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Tau decay in a circularly polarized laser field

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Abstract: Tau τ^- is the most massive lepton. Therefore, it has several decay channels. Unlike other leptons, tau can decay into hadronic and leptonic particles. In this article, we have studied the leptonic ($\tau^- \longrightarrow \nu_\tau \ell^- \bar{\nu}_\ell$) and two-body hadronic ($\tau^- \longrightarrow \nu_\tau h^-$) decays of tau, where ℓ^- represents the electron or muon, and h^- refers to one of the mesons π^- or K^-, in the presence of a circularly polarized monochromatic laser field. Using the Dirac–Volkov formalism for fermions and that of Klein-Gordon for mesons, we have analytically calculated the expression for the total width of the laser-assisted tau decay at the lowest order. We have examined the variation of the total decay width, the multiphoton absorption and emission process, as well as the branching ratios and lifetime as a function of the parameters that characterize the laser field. The results show that the insertion of the laser field reduces the total decay width, and consequently extends the lifetime due to the quantum Zeno effect. More importantly, we found that the application of a laser field with an appropriate frequency and field strength alters the decay width by enhancing some decay modes and suppressing others.

Keywords: Electroweak theory; Tau decay; Laser-assisted; Lifetime; Branching ratio

1. Introduction

The tau lepton τ^- decays via a charged weak current. Unlike other leptons, it can decay into hadrons such as pions and kaons because of its high mass of $m_\tau = 1.777$ GeV. This heavy mass also allows decay in the form of resonances. The tau lepton was discovered in 1974 by M.L. Perl and his team from the SLAC group (Stanford Linear Accelerator Center) during their research on heavy leptons and anomalous leptonic interactions, which constituted a significant discovery for physics. In recognition of this discovery, M.L. Perl shared the Nobel Prize in Physics with F. Reines in 1995 [1]. Since its invention in 1960, laser technology has attracted great interest in the scientific community due to its significant improvements in intensity and pulse duration [2]. This fascination stems from the discovery of previously unknown phenomena resulting from the interaction between radiation and matter, which has led to a better understanding of the atomic and molecular structure of matter, as well as the behavior of high-energy particles and associated phenomena. It is now widely recognized that lasers can modify the physics of unstable particles [3]. Therefore, the study of the laser field effects on the scattering and decay processes of high-energy particles is one of the main ways to obtain information about them and to probe the internal structure of matter. Several researchers have been interested in the decay of the tau lepton in the absence of an external field. In 1971, Tsai assumed the existence of leptons heavier than the muon in his work that investigates the decay correlations of heavy leptons in $e^+ + e^- \longrightarrow l^+ + l^-$ [4]. In 1984, Gilman considered the calculation of exclusive decay modes of the tau [5], and in 1986, Burchat treated the subject of decays of the tau lepton [6]. Since the 1970s until today, the processes of scattering and decay within an intense electromagnetic field have been studied in several works [7–9]. Recently, several studies have focused on laser-assisted scattering processes, such as top-quark pair production by electron-positron annihilation in a circularly polarized laser field [10] and laser-assisted scattering of an electron by a muon [11, 12]. In the context of decay, several works have been published, such as laser-assisted muon decay [13], laser-assisted pion decay [14], and the leptonic and hadronic decay of the W^- boson [15, 16]. The one-meson

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tau decays have been studied lately in [17–21]. For its leptonic decay modes, these have been addressed recently in [22–24]. Regarding decay processes, a controversial discussion arose in 2007 on the possibility that a laser field could affect the lifetime of unstable particles. This began when Liu et al. [13] carried out pioneering work, in which they examined the effect of the laser field on the lifetime of a muon when it decays in the presence of a linearly polarized laser field. As a surprising result, they found that lifetime was considerably reduced; exactly, from its normal value of 2.2×10^{-6} seconds to 5×10^{-7} seconds or even less. This work was controversial and received criticism and comments [25, 26], and was subsequently reviewed twice more by other researchers [27, 28]. In this context, we raised the following question. Can an external electromagnetic field really affect the lifetime of a particle and contribute to its enhancement or reduction? To answer this question, we carried out a series of theoretical studies of the decay processes in the presence of a laser field in an attempt to contribute to enriching the debate on this area of research. The first unstable particle we studied was the pion, in which we discussed this controversial debate in detail [14]. Therefore, we agree that there should be no effect of the laser field on lifetime. However, what we emphasize, and what we have demonstrated through the results obtained in a series of investigations, is that this conclusion is valid only when we reach a number of exchanged photons (called the cutoff number) that satisfies the sum rule. The latter means that the lifetime summed over all exchanged photons (the cutoff number) gives the laser-free lifetime. On the other hand, as long as the number of photons exchanged is smaller than the cutoff number, which varies according to the laser field strengths and frequencies used, the effect of the laser field on the lifetime will be present. As theoretical physicists, until now we do not know if there is a way to experimentally identify and control the number of photons exchanged. We also cannot know exactly how many photons the decay system exchanges with the laser field at a given field strength and frequency. The current work, which investigates tau decay in a circularly polarized laser field, is part of a series of studies we have conducted in an attempt to contribute to enriching the debate on this area of research. The first unstable particle we studied was the pion, in which we discussed this controversial debate in detail [14]. This paper focuses on the leptonic and two-body hadronic decay process of tau τ^-, at the lowest order, in the presence of a circularly polarized laser field, in the framework of the electroweak theory. We analyzed the simultaneous variation of the total decay width as a function of electric field strength and laser frequency. We also examined the multiphoton process and presented the branching ratio (BR) results of the different decay modes as well as the tau lifetime. The main objective of this work is to explore the impact of the laser field on the total decay width of tau from a theoretical point of view. To do so, we studied the behavior of the tau lifetime and branching ratios as a function of the parameters that define the laser field, such as the electric field strength, the frequency and the number of photons exchanged. The next sections of this paper are organized as follows: Sect. 2 deals with the detailed analytical calculation of the total lepton and two-body hadronic decay width of tau in the presence of a circularly polarized laser field. In Sect. 3, we present the numerical results obtained for the total decay width, the multiphoton process, the branching ratios and the tau lifetime. Finally, in Sect. 4, we briefly summarize our conclusions. Throughout our study, we use natural units ($\hbar = c = 1$), we choose the Livi-Civita tensor such that $\epsilon_{0123} = 1$, and we use the Minkowski metric tensor $g^{\mu\nu}$ =diag$(1, -1, -1, -1)$.

2. Theoretical framework

This section is devoted to the analytical calculation of the partial decay width of tau τ^-, which is one of the most important quantities in decay processes. In general, tau τ^- can decay into several leptonic (e^-, μ^-) and hadronic $(\pi^-, K^-, ..)$ channels. However, these decay channels are different in terms of decay width. Indeed, we first calculate the expression of the partial decay width in the case of leptonic channels. Then, we calculate its expression in the case of two-body hadronic channels inside a strong circularly polarized electromagnetic field.

2.1. Leptonic decays of the tau τ^-

We consider the leptonic decays process of tau τ^- into two leptons $\ell^- \equiv (e^-, \mu^-)$. It is expressed as follows:

$$\tau^-(p_1) \to \nu_\tau(k_1) + \ell^-(p_2) + \bar{\nu}_\ell(k_2), \tag{1}$$

where ℓ^- can be an electron or muon, and the arguments are our labels for the associated four-momenta. The leptonic decays of tau τ^- are described in the framework of the standard model by the Feynman diagram illustrated in Fig. 1.

In the presence of the electromagnetic field, the transition matrix element $\mathcal{S}_{fi}$ associated with the decays of tau τ^- into leptons (e^-, μ^-) is written as [29]

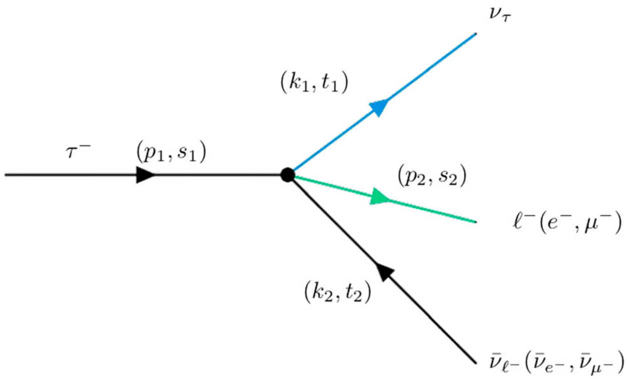

Fig. 1 Lowest order Feynman diagram for the leptonic decay of tau ($\tau^- \rightarrow \nu_\tau \ell^- \bar{\nu}_\ell$)

$$\mathcal{S}_{fi}(\tau^- \rightarrow \nu_\tau \ell^- \bar{\nu}_\ell) = -i \int d^4x \int d^4y \left[\bar{\psi}_{\nu_\tau}(x) \left(\frac{-ig}{2\sqrt{2}}\gamma^\mu(1-\gamma^5)\right) \psi_{\tau^-}(x)\right] W_{\mu\nu}(x-y) \times \left[\bar{\psi}_{\ell^-}(y) \left(\frac{-ig}{2\sqrt{2}}\gamma^\nu(1-\gamma^5)\right) \psi_{\bar{\nu}_\ell}(y)\right], \tag{2}$$

where g is the electroweak coupling constant and $W_{\mu\nu}(x-y)$ is the W^--boson propagator given by [29]:

$$W_{\mu\nu}(x-y) = -i \int \frac{d^4q}{(2\pi)^4} \frac{\mathrm{e}^{-iq.(x-y)}}{q^2 - M_W^2} \left[g_{\mu\nu} - \frac{q_\mu q_\nu}{M_W^2}\right]. \tag{3}$$

