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A Measurement of B$\overline{\text{B}}$ Mixing Using Di-electrons

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Abstract

We have measured $\overline{\chi}$, the probability of a transition between a B^0 and $\overline{B}^0$ state, averaged over B_s and B_d, using di-electron events. $\overline{\chi}$ is determined from the observed ratio of the number of like-sign to all electron pairs, taking into account backgrounds and processes other than mixing which could produce like-sign di-leptons. We report $\overline{\chi} = .181 \pm .061$ (stat.) $\pm .017$ (BK sys.) $\pm .021$ (MC sys.), where the systematic error is broken down into the error due to our background subtraction, and that due to the Monte Carlo simulation used in the analysis.

1 Overview

In the absence of mixing, any $b\bar{b}$ event in which both B mesons decay semi-leptonically ("direct" decays) will necessarily result in a lepton-antilepton pair, that is, a pair of leptons with opposite charge. Clearly, if one of the B's undergoes a transition to its antiparticle first, the two leptons will be of same charge. If χ is the probability of mixing between b and $\bar{b}$, that is

$$\chi = \frac{prob(b \to \overline{B}^0 \to B^0 \to \ell^+ + X)}{prob(b \to B \to \ell^\pm + X)}$$

then in the absence of other processes which could produce di-electrons, the relationship between χ and the number of like-sign (LS) and opposite-sign (OS) di-electrons is

$$X = \frac{LS}{LS + OS} = 2\overline{\chi}(1 - \overline{\chi})$$

Note that because we do not distinguish between B_d and B_s, we ultimately determine $\overline{\chi}$, the average of χ over the two states. If i refers to the type of meson (d or s), f_i is the fraction of mesons per jet, $\frac{l_i}{\langle l \rangle}$ is the ratio of the semi-leptonic branching fraction of mesons per jet with respect to the average semi-leptonic branching fraction, and χ_i is the mixing for mesons, then

$$\overline{\chi} = f_D \frac{l_D}{\langle l \rangle} \chi_D + f_S \frac{l_S}{\langle l \rangle} \chi_S$$

Other processes exist which can result in a like-sign pair of leptons. Perhaps the most significant of these is the semi-leptonic decay of one B meson accompanied by the semi-leptonic decay of a charmed meson produced in an hadronic decay of the other B meson. (These are referred to herein as "sequential" decays.) A sequential decay produces a like-sign di-lepton when either neither b

quark mixes, or both mix. The probability of a sequential decay resulting in a like-sign di-lepton is therefore

$$\overline{\chi}^2 + (1-\overline{\chi})^2$$

Initial states containing four quarks have a non-zero production rate, and contribute to like-sign lepton pairs. For the state $b\bar{b}b\bar{b}$, there are six possible combinations of two quarks which can produce a di-lepton final state: bb, $\bar{b}\bar{b}$, and four different combinations of a $b\bar{b}$ pair. The probability of a like-sign event from these are $\overline{\chi}^2 + (1-\overline{\chi})^2$ for the first two, and $\overline{\chi}(1-\overline{\chi})$ for the remaining four. The total probability for a like-sign di-lepton from this class of events is therefore

$$\frac{2\overline{\chi}(1-\overline{\chi})+1}{3}$$

For the state $b\bar{b}c\bar{c}$ there are two combinations of bc with probability $\overline{\chi}$, two of $b\bar{c}$ with probability $(1-\overline{\chi})$, and one case of $b\bar{b}$ with probability $2\overline{\chi}(1-\overline{\chi})$. The $c\bar{c}$ combination never produce a like-sign di-lepton. The contribution to like-sign pairs from $b\bar{b}c\bar{c}$ initial states is therefore

$$\frac{\overline{\chi}(1-\overline{\chi})+1}{3}$$

Combining all of above, we arrive at the single equation with which we ultimately calculate $\overline{\chi}$:

$$\mathrm{X} = \frac{2\overline{\chi}(1-\overline{\chi}) + (\overline{\chi}^2 + (1-\overline{\chi})^2)\mathrm{S} + (\frac{2\overline{\chi}(1-\overline{\chi})+1}{3})\mathrm{N}_4 + (\frac{\overline{\chi}(1-\overline{\chi})+1}{3})\mathrm{N}_{\mathrm{bc}} + \alpha\mathrm{B}}{1+\mathrm{S}+\mathrm{C}+\mathrm{N}_4+\mathrm{N}_{\mathrm{bc}}+\mathrm{B}} \tag{1}$$

where S, N_4, N_{bc}, C, and B are all ratios with respect to the number of direct $b\bar{b}$ to di-electrons predicted by the Monte Carlo, specifically, S the fraction of sequential decays, N_4, the fraction of $b\bar{b}b\bar{b}$ events, N_{bc}, the fraction of $b\bar{b}c\bar{c}$ events, C, the fraction of $c\bar{c}$ events, and B, the fraction of other events, such as "same b". The factor α is that fraction of these events which result in a like-sign electron pair. Also note that in practice, the ratio X is computed with background subtracted values of OS and LS.

2 Data Sample

In order to determine electron quality cuts appropriate to selecting low p_t di-electron events, we studied pairs of electrons arising from photon conversions in the outer wall of the VTPC. Our method for selecting these events, which differs in some ways from both the Penn approach (CDF-902) and Michael Gold's method (CDF-913), is described more fully in a separate note, CDF-1399. In short, however, it is based on the separation at closest approach (SEP) of candidate electron-track pairs and the radius of the projected conversion point (R_{conv}) of these pairs. For events selected by our modified version of CONVERT, we studied Had/EM, LSHR, E/P, track-strip match $\Delta(r\phi)$, and track-strip match $\Delta(z)$ distributions. This resulted in the following electron quality cuts [1]:

E_t	$<$	20.
E/p	$<$	1.5

Had/EM	$<$	.03
LSHR	$<$	2.0
$\Delta(r\phi)$	$<$	1.5
$\Delta(z)$	$<$	2.5

We made a fiducial cut on the central region to assure good track information, and we required a 3-d track for each electron. In addition, we required that all events satisfy the DIELECTRON_EMC_5 (EE5) trigger. When all quality cuts are applied but no trigger requirement made, over 95 percent of events which pass satisfy the EE5 trigger. From a data sample of approximately 3.7 pb^{-1} we extracted 148 LS and 545 OS di-electron pairs.

To reject electrons produced in photon conversions, we made wide cuts on the SEP and R_{conv} variables defined above. We determine SEP and R_{conv} between each electron in an event and that track of opposite sign for which the polar opening angle between them is a minimum for the event. An electron was rejected as part of a conversion pair if

$\mid$ SEP $\mid$	$<$	0.5 cm.
R_{conv}	$>$	0. cm.
R_{conv}	$<$	50. cm.

These cuts eliminated 79 events from the sample; 34 LS pairs and 45 OS pairs, leaving 114 LS and 500 OS events. Note that LS as well as OS pairs should be rejected by the conversion cut, since we test for conversions by pairing each electron separately with charged tracks.

From the OS effective mass plot (Fig. 1), two sources of non-$b\bar{b}$ backgrounds are apparent; the ψ and the Υ. One further non-visible background are the same b sequential events which always produce OS pairs of mass less than 5.0 GeV. To eliminate the ψ's and same b sequential events, we reject events with a mass less than 5.0 GeV. This selection is applied to both LS and OS sign events. To eliminate the Υ's, we reject events in the range $8.0 < m(ee) < 10.8$ GeV. The resulting sample consists of 95 LS and 235 OS events.

To reduce the number of di-electrons which are produced via the Drell-Yan mechanism, we employ an isolation-type cut on both electrons,

$$\text{Max(DEI)} \quad > \quad 2.4 \text{ GeV},$$

where DEI is defined as the difference between the E_t deposited in a cone of radius .7 around the electron and the E_t deposited in a cone of radius .4. Max() indicates that the cut is made using the electron with the greater value of DEI. A detailed discussion of DEI is presented in the Drell-Yan section under Backgrounds, however we note here that this isolation variable was selected over the standard one to facilitate the analysis of the efficiency and systematic error associated with the Drell-Yan rejection cut. The Drell-Yan rejection cut was applied to all events regardless of the signs of the electrons, and therefore both LS as well as OS pairs were rejected. In all, 17 LS and 101 OS di-electrons were rejected by the Max(DEI) cut. The fraction of OS events rejected as Drell-Yan is 85%.

After all cuts are applied, we are left with a final sample - before corrections due to inefficiencies in our cuts - of 212 events, broken down as follows:

Like-sign:	78 events
Opposite-sign:	134 events .

3 Backgrounds

The measurement of $\overline{\chi}$ presented here is, at its heart, a counting experiment. At the zeroth level, we count the number of like- and opposite-sign electron pairs and form a ratio. However, certain background processes, all independent of $\overline{\chi}$, can alter this ratio and must be accounted for. Hadrons mis-identified as electrons, or a single electron from a photon conversion which occurs in an event with another electron present can produce a fake di-electron pair. Electron pairs produced by the Drell-Yan mechanism still remaining in the sample are another source of background. These backgrounds are discussed in detail below.

3.1 Drell-Yan

Di-electrons produced by the Drell-Yan mechanism will pass our selection criteria for high-quality electrons. The standard method for either selecting or rejecting Drell-Yan pairs is to apply an isolation cut, since Drell-Yan events are characterized by little or no hadronic activity accompanying the electrons. Since it difficult to select a low mass Drell-Yan sample in data, we considered using a Monte Carlo sample to understand the Drell-Yan background. ISAJET, however, is known to simulate poorly the underlying event energy and distribution, and therefore it would be impossible to correctly estimate the number of Drell-Yan events that might still pass any cut we imposed. We could expect to extract this information from a sample of Z events, but the p_t spectrum for Z's is very different from that for the Drell-Yan events which bear on this analysis. In order to remove any bias due to this difference, therefore, we define the variable DEI, described above as the difference between the E_t deposited in the r=.7 and r=.4 cones around the electron. Specifically, we use Max(DEI), the maximum DEI value of the two electrons in each event.

For well-isolated electrons, Max(DEI) is a measure of the underlying event. We tested this variable for electrons from Z's, shown in Fig.2c), and also in a $Z \to \mu^+\mu^-$ sample. In both cases the mean amount of E_t per unit η-ϕ space extracted from the Max(DEI) distribution is in good agreement with results seen in jets events. We therefore take the Max(DEI) distribution for electrons from Z's to be representative of the shape of the same distribution in Drell-Yan events.

