New Journal of Physics
An Institute of Physics and Deutsche Physikalische Gesellschaft Journal

The Bose–Einstein distribution functions and the multiparticle production at high energies

G A Kozlov
Bogoliubov Laboratory of Theoretical Physics, Joint Institute for Nuclear Research, 141980 Dubna, Moscow Region, Russia
E-mail: kozlov@thsun1.jinr.ru

New Journal of Physics **4** (2002) 23.1–23.12 (http://www.njp.org/)
Received 25 January 2002, in final form 14 March 2002
Published 5 April 2002

Abstract. The evolution properties of propagating particles produced at high energies in a randomly distributed environment are studied. The finite size of the phase space of the multiparticle production region as well as the chaoticity can be derived.

1. Introduction

The particle collisions produced by the new generation of high-energy hadron machines—Fermilab's Tevatron for proton–antiproton ($\bar{\mathrm{p}}\mathrm{p}$) collisions, Brookhaven's RHIC for heavy ions and CERN's large hadron collider for proton–proton (pp) collisions and heavy ions—create or will create many secondary particles. The investigation of these collisions with high multiplicity is a central feature of modern particle physics. Interest in (charged) particles 'moving' in an environment of quantum fields, taking into account the relations between quantum fluctuations and chaoticity, attracts particle physicists. One of the most important tasks of (super)high-energy particle studies is to analyse fluctuations and correlations such as the Bose–Einstein (BE) correlation [1, 2] of produced particles. This is a rather instructive tool to study high multiplicity hadron processes in detail. The most recent reviews of the presentation of the BE correlations can be found in [3]. We understand the multiparticle production as the process of colliding particles where the kinetic energy is dissipated into the mass of produced particles [4]. We consider the incident energy $\sqrt{s} \gg \Lambda$, where Λ means the quantum chromodynamics scale. Phenomenological models [5, 6] describing the crucial properties of multiparticle correlations are very useful for systematic investigations of the properties caused by fluctuations and correlations. By considering them, one can obtain the characteristic properties of the internal structure of the disordering of produced particles in order to extract the information on the space-time size of the multiparticle production region, to estimate the lifetime of the particle emitter, etc. The analysis of correlation functions and distribution functions was used in [7, 8] to understand the possible

PII: S1367-2630(02)33028-3
1367-2630/02/000023+12$30.00

view of the quark–gluon plasma formation. In this paper, we present the model to describe the very high multiplicity effects at high energies. The most characteristic point of our model is that both distribution and correlation functions are taken into account on the quantum level (the operators of production and annihilation are used) with the random source contributions coming from the environment. It is well known that the cross section of the production of N particles at a given centre of mass energy $\sqrt{s}$ of two colliding particles with the momenta p and $\bar{p}$ is defined as

$$\sigma_N(s) = \int \mathrm{d}\Omega_N \, \delta^4\Big(p + \bar{p} - \sum_{j=1}^{N} q_j\Big) |A_N(p, q)|^2,$$

where A_N is the N particle production amplitude, q_j are the four-momenta of produced particles and Ω_N is a phase space. In the simple nonrelativistic case, the multiplicity N depends on the mean kinetic energy $\epsilon = \frac{3}{2}kT$ at the temperature T as (see [4] and references therein)

$$\epsilon \frac{N-1}{\sqrt{s} - Nm} = 1,$$

where k is the Boltzmann constant and m is the mass of a particle. We define the average mean multiplicity $\langle \bar{N} \rangle$ (as a natural scale of the produced particle multiplicity N) via the multiparticle correlation function $w(\vec{k})$ as $\langle \bar{N} \rangle = \int \mathrm{d}_3 \vec{k} \, w(\vec{k})$, where $\vec{k}$ is the spatial momentum of a particle. Following a natural way we suppose $\langle \bar{N} \rangle \ll N$, while $N \ll N_0 = \sqrt{s}/m$, where $m \sim \mathrm{O}(0.1 \text{ GeV})$. The main object in this investigation is the multiparticle thermal distribution function $\tilde{W}(k_\mu, k'_\mu)$ related to $\langle N \rangle$ as

$$\tilde{W}(k_\mu) = \langle N \rangle \, f(k_\mu) = \langle N \rangle \langle b^+(k_\mu) \, b(k_\mu) \rangle_\beta, \tag{1.1}$$

where $\langle N \rangle$ is defined as the scale of the multiplicity N at four-momentum k_μ (μ is the Lorenz index), the normalized distribution function $f(k_\mu)$ is finite, i.e. $\int \mathrm{d}_4 k \, f(k_\mu) < \infty$ and the label β in equation (1.1) means the temperature T (of the phase space occupied by operators $b^+(k_\mu)$ and $b(k_\mu)$) inverse. The nature of operators $b^+(k_\mu)$ and $b(k_\mu)$ is clarified in section 2.

At present, for BE correlations no (phenomenological) model can fit the experimental data and no analysis from first principles is in sight. As a new theoretical idea, we use the method combining different fields of physics to describe the BE correlations in multiparticle production processes:

(i) a semiphenomenological transport theory which is formulated by means of an operator-field evolution equation of the Langevin type;

(ii) an axiomatic quantum field theory in terms of distributions (generalized functions);

(iii) a statistical theory.