The four-momentum of the W^- boson propagator is represented by q, while its rest mass is given by M_W. We warn the reader here that the contributions from the longitudinal W-polarizations ($q^\mu q^\nu / M_W^2$ terms in Eq. (3)) are extremely suppressed and numerically irrelevant and are retained in all subsequent expressions only for the sake of completeness. We consider that the decay of tau τ occurs inside a circularly polarized laser field. The classical quadri-potential of this laser field is expressed by A^μ, such that:

$$A^\mu(\phi) = a_1^\mu \cos\phi + a_2^\mu \sin\phi. \tag{4}$$

The variable ϕ represents the phase of the laser field, defined as $\phi = (k.y)$. The two four-vectors, a_1 and a_2, are chosen such that they are orthogonal, with $a_1^\mu = |a|(0,1,0,0)$ and $a_2^\mu = |a|(0,0,1,0)$. This orthogonality property is characterized by the condition $a_1 \cdot a_2 = a_2 \cdot a_1 = 0$. Moreover, the squared norm of these 4-vectors is given by $a_1^2 = a_2^2 = -|a|^2 = -(\xi_0/\omega)^2$, where ξ_0 represents the electric field strength and ω its frequency. We assume that the quadri-potential satisfies the Lorentz gauge condition, $k_\mu A^\mu = 0$, which means $(k \cdot a_1) = (k \cdot a_2) = 0$, indicating that the wave vector $\mathbf{k}$ is chosen to be along the z-axis. Due to its large mass and to the electric field strengths considered here, the tau τ is much less affected by the laser field than the other leptons $\ell^- = (\mathrm{e}^-, \mu^-)$. Therefore, the lepton τ^- is described by a Dirac free wave function as follows:

$$\psi_{\tau^-}(x) = \frac{u(p_1, s_1)}{\sqrt{2p_1^0 V}} \times \mathrm{e}^{-i(p_1 \cdot x)}, \tag{5}$$

where $u(p_1, s_1)$ represents the bispinor for the free lepton τ^- with 4-momentum p_1 and spin s_1 satisfying $\sum_{s_1} u(p_1, s_1)\overline{u}(p_1, s_1) = \not{p}_1 + m_{\tau^-}$. The Dirac–Volkov function $\psi_\ell(y)$ normalized to the volume V represents the state of the lepton ℓ^- in the external electromagnetic field. The form of this function for a circularly polarized laser field is [30]:

$$\psi_\ell(y) = \left[1 + \frac{e}{2(k.p_2)} \not{k}\not{A}\right] \frac{u(p_2, s_2)}{\sqrt{2Q_2 V}} \times e^{iS(q_2, y)}, \tag{6}$$

with

$$S(q_2, y) = -q_2 \cdot y - \frac{e(a_1 \cdot p_2)}{(k \cdot p_2)} \sin(\phi) + \frac{e(a_2 \cdot p_2)}{(k \cdot p_2)} \cos(\phi). \tag{7}$$

The electron charge is denoted by $e = -|e|$. The effective four-momentum of the lepton ℓ^- acquired inside the laser field is defined as $q_2 = (Q_2, \mathbf{q}_2)$, where Q_2 is the effective energy and $\mathbf{q}_2$ is the momentum vector. It is obtained from the corresponding free momentum p_2 via the relation $q_2 = p_2 - \mathrm{e}^2 a^2/(2(k \cdot p_2))k$, where k is the wave four-vector of the laser field. We have $q_2^2 = p_2^2 - \mathrm{e}^2\, a^2 = m_\ell^2 - \mathrm{e}^2\, a^2 = m_\ell^{*2}$, where m_ℓ^* is the effective mass of the lepton considered. The tau neutrino ν_τ and the anti-neutrino $\bar{\nu}_\ell$ are considered as massless particles, with a four-momentum k_i and spin t_i. The Dirac free wave functions that describe these particles are given by:

$$\psi_{\nu_\tau}(x) = \frac{u(k_1, t_1)}{\sqrt{2k_1^0 V}} \times \mathrm{e}^{-i(k_1 \cdot x)}, \qquad \psi_{\bar{\nu}_\ell}(y) = \frac{v(k_2, t_2)}{\sqrt{2k_2^0 V}} \times \mathrm{e}^{i(k_2 \cdot y)}, \tag{8}$$

where $u(k_1, t_1)$ and $v(k_2, t_2)$ represent the Dirac bispinor for the free neutrino and antineutrino with 4-momentum $k_{i=1,2}$ and spin $t_{i=1,2}$, respectively, satisfying $\sum_{t_1} u(k_1, t_1)\overline{u}(k_1, t_1) = \not{k}_1$ and $\sum_{t_2} v(k_2, t_2)\overline{v}(k_2, t_2) = \not{k}_2$. By inserting Eqs. (3–8) into Eq. (2) and after some algebraic manipulations, we find

$$\mathcal{S}_{fi}(\tau^- \to \nu_\tau \ell^- \bar{\nu}_\ell) = \frac{g^2}{8\sqrt{16k_1^0k_2^0p_1^0Q_2V^4}} \int d^4x \int d^4y \, \frac{d^4q}{(2\pi)^4} \, e^{i[(k_1-p_1-q).x]} \, e^{i[(k_2+q).y-S(q_2,y)]} \times \left[\bar{u}(k_1,t_1) \, \gamma^\mu(1-\gamma^5) \, u(p_1,s_1)\right] \times \frac{(g_{\mu\nu} - q_\mu q_\nu/M_W^2)}{q^2 - M_W^2} \times \left[\bar{u}(p_2,s_2) \, \Big(1 + C(p_2) \, \not{a}_1 \not{k} \cos(\phi) + C(p_2) \, \not{a}_2 \not{k} \sin(\phi)\Big) \, \gamma^\nu(1-\gamma^5) \, v(k_2,t_2)\right], \tag{9}$$

where $C(p_2) = e/(2(k\cdot p_2))$. Now we can transform the exponential term in Eq. (9) by introducing the two quantities z and ϕ_0 such that:

$$z = \sqrt{\beta_1^2 + \beta_2^2}; \quad \phi_0 = \arctan(\beta_2/\beta_1), \quad \beta_1 = -e\frac{(a_1\cdot p_2)}{(k\cdot p_2)}; \quad \beta_2 = -e\frac{(a_2\cdot p_2)}{(k\cdot p_2)}. \tag{10}$$

We get the expression as follows:

$$(k_2+q)\cdot y - S(q_2,y) = (k_2+q_2+q)\cdot y - z\sin(\phi-\phi_0). \tag{11}$$

As a result, the S-matrix element $\mathcal{S}_{fi}$ becomes:

$$\mathcal{S}_{fi}(\tau^- \to \nu_\tau \ell^- \bar{\nu}_\ell) = \frac{g^2}{8\sqrt{16k_1^0k_2^0p_1^0Q_2V^4}} \int d^4x \int d^4y \, \frac{d^4q}{(2\pi)^4} \, e^{i[(k_1-p_1-q)\cdot x]} \, e^{i[(k_2+q_2+q)\cdot y]} \times \frac{1}{q^2-M_W^2}\left[\bar{u}(k_1,t_1) \, \gamma^\mu(1-\gamma^5)u(p_1,s_1)\right] e^{-iz\sin(\phi-\phi_0)} \times \left[\bar{u}(p_2,s_2) \, \Big(\Lambda_{0\mu} + \Lambda_{1\mu}\cos(\phi) + \Lambda_{2\mu}\sin(\phi)\Big) \, v(k_2,t_2)\right], \tag{12}$$

where the three quantities $\Lambda_{0\mu}$, $\Lambda_{1\mu}$ and $\Lambda_{2\mu}$ are given by:

$$\Lambda_{0\mu} = \gamma_\mu(1-\gamma^5) - \frac{q_\mu q_\nu}{M_W^2}\gamma^\nu(1-\gamma^5), \quad \Lambda_{1\mu} = C(p_2) \, \not{a}_1\not{k}\Big(\gamma_\mu(1-\gamma^5) - \frac{q_\mu q_\nu}{M_W^2}\gamma^\nu(1-\gamma^5)\Big), \quad \Lambda_{2\mu} = C(p_2) \, \not{a}_2\not{k} \, \Big(\gamma_\mu(1-\gamma^5) - \frac{q_\mu q_\nu}{M_W^2}\gamma^\nu(1-\gamma^5)\Big). \tag{13}$$

