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Effect of torsion on the radiation fields in curved spacetime

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ABSTRACT

The torsion-free connection is one of the important assumptions in Einstein's General theory of Relativity which has not been verified so far from experimental observation. In this article, the effect of torsion on the on-shell action of radiation fields in curved spacetime is reported in order to show certain consequences and possible ways to probe torsion in curved spacetime. In order to describe non-vanishing torsion, we mostly use contorsion tensor in connection which can be written in the combination of torsion tensor and vice versa. We have discussed how does the field equation coming from the minimization of action *with respect to* torsion changes the matter part of the on-shell action of radiation through substitution of the spin tensor. We have also studied here the effect of torsion on the scalar and vector radiation field in curved spacetime. It is shown that the presence of torsion leads to extra polarizations in Gravitational waves or radiations other than the usual two polarizations already present in General Relativity. Further, it is also shown that in the presence of torsion, there exists a non-trivial vacuum configuration that is different from the trivial vacuum in the absence of coupling with torsion. It is also found that in a generic theory of vector field, the presence of torsion breaks the $U(1)$ gauge invariance just like the presence of mass shows the violation of gauge invariance. The observational consequences of these results are also discussed.

1. Introduction

One of the crucial assumptions that were made during the development of General Relativity (GR) is the connection would be torsion-free. This assumption together with the metric compatibility [1] assumption defines the connections used in GR uniquely, known as the Christoffel symbols [2]. These two assumptions made a lot of mathematical simplifications in the theory. However, still a lot of people in the community do not follow the assumption of torsion-free connection [3–5] to see what are the possible effects that torsion can give rise to in terms of the dynamics of the system in GR [6,7].

Stress-energy tensor defined in GR is symmetric from its definition [8] $T^{\mu\nu} = -\frac{2}{\sqrt{-g}}\frac{\delta S_{\text{matter}}}{\delta g_{\mu\nu}}$ and it is sufficient to provide us the solution of metric tensor from the Einstein's equation. However, that information is not sufficient once we include the torsion [9,10] in the geometry because then, the minimization of action also implies minimization of the Einstein-Hilbert action *with respect to* the torsion tensor which is independent of the metric tensor. The metric compatibility is assumed in all stages of calculation.

Working in a theory of gravity with torsion [4], the first question that comes naturally is that would it be possible to determine uniquely all the components of torsion tensor in some way? The next question would be how do the equations of motion or Euler-Lagrange equations from minimizing the Einstein-Hilbert action [1] change if we take into account the effect of torsion in the theory.

Further one can ask, how does the minimization of the Einstein-Hilbert action *with respect to* the torsion tensor components affect the matter action effectively? It is shown here through simple examples that the minimally coupled free-field theories after the minimization of the Einstein-Hilbert action *with respect to* torsion tensor components give rise to effectively self-interacting theories coupled to metric only. In this process, there can be a non-trivial vacuum configuration, arise during the replacement of the torsion degrees of freedom in terms of the metric tensor and matter fields.

With the recent discovery of Gravitational waves from the binary mergers by LIGO collaboration [11–18] it becomes possible to test different models of gravity [19–30] and put constraints on them. Hence, it is natural to ask about whether Gravitational wave can put constraints on Torsion of curved spacetime [31–34], neglected in GR. With the current literature available [35–45], it seems quite difficult

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to put constraints on the form of torsion just from the Gravitation Wave observation, Lorentz violation, and Cosmological observations. This might be because of the current sensitivity limit [46–52] or it seems difficult to put constraints on the torsion of geometries from just one kind of observations. Hence, as an alternate way to test gravity theories with torsion, we look at how does torsion affect the dynamics of radiation fields (massless fields) in curved spacetime, presented in the last two sections in this article. Here we have shown the existence of non-trivial vacuum configurations and breakdown of $U(1)$ Gauge invariance which have profound implications in terms of observations, discussed in the 'Conclusion' section. Since the Sun is the nearest source of such radiations, therefore, it is easier to look at the behavior of radiation fields which enables one to put restrictions on such theories. We have also shown in the next section (where we have shown a modification of Einstein's equation in presence of Torsion) that the presence of torsion leads to new degrees of freedom or extra polarizations in the Gravitational waves or radiations and it also affects the dynamics of Gravitational waves in a non-trivial manner.

2. Inclusion of torsion in General Relativity and presence of extra polarizations in Gravitational waves

In this section, we present how to include torsion mathematically in Einstein's General theory of Relativity, based on the earlier works done in [3,10,53–63] to show the mathematical complexity at a different level without the assumption of torsion-free connection used in GR. This brief review is preliminary of torsion, required for our main results. It also makes the material more transparent to the experimentalists in order to put constraints on torsion from various experiments based on simple mathematical ideas. Further, through this, we present in a more general way that the presence of torsion leads to the extra degrees of freedom or more specifically extra polarizations in Gravitational waves and Gravitational radiations from stars, binary mergers, black holes, etc. which is against the conclusion of [64,65] where torsion is considered only in the form of scalar. On the other hand, the result presented in [31] is consistent with our conclusion.