We assume that the data can be represented as a properly-weighted combination of two samples, one of which has behavior equivalent to that of the DY events, and the other of which is DY-free, and therefore has a DEI distribution typical of di-electrons from B decay. The Z^0 events behave like the DY events in the DEI variable, and the LS data events are taken to be DY-ree. That is,

$$\text{Data(OS)} = \alpha \cdot Z^0 + \beta \cdot \text{Data(LS)}$$

Since there is limited data, a Poisson χ^2 is used to determine the parameters. For each data bin $n_{OS} = [\alpha \cdot n_Z + \beta \cdot n_{LS}]$ where the n's are the observed number of events in each sample. The log of the likelihood w_i for each bin is

$$w_i = (n_{OS} - [\alpha \cdot n_Z + \beta \cdot n_{LS}]) + n_{OS} \cdot ln\left(\frac{\alpha \cdot n_Z + \beta \cdot n_{LS}}{n_{OS}}\right)$$

Summing the above expression over all bins, and determining the minimum yields $\alpha = 1.11 \pm .13$ assuming the error is given by a change of 2 in the log of the likelihood. The input distributions scaled by the fit parameters are given in Fig 3. The error is given by

$$\delta N_{DY} = N_{DY} \cdot \left[\left(\frac{\delta\alpha}{\alpha}\right)^2 + \left(\frac{\delta n_Z}{n_Z}\right)^2\right]^{\frac{1}{2}}$$

The total number of DY events is derived from the 92 Z^0's giving $\alpha \int n_Z = 102.1 \pm 16.0$, and the number that remain in the sample after a cut at DEI > 2.4 is derived from the 14 Z^0's giving 15.5±4.5.

It has been suggested that the DEI distribution for DY events is better modeled by the decay of the Υ. From the $\Upsilon \to \mu^+\mu^-$ events we have obtained a background subtracted DEI distribution of 203 events shown in Fig. 2d). These events have a DEI distribution which is slightly different than the Z's suggesting that processes other than Drell-Yan may be involved. However, we used this sample to provide a check on the DY determination above; instead of removing Υ's with a mass cut, we subtracted the $\Upsilon \to \mu^+\mu^-$ DEI distribution, normalized to our Υ data, from the OS DEI distribution before making the fit to $\alpha \cdot Z^0 + \beta \cdot \text{Data(LS)}$. The normalization was done as follows: THe background in the e^+e^- invariant mass distribution is seen to be flat in the region of the Υ. We observe 23 events in the range $5.2 < m(e^+e^-) < 8.0$ GeV and therefore subtract these from the 92 events in the $8.0 < m(e^+e^-) < 10.8$ GeV range. This leaves 69 Υ events over background before the DEI cut is applied, and we therefore normalize the $\Upsilon \to \mu^+\mu^-$ DEI events to this number before determining the Drell-Yan background. The fit used is then:

$$\text{Data(OS)} = \alpha \cdot Z^0 + \beta \cdot \text{Data(LS)} + (69/203) \cdot \Upsilon$$

This results in an estimate for the total number of DY events of 96.6± 14.9, in good agreement with the above procedure. Note, however, that all results, reported in this note, the Υ mass cut has been imposed.

3.2 Conversions

The source of background to $b\bar{b}$ di-electrons from photon conversions are events in which one electron is produced via b or c decay, and the other is one leg of a conversion. This is true both because typically at least one leg of a conversion will likely be buried inside hadronic (jet) activity and hence not easily identified as an electron, but also because our conversion rejection algorithm will always reject both electrons if identified when either satisfies the rejection criteria. Although we reject conversions with fairly wide cuts, the algorithm is nevertheless not completely efficient, and in addition, we can never identify conversions where the low p_t is below the threshold for tracking.

In order to estimate the fraction of conversions we miss due to the electron momentum region inaccessible to tracking, we extrapolate the momentum spectrum of the second leg of identified conversions down to $P = 0$. We selected a sample of conversions by finding for each electron ("trigger electron") in an event the charged track of opposite sign ("found electron") for which the polar angle between them is an event minimum, and then requiring $|\ SEP\ | < 0.3$ and $20. < R_{conv} < 29$. This selection is seen to be relatively free of non-conversion background by applying the same cuts to like-sign electron-track pairs. We constructed a "toy" Monte Carlo in which π^0's are converted to two electrons, and in which the shape of the momentum distribution of the higher momentum electron is forced to match that of the observed trigger electron. For the lower momentum electron, we normalized the Monte Carlo to data in the region $.5 < P < 5.$ GeV, and thereby extrapolated the found electron momentum into the region below .5 GeV, where it begins to fall off in data. Figure 4) shows the momentum distributions for both the trigger and found electrons, with the Monte Carlo prediction in dashed lines superimposed. From this Monte Carlo analysis, we estimate the fraction of conversions to be $.37 \pm .04$ due to charged tracks below the tracking threshold.

We estimate the overall inefficiency of the conversion rejection from the inefficiency in the SEP variable. We applied a $20. < R_{conv} < 29.$ cut to both like- and opposite-sign trigger electron-found electron pairs, but made no requirement on SEP. We treat the like-sign pairs accepted by this cut as background and subtract the SEP distribution for these events from that of the opposite-sign pairs, with result shown in figure 5). The fraction of events outside our conversion rejection cut of $|\ SEP\ | < 0.5$ is found to be $.09 \pm .01$ due to inefficiencies in our conversion rejection method.

To estimate the total number of conversions not eliminated by rejection cuts, we apply both efficiencies just described to the total number of events identified as conversions in our sample. When all but conversion rejection cuts have been applied to the data, 29 events remain that are eliminated only by the conversion rejection cut. However, we expect that this cut will also falsely reject real $b\bar{b}$ di-electrons. Using the same data sample we find that 18 events also make a pairing with another track of the same sign, pairs that could not be conversions. Hence the real number of conversions in this sample is 29-18 = 11 . The R_{conv} distributions for those pairs rejected as conversions, shown in figure 6) for like- and opposite-sign pairs separately, indicate that the conversion rejection cuts are over-efficient to this level. Applying the inefficiency correction is to the 11 events which have already been removed our estimate of the number of conversions left in our final data sample is therefore $11 \times (1.37 \times 1.09 - 1) = 5.3 \pm 3.3$ events. The total inefficiency is $.49 \pm .05$. These events will be equally distributed between like- and opposite-sign di-electrons.

3.3 Hadrons Mis-identified as Electrons

Although our electron quality cuts are tight, the number of hadrons, or more specifically pions is so large that there is non-zero probability for an event with one good electron - from B decay or not

- to include a pion with all quality variables within our electron cuts. The most likely scenarios for a pion faking an electron are the overlap of a photon with a charged pion and the charge exchange scattering of a pion early in the calorimeter.

The background due to fakes is known to be large in a selection of single electron events. However, our requirement that there be two objects in each events satisfying electron cuts will significantly reduce this background. (The rate at which pions fake electrons is independent of this requirement, but the fraction of events containing a fake will be smaller.) In order to estimate the number of events in our sample attributable to fakes, then, we needed to study an event sample in which the strength of the two-electron requirement is in effect, but which does not eliminate the background entirely. To this end, we selected a sample of single electrons with tight cuts on one electron, and all tight cuts except the $\Delta(r\phi)$ strip match and Had/EM cuts, which were turned off, on the other. By a method explained below, we extracted from these events a sample enriched in fake electrons, and compared them to electrons from our ψ events.

Although the fakes which pass our cuts will necessarily have a good $\Delta(r\phi)$ strip match, we would expect many more lower-quality fakes to have $\Delta(r\phi)$ out in the wings of that distribution. This will be particularly true for overlaps. Based on this, we studied the shape of the Had/EM distribution in the event selection described above as a function of successively narrower wings in $\Delta(r\phi)$, that is, $|\Delta(r\phi)| > 1.$, $|\Delta(r\phi)| > 2.$, etc. In order to get a truer measure of Had/EM shape, we did not include electrons with more than one charged track pointing to the cluster. Figure 7) shows the Had/EM distributions for wing cuts in $\Delta(r\phi)$ from 1. cm to 6. cm. Excluding the first bin, the shape of Had/EM is seen to be constant in the $|\Delta(r\phi)| > 4.$ region and beyond. We interpret this to mean that the fraction of real electrons has essentially vanished for values of $|\Delta(r\phi)| > 4.$ and larger, and we can therefore consider these events to a be a clean sample of fake electrons.

In order to determine the number of fake di-electrons which pass all cuts and are therefore not removed from our final data sample, we use both the sample of fakes together with our known good di-electrons from ψ's. The Had/EM distribution in the data sample is a linear combination of the Had/EM distribution for fakes and that of real electrons. In addition, the shape of the distribution for the fakes is nearly linear; we therefore divide each of the two distributions into the regions above and below our Had/EM < .03 cut, and determine the multiplicative coefficients required to obtain the number of events in observed in data in these two regions. This procedure is sensitive only to the shape but not the absolute level of the distribution for fake electrons. (In this case "data" refers to a selection of di-electrons where tight cuts were applied to one electron, and all tight cuts except for Had/EM were applied to the second electron.) The numbers of events are as follows:

Sample	Had/EM < .03	Had/EM ≥ .03
Fakes	87	133
ψ electrons	83	11
Di-electron	212	66

The linear combination of fakes and electrons is given by the set of simultaneous equations

$$d_i = \alpha f_i + \beta \psi_i$$

$$d_o = \alpha f_o + \beta \psi_o$$

where d, f, and ψ represent numbers of events from di-electrons, fakes, and ψ electrons respectively, and "i" and "o" refer to the regions inside and outside our Had/EM cut respectively. With our measured values the previous equations become:

$$212 = \alpha \cdot 87 + \beta \cdot 83$$

$$66 = \alpha \cdot 133 + \beta \cdot 11$$

The total number of fake events F can be obtained by solving the previous equation for α

$$\mathrm{F} = \alpha \cdot \mathrm{f_i} = \frac{\mathrm{N}}{\mathrm{D}} \cdot \mathrm{f_i}$$

where

$$\mathrm{N} = \mathrm{d_i}\psi_\mathrm{o} - \mathrm{d_o}\psi_\mathrm{i}$$

$$\mathrm{D} = \mathrm{f_i}\psi_\mathrm{o} - \mathrm{f_o}\psi_\mathrm{i}$$

Differentiating with respect to each of the variables, and assuming the error in each variable is the square root of the number of events

$$(\delta\mathrm{F})^2 = \frac{1}{\mathrm{D}^2}[(\psi_\mathrm{o}\mathrm{f_i})^2\mathrm{d_i} + (\psi_\mathrm{i}\mathrm{f_i})^2\mathrm{d_o} + (\mathrm{N} - \mathrm{F}\psi_\mathrm{o})^2\mathrm{f_i} + (\mathrm{F}\psi_\mathrm{i})^2\mathrm{f_o} + (-\mathrm{d_o}\mathrm{f_i} + \mathrm{F}\mathrm{f_o})^2\psi_\mathrm{i} + (\mathrm{d_i}\mathrm{f_i} - \mathrm{F}\mathrm{f_i})^2\psi_\mathrm{o}]$$

The number of fake events which pass our cuts is therefore $\alpha \mathrm{f_i} = 27.2$, or 9.8 percent of the sample of 212+66=278 events. Thus $\mathrm{F} \pm \delta\mathrm{F} = 27.2 \pm 9.2$ and rescaling to the actual number of events (212/278) gives $\mathrm{F} \pm \delta\mathrm{F} = 20.7 \pm 7.0$. The events will be equally distributed between like- and opposite-sign di-electrons.