Using the Jacobi-Anger identity [31], we can convert the spinorial part of Eq. (12) into terms of the functions $\mathbf{B}_n(z)$, $\mathbf{B}_{1n}(z)$ and $\mathbf{B}_{2n}(z)$, the latter being expressed in terms of the Bessel functions $J_n(z)$ as follows:

$$e^{-iz\sin(\phi-\phi_0)} = \sum_{n=-\infty}^{+\infty} \mathbf{B}_n(z)e^{-in\phi}, \tag{14}$$

$$\cos(\phi) \, e^{-iz\sin(\phi-\phi_0)} = \sum_{n=-\infty}^{+\infty} \mathbf{B}_{1n}(z)e^{-in\phi}, \tag{15}$$

$$\sin(\phi) \, e^{-iz\sin(\phi-\phi_0)} = \sum_{n=-\infty}^{+\infty} \mathbf{B}_{2n}(z)e^{-in\phi}, \tag{16}$$

with

$$\begin{bmatrix} \mathbf{B}_n(z) \\ \mathbf{B}_{1n}(z) \\ \mathbf{B}_{2n}(z) \end{bmatrix} = \begin{bmatrix} J_n(z)e^{in\phi_0} \\ \big(J_{n+1}(z)e^{i(n+1)\phi_0} + J_{n-1}(z)e^{i(n-1)\phi_0}\big)/2 \\ \big(J_{n+1}(z)e^{i(n+1)\phi_0} - J_{n-1}(z)e^{i(n-1)\phi_0}\big)/2i \end{bmatrix}, \tag{17}$$

where z is the argument of the Bessel functions given in Eq. (10) and n, their order, is usually interpreted as the number of exchanged photons [32]. Applying these transformations in Eq. (12), $\mathcal{S}_{fi}$ becomes:

$$\mathcal{S}_{fi}(\tau^- \to \nu_\tau \ell^- \bar{\nu}_\ell) = \sum_{n=-\infty}^{+\infty} \frac{g^2}{8\sqrt{16k_1^0k_2^0p_1^0Q_2V^4}} \int d^4x \int d^4y \, \frac{d^4q}{(2\pi)^4} \, e^{i[(k_1-p_1-q)\cdot x]} \, e^{i[(k_2+q_2-nk+q)\cdot y]} \times \frac{1}{q^2-M_W^2}\left[\bar{u}(k_1,t_1) \, \gamma^\mu(1-\gamma^5)u(p_1,s_1)\right] \times \left[\bar{u}(p_2,s_2) \, \Big(\Lambda_{0\mu}\mathbf{B}_n(z) + \Lambda_{1\mu}\mathbf{B}_{1n}(z) + \Lambda_{2\mu}\mathbf{B}_{2n}(z)\Big) \, v(k_2,t_2)\right], \tag{18}$$

where $g^2/8 = G_F\, M_W^2/\sqrt{2}$, with $G_F = (1.166\,37 \pm 0.000\,02)\times 10^{-5}$ GeV^{-2} is the Fermi coupling constant [33]. Integrating over d^4x, d^4y and d^4q, we get:

$$\mathcal{S}_{fi}(\tau^- \to \nu_\tau \ell^- \bar{\nu}_\ell) = \sum_{n=-\infty}^{+\infty} \frac{G_F\, M_W^2}{\sqrt{2}} \frac{(2\pi)^4\delta^4(k_2+q_2+k_1-p_1-nk)}{\sqrt{16k_1^0k_2^0p_1^0Q_2V^4}} \times \mathcal{M}_{fi}^n, \tag{19}$$

where the quantity $\mathcal{M}_{fi}^n$ is defined as follows:

$$\mathcal{M}_{fi}^n = \frac{1}{(k_2+q_2-nk)^2 - M_W^2} \left[\bar{u}(k_1,t_1)\,\gamma^\mu(1-\gamma^5)\,u(p_1,s_1)\right] \times \left[\bar{u}(p_2,s_2)\,\left(\Lambda_{0\mu}\mathbf{B}_n(z)+\Lambda_{1\mu}\mathbf{B}_{1n}(z)+\Lambda_{2\mu}\mathbf{B}_{2n}(z)\right)\,v(k_2,t_2)\right]. \tag{20}$$

The decay width of tau τ^- is determined by multiplying the square of $\mathcal{S}_{fi}$ by the phase space and per unit time T, summing over the spins of the outgoing leptons $(\ell^-, \nu_\tau, \bar{\nu}_\ell)$ and averaging over the spin of the incoming tau τ^-. The laser-assisted leptonic decay width of tau τ^- is given by:

$$\Gamma(\tau^- \to \nu_\tau \ell^- \bar{\nu}_\ell) = \frac{1}{T}\int \frac{Vd^3q_2}{(2\pi)^3}\int \frac{Vd^3k_2}{(2\pi)^3}\int \frac{Vd^3k_1}{(2\pi)^3}\;|\mathcal{S}_{fi}|^2 = \sum_{n=-\infty}^{+\infty}\Gamma^n(\tau^- \to \nu_\tau \ell^- \bar{\nu}_\ell), \tag{21}$$

where the leptonic decay width resolved by the number of photons, $\Gamma^n(\tau^- \to \nu_\tau \ell^-\ \bar{\nu}_\ell)$, is defined by:

$$\Gamma^n(\tau^- \to \nu_\tau \ell^- \bar{\nu}_\ell) = \frac{G_F^2\ M_W^4}{16\ (2\pi)^5\ p_1^0}\int \frac{d^3q_2}{Q_2}\int \frac{d^3k_2}{2k_2^0}\int \frac{d^3k_1}{2k_1^0}\ \delta^4(k_1+k_2-(p_1-q_2+nk)) \times \sum_{s_1 s_2}\sum_{t_1 t_2}|\mathcal{M}_{fi}^n|^2, \tag{22}$$

where

$$\sum_{s_1 s_2}\sum_{t_1 t_2}|\mathcal{M}_{fi}^n|^2 = \frac{k_{1\alpha}\ k_{2\beta}}{(k_2+q_2-nk)^2 - M_W^2}\ \mathrm{Tr}\left[\gamma^\alpha\ \gamma^\mu(1-\gamma^5)\ (\not{p}_1+m_\tau)\gamma^\nu(1-\gamma^5)\right] \times \mathrm{Tr}\left[(\not{p}_2+m_{\ell^-})\ \left(\Lambda_{0\mu}\mathbf{B}_n(z)+\Lambda_{1\mu}\mathbf{B}_{1n}(z)+\Lambda_{2\mu}\mathbf{B}_{2n}(z)\right)\ \gamma^\beta\ \left(\bar{\Lambda}_{0\nu}\mathbf{B}_n^*(z) + \bar{\Lambda}_{1\nu}\mathbf{B}_{1n}^*(z)+\bar{\Lambda}_{2\nu}\mathbf{B}_{2n}^*(z)\right)\ \right]. \tag{23}$$

In Eq. (23), the three quantities $\bar{\Lambda}_{0\nu}$, $\bar{\Lambda}_{1\nu}$ and $\bar{\Lambda}_{2\nu}$ are expressed by:

$$\begin{aligned}
\bar{\Lambda}_{0\nu} &= \gamma_\nu(1-\gamma^5) - \frac{(k_2+q_2-nk)_\nu(k_2+q_2-nk)_\mu}{M_W^2}\gamma^\mu(1-\gamma^5),\\
\bar{\Lambda}_{1\nu} &= C(p_2)\ \left(\gamma_\nu(1-\gamma^5) - \frac{(k_2+q_2-nk)_\nu(k_2+q_2-nk)_\mu}{M_W^2}\gamma^\mu(1-\gamma^5)\right)\ \not{k}\not{a}_1,\\
\bar{\Lambda}_{2\nu} &= C(p_2)\ \left(\gamma_\nu(1-\gamma^5) - \frac{(k_2+q_2-nk)_\nu(k_2+q_2-nk)_\mu}{M_W^2}\gamma^\mu(1-\gamma^5)\right)\not{k}\not{a}_2.
\end{aligned} \tag{24}$$

The summation of the photon number n in Eq. (21) from $-\infty$ to $+\infty$ is mathematically introduced by ordinary Bessel functions to solve certain transformations, as shown in Eqs. (14–16). So there are two cases for the number n: either it is negative, indicating photon emission, or it is positive, which indicates the absorption of photons. In order to calculate the leptonic decay width resolved by the number of exchanged photons $\Gamma^n(\tau^- \to \nu_\tau \ell^-\ \bar{\nu}_\ell)$, we proceed by inserting the expression of $\sum_{s_1 s_2}\sum_{t_1 t_2}|\mathcal{M}_{fi}^n|^2$ (23) into Eq. (22). We use the formula below to evaluate the integration over d^3k_1 and d^3k_2 [29]:

$$\int \frac{Vd^3k_2}{2k_2^0}\int \frac{Vd^3k_1}{2k_1^0}\ k_{1\alpha}\ k_{2\beta}\ \delta^4(k_1+k_2-(p_1-q_2+nk)) = \frac{\pi}{24}\ \left[(p_1-q_2+nk)^2 g_{\alpha\beta} + 2(p_1-q_2+nk)_\alpha(p_1-q_2+nk)_\beta\right]. \tag{25}$$