2.1. Torsion

As with most geometric concepts, there are indeed several ways to define torsion. Following Wald's book [66], we define it in terms of action of commutator of derivative operators which is as follows

$$\nabla_a \nabla_b f - \nabla_b \nabla_a f = -T^c{}_{ab} \nabla_c f, \tag{2.1}$$

for any smooth function f defined over chosen manifold $\mathcal{M}$. $T^c{}_{ab}$ are the components of tensor. From above equation, it follows that

$$\begin{aligned} \nabla_a \nabla_b f - \nabla_b \nabla_a f &= \nabla_a(\nabla_b f) - \nabla_b(\nabla_a f) \\ &= \partial_a \partial_b f - \partial_b \partial_a f + (\Gamma^c{}_{ba} - \Gamma^c{}_{ab}) \nabla_c f \\ &= -T^c{}_{ab} \nabla_c f \\ T^c{}_{ab} &= \Gamma^c{}_{ab} - \Gamma^c{}_{ba} = 2\Gamma^c{}_{[ab]}. \end{aligned} \tag{2.2}$$

Thus, separating the symmetric part of the connection, it can be written as

$$\implies \Gamma^c{}_{ab} = \frac{1}{2} g^{cd} (\partial_a g_{db} + \partial_b g_{ad} - \partial_d g_{ab} - T_{abd} - T_{bad} + T_{dab}). \tag{2.3}$$

2.2. Curvature and Ricci tensor

Let ∇_a be a derivative operator that we choose. Let ω_a be 1-form field and let f be a smooth function then we can show the following equality that without torsion

$$\nabla_a \nabla_b (f\omega_c) - \nabla_b \nabla_a (f\omega_c) = f(\nabla_a \nabla_b - \nabla_b \nabla_a)\omega_c. \tag{2.4}$$

In the same way Wald show in [66] that the tensor $(\nabla_a \nabla_b - \nabla_b \nabla_a)\omega_c$ at point p depends only on the value of ω_c at p which means it defines a tensor of rank $(1, 3)$ in the following way

$$(\nabla_a \nabla_b - \nabla_b \nabla_a)\omega_c = R^d{}_{abc} \omega_d, \tag{2.5}$$

where $R^d{}_{abc}$ Riemann tensor components. Now if we include torsion in the above picture, we obtain the following relation

$$\begin{aligned} \nabla_a \nabla_b (f\omega_c) - \nabla_b \nabla_a (f\omega_c) &= f(\nabla_a \nabla_b - \nabla_b \nabla_a)\omega_c - T^d{}_{ab} \omega_c \nabla_d f \\ (\nabla_a \nabla_b - \nabla_b \nabla_a + T^d{}_{ab} \nabla_d)(f\omega_c) &= f(\nabla_a \nabla_b - \nabla_b \nabla_a + T^d{}_{ab} \nabla_d)\omega_c, \end{aligned} \tag{2.6}$$

which shows $(\nabla_a \nabla_b - \nabla_b \nabla_a + T^d{}_{ab} \nabla_d)$ operation defines rank $(1, 3)$ tensor which is Riemann tensor components $R^d{}_{abc}$, however, it does not have the same properties as it had for torsion-free case.

Let us define the connection $\tilde{\Gamma}^\alpha{}_{\beta\gamma}$ as a sum of symmetric Christoffel symbols $\Gamma^\alpha{}_{\beta\gamma}$ and a quantity $K^\alpha{}_{\beta\gamma}$ as follows

$$\tilde{\Gamma}^\alpha{}_{\beta\gamma} = \Gamma^\alpha{}_{\beta\gamma} + K^\alpha{}_{\beta\gamma}. \tag{2.7}$$

Since $K^\alpha{}_{\beta\gamma}$ is nothing but the difference of two connections, therefore, it is a tensorial quantity of rank $(1, 2)$. We consider that connections $\tilde{\Gamma}^\alpha{}_{\beta\gamma}$ have non-zero torsion components defined the way we defined earlier. Therefore, (following metric compatibility of derivative *with respect to* ˜ connection) we can write the following relation

$$K^\alpha{}_{\beta\gamma} = \frac{1}{2}(T^\alpha{}_{\beta\gamma} - T_\beta{}^\alpha{}_\gamma - T_\gamma{}^\alpha{}_\beta). \tag{2.8}$$

From the above equation, it is quite clear that

$$K_{\alpha\beta\gamma} = -K_{\beta\alpha\gamma}, \tag{2.9}$$

and $K^{\alpha}{}_{\beta\gamma}$ is known as contorsion tensor. According to our identification,

$$[\tilde{\nabla}_\alpha, \tilde{\nabla}_\beta]\varphi = -(K^{\lambda}{}_{\alpha\beta} - K^{\lambda}{}_{\beta\alpha})\tilde{\nabla}_\lambda\varphi, \tag{2.10}$$

where φ is a scalar function. Through a simple algebraic calculation, it can be shown that

$$[\tilde{\nabla}_\alpha, \tilde{\nabla}_\beta]P^\lambda = -T^{\tau}{}_{\alpha\beta}\tilde{\nabla}_\tau P^\lambda + \tilde{R}^{\lambda}{}_{\tau\alpha\beta}P^\tau, \tag{2.11}$$

where

$$\tilde{R}^{\lambda}{}_{\tau\alpha\beta} = \partial_\alpha\tilde{\Gamma}^{\lambda}{}_{\beta\tau} - \partial_\beta\tilde{\Gamma}^{\lambda}{}_{\alpha\tau} + \tilde{\Gamma}^{\lambda}{}_{\alpha\gamma}\tilde{\Gamma}^{\gamma}{}_{\beta\tau} - \tilde{\Gamma}^{\lambda}{}_{\beta\gamma}\tilde{\Gamma}^{\gamma}{}_{\alpha\tau}. \tag{2.12}$$