In this paper, we claim that the observation of the size effect in a multiparticle production is derived via the multiparticle correlation and distribution functions as well as the so-called chaoticity which is introduced in section 3. The multiparticle correlation function formalism concerns the statistical physics based on the Langevin-type equations. The Langevin equation, introduced in section 3, is considered as a basis for studying the approach to equilibrium of the particle(s). It is assumed that the heat bath being in essence infinite in size remains for all times in equilibrium as well. We use the method applied to the model where a relativistic particle moving in the Fock space is described by the number representation underlying the second quantization formulation of the canonical field theory. We deal with the microscopic look at the problem with the elements of quantum field theory at the stochastic level with the semiphenomenological noise embedded into the evolution dissipative equation of motion. The statistical distribution of the particles is discussed in section 4. We conclude in section 5.

2. Stochastic model. Langevin equation

As was pointed out in the introduction, to derive the characteristic features of multiparticle production physics at high energies, one should specify the model on the quantal level. Let us assume that only the particles p_i of the same kind of statistics labelled by index i are produced just after the collision process occurred, e.g., $\mathrm{pp}, \bar{\mathrm{p}}\mathrm{p} \to \mathrm{p}_i$. In order to extend the method of stochastically distributed particles in the environment, we propose that the rather complicated real physical processes which happened in the multiparticle formation region should be replaced by a single-constituent propagation of particles provided by a special kernel operator (in the stochastic evolution equation) considered as an input of the model and disturbed by the random force F [7, 8]. We assume that F can be the external source being both a c-number function and an operator. In such a hypothetical system of excited (thermal) local phase we deal with the canonical operator $a(\vec{k},t)$ and its Hermitian conjugate $a^+(\vec{k},t)$. We formulate distribution functions of produced particles in terms of a point-to-point equal-time temperature-dependent thermal correlation functions of two operators

$$w(\vec{k},\vec{k}',t;T) = \langle a^+(\vec{k},t)\, a(\vec{k}',t)\rangle_\beta = \mathrm{Tr}[a^+(\vec{k},t)\, a(\vec{k}',t)\mathrm{e}^{-H\beta}]/\,\mathrm{Tr}(\mathrm{e}^{-H\beta}). \tag{2.1}$$

Here, $\langle\cdots\rangle_\beta$ means the procedure of thermal statistical averaging, $\vec{k}$ and t are, respectively, momentum and time variables, $\mathrm{e}^{-H\beta}/\,\mathrm{Tr}(\mathrm{e}^{-H\beta})$ denotes the standard density operator in equilibrium, and the Hamiltonian H is given by the squared form of the annihilation a_p and creation a_p^+ operators for Bose and Fermi particles, $H = \sum_p \epsilon_p a_p^+ a_p$ (the energy ϵ_p and operators a_p, a_p^+ carry some index p, where $p_\alpha = 2\pi\, n_\alpha/L, n_\alpha = 0, \pm 1, \pm 2, \ldots; V = L^3$ is the volume of the system considered). We define the thermal boson field as

$$\Phi_B(x_\mu) = \frac{1}{\sqrt{2}}[\phi(x_\mu) + \phi^+(x_\mu)], \tag{2.2}$$

where

$$\phi(x_\mu) = \int \mathrm{d}^3\vec{k}\, v_k a(\vec{k},t), \qquad v_k = \frac{\mathrm{e}^{\mathrm{i}\vec{k}\vec{x}}}{[(2\pi)^3\Delta(\vec{k})]^{1/2}},$$

$$\phi^+(x_\mu) = \int \mathrm{d}^3\vec{k}\, v_k^+ a^+(\vec{k},t), \qquad v_k^+ = \frac{\mathrm{e}^{-\mathrm{i}\vec{k}\vec{x}}}{[(2\pi)^3\Delta(\vec{k})]^{1/2}}$$

and $\Delta(\vec{k})$ is an element of the invariant phase volume.

The standard canonical commutation relation

$$[a(\vec{k},t), a^+(\vec{k}',t)]_\pm = \delta^3(\vec{k}-\vec{k}') \tag{2.3}$$

at every time t is used as usual for Bose $(-)$ and Fermi $(+)$ operators.

The probability of finding the particles in the multiparticle production region with momenta $\vec{k}$ and $\vec{k}'$ in the same event at the time t is

$$R(\vec{k},\vec{k}',t) = W(\vec{k},\vec{k}',t)/[W(\vec{k},t)\cdot W(\vec{k}',t)], \tag{2.4}$$

where the multiparticle distribution function $W(\vec{k},t)$ in the simple version fluctuates only in its normalization, e.g., the mean multiplicity $\langle N\rangle$.

Here, the one-particle thermal distribution function looks like

$$W(\vec{k},t) = \langle N\rangle \cdot f(\vec{k},t),$$

defining the single spectrum, while

$$W(\vec{k},\vec{k}',t) = \langle N(N'-\delta_{ij})\rangle \cdot f(\vec{k},\vec{k}',t)$$

for i- and j-types of particles. Here, $\delta_{ij}=1$ if $i=j$ and 0 otherwise. Distribution functions $f(\vec{k},t)$ and $f(\vec{k},\vec{k}',t)$ look like (hereafter the label β will be omitted in the sense of equations (1.1) and (2.1))

$$f(\vec{k},t) = \langle b^{+}(\vec{k},t)b(\vec{k},t)\rangle,$$

$$f(\vec{k},\vec{k}',t) = \langle b^{+}(\vec{k},t)\, b^{+}(\vec{k}',t)\, b(\vec{k},t)\, b(\vec{k}',t)\rangle,$$

where

$$b(\vec{k},t) = a(\vec{k},t) + R(\vec{k},t)$$

under the assumption of the random source function $R(\vec{k},t)$ being an operator, in general. One can rewrite equation (2.4) in the following form

$$R(\vec{k},\vec{k}',t) = \xi(N)\frac{f(\vec{k},\vec{k}',t)}{f(\vec{k},t)f(\vec{k}',t)},$$

where

$$\xi(N) = \frac{\langle N(N'-\delta_{ij})\rangle}{\langle N\rangle\langle N'\rangle}.$$