Finally, $\Gamma^n(\tau^- \to \nu_\tau \ell^- \bar{\nu}_\ell)$ can be written as:

$$\Gamma^n(\tau^- \to \nu_\tau \ell^- \bar{\nu}_\ell) = \frac{G_F^2\ M_W^4\ \pi^2}{192\ (2\pi)^5\ p_1^0}\int_0^\pi \int_{m_\ell^*}^{Q_{2\max}} \mathrm{d}Q_2\ |\mathbf{q}_2|\ \sin(\theta)\ \mathrm{d}\theta\ \times |\mathcal{H}_{fi}^n|^2, \tag{26}$$

where the quantity $|\mathcal{H}_{fi}^n|^2$ is written as:

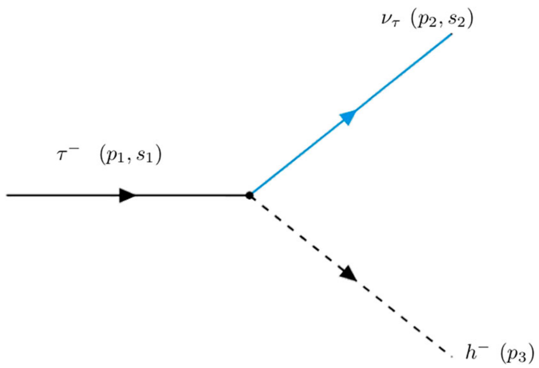

Fig. 2 Lowest order Feynman diagram for the two-body hadronic decays of tau: $\tau^- \to \nu_\tau h^-$

leptonic and hadronic currents, which can both be expressed as follows:

$$\mathcal{J}^{\tau}_{\mu}(y) = \bar{\psi}_{\tau}(y,t)\gamma_{\mu}(1-\gamma_5)\psi_{\nu_\tau}(y,t), \tag{31}$$

and

$$\mathcal{J}^{\mu}_{h^-}(y) = i\sqrt{2}\, f_{h^-} p_3^{\mu}\ \psi_{h^-}(y), \tag{32}$$

where $f_{h^-} = f_\pi = 90.8$ MeV and $f_{h^-} = f_K = 24.9$ MeV are called, respectively, the decay constants of the charged pion π^- and kaon K^- [29]. To be clear, this choice in the values of the pion and kaon decay constants is such that: $\sqrt{2}f_\pi \simeq \mathcal{F}_\pi |V_{ud}|$ and $\sqrt{2}f_K \simeq \mathcal{F}_K |V_{us}|$, where $\mathcal{F}_\pi \simeq 130$ MeV, $\mathcal{F}_K \simeq 155$ MeV and the CKM matrix elements $V_{ud} = 0.97417$ and $V_{us} = 0.2176$ [35]. The tau τ^- inside

$$\begin{aligned}
|\mathcal{H}^n_{fi}|^2 = & \frac{1}{(k_2+q_2-nk)^2 - M_W^2}\Big[(p_1-q_2+nk)^2 g_{\alpha\beta} + 2(p_1-q_2+nk)_\alpha (p_1-q_2+nk)_\beta\Big] \\
& \times \mathrm{Tr}\Big[\gamma^\alpha\ \gamma^\mu (1-\gamma^5)\ (\not{p}_1 + m_\tau)\gamma^\nu (1-\gamma^5)\Big]\ \times \mathrm{Tr}\Big[(\not{p}_2 + m_{\ell^-})\ \Big(\Lambda_{0\mu}\mathbf{B}_n(z) \\
& + \Lambda_{1\mu}\mathbf{B}_{1n}(z) + \Lambda_{2\mu}\mathbf{B}_{2n}(z)\Big)\ \gamma^\beta\ \Big(\bar{\Lambda}_{0\nu}\mathbf{B}^*_n(z) + \bar{\Lambda}_{1\nu}\mathbf{B}^*_{1n}(z) + \bar{\Lambda}_{2\nu}\mathbf{B}^*_{2n}(z)\Big)\ \Big].
\end{aligned} \tag{27}$$

In a particular case, using the same steps, the leptonic decay width of the tau τ^-, in the absence of any external electromagnetic field, is written as follows [4–6, 34]:

$$\Gamma(\tau^- \to \nu_\tau \ell^- \bar{\nu}_\ell) = \frac{G_F^2\ m_\tau^5}{192\ \pi^3} \times \mathcal{F}\left(\frac{m_\ell}{m_\tau}\right), \tag{28}$$

where $\mathcal{F}(\Delta) = 1 - 8\ \Delta^2 + 8\ \Delta^6 - \Delta^8 - 12\ \Delta^4\ \ln(\Delta^2)$.

2.2. Two-body hadronic decays of the tau τ^-

We will now consider the decay process of tau τ^- into neutrino ν_τ and hadrons h^-, which is illustrated as follows:

$$\tau^-(p_1) \to \nu_\tau(p_2) + h^-(p_3), \qquad h^- \equiv (\pi^-, K^-) \tag{29}$$

where h^- can be a pion or kaon, and the arguments are our labels for the associated four-momenta. The two-body hadronic decays of tau τ^- are described in the framework of the standard model by the Feynman diagram shown in Fig. 2.

The two-body hadronic decays process of tau in a circularly polarized external electromagnetic field is a weak interaction process, its classical quadri-potential A^μ is described by the same formula given in Eq. (4). In the first Born approximation, the $\mathcal{S}_{fi}$-matrix element for the laser-assisted two-body hadronic decays of τ^- can be written as a product of two currents such that [29]:

$$\mathcal{S}_{fi}(\tau^- \to \nu_\tau h^-) = \frac{-iG_F}{\sqrt{2}} \int d^4y\ \mathcal{J}^{\tau\dagger}_{\mu}(y)\ \mathcal{J}^{\mu}_{h^-}(y). \tag{30}$$

The two currents $\mathcal{J}^{\tau}_{\mu}(y)$ and $\mathcal{J}^{\mu}_{h^-}(y)$ are, respectively, the

the laser field is described by the relativistic Dirac–Volkov function normalized to the volume V such that [30]:

$$\psi_{\tau^-}(y) = \left[1 + \frac{e\not{k}\not{A}}{2k.p_1}\right]\frac{u(p_1,s_1)}{\sqrt{2Q_1 V}}e^{iS(q_1,y)}, \tag{33}$$

where

$$S(q_1,y) = -q_1\cdot y - e\frac{(a_1\cdot p_1)}{(k\cdot p_1)}\sin\phi + e\frac{(a_2\cdot p_1)}{(k\cdot p_1)}\cos\phi. \tag{34}$$

The four-vector $q_1 = (Q_1, \mathbf{q}_1)$ is the effective four-momentum that the tau τ^- acquires in the presence of an external electromagnetic field,

$$q_1^\mu = p_1^\mu - \frac{\mathrm{e}^2 a^2}{2(k\cdot p_1)}k^\mu, \tag{35}$$

while the zero component Q_1 is its effective energy. The square of this effective four-momentum is given by:

$${q_1}^2 = {p_1}^2 - \mathrm{e}^2 a^2 = m_\tau^2 - \mathrm{e}^2 a^2 = m_\tau^{*2}, \tag{36}$$

where m_τ^* is the effective mass of tau. Inside the laser field, the charged pion and kaon are described by the Klein-Gordon equation of a charged scalar (spinless particle) coupled to an electromagnetic field,[1] which is given by [36]:

[1] The effect of the laser field on the tau lifetime and branching ratios studied in this paper is more significant at low frequencies, where the point-like approximation for the pion/kaon is best justified.

$$\Big[(i\partial - e\,A)^2 - m_{h^-}\Big]\psi_{h^-}(y) = 0\,, \tag{37}$$

where $m_{h^-} = (m_{\pi^-}$ or $m_{K^-})$ is the rest mass of the charged pion or kaon. From Eq. (37), we can obtain the following wave function:

$$\psi_{h^-}(y) = \frac{1}{\sqrt{2Q_3V}} \times \mathrm{e}^{-iS(q_3,y)}\,, \tag{38}$$

where

$$S(q_3,y) = -q_3\cdot y - \frac{e(a_1\cdot p_3)}{(k\cdot p_3)}\sin(\phi) + \frac{e(a_2\cdot p_3)}{(k\cdot p_3)}\cos(\phi). \tag{39}$$

The quasi-momentum $q_3 = (Q_3, \mathbf{q}_3)$ and the effective mass m_{h^-*} are expressed by:

$$q_3^\mu = p_3^\mu - \frac{\mathrm{e}^2a^2}{2(k\cdot p_3)}k^\mu, \quad m^2_{h^-*} = m^2_{h^-} - \mathrm{e}^2a^2. \tag{40}$$

