RESOLUTION OF TRANSVERSE ELECTRON BEAM MEASUREMENTS USING OPTICAL TRANSITION RADIATION[*]

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Abstract

In the plasma wakefield acceleration experiment E-167, optical transition radiation is used to measure the transverse profile of the electron bunches before and after the plasma acceleration. The distribution of the electric field from a single electron does not give a point-like distribution on the detector, but has a certain extension. Additionally, the resolution of the imaging system is affected by aberrations. The transverse profile of the bunch is thus convolved with a point spread function (PSF). Algorithms that deconvolve the image can help to improve the resolution. Imaged test patterns are used to determine the modulation transfer function of the lens. From this, the PSF can be reconstructed. The Lucy-Richardson algorithm is used to deconvolute this PSF from test images.

INTRODUCTION

The plasma wakefield acceleration experiment E-167 [1] relies on optical transition radiation (OTR) monitors to determine the transverse size of the electron bunch on a shot-to-shot basis. Before the bunch enters the plasma, its profile is determined to tune the beam and to ascertain the symmetry of the profiles. Asymmetries are typically a hint for tilted incoming beams; in this case, the tail of the bunch is not centered with respect to the ion column created by the head of the bunch, which leads to oscillations. After the plasma, another OTR setup is used to determine deflections and the focusing properties of the plasma channel [2, 3]. The optical part of the spectrum is imaged onto a CCD detector. In the present setup, a commercial photography lens[1] and a CCD with a 12-bit readout[2] are used. A polarizer and a color filter are used to improve the resolution.

The electron bunch passes a titanium foil of 1 μm thickness, where it emits transition radiation [4]. The electric field on the foil is radially polarized and its $\vec{r}$ component is proportional to

$$E_r(r) \propto \frac{2\pi}{\gamma\lambda} \cdot K_1\left(\frac{2\pi}{\gamma\lambda} \cdot r\right)$$

where γ is the relativistic factor of the electrons and λ is the wavelength of the radiation. (For the present discussion of imaging properties, we are only interested in relative distributions of the light intensity.) Imaging this by a perfect lens with an aperture gives rise to the following intensity pattern on the detector, which can be found by numerical integration [5]:

$$I(r) \propto \left[\int_0^{\vartheta_{max}} \frac{\vartheta^2(1+\sqrt{1-\vartheta^2})}{\vartheta^2 + (\beta\gamma)^{-2}} \cdot J_1\left(\frac{2\pi\vartheta r}{\lambda M}\right) d\vartheta\right]^2 \tag{1}$$

where M is the magnification and $\beta = v/c$ the speed of the electrons. The integration is done over the opening angle of the radiation ϑ and is limited by the aperture of the lens a (in the present setup, $a = 105\text{mm}/2.8$):

$$\vartheta_{max} = \arctan\left(\frac{a/2}{g}\right)$$

where g is the distance between the lens and the image. Since the electrons emit incoherently, the image is a convolution of the particle distribution in the bunch with the single-electron radiation distribution. For the present setup, this distribution is shown in Figure 1. It is approximately four times broader than the radiation pattern from a uniformly emitting source, which has the following well-known intensity distribution:

$$I(r) \propto \left[\frac{J_1\left(\frac{\pi a}{\lambda b}\right)}{\frac{\pi a}{\lambda b}}\right]^2$$

where b is the distance between the lens and the image. The above equations are valid for ideal lenses. In reality, spherical aberrations in the lens broaden the distribution, and the point spread function has to be determined experimentally.

MEASUREMENT OF THE RESOLUTION OF THE IMAGING SYSTEM

By imaging system, we understand the combination of the lens, the photo-sensitive chip (typically a charge-coupled device CCD), analog signal processing, the