For simplicity, we deal with operators a and b as if they are the single boson or fermion operators. Considering the ‘propagation’ of a particle with the momentum $\vec{k}$ in the quantum equilibrium phase space under the influence of a random force coming from surrounding particles, the dissipative dynamics of the relevant system is described by the equation containing only the first-order time derivatives of the dynamic degrees of freedom, the operators $b(\vec{k},t)$ and $b^{+}(\vec{k},t)$ [7]:

$$\mathrm{i}\,\partial_t b(\vec{k},t) = F(\vec{k},t) - A(\vec{k},t) + P, \tag{2.5}$$

$$\mathrm{i}\,\partial_t b^{+}(\vec{k},t) = A^{*}(\vec{k},t) - F^{+}(\vec{k},t) - P. \tag{2.6}$$

Here, the interaction of particles under consideration with surroundings as well as providing the propagation is given by the operator $A(\vec{k},t)$ defined as the one closely related to the dissipation force:

$$A(\vec{k},t) = \int_{-\infty}^{+\infty} K(\vec{k},t-\tau)b(\vec{k},\tau)\,\mathrm{d}\tau. \tag{2.7}$$

The particle transitions are provided by the random source operator $F(\vec{k},t)$ while P represents a stationary external force. An interplay of particles with surroundings is embedded into the interaction complex kernel $K(\vec{k},t)$, while the real physical transitions are provided by the random source operator $F(\vec{k},t)$ with the zeroth value of the statistical average, $\langle F\rangle = 0$.

The random evolution field operator $K(\vec{k}, t)$ in equation (2.7) denotes the random noise and it is assumed to vary stochastically with a δ-like equal time correlation function

$$\langle K^{+}(\vec{k}, \tau)\ K(\vec{k}', \tau)\rangle = 2(\pi\rho)^{1/2}\kappa\delta(\vec{k} - \vec{k}'),$$

where both the strength of the noise κ and the positive constant $\rho \to \infty$ define the effect of the Gaussian noise on the evolution of particles in the thermalized environment.

The formal solutions of equations (2.5) and (2.6) in the operator form in the four-momentum space-time $S(\Re_4)$ ($k^{\mu} = (\omega = k^0, k_j)$) are respectively

$$\tilde{b}(k_\mu) = \tilde{a}(k_\mu) + \tilde{R}(k_\mu), \qquad \tilde{b}^{+}(k_\mu) = \tilde{a}^{+}(k_\mu) + \tilde{R}^{*}(k_\mu),$$

where the operator $\tilde{a}(k_\mu)$ is expressed via the Fourier transformed operator $\tilde{F}(k_\mu)$ and the Fourier transformed kernel function $\tilde{K}(k_\mu)$ (coming from equation (2.7)) as

$$\tilde{a}(k_\mu) = \tilde{F}(k_\mu) \cdot [\tilde{K}(k_\mu) - \omega]^{-1},$$

while the function $\tilde{R}(k_\mu) \sim P \cdot [\tilde{K}(k_\mu) - \omega]^{-1}$. In our model, we suppose that a heat bath (an environment) is an assembly of damped oscillators coupled to the produced particles which in turn are distributed by the random force $\tilde{F}(k_\mu)$. In addition, there is the assumption that the heat bath is statistically distributed. The random force operator $F(\vec{k}, t)$ can be expanded by using the Fourier integral

$$F(\vec{k}, t) = \int_{-\infty}^{+\infty} \frac{d\omega}{2\pi} \psi(k_\mu)\hat{c}(k_\mu)\, e^{-i\omega t}, \tag{2.8}$$

where the form $\psi(k_\mu) \cdot \hat{c}(k_\mu)$ is just the Fourier operator $\tilde{F}(k_\mu) = \psi(k_\mu) \cdot \hat{c}(k_\mu)$, and the canonical operator $\hat{c}(k_\mu)$ obeys the commutation relation

$$[\hat{c}(k_\mu), \hat{c}^{+}(k'_\mu)]_{\pm} = \delta^4(k_\mu - k'_\mu).$$

The function $\psi(k_\mu)$ in equation (2.8) is determined by the condition (the canonical commutation relation (2.3) is taken into account)

$$\int_{-\infty}^{+\infty} \frac{d\omega}{2\pi} \left[\frac{\psi(k_\mu)}{\tilde{K}(k_\mu) - \omega}\right]^2 = 1.$$