The outgoing tau neutrino ν_τ is described by a Dirac free wave function given in Eq. (8), with a four-momentum p_2 and spin s_2. By inserting Eqs. (31) and (32) into Eq. (30) and substituting wave functions, we find that the S-matrix element $\mathcal{S}_{fi}$ for the laser-assisted two-body hadronic decays of τ^- can be written as:

$$\mathcal{S}_{fi}(\tau^-\to\nu_\tau h^-) = \frac{G_F\,f_{h^-}}{2\sqrt{2}}\int d^4y\frac{e^{i\big(S(q_1,y)-S(q_3,y)+p_2\cdot y\big)}}{\sqrt{Q_1p_2^0Q_3V^3}} \times\Big[\bar{u}(p_2,s_2)\gamma_\mu(1-\gamma^5)\Big(1 + C(p_1)\not{k}\not{a}_1\cos(\phi) + C(p_1)\not{k}\not{a}_2\sin(\phi)\Big)u(p_1,s_1)\Big]\;p_3^\mu, \tag{41}$$

where $C(p_1) = e/(2(k\cdot p_1))$. Now, we will transform the exponential part in Eq. (41) by introducing the two quantities z_h and ϕ_{0h}, such that:

$$z_h = e\sqrt{\Big(\frac{(a_1\cdot p_1)}{(k\cdot p_1)} - \frac{(a_1\cdot p_3)}{(k\cdot p_3)}\Big)^2 + \Big(\frac{(a_2\cdot p_1)}{(k\cdot p_1)} - \frac{(a_2\cdot p_3)}{(k\cdot p_3)}\Big)^2},$$
$$\phi_{0h} = \arctan\Big(\frac{(a_2\cdot p_1)(k\cdot p_3) - (a_2\cdot p_3)(k\cdot p_1)}{(a_1\cdot p_1)(k\cdot p_3) - (a_1\cdot p_3)(k\cdot p_1)}\Big). \tag{42}$$

We get

$$S(q_1,y) - S(q_3,y) + p_2\cdot y = (p_2 + q_3 - q_1).\,y - z_h\sin(\phi-\phi_{0h}). \tag{43}$$

Therefore, the S-matrix element $\mathcal{S}_{fi}$ becomes:

$$\mathcal{S}_{fi}(\tau^-\to\nu_\tau h^-) = \frac{G_F\,f_{h^-}}{2\sqrt{2}}\sum_{n=-\infty}^{+\infty}\frac{(2\pi)^4\delta^4\big(p_2+q_3-q_1-nk\big)}{\sqrt{Q_1p_2^0Q_3V^3}}\times\mathcal{M}_{fi}^{hn}, \tag{44}$$

where the quantity $\mathcal{M}_{fi}^{hn}$ is defined by:

$$\mathcal{M}_{fi}^{hn} = \Big[\bar{u}(p_2,s_2)\,\Big(\Delta_0\,\mathbf{B}_n(z_h) + \Delta_1\,\mathbf{B}_{1n}(z_h) + \Delta_2\,\mathbf{B}_{2n}(z_h)\Big)u(p_1,s_1)\Big], \tag{45}$$

where

$$\Delta_0 = \not{p}_3(1-\gamma^5),$$
$$\Delta_1 = C(p_1)\not{p}_3(1-\gamma^5)\not{k}\not{a}_1,$$
$$\Delta_2 = C(p_1)\not{p}_3(1-\gamma^5)\not{k}\not{a}_2. \tag{46}$$

To calculate the two-body hadronic decay width of tau in the presence of an electromagnetic field, we follow the same procedure as for the lepton decay width. We obtain

$$\Gamma_h(\tau^-\to\nu_\tau h^-) = \sum_{n=-\infty}^{+\infty}\Gamma_h^n(\tau^-\to\nu_\tau h^-), \tag{47}$$

where the two-body hadronic decay width resolved by the number of photons, $\Gamma_h^n(\tau^-\to\nu_\tau h^-)$, is defined by:

$$\Gamma_h^n(\tau^-\to\nu_\tau h^-) = \frac{G_F^2\,f_{h^-}^2}{8(2\pi)^2Q_1}\int\frac{d^3q_3}{Q_3}\int\frac{d^3p_2}{p_2^0}\,\delta^4\big(p_2+q_3-q_1-nk\big)\times\frac{1}{2}\sum_{s_1}\sum_{s_2}|\mathcal{M}_{fi}^{hn}|^2, \tag{48}$$

where $\delta^4\big(p_2+q_3-q_1-nk\big) = \delta^3\big(\mathbf{p}_2+\mathbf{q}_3-\mathbf{q}_1-n\mathbf{k}\big)\delta\big(p_2^0+Q_3-Q_1-n\omega\big)$. After integrating over d^3p_2, we get:

$$\Gamma_h^n(\tau^- \to \nu_\tau h^-) = \frac{G_F^2 f_{h^-}^2}{8(2\pi)^2 Q_1} \int \frac{d^3 q_3}{Q_3 p_2^0} \delta\left(p_2^0 + Q_3 - Q_1 - n\omega\right) \times \frac{1}{2}\sum_{s_1}\sum_{s_2} |\mathcal{M}_{fi}^{hn}|^2. \quad (49)$$

We use the relation $d^3q_3 = |\mathbf{q}_3|^2\, d|\mathbf{q}_3|\, \mathrm{d}\Omega_{h^-}$, where $\mathrm{d}\Omega_{h^-} = \int_0^{2\pi} \mathrm{d}\varphi \int_0^{\pi} \sin(\theta)\, \mathrm{d}\theta$ is the solid angle associated with the charged pion or kaon. The remaining integral over $d|\mathbf{q}_3|$ can be evaluated by using the usual formula [29]:

$$\int dy \mathbf{F}(y)\delta(\mathbf{G}(y)) = \frac{\mathbf{F}(y)}{|\mathbf{G}'(y)|}\Bigg|_{\mathbf{G}(y)=0} \quad \text{with} \quad \mathbf{G}(|\mathbf{q}_3|) = p_2^0 + Q_3 - Q_1 - n\omega. \quad (50)$$

Thus, we get:

$$\Gamma_h^n(\tau^- \to \nu_\tau h^-) = \frac{G_F^2 f_{h^-}^2}{16\pi Q_1} \int_0^{\pi} \frac{|\mathbf{q}_3|^2\ \sin(\theta)\ \mathrm{d}\theta}{Q_3 p_2^0\ |\mathbf{G}'(|\mathbf{q}_3|)|} \times \frac{1}{2}\sum_{s_1}\sum_{s_2} |\mathcal{M}_{fi}^{hn}|^2\Bigg|_{\mathbf{G}(|\mathbf{q}_3|)=0}, \quad (51)$$

where $p_2^0 = |\mathbf{p}_2| = |\mathbf{q}_1 - \mathbf{q}_3 + n\mathbf{k}|$ and $Q_3 = \sqrt{|\mathbf{q}_3|^2 + m_{h^-}^{*2}}$. The absolute value of $\mathbf{G}'(|\mathbf{q}_3|)$ is obtained from the first derivative of $\mathbf{G}(|\mathbf{q}_3|)$ with respect to $|\mathbf{q}_3|$. It is given by:

$$|\mathbf{G}'(|\mathbf{q}_3|)| = \frac{|\mathbf{q}_3| - \omega\cos(\theta) \times \left(n - \frac{\mathrm{e}^2 a^2}{2(k\cdot p_1)}\right)}{\sqrt{|\mathbf{q}_3|^2 - 2\omega|\mathbf{q}_3|\cos(\theta) \times \left(n - \frac{\mathrm{e}^2a^2}{2(k\cdot p_1)}\right) + \omega^2\left(n - \frac{\mathrm{e}^2a^2}{2(k\cdot p_1)}\right)^2}} + \frac{|\mathbf{q}_3|}{\sqrt{|\mathbf{q}_3|^2 + m_{h^-}^{*2}}}, \quad (52)$$

and

$$\frac{1}{2}\sum_{s_1}\sum_{s_2} |\mathcal{M}_{fi}^{hn}|^2 = \frac{1}{2}\ \mathrm{Tr}\Big[\not{p}_2\Big(\Delta_0\ \mathbf{B}_n(z_h) + \Delta_1\ \mathbf{B}_{1n}(z_h) + \Delta_2\ \mathbf{B}_{2n}(z_h)\Big)(\not{p}_1 + m_\tau) \times \Big(\overline{\Delta}_0\ \mathbf{B}_n^*(z_h) + \overline{\Delta}_1\ \mathbf{B}_{1n}^*(z_h) + \overline{\Delta}_2\ \mathbf{B}_{2n}^*(z_h)\Big)\Big], \quad (53)$$

with

$$\overline{\Delta}_0 = \not{p}_3(1-\gamma^5),\quad \overline{\Delta}_1 = C(p_1)\not{a}_1\not{k}\not{p}_3(1-\gamma^5),\quad \overline{\Delta}_2 = C(p_1)\not{a}_2\not{k}\not{p}_3(1-\gamma^5). \quad (54)$$

In the absence of the electromagnetic field, the two-body hadronic decay width is given by [5, 6, 34]:

$$\Gamma(\tau^- \to \nu_\tau h^-) = \frac{G_F^2 f_{h^-}^2}{16\ \pi} m_\tau^3 \left(1 - \frac{m_{h^-}^2}{m_\tau^2}\right)^2. \quad (55)$$