4 Monte Carlo Sample

A Monte Carlo sample of di-electron events is used to estimate the fraction of certain contributions to the LS to all events ratio that do not arise from mixing. It should be noted from the method described in the Overview section that since each contribution enters into the computation as a ratio with respect to the number of direct $b\bar{b}$ decays, it is not necessary that the luminosities of the data and Monte Carlo samples be the same. We generated a 2.2 pb^{-1} event sample using ISALEP combined with ISAJET 6.21. Events were simulated in the detector and then reconstructed using CDFSIM.

The 2.2 pb^{-1} were generated with ISALEP by requiring two-jet events, with the hard interaction $\mathrm{p_t}$ divided into four ranges. For all $b\bar{b}$ or $c\bar{c}$ processes, we generated in the ranges 7.5 GeV $< \mathrm{p_t} <$ 20 GeV, 20 GeV $< \mathrm{p_t} <$ 40 GeV and 40 GeV $< \mathrm{p_t} <$ 160 GeV. In addition, we also generated $c\bar{c}$ events only in the range 15 GeV $< \mathrm{p_t} <$ 20 GeV. In all cases, parton evolution was run 10 times for each hard interaction, and fragmentation was run 10 times per accepted pair of partons. We required the b or c quark to have at least 7.5 GeV $\mathrm{p_t}$, and each final state lepton was required to have $\mathrm{p_t} >$ 4 GeV for the fragmentation results to be accepted.

The Monte Carlo events were simulated and reconstructed, and required to pass all cuts applied to the data. A simulated DIELECTRON_5 trigger requirement was made as well. From the 2.2 pb^{-1} we obtained 206 events.

The primary information obtained from the Monte Carlo is the ratio S:

$$S = \frac{\text{Sequential b decays}}{\text{Direct b decays}} = \frac{g_s}{g_d \cdot r} \tag{2}$$

where g_s and g_d are the number of generated sequential and direct decays respectively. The parameter r is the true semi-leptonic branching ratio compared to the value of 12% used during generation. The best value for this number comes from CLEO = $10.1 \pm 0.8\%$, ARGUS = $10.3 \pm 0.7\%$, and CUSB = $11.1 \pm 0.7\%$. This gives a weighted average of $10.5 \pm 0.4\%$ where we have enlarged the errors to reflect the inconsistency of the measurements. Hence $r = .875 \pm .05$ will be used to correct the lepton decay rate. The numbers of generated and corrected events for each category are given below:

Event Type	N_{events}	Corrected N_{events}
Direct Di-electron	112	85.8
Sequential	25	21.9
$b\bar{b}b\bar{b}$	4	3.1
$b\bar{b}c\bar{c}$	4	3.5
$c\bar{c}$	2.0	2.0
Other	7	7.0

We note that normalizing the total corrected number of Monte Carlo events to the luminosity of the data sample yields 207 events. This compares well with the number of events observed. (Although we note that this number cannot be compared directly to the 212 data events, since certain backgrounds not present in the MC are included in that number.)

The most critical number we extract from the Monte Carlo is the ratio of sequential to direct decays of b quarks. This is true first because of the relatively high rate of sequential decays, but also because sequentials contribute only to the number of LS events in the absence of mixing, and therefore have a significant effect on the observed fraction X. We therefore attempted to substantiate the value of S above, by studying the $\Delta(\phi)$ distributions for like- and opposite-sign di-electron pairs in data. This study is sufficiently complicated that we present it separately, in memo entitled "Monte Carlo Independent Determination of $\overline{\chi}$ "(CDF1375). We state here the conclusion of that study, that the ratio of sequential to direct decays predicted by the Monte Carlo agrees well with the data.

The method for determining $\overline{\chi}$ as outlined above cannot succeed unless one of two conditions is met. Either the efficiency, as a function of p_t, is the same for detecting electrons in both the Monte Carlo and data, or the p_t spectra of the electrons, and most importantly, the spectra of the second or lower p_t electrons, must be the same in both Monte Carlo and data. If neither was true, then we could not, for example, use the Monte Carlo ratio of sequential to direct decays to predict that ratio in data. Figure 8) shows the E_t spectra from the Monte Carlo for the second electron in both direct and sequential decays. Within statistics, the shape of the spectra are seen to be the same in the p_t region of interest.

On the other hand, we note that the b cross-section - a number not well known - need not be the correct in the Monte Carlo for the method we employ to be successful. Again, since we rely on ratios internal to the Monte Carlo sample, the b cross-section drops out everywhere. The method we employ is sensitive to branching fractions, most directly the b semi-leptonic branching fractions, but not to cross-sections.

5 Results

The experimentally measured R, the ratio of like-sign to opposite-sign di-electrons, must be corrected for the backgrounds discussed previously. One-half of the conversion and fake electron background events are subtracted from the LS subsample and an equal number from the OS, while all of the Drell-Yan events are removed from the opposite-sign subsample:

$$R' = \frac{L'}{O'} = \frac{L - \frac{F}{2} - \frac{C}{2}}{L - \frac{F}{2} - \frac{C}{2} - DY} \tag{3}$$

where F is the number of fakes, C is the number of conversions, and DY is the number of Drell-Yan events. For our data,

$$R' = 64.4/105.0 = .613 \pm .087$$

where the error in the ratio is statistical. The errors on the background subtractions are treated as systematic errors in our determination of $\overline{\chi}$.

A related parameter is X':

$$X' = \frac{L'}{T'} = \frac{L'}{L' + O'} == \frac{L - \frac{F}{2} - \frac{C}{2}}{T - F - C - DY} \tag{4}$$

For our data, $X' = .380 \pm .037$ where the statistical error in X' is given by

$$\Delta X' = X' \sqrt{\left(\frac{1 - X'}{L'}\right)^2 (\delta L)^2 + \left(\frac{1}{T'}\right)^2 (\delta O)^2}$$

The value of $\overline{\chi}$ is derived by re-writing equation 1 as a quadratic of the form:

$$a\overline{\chi}^2 + b\overline{\chi} + c - X' = 0 \tag{5}$$

where

$$a = (2\mathrm{S} - \frac{2}{3}\mathrm{N}_4 - \frac{1}{3}\mathrm{N}_{bc} - 2) \cdot f$$

$$b = -a$$

$$c = (\mathrm{S} + \frac{1}{3}\mathrm{N}_4 + \frac{1}{3}\mathrm{N}_{bc} + \alpha \mathrm{B} - X') \cdot f$$

and

$$f = \frac{1}{(1 + \mathrm{S} + \mathrm{C} + \mathrm{N}_4 + \mathrm{N}_{bc} + \mathrm{B})}$$

The sensitivity of the measured values X' and S versus DEI is shown in Fig. 10. Both X' and S are constant beyond DEI=2.4, and this point is chosen to measure the mixing. The variables are defined in the Overview and their values taken from the Monte Carlo. The result is $\overline{\chi} = .181$. The statistical error is determined by the error on X'

$$\Delta\overline{\chi} = \frac{\partial\overline{\chi}}{\partial X'}\Delta X' \tag{6}$$

where

$$\frac{\partial\overline{\chi}}{\partial X'} = \frac{1}{a(2\overline{\chi}-1)}$$

resulting in a statistical uncertainty on $\overline{\chi}$ of .061, hence

$$\overline{\chi} = .181 \pm .061(\text{stat.})$$

6 Systematic Errors

The systematic errors are assumed to arise from two sources, the uncertainty in the number of background events remaining in the data sample after cuts have been made and the error associated with distribution of Monte Carlo events.

6.1 Systematic Errors Due to Background Subtraction

The error in X' in equation 4 is given by:

$$(\delta X')^2 = \left[\left(\frac{\partial X'}{\partial F}\right)^2 \delta F^2 + \left(\frac{\partial X'}{\partial C}\right)^2 \delta C^2 + \left(\frac{\partial X'}{\partial DY}\right)^2 \delta(DY)^2 + \left(\frac{\partial X'}{\partial \Upsilon}\right)^2 \delta(\Upsilon)^2\right]$$

where

$$\frac{\partial X'}{\partial F} = \frac{\partial X'}{\partial C} = \frac{(X'-\frac{1}{2})}{T'}$$

and

$$\frac{\partial X'}{\partial DY} = \frac{\partial X'}{\partial \Upsilon} = \frac{X'}{T'}$$

where the errors on individual terms are described in the Backgrounds section.

Using equation 6, the systematic error in X' can be converted into an error in $\overline{\chi}$. Combining all these results gives a total systematic error in $\overline{\chi}$ of .017.

