The lower (upper) horizontal axis shows the value of λ_+(m_V in GeV). Curves were obtained by using Eq.(5), or a suitably modified form for the case of m_V.

Table I

Experiment	Input	χ^2/d.f.	α in $(GeV/c)^{-2}$
Ref. 8	$\lambda_+ = 0.337 \pm 0.026$ (entire t-range fit)	4.2	0.20 ± 0.09
Ref. 3	$\lambda_+ = 0.0312 \pm 0.0025$	456/483	$0.12^{+0.08}_{-0.09}$
	$m_V = 840 \pm 35$	455/483	0.14 ± 0.09
Ref. 9	$\lambda_+ = 0.0348 \pm 0.0044$ (Dalitz plot)	151/146	$0.23^{+0.15}_{-0.14}$
	$= 0.0335 \pm 0.0042$ (each t-bin)	15/11	0.19 ± 0.14



0.90
0.80
0.70
M_V(GeV)
Buchanan et al.
Gjesdal et al.
α
$(GeV/c)^{-2}$
(M_V)
(λ_+)
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
Buchanan et al.
Blumenthal et al.
Gjesdal et al.
Hill et al.
0.020
0.030
0.040
λ_+