2.3. Lifetime and branching ratios

In the previous subsections, we developed theoretical expressions for the leptonic and hadronic decay widths under the influence of a circularly polarized laser field. The inverse of the total decay width, called lifetime, is the most important quantity that depends on the decay widths. The total decay width of the tau lepton τ^- is the sum of the leptonic and hadronic decay widths. Therefore, the lifetime of the tau lepton in the presence of an electromagnetic field can be expressed as follows:

$$\tau_{\tau^-} = \frac{1}{\Gamma_{\text{total}}^{\tau^-}}, \quad (56)$$

where

$$\Gamma_{\text{total}}^{\tau^-} = \Gamma(\tau^- \to \nu_\tau \ell^- \bar{\nu}_\ell) + \Gamma(\tau^- \to \nu_\tau + \text{Hadrons}). \quad (57)$$

$\Gamma(\tau^- \to \nu_\tau \ell^- \bar{\nu}_\ell)$ is the laser-assisted leptonic decay width of tau given by (21) and $\Gamma(\tau^- \to \nu_\tau + \text{Hadrons})$ represents the sum of all the laser-assisted two-body hadronic decay widths of tau τ^-. That is:

$$\Gamma(\tau^- \to \nu_\tau \ell^- \bar{\nu}_\ell) = \Gamma(\tau^- \to \nu_\tau \mathrm{e}^- \bar{\nu}_{\mathrm{e}^-}) + \Gamma(\tau^- \to \nu_\tau \mu^- \bar{\nu}_{\mu^-}),$$
$$\Gamma(\tau^- \to \nu_\tau + \text{Hadrons}) = \Gamma_h(\tau^- \to \nu_\tau h^-) + \Gamma(\tau^- \to \text{Other modes}), \quad (58)$$

where $\Gamma(\tau^- \to \text{Other modes})$ is the sum of the decay widths that correspond to the other channels and $\Gamma(\tau^- \to \nu_\tau h^-)$ is the laser-assisted two-body hadronic decay width given by (47), its expression is:

Table 1 Numerical values, in natural units, of the physical constants and masses [33]

Constant	Value	Constant	Value
m_e	0.511×10^{-3} GeV	G_F	1.16637×10^{-5} GeV^{-2}
M_W	80.379 GeV	m_μ	0.10566 GeV
m_τ	1.777 GeV	m_π	0.13957 GeV
$e = -\lvert e \rvert$	-8.542454×10^{-2}	m_K	0.4937 GeV

$$\Gamma(\tau^- \longrightarrow \nu_\tau h^-) = \Gamma(\tau^- \longrightarrow \nu_\tau + \pi^-) + \Gamma(\tau^- \longrightarrow \nu_\tau + K^-). \tag{59}$$

After giving the definition of the lifetime, we can establish the branching ratios for the different decay channels of the tau lepton (τ^-) under the influence of an electromagnetic field using the following definitions:

$$\mathrm{BR}(\tau^- \rightarrow \nu_\tau \ell^- \bar{\nu}_\ell) = \frac{\Gamma(\tau^- \longrightarrow \nu_\tau \ell^- \bar{\nu}_\ell)}{\Gamma^{\tau^-}_{\text{total}}}, \tag{60}$$

$$\mathrm{BR}(\tau^- \rightarrow \nu_\tau h^-) = \frac{\Gamma(\tau^- \longrightarrow \nu_\tau h^-)}{\Gamma^{\tau^-}_{\text{total}}}, \tag{61}$$

$$\mathrm{BR}(\tau^- \longrightarrow \text{Other modes}) = \frac{\Gamma(\tau^- \longrightarrow \text{Other modes})}{\Gamma^{\tau^-}_{\text{total}}}. \tag{62}$$

The experimental values of the lifetime and branching ratios in the absence of the electromagnetic field are given as follows [33]:

$$\begin{aligned} \tau_{\tau^-} &= (2.903 \pm 0.5) \times 10^{-13} s, \\ \mathrm{BR}(\tau^- \longrightarrow \nu_\tau \ell^- \bar{\nu}_\ell) &= (35.21 \pm 0.04)\%, \\ \mathrm{BR}(\tau^- \longrightarrow \nu_\tau h^-) &= (11.51 \pm 0.05)\%, \\ \mathrm{BR}(\tau^- \longrightarrow \text{Other modes}) &= (53.28 \pm 0.09)\%. \end{aligned} \tag{63}$$

3. Numerical results and discussion

After the theoretical calculation of the tau decay to the leptonic and hadronic modes in an intense external electromagnetic field, we will now discuss and analyze the different numerical results obtained. We note here that the various integrals and traces given in Eqs. (26), (27), (51) and (53) were calculated numerically. Similarly, the figures presented in this section are obtained using Mathematica software and FeynCalc-9.3.1 package [37, 38]. We

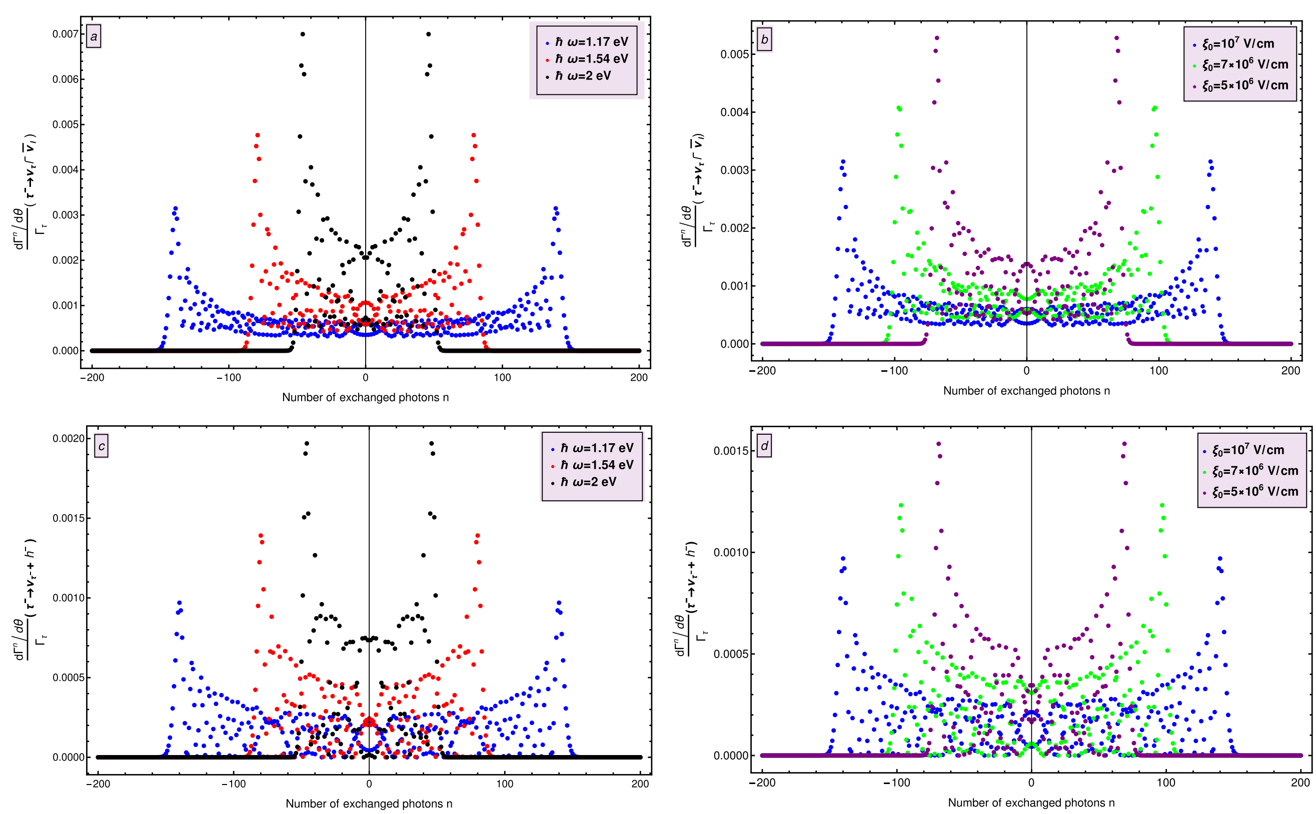

Fig. 3 Laser-assisted differential partial decay width $\mathrm{d}\Gamma^n/\mathrm{d}\theta(\tau^- \rightarrow \nu_\tau \ell^- \bar{\nu}_\ell)/\Gamma_\tau$ (26) and $\mathrm{d}\Gamma^n/\mathrm{d}\theta(\tau^- \rightarrow \nu_\tau h^-)/\Gamma_\tau$ (51) as a function of the number of exchanged photons n for different values of ξ_0 and $\hbar\omega$, choosing the spherical coordinates such that $\varphi = 0°$ and $\theta = 90°$. The other parameters are: **a** and **c** $\xi_0 = 10^7$ V/cm; **b** and **d** $\hbar\omega = 1.17$ eV