6.2 Systematic Errors Due to Monte Carlo Uncertainties

The error in the Monte Carlo is presumed to arise primarily from the statistical fluctuations in the distribution of events generated by ISALEP, and the uncertainty in the leptonic branching ratio. The error in $\overline{\chi}$ arises primarily from $\frac{\partial\overline{\chi}}{\partial S}\Delta S$, since the errors due to other processes are small. Solving equation 5 for $\overline{\chi}$ and differentiating with respect to S yields:

$$\frac{\partial\overline{\chi}}{\partial S} = \frac{1}{a(2\overline{\chi}-1)}\left(-\frac{\partial c}{\partial S} + \left[\frac{(c-X')}{a}\right]\frac{\partial a}{\partial S}\right)$$

where

$$\frac{\partial c}{\partial S} = (1-c)\cdot f$$
$$\frac{\partial a}{\partial S} = (2-a)\cdot f$$

The dominant error is the uncertainty of the semi-leptonic branching ratio, which when divided by the nominal 12% used in ISALEP yields r=.875 $\pm$ .05. The error in S is derived from this error combined with the statistical errors associated with the other terms in equation 2. Our MC final MC sample contains 112 direct decays and 25 sequential decays, yielding S = .255 $\pm$ .058. This results in a systematic error in $\overline{\chi}$ due to the Monte Carlo to be $\Delta\overline{\chi}$ = .021. In Fig. 11) we show the relationship between R' and $\overline{\chi}$ and its dependence on the value of S.

7 Data Summary

DATA

Like Sign Events	78
Opposite-Sign Events	134
Conversions Rejected - Correct Pairing	29
Conversions Rejected - Wrong Pairing	18
Events Outside HAD/EM Cut	66

BACKGROUNDS

Conversions	5.3 ± 3.3
Mis-Identified Hadrons	20.7 ± 7.0
Drell-Yan Events	15.4 ± 4.6

8 Conclusion

Combining all the errors on $\overline{\chi}$, then yields our final value

$$\overline{\chi} = .181 \pm .061(\text{stat.}) \pm .017(\text{BK}) \pm .021(\text{MC})$$

where BK and MC are the systematic errors due to background subtraction and Monte Carlo respectively.

The memo supercedes CDF1314/Version 2 and CDF1374.

References

[1] In a previously reported version of this analysis, a cut on Border Tower E_t was used in event

selection. The distribution of this variable for our events was incorrectly reported; the correct distribution is shown in figure 12). The primary effect of the Border Tower cut was found to be a reduction in di-electrons from $b\bar{b}$ and this cut was therefore eliminated from the analysis.

9 Figures

1) e^+e- effective mass.

2a) MAX(DEI) for opposite-sign events.

2b) MAX(DEI) for like-sign events.

2c) Max(DEI) for $Z \rightarrow e^+e-$.

2d) Max(DEI) for $\Upsilon \rightarrow \mu^+\mu-$.

3) Fit of DEI distribution for Drell-Yan events

4) Momentum distribution of "trigger" and "found" electrons from conversions.

5) Separation distribution for conversions.

6) Radius of Conversion for events rejected as Conversions - LS and OS

7) HAD/EM for various r$\Delta(\phi)$ selections.

8) E_t of 2nd Monte Carlo electron.

9) Opening angle ϕ between di-electrons.

10) X' and S versus DEI

11) R' vs. $\overline{\chi}$

12) Border Tower E_t

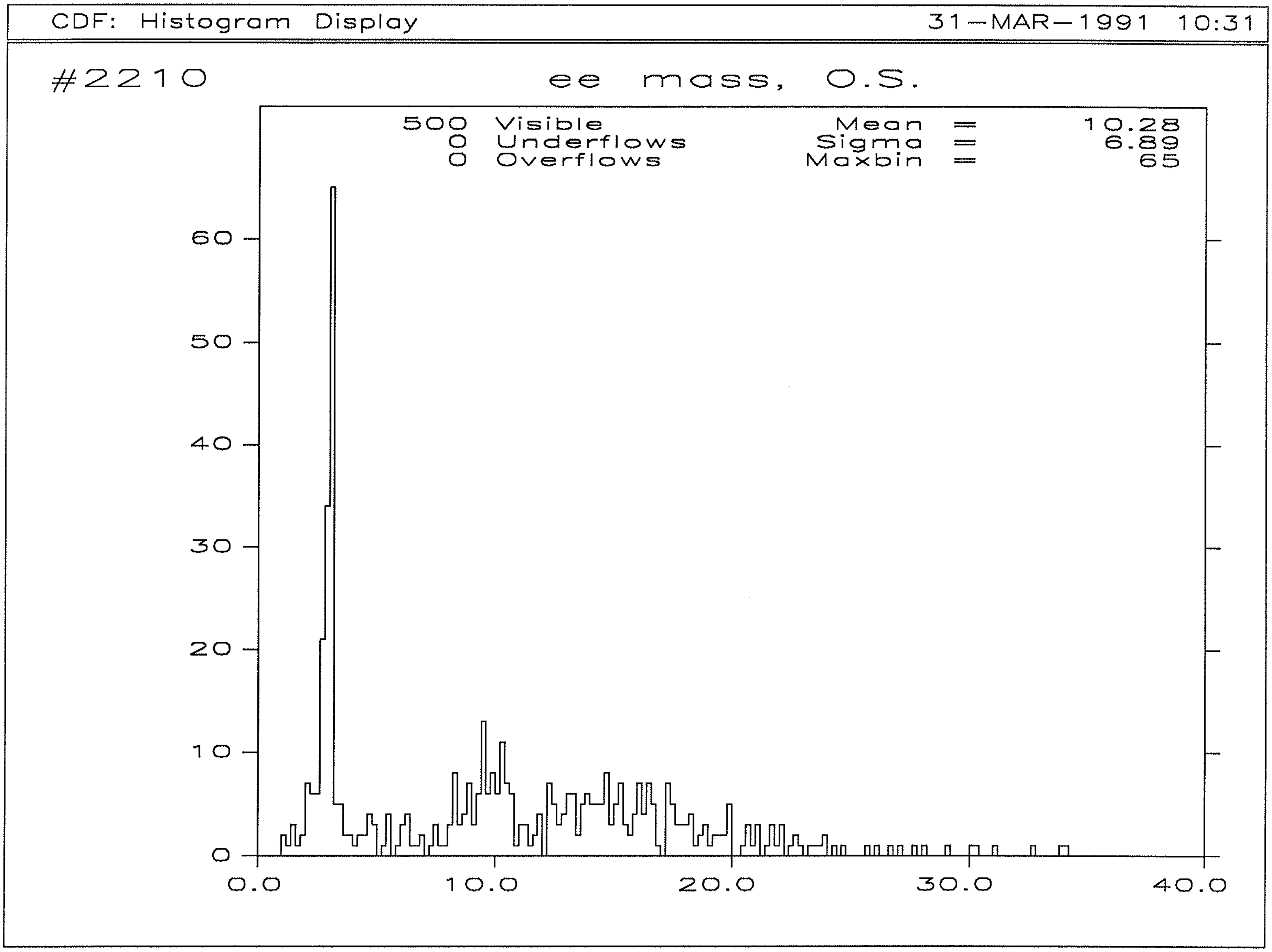

CDF: Histogram Display
31-MAR-1991 10:31
#2210
ee mass, O.S.
500 Visible
0 Underflows
0 Overflows
Mean = 10.28
Sigma = 6.89
Maxbin = 65
60
50
40
30
20
10
0
0.0
10.0
20.0
30.0
40.0