In terms of torsion-less covariant derivative and contorsion, we obtain the following relations

$$\begin{aligned}
&\tilde{R}^{\lambda}{}_{\tau\alpha\beta} = R^{\lambda}{}_{\tau\alpha\beta} + \nabla_\alpha K^{\lambda}{}_{\beta\tau} - \nabla_\beta K^{\lambda}{}_{\alpha\tau} + K^{\lambda}{}_{\alpha\gamma}K^{\gamma}{}_{\beta\tau} - K^{\lambda}{}_{\beta\gamma}K^{\gamma}{}_{\alpha\tau} \\
\implies &\tilde{R}_{\tau\beta} = \tilde{R}^{\lambda}{}_{\tau\lambda\beta} = R_{\tau\beta} + \nabla_\lambda K^{\lambda}{}_{\beta\tau} - \nabla_\beta K^{\lambda}{}_{\lambda\tau} + K^{\lambda}{}_{\lambda\gamma}K^{\gamma}{}_{\beta\tau} - K^{\lambda}{}_{\beta\gamma}K^{\gamma}{}_{\lambda\tau} \\
&\qquad = R_{\tau\beta} + \nabla_\lambda K^{\lambda}{}_{\beta\tau} - K^{\lambda}{}_{\beta\gamma}K^{\gamma}{}_{\lambda\tau} \\
\implies &\tilde{R} = g^{\tau\beta}\tilde{R}_{\tau\beta} = R - \nabla^\lambda K^{\tau}{}_{\lambda\tau} + K^{\lambda\tau\gamma}K_{\lambda\gamma\tau},
\end{aligned} \tag{2.13}$$

where we have used the fact that $K^{\lambda}{}_{\lambda\rho} = 0$ since $K_{\lambda\alpha\beta} = -K_{\alpha\lambda\beta}$.

2.3. Einstein's equations with torsion

We start by writing down the Einstein-Hilbert action

$$S_{EH} = \int d^4x\sqrt{-g}\tilde{R}. \tag{2.14}$$

Taking variation of metric, we obtain

$$\begin{aligned}
&\delta_g S_{EH} = \int d^4x\Big[(\delta_g\sqrt{-g})\tilde{R} + \sqrt{-g}\tilde{R}_{\mu\nu}\delta_g g^{\mu\nu} + \sqrt{-g}g^{\rho\lambda}\delta_g\tilde{R}_{\rho\lambda}\Big] \\
&\delta_g\sqrt{-g} = -\frac{1}{2}\sqrt{-g}g_{\mu\nu}\delta_g g^{\mu\nu} \\
\implies &\delta_g S_{EH} = \int d^4x\sqrt{-g}\Big[(\tilde{R}_{\mu\nu} - \frac{1}{2}g_{\mu\nu}\tilde{R})\delta_g g^{\mu\nu} + g^{\rho\lambda}\delta_g\tilde{R}_{\rho\lambda}\Big].
\end{aligned} \tag{2.15}$$

It can be shown that

$$\begin{aligned}
&\delta_g\tilde{R}_{\rho\lambda} = \tilde{\nabla}_\alpha(\delta_g\tilde{\Gamma}^{\alpha}{}_{\lambda\rho}) - \tilde{\nabla}_\lambda(\delta_g\tilde{\Gamma}^{\alpha}{}_{\alpha\rho}) + T^{\beta}{}_{\alpha\lambda}(\delta_g\tilde{\Gamma}^{\alpha}{}_{\beta\rho}) \\
&T^{\beta}{}_{\alpha\lambda} = K^{\beta}{}_{\alpha\lambda} - K^{\beta}{}_{\lambda\alpha}.
\end{aligned} \tag{2.16}$$

Hence,

$$\begin{aligned}
\int d^4x\sqrt{-g}g^{\rho\lambda}\delta_g\tilde{R}_{\rho\lambda} &= \int d^4x\sqrt{-g}g^{\rho\lambda}\Big[\tilde{\nabla}_\alpha(\delta_g\tilde{\Gamma}^{\alpha}{}_{\lambda\rho}) - \tilde{\nabla}_\lambda(\delta_g\tilde{\Gamma}^{\alpha}{}_{\alpha\rho}) + T^{\beta}{}_{\alpha\lambda}(\delta_g\tilde{\Gamma}^{\alpha}{}_{\beta\rho})\Big] \\
&= \int d^4x\sqrt{-g}\Big[\tilde{\nabla}_\alpha(g^{\lambda\rho}\delta_g\tilde{\Gamma}^{\alpha}{}_{\lambda\rho} - g^{\rho\alpha}\delta_g\tilde{\Gamma}^{\beta}{}_{\beta\rho}) + g^{\lambda\rho}T^{\beta}{}_{\alpha\lambda}\delta_g\tilde{\Gamma}^{\alpha}{}_{\beta\rho}\Big] \\
&= -\int d^4x\sqrt{-g}T^{\mu\nu}{}_{\alpha}\delta_g\tilde{\Gamma}^{\alpha}{}_{\mu\nu}.
\end{aligned} \tag{2.17}$$

Since we consider metric compatibility throughout the variation, hence,

$$\begin{aligned}
0 &= \overset{\delta}{\tilde{\nabla}}_\rho\,(g_{\mu\nu} + \delta_g g_{\mu\nu}) \\
&= \tilde{\nabla}_\rho(g_{\mu\nu} + \delta_g g_{\mu\nu}) - \delta_g\tilde{\Gamma}^{\lambda}{}_{\rho\mu}((g_{\lambda\nu} + \delta_g g_{\lambda\nu})) - \delta_g\tilde{\Gamma}^{\lambda}{}_{\rho\nu}(g_{\mu\lambda} + \delta_g g_{\mu\lambda}) \\
&= \tilde{\nabla}_\rho(\delta_g g_{\mu\nu}) - g_{\lambda\nu}\delta_g\tilde{\Gamma}^{\lambda}{}_{\rho\mu} - g_{\mu\lambda}\delta_g\tilde{\Gamma}^{\lambda}{}_{\rho\nu}.
\end{aligned} \tag{2.18}$$