[1]Nikkor 105mm f/2.8
[2]Photometrics SenSys KAF 1400, will be replaced with PointGrey Flea for 10Hz readout
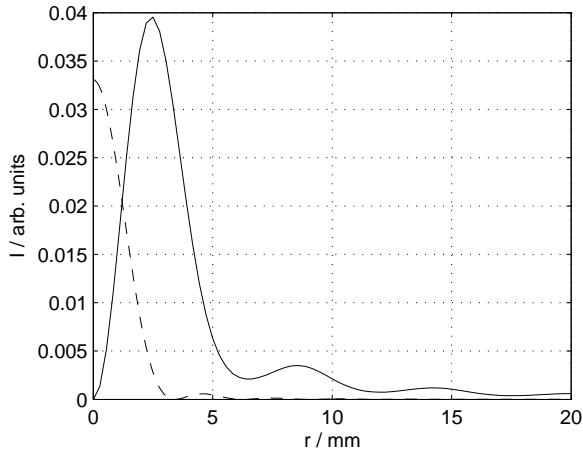
Figure 1: Point spread function of an ideal lens for the present setup, assuming a homogeneously emitting point-like source(- - -) and a transition radiation source (—)
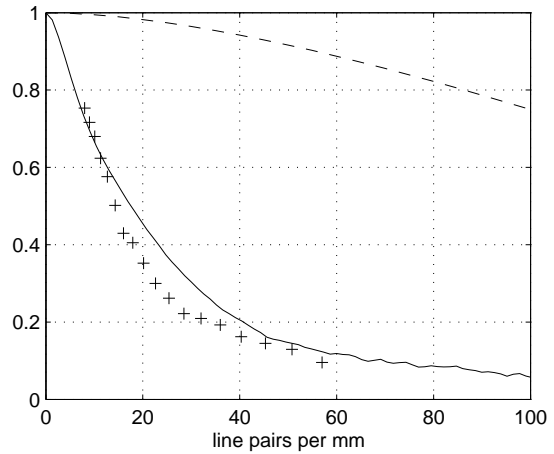

Figure 2: Resolution of the Nikkor 105mm f/2.8 lens, in 1:1 imaging mode. The CTF, determined using the USAF1951 test target (+ + +) and the MTF, determined by analyzing the image of a slanted edge (—) as well as the MTF for an ideal lens (- - -) are shown.

analog-to-digital converter (ADC) and all image post-processing. Several methods can be used to characterize the resolution of the imaging system. The point spread function can be measured directly by imaging a small pin-hole. However, only a few pixels are being used for this measurement, making the result prone to pixel errors. Furthermore, the image of the pinhole is generally not centered on a pixel, which leads to a deformation of the distribution. If it can be assumed that the imaging system is rotationally symmetric, it is better to measure the modulation transfer function and to use this to reconstruct a PSF.

Any image can be decomposed as a sum of trigonometric functions. For each of the spatial frequencies, a modulation can be defined from the intensity in the maximum I_{max} and the minimum I_{min}:

$$\mathcal{V} = \frac{I_{max} - I_{min}}{I_{max} + I_{min}}$$

The modulation transfer function (MTF) is the ratio between the contrast (modulation) of an image and the modulation of the actual object, as a function of spatial frequency:

$$MTF = \frac{\mathcal{V}_{image}}{\mathcal{V}_{object}}$$

In practice, sine-like intensity distributions are difficult to obtain, and rectangular distributions, i.e. black lines on a white background are used. The pattern contains higher harmonics in spatial frequency and; the modulation of an image of such a pattern is called contrast transfer function (CTF). Several standardized test patterns are available to determine the CTF directly, such as the USAF1951, or the ISO 12233 test pattern [6]. The latter contains also a sharp edge that is slanted by a small angle to the pixel rows, which can be used to determine the edge transfer function (ETF). Since the edge is slanted, neighboring columns are shifted by a fraction of a pixel size and the imaged step function can be recorded with sub-pixel accuracy. Aliasing

effects cannot affect the measurement as with CTF targets. The derivative of the ETF is the PSF; this method presents thus a convenient and stable way to determine the resolution of the imaging system [7].

However, image processing artifacts that result in artificial contrast enhancement a spatial frequencies not present in the original image can result in a contrast measurement that exceeds the actual performance of the system. The two methods are thus complementary.

The resolution of the present optical system has been determined with the USAF1951 target and with a slanted edge. The resulting CTF and MTF are shown in Figure 2.

DECONVOLUTION ALGORITHMS

In certain cases, the resolution of a digitally recorded image can be improved by deconvolution algorithms. The image that is recorded by the CCD is a scaled convolution of the object intensity with the point spread function of the lens. If the image were not affected by noise, it would be possible to divide the Fourier transform of the image by the MTF. However, for large spatial frequencies, the MTF is small and the division amplifies the noise significantly. In the back-transformed image, negative intensities will appear, which have no physical justification.