3. The ratio R in $S(\Re_4)$

The enhanced probability for emission of identical particles is given by the ratio R of distribution functions in $S(\Re_4)$ as follows:

$$R(k_\mu, k'_\mu; T) = \xi(N) \frac{\tilde{f}(k_\mu, k'_\mu; T)}{\tilde{f}(k_\mu) \cdot \tilde{f}(k'_\mu)}, \tag{3.1}$$

where $\tilde{f}(k_\mu, k'_\mu; T) = \langle \tilde{b}^{+}(k_\mu)\tilde{b}^{+}(k'_\mu)\tilde{b}(k_\mu)\ \tilde{b}(k'_\mu)\rangle$ and $\tilde{f}(k_\mu) = \langle \tilde{b}^{+}(k_\mu)\tilde{b}(k_\mu)\rangle$. Using Fourier solutions of equations (2.5) and (2.6) in $S(\Re_4)$, one can get

$$R(k_\mu, k'_\mu; T) = \xi(N)[1 + D(k_\mu, k'_\mu; T)], \tag{3.2}$$

where

$$D(k_\mu, k'_\mu; T) = \frac{\Xi(k_\mu, k'_\mu)[\Xi(k'_\mu, k_\mu) + \tilde{R}^{+}(k'_\mu)\tilde{R}(k_\mu)] + \Xi(k'_\mu, k_\mu)\tilde{R}^{+}(k_\mu)\tilde{R}(k'_\mu)}{\tilde{f}(k_\mu) \cdot \tilde{f}(k'_\mu)} \tag{3.3}$$

and the BE correlation function $\Xi(k_\mu, k'_\mu)$ looks like

$$\Xi(k_\mu, k'_\mu) = \langle \tilde{a}^+(k_\mu)\, \tilde{a}(k'_\mu)\rangle = \frac{\psi^*(k_\mu)\cdot\psi(k'_\mu)}{[\tilde{K}^*(k_\mu)-\omega]\cdot[\tilde{K}(k'_\mu)-\omega']}\cdot\langle \hat{c}^+(k_\mu)\,\hat{c}(k'_\mu)\rangle. \tag{3.4}$$

Inserting equation (3.4) into (3.3) and taking into account the trick with the $\delta^4(k_\mu - k'_\mu)$-function to be replaced by the δ-like consequence such as $\Omega(r)\exp[-(k-k')^2r^2]$ [9], one can obtain the following expression for the D-function instead of equation (3.3)

$$D(k_\mu, k'_\mu; T) = \lambda(k_\mu, k'_\mu; T)\exp(-q^2/2) \\ \times [n(\bar{\omega}, T)\Omega(r)\exp(-q^2/2) + \tilde{R}^*(k'_\mu)\tilde{R}(k_\mu) + \tilde{R}^*(k_\mu)\tilde{R}(k'_\mu)], \tag{3.5}$$

where

$$\lambda(k_\mu, k'_\mu; T) = \frac{\Omega(r)}{\tilde{f}(k_\mu)\cdot\tilde{f}(k'_\mu)} n(\bar{\omega}, T),\ \ \bar{\omega} = \frac{1}{2}(\omega+\omega'),$$

while $q^2 \equiv Q^2r^2$ and the function $\Omega(r)n(\omega; T)\exp(-q^2/2)$ in (3.5) describes the space-time size of the multiparticle production region. Choosing the z-axis along the pp or $\bar{\mathrm{p}}\mathrm{p}$ collision axis one can put

$$Q_\mu = (k-k')_\mu, \qquad Q_0 = \epsilon_{\vec{k}} - \epsilon_{\vec{k}'}, \qquad Q_z = k_z - k'_z,$$

$$Q_t = \left[(k_x - k'_x)^2 + (k_y - k'_y)^2\right]^{1/2}, \qquad \Omega(r) = \frac{1}{\pi^2} r_0 r_z r_t^2,$$

where r_0, r_z and r_t are time-like, longitudinal and transverse 'size' components of the multiparticle production region. To derive equation (3.5), the Kubo–Martin–Schwinger condition (μ is the chemical potential)

$$\langle a(\vec{k}', t')\, a^+(\vec{k}, t)\rangle = \langle a^+(\vec{k}, t)\, a(\vec{k}', t - \mathrm{i}\beta)\rangle \cdot \exp(-\beta\,\mu)$$

has been used, and the thermal statistical averages for the $\hat{c}(k_\mu)$-operator should be represented in the following form:

$$\langle \hat{c}^+(k_\mu)\,\hat{c}(k'_\mu)\rangle = \delta^4(k_\mu - k'_\mu)\cdot n(\omega, T), \tag{3.6}$$

$$\langle \hat{c}(k_\mu)\,\hat{c}^+(k'_\mu)\rangle = \delta^4(k_\mu - k'_\mu)\cdot[1 \pm n(\omega, T)] \tag{3.7}$$

for Bose $(+)$ and Fermi $(-)$ statistics, where $n(\omega, T) = \{\exp[(\omega-\mu)\beta] \pm 1\}^{-1}$. Formula (3.5) indicates that the chaotic multiparticle source emanating from the thermalized multiparticle production region exists. Taking into account equations (3.6) and (3.7), it is easy to see that the correlation functions containing the random force functions $F(\vec{k}, t)$ (2.8) carry the quantum features in the thermalized stationary equilibrium, namely

$$\langle F(\vec{k}, t)F^+(\vec{k}', t')\rangle = \delta^3(\vec{k} - \vec{k}'))\Gamma_1(\vec{k}, -\Delta t),$$

$$\Gamma_1(\vec{k}, -\Delta t) = \int \frac{\mathrm{d}\omega}{2\pi}|\psi(k_\mu)|^2[1 \pm n(\omega, \beta)]\exp(-\mathrm{i}\omega\Delta t), \qquad \Delta t = t - t';$$

$$\langle F^+(\vec{k}, t)F^+(\vec{k}', t')\rangle = \delta^3(\vec{k} - \vec{k}'))\Gamma_2(\vec{k}, \Delta t),$$

$$\Gamma_2(\vec{k}, \Delta t) = \int \frac{\mathrm{d}\omega}{2\pi}|\psi(k_\mu)|^2 n(\omega, \beta)\exp(\mathrm{i}\omega\Delta t).$$