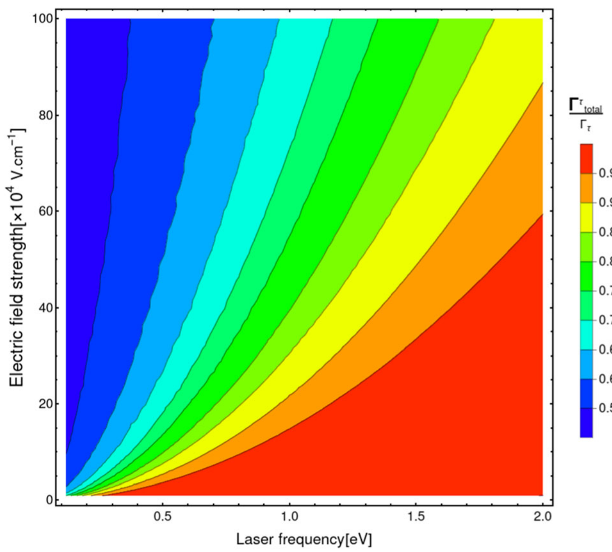

Fig. 4 Variation of the total decay width $\Gamma^{\tau^-}_{total}/\Gamma_\tau$ (57) as a function of the electric field strength ξ_0 and frequency. The number of exchanged photons chosen is $n = \pm 5$

will discuss the effect of each laser parameter (frequency and electric field strength) on laser-assisted partial differential and total tau decay widths, branching ratios and lifetime. The different numerical results are obtained in the natural unit system ($\hbar = c = 1$). We give the various numerical values used in our program in Table 1. Note here that we have adopted, for the electric charge e, a conventional notation in natural system units that sets $|e| = \sqrt{\alpha}$ (with $\alpha \simeq 1/137$ is the fine structure constant), and this should not cause any confusion.

We begin our discussion by analyzing the variations of the laser-assisted partial differential decay width $d\Gamma^n/d\theta(\tau^- \to \nu_\tau\ \ell^-\ \bar{\nu}_\ell)/\Gamma_\tau$ (26) and $d\Gamma^n/d\theta(\tau^- \to \nu_\tau\ h^-)/\Gamma_\tau$ (51) as a function of the number of exchanged photons n (emitted for $n<0$ or absorbed for $n>0$) in order to introduce the notion of cutoff for different frequencies $\hbar\omega$ and electric field strengths ξ_0.

Figure 3a, c show the laser-assisted partial differential decay width $d\Gamma^n/d\theta(\tau^- \to \nu_\tau\ \ell^-\ \bar{\nu}_\ell)/\Gamma_\tau$ (26) and $d\Gamma^n/d\theta(\tau^- \to \nu_\tau\ h^-)/\Gamma_\tau$ (51), respectively, for different laser frequencies Nd:YAG ($\hbar\omega = 1.17$ eV), Ti:sapphire ($\hbar\omega = 1.54$ eV) and He:Ne ($\hbar\omega = 2$ eV) with a fixed electric field strength at 10^7 V/cm. We observe that the envelopes are symmetric with respect to $n = 0$ and that the partial differential decay width becomes zero for a cutoff number. It also appears that all spectra are symmetric

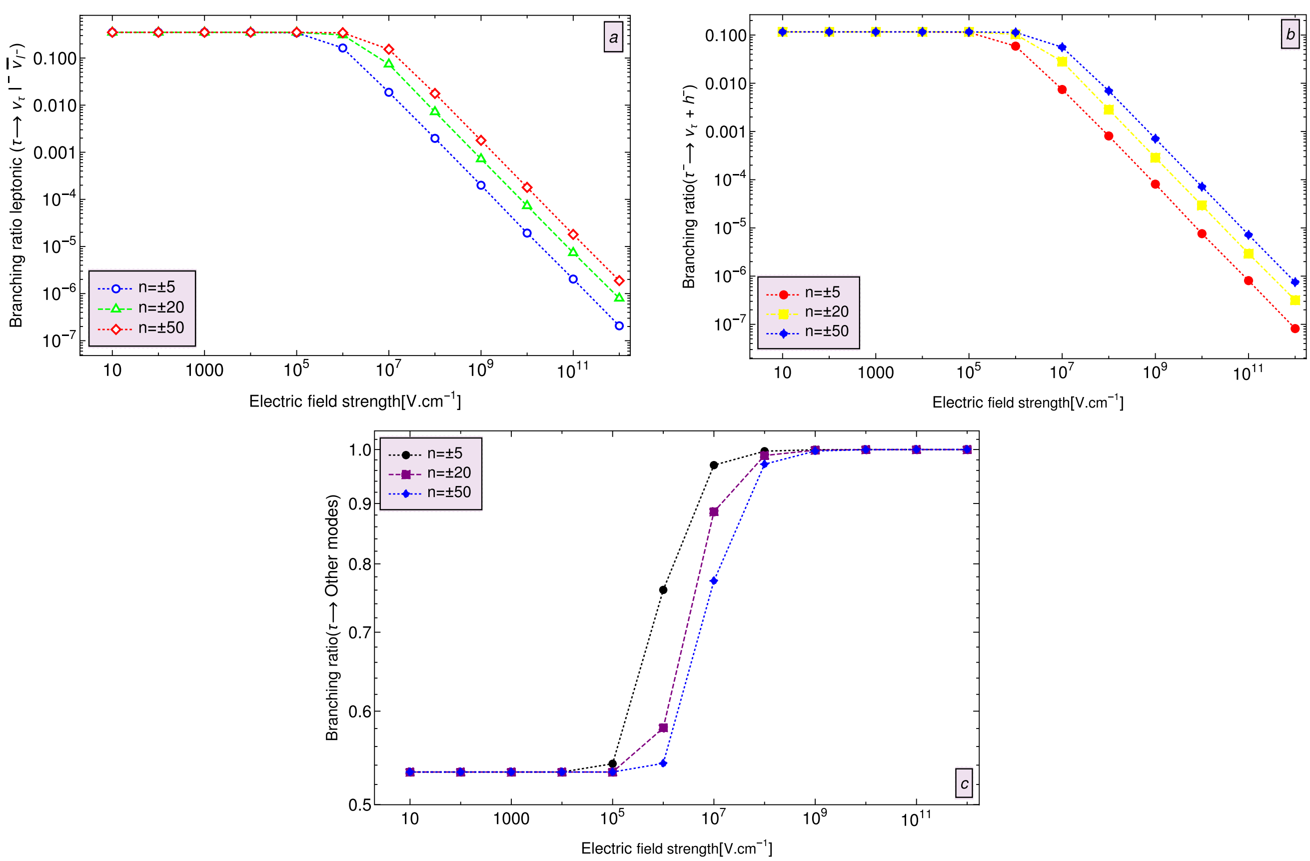

Fig. 5 The branching ratio behavior as a function of the electric field strength ξ_0, for different numbers of exchanged photons. (**a**) for $BR(\tau^- \to \nu_\tau \ell^- \bar{\nu}_\ell)$, (**b**) for $BR(\tau^- \to \nu_\tau h^-)$ and (**c**) for $BR(\tau^- \to$ Other modes). The laser frequency is $\hbar\omega = 1.17$ eV

Table 2 The numerical values of the tau lifetime τ_{τ^-} (56), as a function of the electric field strength ξ_0 for three numbers of exchanged photons n

ξ_0 (V cm^{-1})	Tau lifetime τ_{τ^-} (s)		
	$-5 \leqslant n \leqslant +5$	$-20 \leqslant n \leqslant +20$	$-50 \leqslant n \leqslant +50$
10^1	2.903×10^{-13}	2.903×10^{-13}	2.903×10^{-13}
10^2	2.903×10^{-13}	2.903×10^{-13}	2.903×10^{-13}
10^3	2.903×10^{-13}	2.903×10^{-13}	2.903×10^{-13}
10^4	2.903×10^{-13}	2.903×10^{-13}	2.903×10^{-13}
10^5	2.950×10^{-13}	2.903×10^{-13}	2.903×10^{-13}
10^6	4.142×10^{-13}	3.164×10^{-13}	2.952×10^{-13}
10^7	5.285×10^{-13}	4.825×10^{-13}	4.216×10^{-13}
10^8	5.431×10^{-13}	5.385×10^{-13}	5.295×10^{-13}
10^9	5.447×10^{-13}	5.442×10^{-13}	5.433×10^{-13}
10^{10}	5.448×10^{-13}	5.448×10^{-13}	5.447×10^{-13}
10^{11}	5.449×10^{-13}	5.449×10^{-13}	5.448×10^{-13}
10^{12}	5.449×10^{-13}	5.449×10^{-13}	5.449×10^{-13}

The laser frequency is fixed at $\hbar\omega = 1.17$ eV

envelopes with respect to the positive and negative sides, which means that the photon absorption ($n > 0$) is equal to the photon emission ($n<0$). The cutoff number can be explained by the properties of the Bessel function, which decreases when its argument (z) is close to its order (n). We also note that there is an inverse proportionality between the number of photons exchanged and the laser frequency, which means that if the frequency increases, the number exchanged of photons between the decay process and the laser field decreases. For example, the cutoff numbers are approximately $n = \pm 150$, $n = \pm 90$ and $n = \pm 55$ in the cases of $\hbar\omega = 1.17$ eV, $\hbar\omega = 1.54$ eV and $\hbar\omega = 2$ eV, respectively.