Iterative techniques, such as the Lucy-Richardson algorithm [8, 9], do not share these difficulties. They maximize the likelihood that the convolution of the reconstructed intensity distribution with the PSF resembles the image, while imposing constraints on the non-negativity and smoothness of the image.

To test the algorithm before applying it to beam images Figure 3 shows the reconstruction of the central part USAF1951 pattern, when imaged with the Nikkor 105mm f/2.8 lens at f/11. The accelerated, damped Lucy-
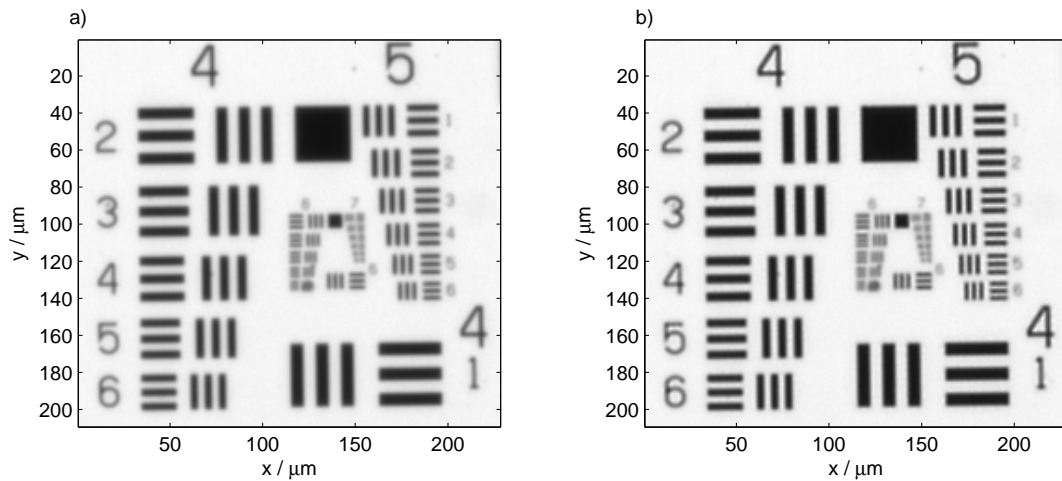
Figure 3: a) central part of the USAF1951 pattern, imaged by the Nikkor 105mm f/2.8 lens at a magnification of 1:1 and an aperture of f/11. Only the central area, measuring $0.2 \cdot 0.2\text{mm}^2$ is shown. b) The image has been deconvoluted with the accelerated, damped Lucy-Richardson algorithm, using a point spread function calculated under the assumption of a diffraction-limited lens.
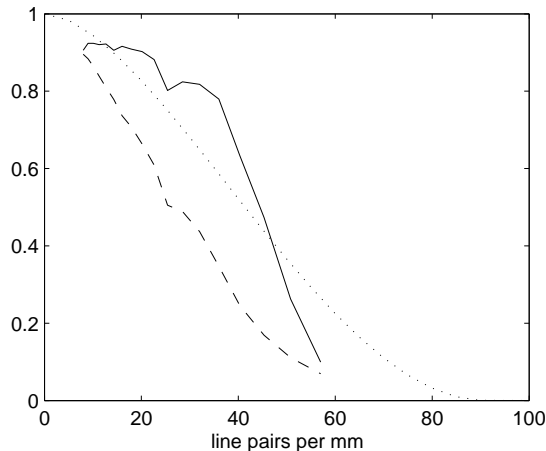

Figure 4: CTF, determined from the USAF1951 patterns shown in Figure 3. The CTF of the original image is shown as a dashed line (- - -). The application of the Lucy-Richardson algorithm increases the CTF significantly (—); it is actually higher than the MTF of an ideal lens ($\cdots$).

Richardson algorithm [10] in the MATLAB Image Processing Toolbox [11] is used. As a result, the visual sharpness of the image is significantly improved (Figure 3). The CTF of the reconstructed image is partially increased to a value that is higher than the MTF of an ideal lens (Figure 4). This is however no contradiction; with a sufficient signal-to-noise ratio and accurate knowledge of the point spread function, there is no a-priori limit to the resolution of an optical system (see also [12]).

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