Fig. 1

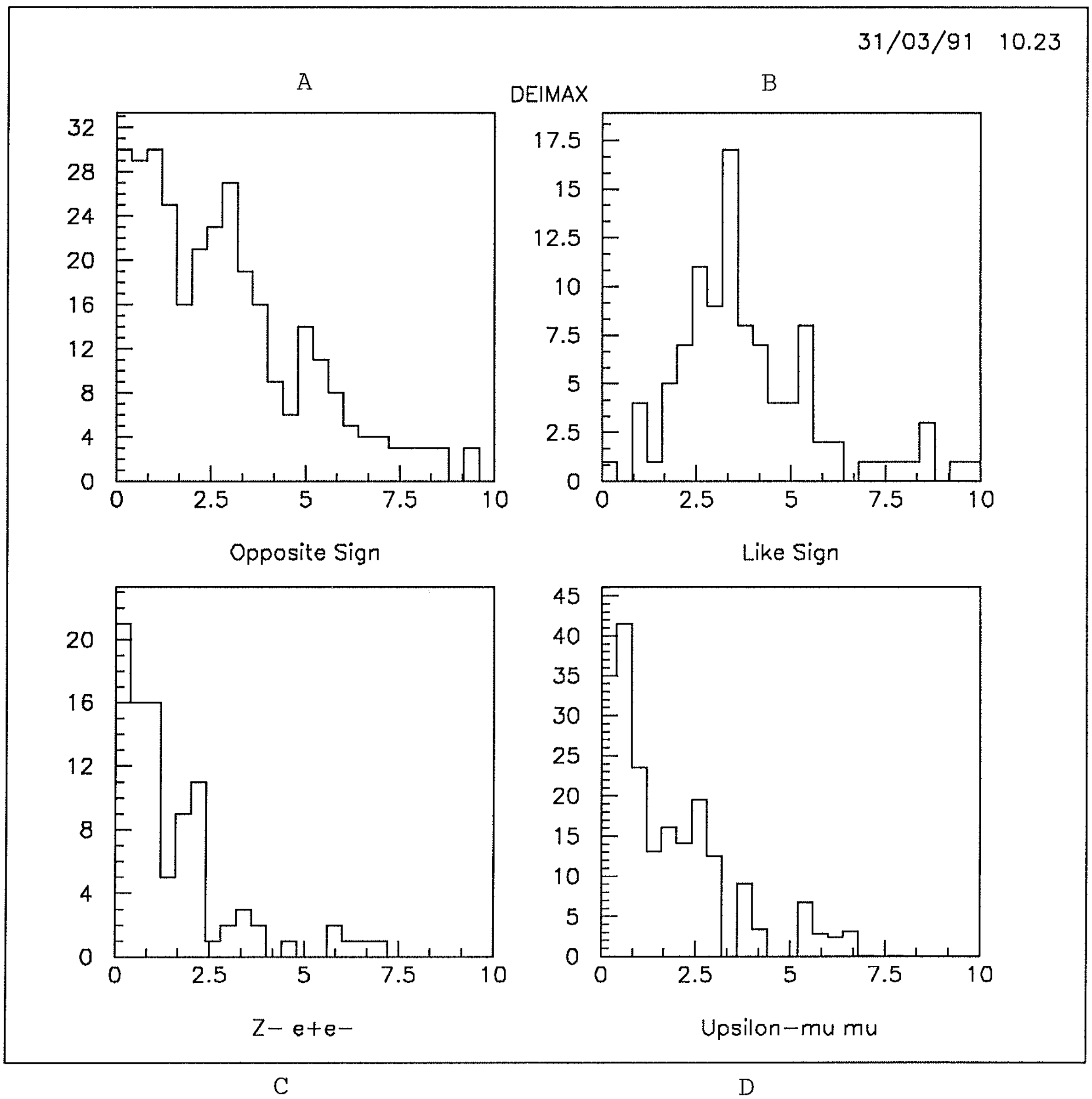

31/03/91 10.23
A
DEIMAX
B
Opposite Sign
Like Sign
Z- e+e-
Upsilon-mu mu
C
D

Fig 2

$\alpha = 1.11 \pm .013$

OS DATA

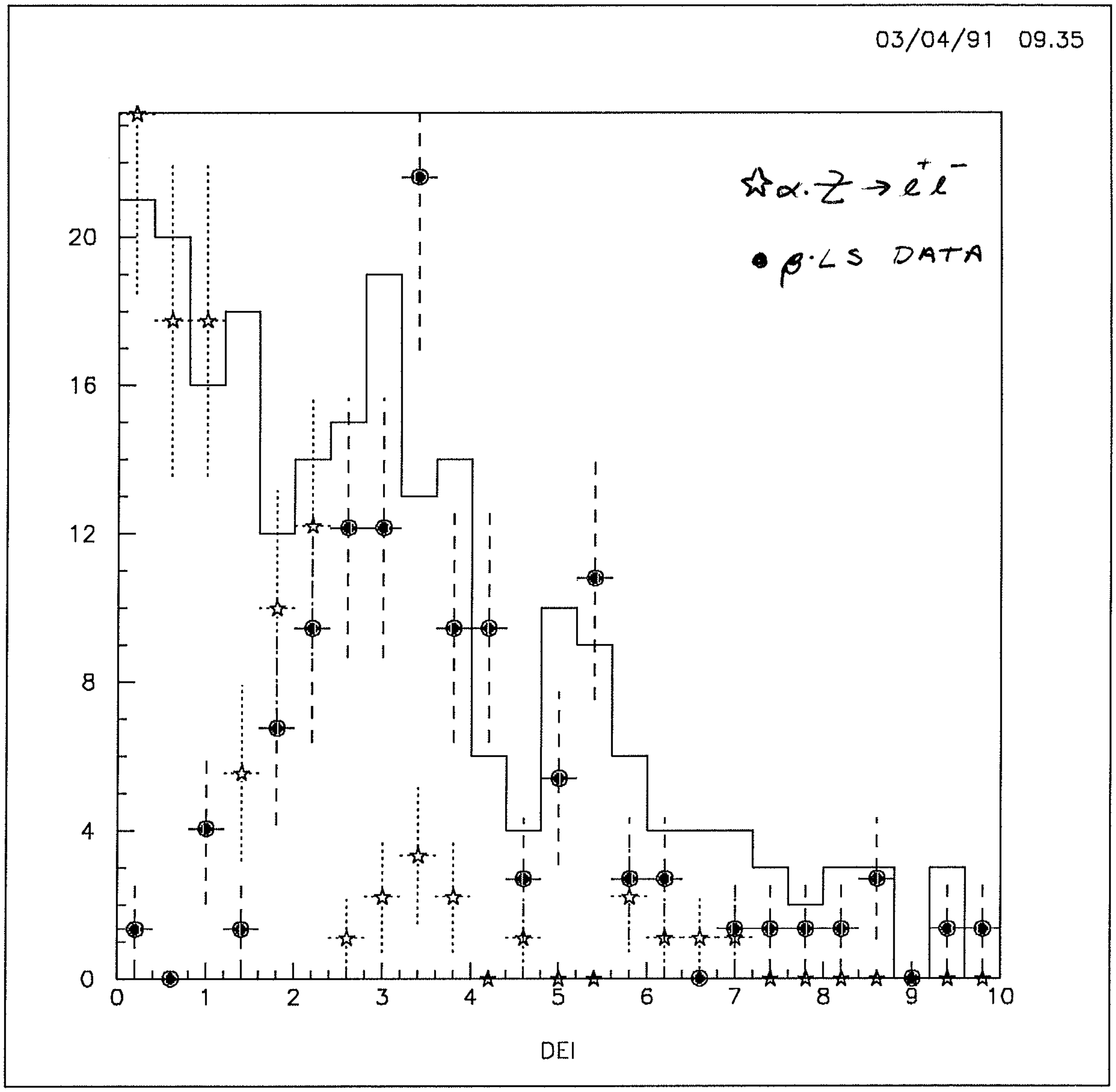


Fig. 3

NORM $0.5 < P_f < 5.0$

	PRED	ACTUAL	EXEESS
0.→.5	133	18	115
.5→5.0	268	268	
		286	

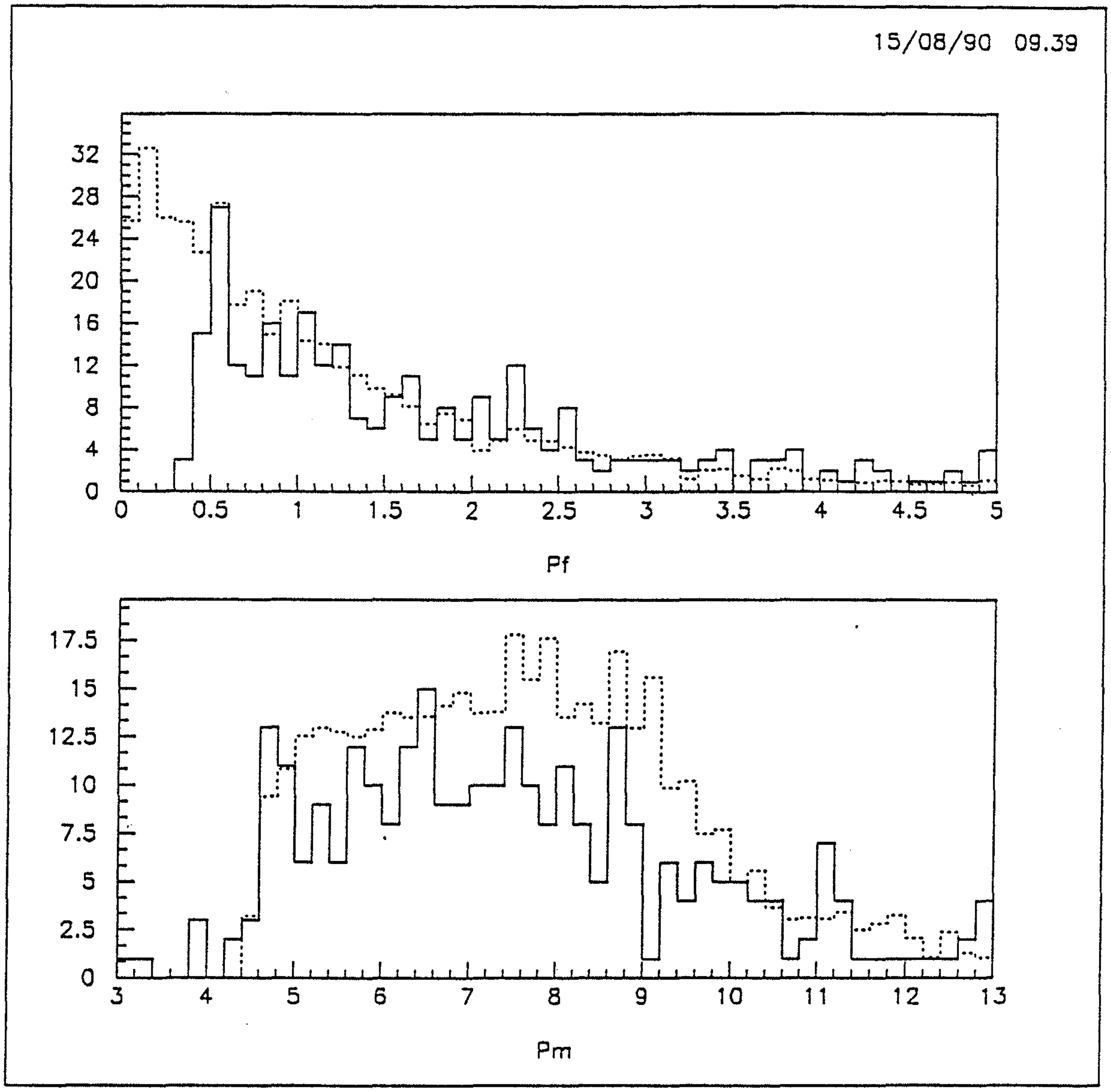


Fig. 4

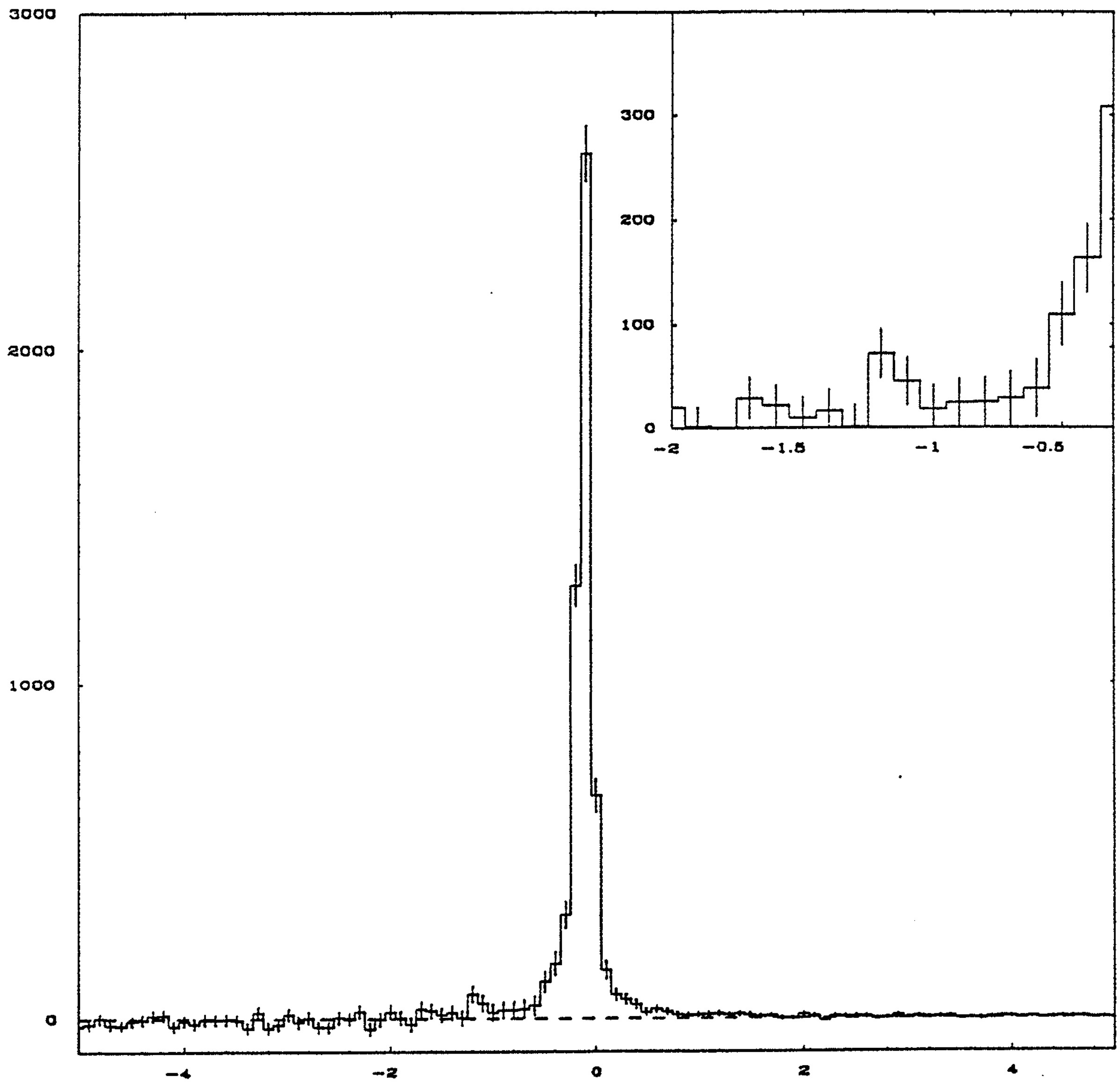
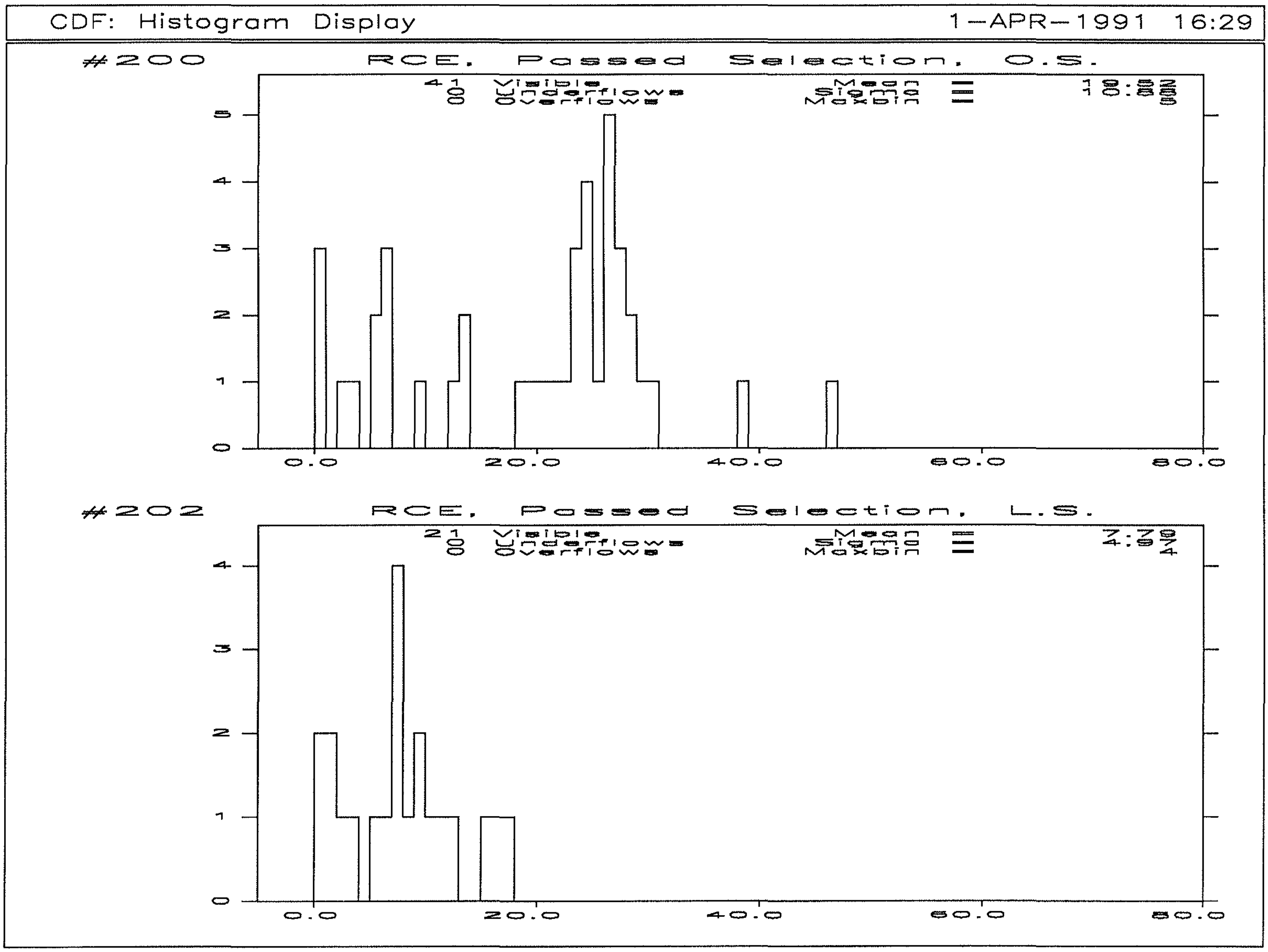


Fig. 5



CDF: Histogram Display
1-APR-1991 16:29