In Fig. 3b, d, we present the same decay widths of tau for a fixed frequency of 1.17 eV and for different electric field strengths. It is observed that a direct proportionality appears and that the number of exchanged photons increases with an increase in the electric field strength. We can read from these figures that the numbers of exchanged photons are approximately $n = \pm 80$, $n = \pm 110$ and $n = \pm 155$ for the cases of $\xi_0 = 5 \times 10^6$ V/cm, $\xi_0 = 7 \times 10^6$ V/cm and $\xi_0 = 10^7$ V/cm, respectively. The coupling between the particles involved in the tau decay process and the laser field depends on the argument z (and z_h)$\propto \xi_0/\omega^2$. Thus, an increase in the electric field strength or a decrease in the laser frequency will result in an increase in the "cutoff" and an increased exchange between the decay process and the laser field. It is important to note that the partial decay width of tau is influenced by the laser field. We will now investigate in Fig. 4 the simultaneous

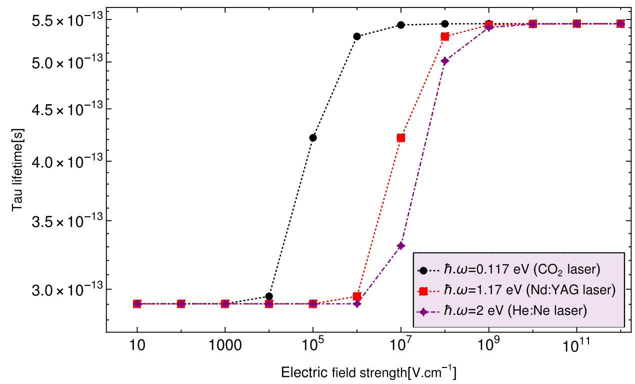

Fig. 6 Laser-assisted tau lifetime τ_{τ^-} (56) as a function of the electric field strength ξ_0 for three frequencies: $\hbar\omega = 0.117$ eV(CO_2 laser), $\hbar\omega = 1.17$ eV(Nd:YAG laser) and $\hbar\omega = 2$ eV(He:Ne laser). The chosen number of exchanged photons is $n = \pm 50$

dependence of the laser-assisted total width $\Gamma^{\tau^-}_{\text{total}}/\Gamma_\tau$ (57) on different values of the electric field strength ξ_0 and frequency $\hbar\omega$, for the same number of exchanged photons $n = \pm 5$.

We obtain contour-plots that reinforce the results of Fig. 3. They show that the total decay width decreases with the electric field strength and increases with the laser frequency. To be more precise, we emphasize here that the choice of a certain number of photons and the truncation of the sum over this number is a random choice to give an illustrative result of the effect of the laser field on the measured quantities, since we cannot know exactly how number of photons the decaying system actually exchanges with the laser at a given field strength and frequency.

In the next part, we will discuss another important quantity, which is the branching ratio of the different decay modes. Figure 5 illustrates the variation of $\text{BR}(\tau^- \to \nu_\tau \ell^- \bar{\nu}_\ell)$ (60), $\text{BR}(\tau^- \to \nu_\tau h^-)$ (61) and $\text{BR}(\tau^- \to \text{Other modes})$ (62) as a function of the electric field strength ξ_0 for different numbers of exchanged photons.

We find that, for low electric field strengths ($10 - 10^4$ V/cm), the BR curves do not change and remain constant, whatever the number of exchanged photons. In this case, the BRs are equal to $\text{BR}(\tau^- \to \nu_\tau \ell^- \bar{\nu}_\ell) = 0.35$, $\text{BR}(\tau^- \to \nu_\tau h^-) = 0.11$ and $\text{BR}(\tau^- \to \text{Other modes}) = 0.53$, which are the same as without laser field. Above the threshold of 10^5 V/cm, we observe that the leptonic branching ratio $\text{BR}(\tau^- \to \nu_\tau \ell^- \bar{\nu}_\ell)$ and the hadronic one $\text{BR}(\tau^- \to \nu_\tau h^-)$ start to decrease until they reach a value close to 0. On the other hand, the branching ratio for the other hadronic modes $\text{BR}(\tau^- \to \text{Other modes})$, which is the most probable in the absence of the laser field, starts to increase until it reaches a value close to 1, for an electric field strength of 10^9 V/cm. Regarding the dependence on the number of exchanged photons, as n increases, the improvement degree in the case of Fig. 5c and the suppression degree in Fig. 5a, b is delayed. It is reported that the laser field helped to enhance the ($\tau^- \to$ Other modes) to be more predominant and to reduce the decay modes ($\tau^- \to \nu_\tau \ell^- \bar{\nu}_\ell$) and ($\tau^- \to \nu_\tau h^-$), which are attenuated and reduced. The results also show that the three branching ratios are complementary because their sum equals 1. Therefore, an increase in the branching ratio BR($\tau^- \to$ Other modes) will be compensated by a decrease in the BR($\tau^- \to \nu_\tau \ell^- \bar{\nu}_\ell$) and BR($\tau^- \to \nu_\tau h^-$).

Now, let us examine how the lifetime is affected by the laser field. In Table 2, we illustrate the behavior of the tau lifetime τ_{τ^-} given by Eq. (56) as a function of the electric field strength ξ_0 for three numbers of exchanged photons. From this table, we can see that, for low fields, the lifetime remains unchanged from its value in the absence of the laser field. When we increase the electric field strength ξ_0, the lifetime of the tau also increases. This effect is most visible when the electric field strength exceeds a given threshold, namely 10^5 V/cm for $-5 \leqslant n \leqslant +5$, and 10^6 V/cm for $-20 \leqslant n \leqslant +20$ and $-50 \leqslant n \leqslant +50$. Regarding the effect of the number of photons exchanged n on the lifetime of tau, we observe that when n increases, the impact of the laser field on the lifetime of tau decreases, until a given threshold is achieved where the influence disappears. However, due to our limited computational capabilities, we cannot sum over a high number of photons to include this result. These results are comparable to those found in other studies, such as the two-body top-quark decay in a circularly polarized laser field [39], two-body hadronic kaon $K^\pm$ decay in a circularly polarized laser field [40], and the leptonic and hadronic decay of the boson W^- in the presence of a circularly polarized laser field [15, 16]. To understand these results, we refer to the Turing paradox discovered in 1954 by Alan Turing. This phenomenon was reformalized in 1974 by Degasperis, Misra et al. They called it the quantum Zeno effect [41]. It means that the coupling to a laser field acts as a kind of measurement and that the repeated application of measurements blocks the evolution of the decay. It was not until 1989 that the Zeno effect was observed experimentally with laser-cooled ions trapped by magnetic and electric fields [42].

In addition to the effect of the number of exchanged photons n and the electric field strength on the lifetime of the tau τ_{τ^-}, we will also explore the dependence of the lifetime on laser frequency. In Fig. 6, we have represented the variations of the tau lifetime τ_{τ^-} as a function of the electric field strength ξ_0, for three different frequencies: $\hbar\omega = 0.117$ eV (CO_2 laser), $\hbar\omega = 1.17$ eV (Nd:YAG laser) and $\hbar\omega = 2$ eV (He:Ne laser). All graphs were plotted with a fixed number of exchanged photons equal to ± 50.

We observed in Fig. 6 that the laser field globally increases the lifetime of tau for very high electric field strengths ($\xi_0 > 10^3$ V/cm). This increase in lifetime is expected and acceptable because the lifetime is inversely proportional to the total decay width of tau $\Gamma^{\tau^-}_{\text{total}}$. For example, for the same frequency $\hbar\omega = 0.117$ eV, the lifetime of tau has increased from $2.903 \times 10^{-13} s$ to $5.449 \times 10^{-13} s$. It is important to note that the tau lifetime τ_{τ^-} also depends on the laser frequency because the effect only appears at certain electric field strength threshold values. Moreover, this threshold value increases as the laser frequency increases. For example, it is 10^3 V/cm for the CO_2 laser and 10^5 V/cm for the Nd:YAG and He:Ne lasers. This means that the effect of the laser field decreases at higher frequencies.

4. Conclusions

In this paper, we have theoretically investigated the leptonic and hadronic tau τ^- decay assisted by a circularly polarized laser field. We have presented analytically and numerically the total tau decay width, branching ratios and lifetime. We have observed that the laser field contributed strongly to the reduction of the total decay width of tau, which significantly extends the lifetime of tau. In addition, the laser field enhanced the decay mode ($\tau^- \to$ Other modes) to be more dominant and suppressed the two decay modes ($\tau^- \to \nu_\tau \ell^- \bar{\nu}_\ell$) and ($\tau^- \to \nu_\tau h^-$), which were attenuated and reduced. Finally, we hope that the results presented here will contribute to paving the way for possible future experiments, in parallel with the rapid advances that laser technology has recently achieved.

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