Therefore, one can say that every dynamical quantity can be written in terms of only operators $\hat{c}$ and $\hat{c}^+$ in the stationary state or thermal equilibrium. The quantitative information (longitudinal r_z and transverse r_t components of the multiparticle production region, the temperature T of the environment) could be extracted by fitting the theoretical formula (3.5) to the measured D-function and estimating the errors of the fit parameters. Hence, the measurement of the space-time evolution of the multiparticle source would provide information of the multiparticle process and the general reaction mechanism. The temperature of the environment enters into formula (3.5) through the two-particle correlation function $\Xi(k_\mu, k'_\mu; T)$. The function R (3.1) is temperature-dependent because of the T-dependence of the two-particle distribution function $\tilde{f}$ which, in fact, can be considered as an effective density of the multiparticle source. Formula (3.1) looks like the fitting R-ratio using a source parameterization:

$$R_F(r) = \mathrm{const}[1 + \lambda_F(r)\exp(-r_t^2 Q_t^2/2 - r_z^2 Q_z^2/2)],$$

where $r_t(r_z)$ is the transverse (longitudinal) radius parameter of the source with respect to the beam axis, and λ_F denotes the effective intercept parameter (chaoticity parameter) which has a general dependence on the mean momentum of the observed particle pair. Here, the dependence on the source lifetime is omitted. The chaoticity parameter λ_F is temperature-dependent and the positive one defined by

$$\lambda_F(r) = \frac{|\Omega(r) n(\bar{\omega}; T)|^2}{\tilde{f}(k_\mu) \cdot \tilde{f}(k'_\mu)}.$$

Comparing equations (3.3) and (3.4) one can identify

$$\Xi(k_\mu, k'_\mu) = \Omega(r) n(\bar{\omega}; T) \exp(-q^2/2).$$

Hence, the correlation function $\Xi(k_\mu, k'_\mu)$ defines uniquely the size r of the multiparticle production region. There is no satisfactory tool to derive the precise analytic form of the random source function $\tilde{R}(k_\mu)$ in equation (3.3), but one can put (see equation (3.4) and taking into account $\tilde{R}(k_\mu) \sim P[\tilde{K}(k_\mu) - \omega]^{-1}$) [7, 8, 10] that

$$\tilde{R}(k_\mu) = [\alpha \cdot \Xi(k_\mu)]^{1/2},$$

where α is of the order $\mathrm{O}(P^2/n(\omega, T) \cdot |\psi(k_\mu)|^2)$. Thus,

$$D(q^2; T) = \frac{\tilde{\lambda}^{1/2}(\bar{\omega}; T)}{(1+\alpha)(1+\alpha')} \mathrm{e}^{-q^2/2} [\tilde{\lambda}^{1/2}(\bar{\omega}; T)\mathrm{e}^{-q^2/2} + 2(\alpha\alpha')^{1/2}], \tag{3.8}$$

where

$$\tilde{\lambda}(\bar{\omega}; T) = \frac{n^2(\bar{\omega}; T)}{n(\omega; T) n(\omega'; T)}.$$

It is easy to see that, in the vicinity of $q^2 \approx 0$, one can obtain the full correlation if $\alpha = \alpha' = 0$ and $\tilde{\lambda}(\bar{\omega}; T) = 1$. Putting $\alpha = \alpha'$ in equation (3.8), we find the formal lower bound on the space-time dimensionless size of the multiparticle production region of the bosons

$$q^2 \geq \ln \frac{\tilde{\lambda}(\bar{\omega}; T)}{[\sqrt{(\alpha+1)^2 + \alpha^2} - \alpha]^2}.$$

In fact, the function $D(k_\mu, k'_\mu; T)$ in equation (3.5) could not be observed because of some model uncertainties. In the real world, the D-function has to contain background contributions which have not been included in the calculation. To be close to the experimental data, one has to expand the D-function as projected on some well-defined function (in $S(\Re_4)$) of the relative momentum of bosons produced $D(k_\mu, k'_\mu; T) \to D(Q^2_\mu; T)$. Thus, it will be very instructive to use the polynomial expansion which is suitable to avoid any uncertainties as well as to characterize the degree of deviation from the Gaussian distribution, for example. The complete orthogonal set of functions can be obtained with the help of the Hermite polynomials in the Hilbert space of the square integrable functions with the measure $\mathrm{d}\mu(z) = \exp(-z^2/2)\,\mathrm{d}z$. The function D corresponds to this class if

$$\int_{-\infty}^{+\infty} \mathrm{d}q\, \exp(-q^2/2)\, |D(q)|^n < \infty, \qquad n = 0, 1, 2, \ldots .$$