Rearranging indices, we obtain

$$\begin{aligned}
&\tilde{\nabla}_\mu\delta_g g_{\nu\rho} + \tilde{\nabla}_\nu\delta_g g_{\mu\rho} - \tilde{\nabla}_\rho\delta_g g_{\mu\nu} = 2g_{\lambda\rho}\delta_g\tilde{\Gamma}^{\lambda}{}_{\mu\nu} + g_{\lambda\nu}\delta_g T^{\lambda}{}_{\mu\rho} + g_{\lambda\mu}\delta_g T^{\lambda}{}_{\nu\rho} - g_{\lambda\rho}\delta_g T^{\lambda}{}_{\mu\nu} \\
\implies &\delta_g\tilde{\Gamma}^{\alpha}{}_{\mu\nu} = \frac{1}{2}g^{\alpha\rho}(\tilde{\nabla}_\mu\delta_g g_{\nu\rho} + \tilde{\nabla}_\nu\delta_g g_{\mu\rho} - \tilde{\nabla}_\rho\delta_g g_{\mu\nu} - g_{\lambda\nu}\delta_g T^{\lambda}{}_{\mu\rho} - g_{\lambda\mu}\delta_g T^{\lambda}{}_{\nu\rho} + g_{\lambda\rho}\delta_g T^{\lambda}{}_{\mu\nu}).
\end{aligned} \tag{2.19}$$

Now if we consider variation of metric only then,

$$\delta_g \tilde{\Gamma}^{\alpha}_{\mu\nu} = \frac{1}{2} g^{\alpha\beta} (\tilde{\nabla}_\mu \delta_g g_{\nu\beta} + \tilde{\nabla}_\nu \delta_g g_{\mu\beta} - \tilde{\nabla}_\beta \delta_g g_{\mu\nu}). \tag{2.20}$$

Substituting above into (2.17), we obtain

$$\begin{aligned}
\int d^4x \sqrt{-g} g^{\rho\lambda} \delta_g \tilde{R}_{\rho\lambda} &= -\frac{1}{2} \int d^4x \sqrt{-g} g^{\alpha\beta} T^{\mu\nu}_{\ \ \alpha} (\tilde{\nabla}_\mu \delta_g g_{\nu\beta} + \tilde{\nabla}_\nu \delta_g g_{\mu\beta} - \tilde{\nabla}_\beta \delta_g g_{\mu\nu}) \\
&= -\frac{1}{2} \int d^4x \sqrt{-g} (T^{\mu\nu\beta} \tilde{\nabla}_\mu \delta_g g_{\nu\beta} - 2T^{\mu\nu\beta} \tilde{\nabla}_\beta \delta_g g_{\mu\nu}) \\
&= -\int d^4x \sqrt{-g} \tilde{\nabla}_\beta T_{(\mu\nu)}^{\ \ \ \ \beta} \delta_g g^{\mu\nu}.
\end{aligned} \tag{2.21}$$

Therefore, the minimization of action *with respect to* variation of metric gives the following modified equation

$$\tilde{G}_{(\mu\nu)} - \tilde{\nabla}_\beta T_{(\mu\nu)}^{\ \ \ \ \beta} = 8\pi T_{\mu\nu}, \tag{2.22}$$

where $T_{\mu\nu}$ is the matter stress-energy tensor.

Note that

$$\begin{aligned}
\tilde{G}_{\mu\nu} = \tilde{R}_{\mu\nu} - \frac{1}{2} g_{\mu\nu} \tilde{R} &= G_{\mu\nu} + \nabla_\lambda \left(K^{\lambda}_{\ \nu\mu} - \frac{1}{2} g_{\mu\nu} K^{\lambda\tau}_{\ \ \ \tau} \right) - \left(K^{\lambda}_{\ \nu\gamma} K^{\gamma}_{\ \lambda\mu} + \frac{1}{2} g_{\mu\nu} K^{\lambda\tau\gamma} K_{\lambda\gamma\tau} \right) \\
\implies \tilde{G}_{(\mu\nu)} &= G_{\mu\nu} + \nabla_\lambda \left(K^{\lambda}_{\ (\nu\mu)} - \frac{1}{2} g_{\mu\nu} K^{\lambda\tau}_{\ \ \ \tau} \right) - \left(K^{\lambda}_{\ (\nu|\gamma} K^{\gamma}_{\ \lambda|\mu)} + \frac{1}{2} g_{\mu\nu} K^{\lambda\tau\gamma} K_{\lambda\gamma\tau} \right).
\end{aligned} \tag{2.23}$$

Denoting $T_{(\mu\nu)\lambda} = S_{\lambda\mu\nu}$ leads to the following constraint

$$\implies S_{\lambda\mu\nu} + S_{\mu\nu\lambda} + S_{\nu\lambda\mu} = 0. \tag{2.24}$$

Furthermore,

$$K^{\lambda}_{\ (\nu\mu)} = \frac{1}{2} (T^{\lambda}_{\ (\nu\mu)} - T_{(\nu\ \ \mu)}^{\ \ \lambda} - T_{(\mu\ \ \nu)}^{\ \ \lambda}) = -T_{(\mu\ \ \nu)}^{\ \ \lambda} = T_{(\mu\nu)}^{\ \ \ \ \lambda}, \tag{2.25}$$

and

$$\begin{aligned}
\tilde{\nabla}_\lambda K^{\lambda}_{\ \nu\mu} &= \nabla_\lambda K^{\lambda}_{\ \nu\mu} + \underbrace{K^{\lambda}_{\ \lambda\rho}}_{=0} K^{\rho}_{\ \nu\mu} - K^{\rho}_{\ \lambda\nu} K^{\lambda}_{\ \rho\mu} - K^{\rho}_{\ \lambda\mu} K^{\lambda}_{\ \nu\rho} \\
\tilde{\nabla}_\lambda K^{\lambda}_{\ (\nu\mu)} &= \nabla_\lambda K^{\lambda}_{\ (\nu\mu)} - K^{\rho}_{\ \lambda(\nu} K^{\lambda}_{\ \rho\mu)} - K^{\rho}_{\ \lambda(\mu} K^{\lambda}_{\ \nu)\rho} \\
\implies \nabla_\lambda K^{\lambda}_{\ (\nu\mu)} - K^{\lambda}_{\ (\nu|\rho} K^{\rho}_{\ \lambda|\mu} &= \tilde{\nabla}_\lambda K^{\lambda}_{\ (\nu\mu)} + K_{\rho\lambda(\nu} K^{\lambda\rho}_{\ \ \ \mu)},
\end{aligned} \tag{2.26}$$