#200
RCE, Passed Selection, O.S.
41 Visible
0 Underflows
0 Overflows
Mean = 19.52
Sigma = 10.88
Maxbin = 5
0
1
2
3
4
5
0.0
20.0
40.0
60.0
80.0
#202
RCE, Passed Selection, L.S.
21 Visible
0 Underflows
0 Overflows
Mean = 7.79
Sigma = 4.97
Maxbin = 4
0
1
2
3
4
0.0
20.0
40.0
60.0
80.0

Fig. 6

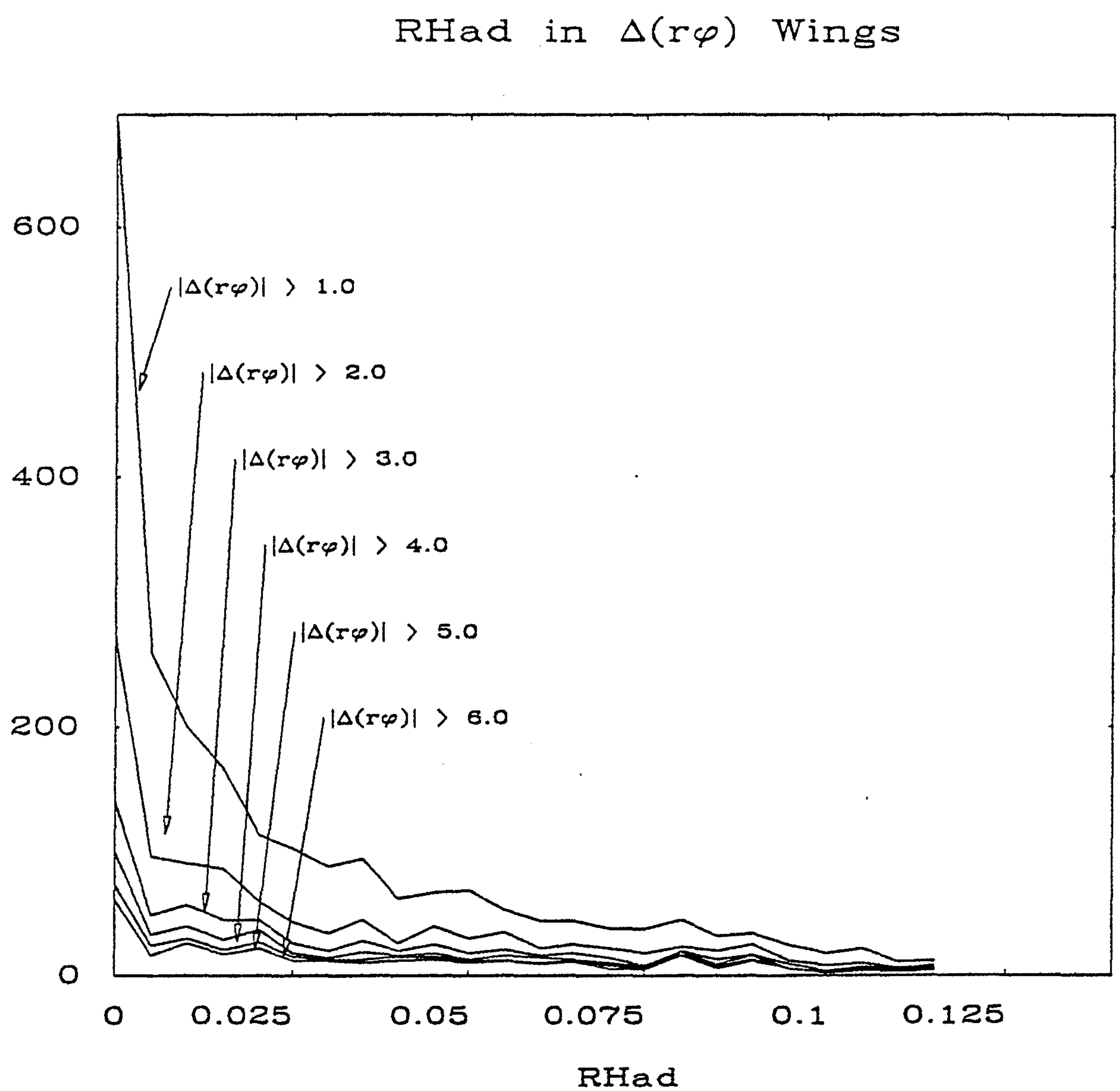

RHad in Δ(rφ) Wings
600
400
200
0
|Δ(rφ)| > 1.0
|Δ(rφ)| > 2.0
|Δ(rφ)| > 3.0
|Δ(rφ)| > 4.0
|Δ(rφ)| > 5.0
|Δ(rφ)| > 6.0
0
0.025
0.05
0.075
0.1
0.125
RHad