The expansion in terms of the Hermite polynomials $H_n(q)$

$$D(q) = \lambda \sum_n c_n H_n(q) \exp(-q^2/2) \tag{3.9}$$

is well suited for the study of possible deviations both from the experimental shape and from the exact theoretical form of the function D (3.5). The coefficients c_n in equation (3.9) are defined via the integrals over the expanded functions D because of the orthogonality condition

$$\int_{-\infty}^{+\infty} H_n(x)\, H_m(x) \exp(-x^2/2)\,\mathrm{d}x = \delta_{n,m}.$$

Thus, the observation of the multiparticle correlation enables one to extract the properties of the structure of q^2, i.e. the space-time size of multiparticle production region. The other possibility is related to the replacement of the R-function (3.2) with respect to the cylindrical symmetry angles θ and ϕ which are non-observable ones at fixed Q_t

$$R(k_\mu, k'_\mu; T) \to \bar{R}(Q_t; T) = C_N^{-1}\, \xi(N) \int \mathrm{d}q_t\, \mathrm{d}Q_z\, \mathrm{d}\theta\, \mathrm{d}\phi\, \tilde{f}(k_\mu, k'_\mu; T),$$

where

$$C_N = \int \mathrm{d}q_t\, \mathrm{d}Q_z\, \mathrm{d}\theta\, \mathrm{d}\phi\, \tilde{f}(k_\mu)\, \tilde{f}(k'_\mu),$$

$$q_t = \frac{1}{\cos\theta + \sin\theta} \left\{ k_x + k_y \mp \frac{1}{2} Q_t [\cos(\theta+\phi) + \sin(\theta+\phi)] \right\}.$$

Then, $\bar{R}(Q_t; T) = \xi(N)\,[1 + \bar{D}(Q_t; T)]$ with

$$\bar{D}(Q_t; T) = \frac{\bar{C_N}^{-1}(T)}{(1+\alpha)(1+\alpha')} \exp[-(r_t^2\, Q_t^2)]\, F(Q_t; T),$$

$$F(Q_t; T) = \int \mathrm{d}q_t\, \mathrm{d}Q_z\, \mathrm{d}\theta\, \mathrm{d}\phi\, n^2(\bar{\omega}; T)\, \mathrm{e}^{-\beta_{0z}} \left[1 + 2\sqrt{\alpha\alpha' \tilde{\lambda}^{-1}(\bar{\omega}; T)}\, \mathrm{e}^{q^2/2} \right],$$

$$\beta_{0z} \equiv r_0^2\, Q_0^2 + r_z^2\, Q_z^2, \qquad \bar{C_N}(T) = \int \mathrm{d}q_t\, \mathrm{d}Q_z\, \mathrm{d}\theta\, \mathrm{d}\phi n(\omega; T)\, n(\omega'; T).$$

It remains to show how one goes about calculating the thermodynamical quantities in a local thermalized system of produced particles. Taking into account the positive- and negative-frequency parts of the boson field operator (2.2) to be applied to the energy–momentum tensor $T_{\mu\nu}(x) =: \Phi_B(x)\, \overleftrightarrow{k_\mu} \overleftrightarrow{k_\nu}\, \Phi_B(x)$: and the particle flow operator $\Pi_\mu(x) =: \Phi_B(x)\, \overleftrightarrow{k_\mu}\, \Phi_B(x)$: we can calculate the energy density $E(\beta)$, the pressure $V(\beta)$ and the entropy density $S(\beta)$ in the local system of the volume v for bosons in the equilibrium thermalized phase space. The simple straightforward calculations give (see also [11])

$$E(\beta) = \frac{1}{(2\,\pi)^2\,v}\int \mathrm{d}_3\vec{k}\,\mathrm{d}\omega\,\omega^2\,M(k_\mu,\beta),$$

$$V(\beta) = \frac{1}{6\,\pi^2\,v}\int \mathrm{d}_3\vec{k}\,\vec{k}^2\,\mathrm{d}\omega\,M(k_\mu,\beta),$$

$$S(\beta) = \frac{1}{2\,\pi\,v}\int \mathrm{d}_3\vec{k}\,\{[1+\Pi_0(\vec{k},\beta)]\,\ln[1+\Pi_0(\vec{k},\beta)] - \Pi_0(\vec{k},\beta)\,\ln\Pi_0(\vec{k},\beta)\},$$

where

$$\Pi_0(\vec{k},\beta) = \tfrac{1}{2}\int_{-\infty}^{+\infty}\mathrm{d}^2\omega\,M(k_\mu,\beta),$$

$$M(k_\mu,\beta) = \frac{\psi^2(k_\mu)}{|\tilde{K}(k_\mu)-\omega|^2}n(\omega,\beta), \qquad \mathrm{d}_3\vec{k} \equiv \frac{\mathrm{d}^3\vec{k}}{\sqrt{(2\,\pi)^3\,\Delta(\vec{k})}},$$

$$\Phi_B(x)\,\overset{\leftrightarrow}{\tilde{k}}_\mu\,\Phi_B(x) \equiv \tfrac{1}{2}[\Phi_B(x)\,(k_\mu\Phi_B(x)) - (k_\mu\Phi_B(x))\,\Phi_B(x)].$$

Here, we suppose that the thermalized multiparticle production region is isotropic, and one can use the space-averaged operators normalized to the volume v, taking ensemble averages (3.6) and (3.7). It is easy to see that both $E(\beta)$ and $V(\beta)$ tend to their maximum values with rising T, while the entropy $S(\beta)$ does not change so much essentially even if $T\to\infty$.