which means we can rewrite (2.23) in the following form

$$\tilde{G}_{(\mu\nu)} = G_{\mu\nu} + \tilde{\nabla}_\lambda K^{\lambda}_{\ (\nu\mu)} - \frac{1}{2} g_{\mu\nu} \nabla_\lambda K^{\lambda\tau}_{\ \ \ \tau} + (K_{\rho\lambda(\nu} K^{\lambda\rho}_{\ \ \ \mu)} - \frac{1}{2} g_{\mu\nu} K^{\lambda\tau\gamma} K_{\lambda\gamma\tau}). \tag{2.27}$$

Together with above information, we can write modified Einstein equation in the following form

$$G_{\mu\nu} - \frac{1}{2} g_{\mu\nu} \nabla_\lambda K^{\lambda\tau}_{\ \ \ \tau} + (K_{\rho\lambda(\nu} K^{\lambda\rho}_{\ \ \ \mu)} - \frac{1}{2} g_{\mu\nu} K^{\lambda\tau\gamma} K_{\lambda\gamma\tau}) = 8\pi T_{\mu\nu}. \tag{2.28}$$

The above equation further can be rearranged in the following form

$$G_{\mu\nu} = \frac{1}{2} g_{\mu\nu} \nabla_\lambda K^{\lambda\tau}_{\ \ \ \tau} - (K_{\rho\lambda(\nu} K^{\lambda\rho}_{\ \ \ \mu)} - \frac{1}{2} g_{\mu\nu} K^{\lambda\tau\gamma} K_{\lambda\gamma\tau}) + 8\pi T_{\mu\nu}. \tag{2.29}$$

Note that in the weak gravity limit where $g_{\mu\nu} = \eta_\mu + h_{\mu\nu}$ is considered, the *left-hand side* of the above equation is invariant under the infinitesimal diffeomorphisms of the form $h_{\mu\nu} \to h_{\mu\nu} + \partial_\mu \xi_\nu + \partial_\nu \xi_\mu$ which is also seen as gauge transformations. On the other hand, *right-hand side* of the above equation is not invariant under the infinitesimal diffeomorphism for obvious reasons. Furthermore, on the *right-hand side*, each term has two contractions at least which shows the presence of $h_{\mu\nu} h^{\mu\nu}$ or $h_{\mu\lambda} h^{\lambda}_{\ \nu}$ or other possible terms that can be found out in detail analysis. This shows the presence of mass terms which also appear in massive graviton like theories [67–71]. Observation or any evidence of these extra polarizations of Gravitational waves not only suggest IR modifications of GR but also opens the possibility of looking for torsion related features of spacetimes.

2.4. Spin tensor

Note that in the Einstein Hilbert action, we can also take the variation *with respect to* the torsion tensor and we find

$$\begin{aligned}\delta S_{EH} &= \frac{1}{2}\int d^4x\sqrt{-g}T^{\mu\nu}_{\ \ \alpha}g^{\alpha\rho}\Big[g_{\lambda\nu}\delta_g T^{\lambda}_{\ \mu\rho} + g_{\lambda\mu}\delta_g T^{\lambda}_{\ \nu\rho} - g_{\lambda\rho}\delta_g T^{\lambda}_{\ \mu\nu}\Big]\\ \implies \Pi^{\mu\nu}_{\ \ \lambda} &= \frac{1}{\sqrt{-g}}\frac{\delta S_{EH}}{\delta T^{\lambda}_{\ \mu\nu}} = \frac{1}{2}g_{\lambda\rho}\Big[T^{[\mu|\rho|\nu]} + T^{\rho[\mu\nu]} - T^{[\mu\nu]\rho}\Big] = g_{\lambda\rho}\Big[T^{[\mu|\rho|\nu]} + \frac{1}{2}T^{\rho\mu\nu}\Big].\end{aligned} \tag{2.30}$$

Hence, from minimization of total action, Spin tensor denoted by $\Pi_{\lambda}^{\ \mu\nu}$ can also be defined in the following way

$$\Pi^{\mu\nu}_{\ \ \lambda} \equiv \frac{1}{\sqrt{-g}}\frac{\delta S_m}{\delta T^{\lambda}_{\ \mu\nu}}, \tag{2.31}$$

and by definition $\Pi_{\lambda}^{\ \mu\nu} = -\Pi_{\lambda}^{\ \nu\mu}$. Then, we have the following equation

$$\begin{aligned} g_{\lambda\rho}\Big[T^{[\mu|\rho|\nu]} + \frac{1}{2}T^{\rho\mu\nu}\Big] &= \Pi^{\mu\nu}_{\ \ \lambda}\\ \implies T^{[\mu|\rho|\nu]} + \frac{1}{2}T^{\rho\mu\nu} &= \Pi^{\mu\nu\rho}.\end{aligned} \tag{2.32}$$

Note that knowing torsion tensor components $T^{\lambda}_{\ \mu\nu}$ is equivalent to knowing $K^{\lambda}_{\ \mu\nu}$ components, hence, the number of $K^{\lambda}_{\ \mu\nu}$ components is the same as the number of torsion components $T^{\lambda}_{\ \mu\nu}$. Further, if torsion does not couple with the matter field then $\Pi^{\rho\mu\nu}$ becomes identically zero and all 24 components of the torsion tensor vanish identically. The actual way to do the calculation is to first estimate torsion tensor from spin tensors and then plug those into modified Einstein's equation to get metric tensor.