Fig. 7

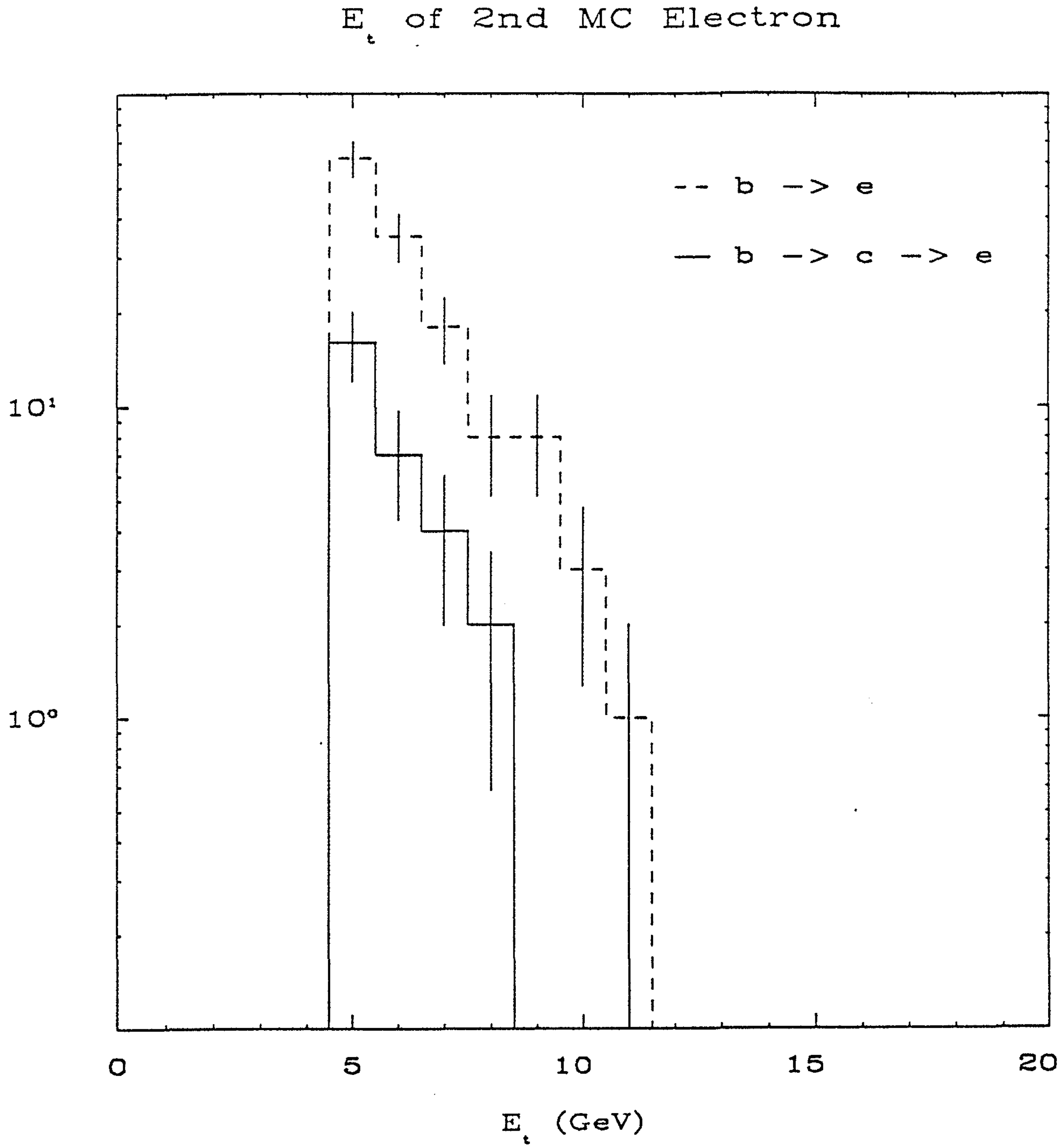
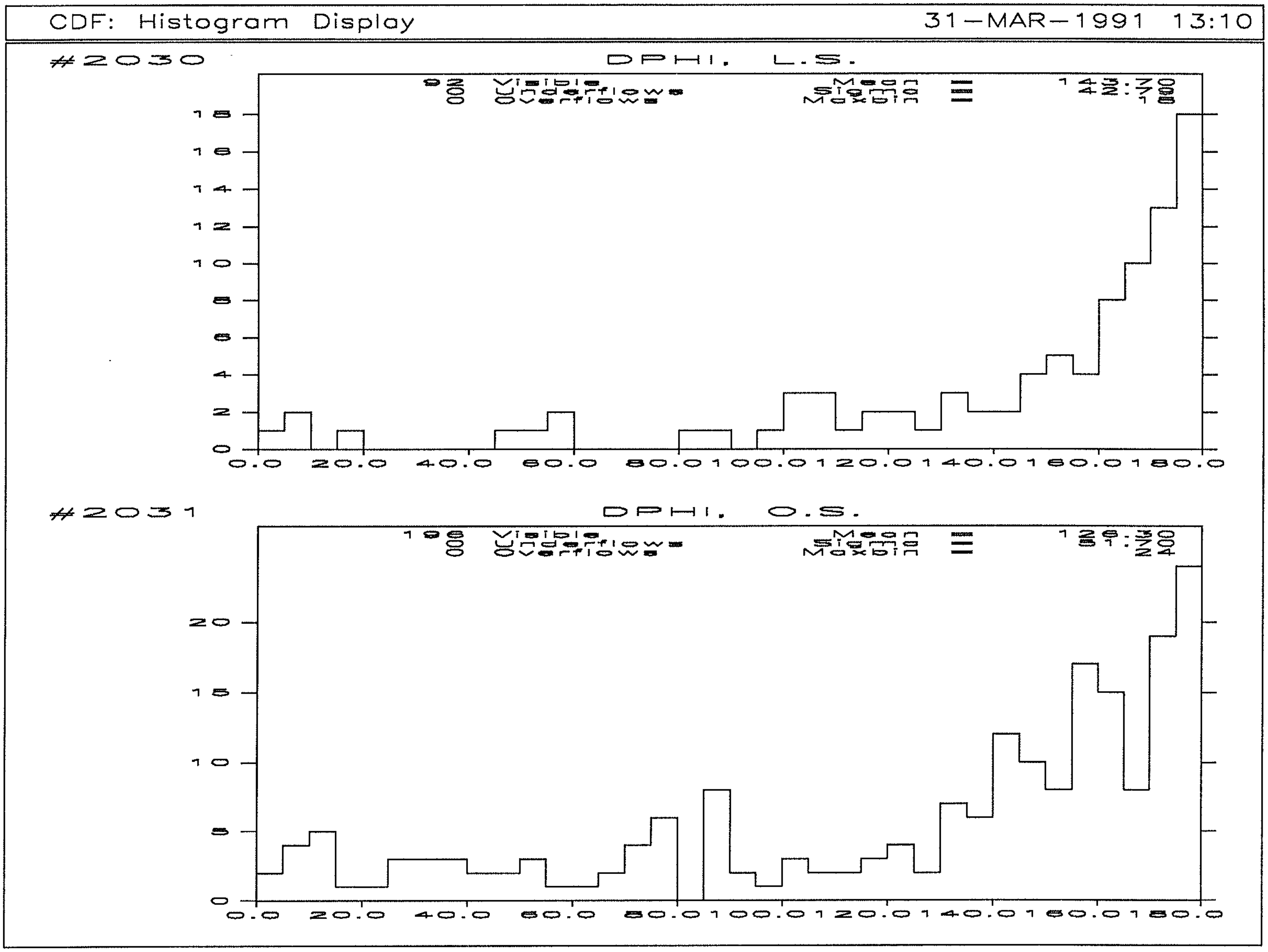


Fig. 8



CDF: Histogram Display
31-MAR-1991 13:10
#2030
DPHI, L.S.
#2031
DPHI, O.S.