4. Statistical distributions

From a widely accepted point of view, at high energies, there are two channels, at least, for multiparticle production where produced particles occupy the multiparticle production region consisting of i elementary cells. These main channels are (a) a direct channel assuming that all particles p_j are produced directly within the quark (q)-antiquark ($\bar{\mathrm{q}}$) annihilation or the gauge–boson fusion, e.g., $\mathrm{q}\bar{\mathrm{q}}\to\mathrm{p}_j\mathrm{p}_j\ldots$; (b) an indirect channel which means that the particles are produced via the decays of intermediate vector bosons χ^* in both heavy and light sectors in the kinematically allowed region, e.g., $\mathrm{q}\bar{\mathrm{q}}\to\chi^*\chi^*\ldots\to\mathrm{p}_j\mathrm{p}_j\ldots$. All the particles produced are classified by the like-sign constituents that are labelled as p^+, p^-, p^0 subsystems, where p: μ, π, $K\ldots$. The mean multiplicity $\langle N\rangle$ and the mean energy $\langle E\rangle$ of the p_j subsystem are defined as [5]

$$\langle N\rangle = \sum_j\sum_{m_j} m_j\,\zeta_j^{(m_j)}, \qquad \langle E\rangle = \sum_j\sum_{m_j} m_j\,\epsilon_j\,\zeta_j^{(m_j)},$$

where ϵ_j is the energy of a p-particle in the jth elementary cell and $\zeta_j^{(m_j)}$ represents the probability of finding m_j p-particles in the jth cell and is normalized as

$$\sum_{m_j=0}^{\infty}\zeta_j^{(m_j)} = 1.$$

In the direct channel, for charged produced mesons $\langle N\rangle$ is defined uniquely for a given β as

$$\langle N\rangle = 2\sum_j[\exp(\epsilon_j\,\beta)-1]^{-1},$$

while $\langle E\rangle$ is

$$\langle E\rangle = \frac{1}{3}\sqrt{s} = \sum_j \frac{\epsilon_j}{\exp(\epsilon_j\,\beta) - 1}.$$

Going into y-rapidity space in the longitudinal phase space with many cells of equal size δy, the energy ϵ_j should be expressed in terms of the transverse mass $m_t = \sqrt{\langle k_t\rangle^2 + m_p^2}$ (where $\langle k_t\rangle$ and m_p are the transverse average momentum and the mass of a p-particle):

$$\epsilon_j(s) = \frac{m_t}{2}\,[g_j(\tilde{s}) + g_j^{-1}(\tilde{s})], \qquad \tilde{s} = \frac{s}{4\,m_t^2},$$

$$g_j(\tilde{s}) = \left(\sqrt{\tilde{s}} + \sqrt{\tilde{s}-1}\right)\,\exp[-(j-1/2)\,\delta y].$$

Here, the four-momentum of a p-particle is given as

$$k^\mu = \left(\sqrt{\langle k_t\rangle^2 + m_p^2}\,\cosh y, k_t\,\cos\varphi, k_t\,\sin\varphi, \sqrt{\langle k_t\rangle^2 + m_p^2}\,\sinh y\right),$$

where the azimuthal angle of k_t is in the range $0 < \varphi < 2\,\pi$. Our model produces an enhancement of $R(Q,\beta)$ in the small enough region of Q where R is defined only by the model parameter α and the mean multiplicity $\langle N(s)\rangle$ at a fixed value of β, namely

$$R(Q,\beta) \simeq \xi(\langle N\rangle)\left\{1 + \frac{\sqrt{\tilde{\lambda}(\bar{\omega},\beta)}}{(1+\alpha)^2}\left[\sqrt{\tilde{\lambda}(\bar{\omega},\beta)} + 2\,\alpha - \left(\sqrt{\tilde{\lambda}(\bar{\omega},\beta)} + \alpha\right)Q^2\,r^2\right]\right\}. \tag{4.1}$$

It is clear that the $R(Q,\beta)$-function at $Q^2 = 0$

$$R(Q,\beta) \simeq \xi(\langle N(s)\rangle)\left[2 - \left(\frac{\alpha}{1+\alpha}\right)^2\right] \tag{4.2}$$

cannot exceed 2 because $\alpha \neq 0$ and $\xi(N(s)) < 1$ even at large multiplicity. The Boltzmann behaviour should be realized in the case when $\alpha \to \infty$, i.e. the main contribution to the fluctuating behaviour of the $R(Q,\beta)$-function should come from the random source contribution (see equations (3.5) and (3.8)). We found that the enhancement of the $R(Q,\beta)$-function, mainly, the shape of this function, strongly depends on the transverse size r_t of the phase space and has a very weak dependence of the δy size of a separate elementary cell. The increase of r_t makes the shape of the $R(Q,\beta)$-function become more crucial.