The equation (2.30) implies

$$\begin{aligned} T^{\mu\rho\nu} + T^{\rho\mu\nu} - T^{\nu\rho\mu} &= 2\Pi^{\mu\nu\rho}\\ T^{\rho\nu\mu} + T^{\nu\rho\mu} - T^{\mu\nu\rho} &= 2\Pi^{\rho\mu\nu}\\ T^{\nu\mu\rho} + T^{\mu\nu\rho} - T^{\rho\mu\nu} &= 2\Pi^{\nu\rho\mu}\end{aligned} \tag{2.33}$$

After inverting the above relations, we obtain the following set of equations

$$\begin{aligned} T^{\mu\rho\nu} &= \Pi^{\rho\mu\nu} + \Pi^{\mu\nu\rho}\\ T^{\rho\nu\mu} &= \Pi^{\rho\mu\nu} + \Pi^{\nu\rho\mu}\\ T^{\nu\mu\rho} &= \Pi^{\nu\rho\mu} + \Pi^{\mu\nu\rho},\end{aligned} \tag{2.34}$$

which is equivalent to only one equation which is the following

$$T^{\mu\rho\nu} = \Pi^{\rho\mu\nu} + \Pi^{\mu\nu\rho} = \Pi^{\rho\mu\nu} - \Pi^{\nu\mu\rho}. \tag{2.35}$$

3. Effect of torsion tensor

3.1. Effect of torsion in modified Einstein equation

Once we get the torsion tensor components, we put it back to the modified Einstein equation and we can write

$$G_{\mu\nu} = 8\pi T_{\mu\nu} + \chi_{\mu\nu}, \tag{3.1}$$

where

$$\chi_{\mu\nu} = \frac{1}{2}g_{\mu\nu}\nabla_{\lambda}K^{\lambda\tau}_{\ \ \tau} - K_{\rho\lambda(\nu}K^{\lambda\rho}_{\ \ \ \mu)} + \frac{1}{2}g_{\mu\nu}K^{\lambda\tau\gamma}K_{\lambda\gamma\tau}. \tag{3.2}$$

Hence, $\chi_{\mu\nu}$, made out of contorsion tensor or in a way of torsion tensor modifies the net stress-energy tensor locally at each spacetime point. Further, if in (2.33), we find that $\Pi_{\lambda}^{\ \mu\nu} = 0$ then, all the components of torsion tensor and similarly contorsion tensor become zero.

3.2. Example of a scalar field theory coupled to torsion

Let us consider a scalar field theory, described by the following action

$$S_0 = \int \sqrt{-g}d^4x\Big[\frac{1}{2}\varphi\tilde{\nabla}^2\varphi - V(\varphi)\Big], \tag{3.3}$$

where we can write

$$\begin{aligned}\tilde{\nabla}^2\varphi &= g^{\mu\nu}\tilde{\nabla}_\mu\tilde{\nabla}_\nu\varphi = g^{\mu\nu}\Big[\nabla_\mu\nabla_\nu\varphi - K^{\rho}_{\mu\nu}\partial_\rho\varphi\Big] \\ &= g^{\mu\nu}\Big[\nabla_\mu\nabla_\nu\varphi + K_{\mu\ \nu}^{\ \rho}\partial_\rho\varphi\Big] = \Box\varphi + T^\rho\partial_\rho\varphi.\end{aligned} \tag{3.4}$$

If potential part $V(\varphi)$ does not couple with torsion tensor then, we can write spin tensor corresponding to this scalar matter as follows

$$\Pi^{\mu\rho}_{\ \ \sigma} = 2g^{[\mu\nu}\delta^{\rho]}_\sigma\varphi\partial_\nu\varphi = (g^{\mu\nu}\delta^\rho_\sigma - g^{\rho\nu}\delta^\mu_\sigma)\varphi\partial_\nu\varphi. \tag{3.5}$$

By doing integration by parts, we can also write the action in the following way

$$\begin{aligned}S_0 &= \int\sqrt{-g}d^4x\Big[\frac{1}{2}g^{\mu\nu}\varphi\nabla_\mu\nabla_\nu\varphi + \frac{1}{2}\varphi T^\rho\partial_\rho\varphi - V(\varphi)\Big] \\ &= -\int\sqrt{-g}d^4x\Big[\frac{1}{2}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - \frac{1}{2}\varphi T^\rho\partial_\rho\varphi + V(\varphi)\Big].\end{aligned} \tag{3.6}$$

Further, the above action can be expressed in the following form

$$\begin{aligned}S_0 &= -\int\sqrt{-g}d^4x\Big[\frac{1}{2}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - \frac{1}{2}\varphi T^\rho\partial_\rho\varphi + V(\varphi)\Big] \\ &= -\int\sqrt{-g}d^4x\Big[\frac{1}{2}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - \frac{1}{4\sqrt{-g}}\partial_\rho(\sqrt{-g}T^\rho\varphi^2) + \frac{1}{4}\partial_\rho T^\rho\varphi^2 + V(\varphi)\Big] \\ &= -\int\sqrt{-g}d^4x\Big[\frac{1}{2}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi + \frac{1}{4}\partial_\rho T^\rho\varphi^2 + V(\varphi)\Big] + \text{boundary term},\end{aligned} \tag{3.7}$$

where $\frac{1}{2}\partial_\rho T^\rho$ effectively acts as a square of mass term. This implies that we can write the stress-energy tensor as follows

$$T_{\mu\nu} = \partial_\mu\varphi\partial_\nu\varphi - \varphi T_{(\mu}\partial_{\nu)}\varphi - g_{\mu\nu}\Big[\frac{1}{2}g^{\rho\sigma}\partial_\rho\varphi\partial_\sigma\varphi - \frac{1}{2}\varphi T^\rho\partial_\rho\varphi + V(\varphi)\Big]. \tag{3.8}$$