Fig. 9

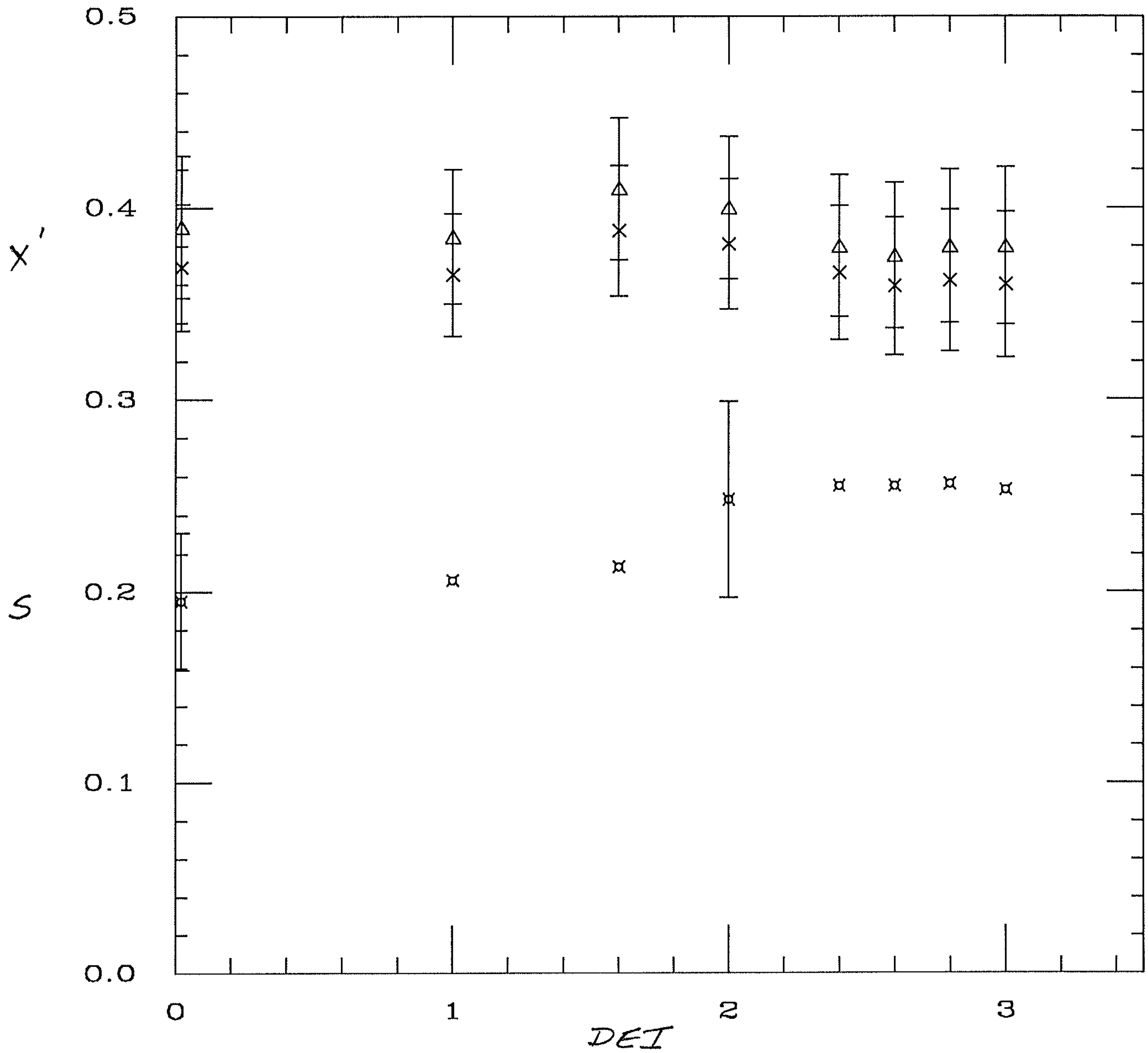


Fig. 10

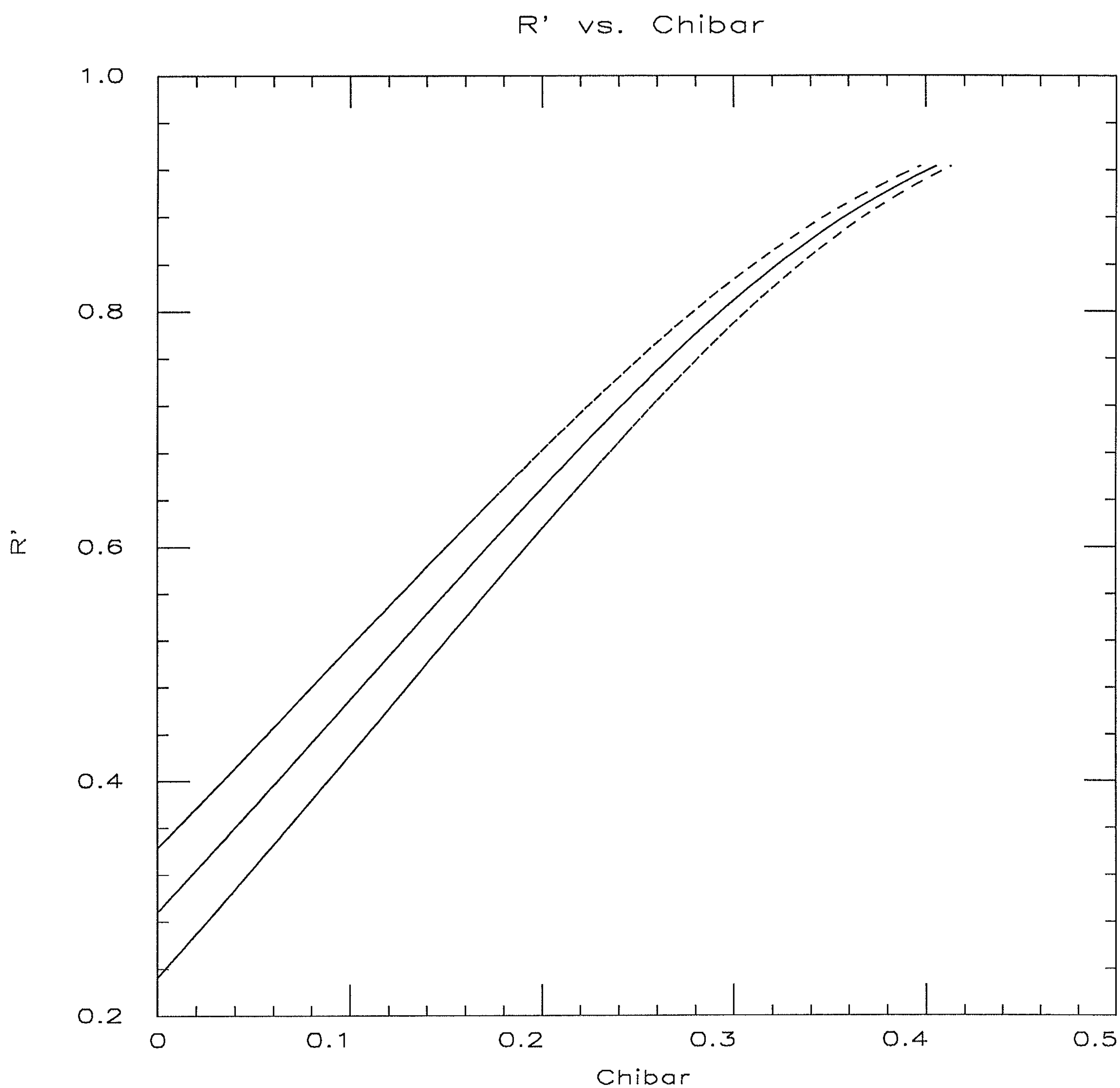


Fig. 11

CONVERSION ELECTRONS

$p_t < 15$ GeV

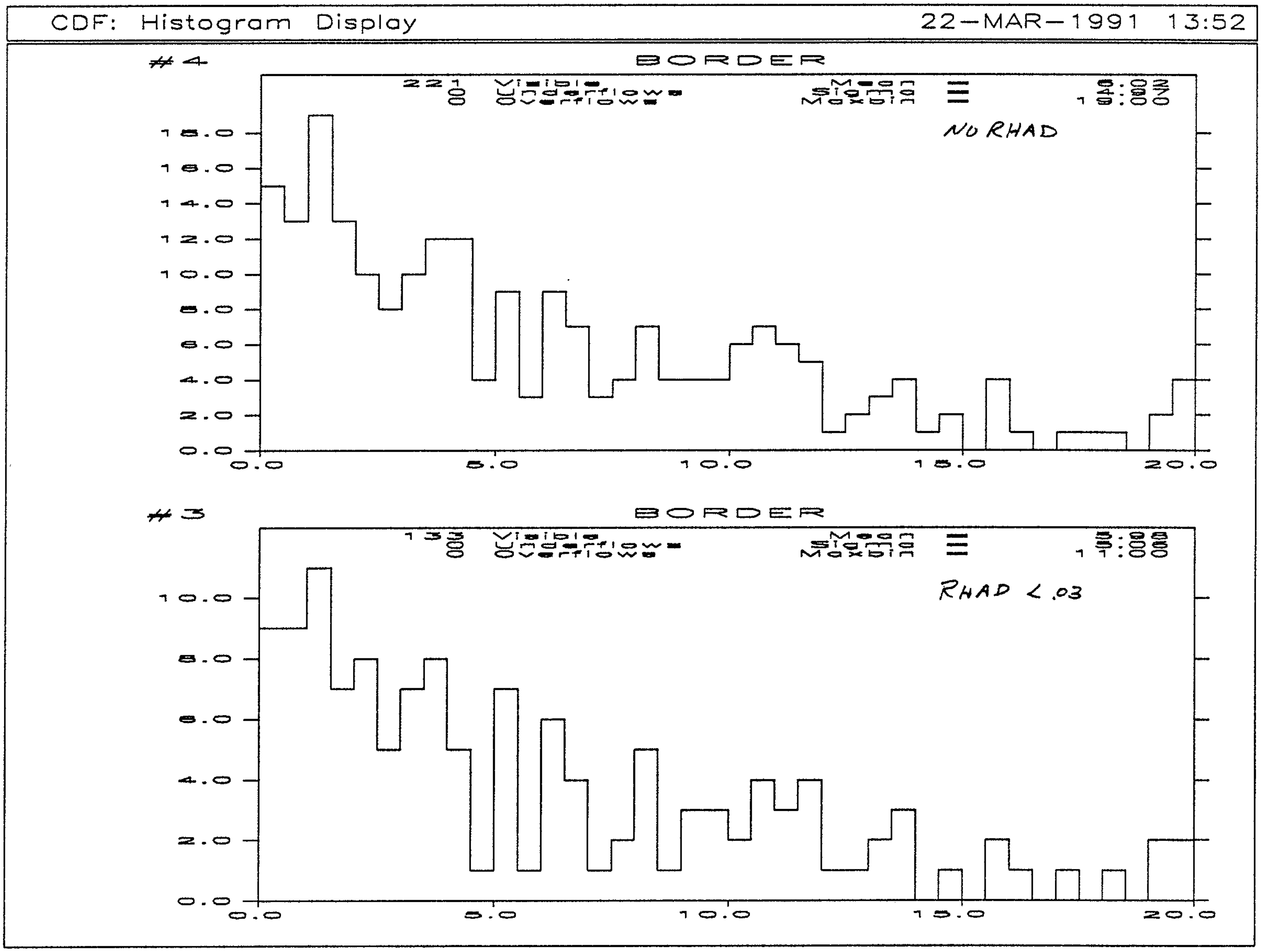


Fig. 12