Obviously, $\xi(\langle N(s)\rangle)$ is the normalization constant in equation (3.2), where $\langle N(s)\rangle$ should be derived at the origin of Q^2 precisely from $R(Q=0,\beta) \equiv R_0(s)$ as

$$\langle N(s)\rangle \simeq \frac{1}{\varepsilon},$$

where

$$\varepsilon = 1 - \frac{R_0(s)}{2 - (\frac{\alpha}{1+\alpha})^2}$$

can be extracted from the experiment at some chosen value of α ($\alpha = 0$ should also be taken into account). On the other hand, the $R(Q,\beta)$-function allows one to measure $\alpha = \alpha'$ which parameterizes the random source contribution as well as the splitting between α and α'. Neglecting the random source contribution (i.e., putting $\alpha = \alpha' = 0$) we can estimate the chaoticity $\tilde{\lambda}(\bar{\omega},\beta)$ by measuring $R(Q,\beta)$ as $Q^2 \to 0$.

In fact, the theoretical prediction that $D(Q, \beta) > 1$ means that in the multiparticle production region one should select the single boson 'dressing' of some quantum numbers, and the particles suited near it in the phase space are 'dressed' with the same set of quantum numbers. The amount of such neighbour particles has to be as many as possible. This allows a cell to form in the space-time occupied by equal-statistics particles only. Such a procedure can be repeated while all the particles will occur in the multiparticle production region. This leads to the space-time BE distribution of produced particles in the phase-space cells formed only for bosons. In fact, there is no restriction of the number of bosons occupying the chosen elementary cells. This means that the $D(Q, \beta)$-functions are defined for all orders.

5. Summary and discussion

We have investigated the finite temperature BE correlations of identical particles in the multiparticle production using the solutions of the operator field Langevin-type equation in $S(\Re_4)$, the quantum version of the Nyquist theorem and the quantum statistical methods. The model considered states that all the particles are produced directly from a high-energy collision process. We presented the crucial role of the model in describing the BE correlations via calculations of distribution functions as the functions of the mean multiplicity and chaoticity at each four-momentum $\sqrt{Q_\mu^2}$. Based on this model, one can compare the effects on single particle spectra and multiparticle distribution caused by multiparticle correlations. There are several parameters in the model: $\beta, \delta y, \alpha(\alpha')$. One can focus on the statement that the deviation of the $D(Q, \beta)$-function from zero at finite values of the physical variables q^2 and the model parameter α indicates that the multiparticle production region should be considered as the phase-space consisting of the elementary cells (each with the size δy) which are occupied by the particles of identical statistics. The Boltzmann behaviour of the R-function is available only at large enough values of α which means the leading role of the random source contribution to the distribution function. An important feature of the model is getting the information on the space-time structure of the multiparticle production region. We are able to predict the source size and the intercept parameter—the chaoticity λ as well. We have found that the distribution function $R(Q, \beta)$ depends on the number of elementary cells defined by the equal size δy in the rapidity y-space.

Of course, the best check of any model could be done if various kinds of high-energy experimental data on the multiparticle correlations would be well reproduced by the model in consideration. The model considered in this work can be applied to the experimental data. This will be our task in our next work. Here, we are just going to give a brief example. The ALEPH data [12] at $\sqrt{s} = 91.2$ GeV applied to the $R(Q)$-function (4.1) in the region $0.1 \leq Q \leq 1$ GeV have an essential nonmonotonic behaviour. This means that the sign of the slope parameter of $R(Q)$ changes in this region, and the values of $R(Q)$ are <1. In accordance with our model, this effect could be interpreted as the fact that in the domain $0.1 \leq Q \leq 1$ GeV the mean multiplicity $\langle N(s) \rangle$ has a small value which gives the strong suppression for the $R(Q)$-function normalized to $\xi(\langle N(s) \rangle)$. In the case of higher energies of colliding particles (Tevatron, LHC), the expected (from our model) averaged multiplicity should be very large $\sim$300–400, and the nonmonotonic effect will not occur. We hope that experiments at the Tevatron's Run II and the LHC will measure observables such as $R(Q)$ more precisely.

References

[1] Hanbury-Brown R and Twiss R Q 1956 *Nature* **178** 1046
[2] Goldhaber G *et al* 1960 *Phys. Rev.* **120** 300
[3] Wiedermann U A and Heinz U 1999 *Phys. Rep.* **319** 145
Weiner R 2000 *Phys. Rep.* **327** 249
[4] For a recent review see, for example, Manjavidze J and Sissakian A 2001 *Phys. Rep.* **346** 1
[5] Osada T, Maruyama M and Tagaki F 1998 *Phys. Rev.* D **59** 014024
[6] Utyuzh O V, Wilk G and Wlodarczyk Z 2001 *Preprint* hep-ph/0102275
Utyuzh O V, Wilk G and Wlodarczyk Z 2001 *Phys. Rev.* C **64** 027901
Utyuzh O V, Wilk G and Wlodarczyk Z 2001 *Phys. Lett.* B **522** 273
[7] Kozlov G A 1998 *Phys. Rev.* C **58** 1188
[8] Kozlov G A 2001 *J. Math. Phys.* **42** 4749
[9] Gelfand M and Shilov G E 1964 *Generalized Functions* vol 1 (New York: Academic)
[10] Namiki M and Muroya S 1992 Theory of particle distribution-correlation in high energy nuclear collisions *Proc. RIKEN Symp. on Physics of High Energy Heavy Ion Collisions (Saitama, Japan)* ed S Date and S Ohta p 91
[11] Mizutani M, Muroya S and Namiki M 1988 *Phys. Rev.* D **37** 3033
[12] Decamp D *et al* (ALEPH Collaboration) 1992 *Z. Phys.* C **54** 75