Using (2.34), we obtain

$$T^{\mu\rho\nu} = \varphi\partial^\rho\varphi g^{\mu\nu} - \varphi\partial^\nu\varphi g^{\mu\rho}, \tag{3.9}$$

and this implies

$$T_\rho = 3\varphi\partial_\rho\varphi. \tag{3.10}$$

Substitute this into the action, we obtain

$$\begin{aligned}S_0 &= -\int\sqrt{-g}d^4x\Big[\frac{1}{2}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - \frac{1}{2}\varphi T^\rho\partial_\rho\varphi + V(\varphi)\Big] \\ &= -\int\sqrt{-g}d^4x\Big[\frac{1}{2}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - \frac{3}{2}\varphi^2\partial^\rho\varphi\partial_\rho\varphi + V(\varphi)\Big].\end{aligned} \tag{3.11}$$

Now as we can see even in the absence of potential $V(\varphi)$, torsion leads to an interacting minimally coupled scalar field theory from a simple looking non-minimally coupled scalar field theory. Further, this theory indeed has a non-zero vacuum expectation value which is $\varphi_0 = \pm\frac{1}{\sqrt{3}}$. It is important to note that we have put $G = 1$, a dimensionful quantity of dimension $[\text{mass}]^{-2}$ in $\hbar = c = 1$. Considering $G \neq 1$, we obtain the following action

$$S_0 = -\int\sqrt{-g}d^4x\Big[\frac{1}{2}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - \frac{3G}{2}\varphi^2\partial^\rho\varphi\partial_\rho\varphi\Big], \tag{3.12}$$

which takes care of dimension of Lagrangian density appropriately and the vacuum expectation value becomes $\varphi_0 = \pm\frac{1}{\sqrt{3G}}$. The stress-energy tensor becomes the following

$$T_{\mu\nu} = \partial_\mu\varphi\partial_\nu\varphi - 3G\phi^2\partial_\mu\varphi\partial_\nu\varphi - g_{\mu\nu}\Big[\frac{1}{2}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - \frac{3G}{2}\varphi^2\partial^\rho\varphi\partial_\rho\varphi\Big]. \tag{3.13}$$

If one wants to study the excitations around vacuum then, we can write $\varphi = \varphi_0 + \tilde{\varphi}$ and we obtain the following action

$$\begin{aligned}S_0 &= -\int\sqrt{-g}d^4x\Big[\frac{1}{2}g^{\mu\nu}(1 - 3G\varphi_0^2)\partial_\mu\tilde{\varphi}\partial_\nu\tilde{\varphi} - \frac{3G}{2}\tilde{\varphi}^2\partial^\rho\tilde{\varphi}\partial_\rho\tilde{\varphi} - 3G\varphi_0\tilde{\varphi}\partial^\rho\tilde{\varphi}\partial_\rho\tilde{\varphi}\Big] \\ &= \int\sqrt{-g}d^4x\Big[\frac{3G}{2}\tilde{\varphi}^2\partial^\rho\tilde{\varphi}\partial_\rho\tilde{\varphi} + 3G\varphi_0\tilde{\varphi}\partial^\rho\tilde{\varphi}\partial_\rho\tilde{\varphi}\Big].\end{aligned} \tag{3.14}$$

Recall that we can write the generating functional for a scalar field theory in the following way

$$\mathcal{Z}[J] = \int\mathcal{D}\tilde{\varphi}\ e^{iS_0[\tilde{\varphi}] + i\int d^4x\sqrt{-g(x)}J(x)\tilde{\varphi}(x)}. \tag{3.15}$$

The action in (3.14) can further be expressed as

$$S_0[\tilde{\varphi}] = \int \sqrt{-g}d^4x\Big[\frac{3G}{8}\partial^\rho\tilde{\varphi}^2\partial_\rho\tilde{\varphi}^2 + 3G\varphi_0\tilde{\varphi}\partial^\rho\tilde{\varphi}\partial_\rho\tilde{\varphi}\Big]$$
$$= \int \sqrt{-g}d^4x\Big[\frac{1}{2}\partial^\rho\bar{\varphi}^2\partial_\rho\bar{\varphi}^2 + (192G)^{\frac{1}{4}}\varphi_0\bar{\varphi}\partial^\rho\bar{\varphi}\partial_\rho\bar{\varphi}\Big],\ \bar{\varphi} = \tilde{\varphi}\left(\frac{3G}{4}\right)^{\frac{1}{4}} \tag{3.16}$$

Since the constant Jacobian due to above scaling of field is irrelevant for practical computations, hence, the generating function can be expressed as

$$\mathcal{Z}[J] = \int \mathcal{D}\bar{\varphi}\ e^{iS_0[\bar{\varphi}]+i\int d^4x\sqrt{-g(x)}J(x)\bar{\varphi}(x)}$$
$$= \int \mathcal{D}\chi\ e^{i\int\sqrt{-g}d^4x\Big[\frac{1}{2}\partial^\rho\chi\partial_\rho\chi+\left(\frac{192G}{\chi^2}\right)^{\frac{1}{4}}\varphi_0\partial^\rho\chi\partial_\rho\chi+\frac{i}{2}\log\chi\Big]+i\int d^4x\sqrt{-g(x)}J(x)\sqrt{\chi(x)}}, \tag{3.17}$$

where $\chi(x) = \bar{\varphi}^2(x)$. The above generating functional clearly shows that torsion leads to a strongly coupled field theory since $G^{\frac{1}{4}}\varphi_0 \propto \frac{1}{G^{\frac{1}{4}}} \gg 1$. Hence, in this theory, the perturbative techniques are not applicable. It is important here to note that the term $\frac{i}{2}\log\chi$ in (3.17) comes from the Jacobian of transformation in the measure of the functional integral. A generic n-point function $\langle\bar{\varphi}(x_1)\ldots\bar{\varphi}(x_n)\rangle$ can be expressed as $\langle\sqrt{\chi(x_1)}\ldots\sqrt{\chi(x_n)}\rangle$. Since the kinetic term in the effective action in (3.17) is not decoupled, hence, we follow a different approach mentioned below to decouple the kinetic term and the potential term in the effective action (exponent in the generating functional).

Note that the initial action S_0 can be written in following form

$$S_0[\varphi] = -\int\sqrt{-g}d^4x\ f(\varphi)g^{\mu\nu}\frac{1}{2}\partial_\mu\varphi\partial_\nu\varphi, \tag{3.18}$$

where $f(\varphi)$ is a polynomial of degree two in φ. Then, the field equation for matter field becomes

$$\partial_\mu\Big[\sqrt{-g}f(\varphi)g^{\mu\nu}\partial_\nu\varphi\Big] = f'(\varphi)\sqrt{-g}\frac{1}{2}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi$$
$$\implies f'(\varphi)g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi + f(\varphi)\Box\varphi = f'(\varphi)\frac{1}{2}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi$$
$$2f(\varphi)\Box\varphi + f'(\varphi)\nabla_\mu\varphi\nabla^\mu\varphi = 0$$
$$\implies f(\varphi)\Box\varphi + \nabla_\mu(f(\varphi)\nabla^\mu\varphi) = 0. \tag{3.19}$$

The above equation suggests that if $\Box\varphi = 0$ then, $f(\varphi(x))\nabla^\mu\varphi(x) = J^\mu(x)$ such that $\nabla_\mu J^\mu(x) = 0$. This vector must be null $J^\mu(x)J_\mu(x)$ since $\Box\varphi = 0 \implies ||\partial\varphi(x)||^2 = \nabla_\mu\varphi\nabla^\mu\varphi = 0$. J_μ is also a closed one-form $\nabla_{[\mu}J_{\nu]} = 0$ since it is exact. In general, we do not expect this null current to be existent. If we restrict ourselves to the case where $||\partial\varphi||^2 \neq 0$ then, we can write the above equation as follows

$$2\Box\varphi + \frac{f'(\varphi)}{f(\varphi)}||\partial\varphi||^2 = 0,\ \text{if } f(\varphi) \neq 0. \tag{3.20}$$

In our case, $f(\varphi) = \left(1 - \frac{\varphi^2}{\varphi_0^2}\right)$. Further, given an action of the form in (3.18), we can write the generating functional as follows

$$\mathcal{Z}[J] = \int \mathcal{D}\varphi\ e^{-i\int\sqrt{-g}d^4x\ f(\varphi)g^{\mu\nu}\frac{1}{2}\partial_\mu\varphi\partial_\nu\varphi + i\int\sqrt{-g}d^4x\ J(x)\varphi(x)}$$
$$= \int \mathcal{D}\Psi\ e^{-i\int\sqrt{-g(x)}d^4x\Big[g^{\mu\nu}\frac{1}{2}\partial_\mu\Psi(x)\partial_\nu\Psi(x)+i\mathcal{K}'(\Psi(x))\Big]+i\int\sqrt{-g(x)}d^4x\ J(x)\mathcal{K}(\Psi(x))}, \tag{3.21}$$

where $\varphi(x) = \mathcal{K}(\Psi(x))$ is a non-trivial function such that $\sqrt{f(\varphi)}d\varphi = d\Psi$ and $\mathcal{K}'(\Psi) = \frac{\delta\mathcal{K}(\Psi)}{\delta\Psi}$. The effective action $S_{\text{eff}}[\Psi] = -\int\sqrt{-g}d^4x\Big[g^{\mu\nu}\frac{1}{2}\partial_\mu\Psi\partial_\nu\Psi + i\mathcal{K}'(\Psi(x))\Big]$ is a complex action (see examples in [72,73]), hence, the Ψ excitations are unstable or in other words these excitations have finite lifetime [74]. We can further add an exponent in (3.21) to define the following generating functional

$$\mathcal{Z}[J,K] = \int \mathcal{D}\Psi\ e^{-i\int\sqrt{-g(x)}d^4x\Big[g^{\mu\nu}\frac{1}{2}\partial_\mu\Psi(x)\partial_\nu\Psi(x)+i\mathcal{K}'(\Psi(x))\Big]+i\int\sqrt{-g}d^4x\ J(x)\mathcal{K}(\Psi(x))+i\int\sqrt{-g}d^4x\ K(x)\Psi(x)}, \tag{3.22}$$

such that

$$\langle\varphi(x_1)\ldots\varphi(x_n)\rangle = \frac{1}{i^n}\left(\prod_{i=1}^n\frac{1}{\sqrt{-g(x_i)}}\right)\frac{\delta}{\delta J(x_1)}\cdots\frac{\delta}{\delta J(x_n)}\mathcal{Z}[J,K]\Big|_{J,K=0}$$
$$\langle\Psi(x_1)\ldots\Psi(x_n)\rangle = \frac{1}{i^n}\left(\prod_{i=1}^n\frac{1}{\sqrt{-g(x_i)}}\right)\frac{\delta}{\delta K(x_1)}\cdots\frac{\delta}{\delta K(x_n)}\mathcal{Z}[J,K]\Big|_{J,K=0}. \tag{3.23}$$

Since $f(\varphi) = \left(1 - \frac{\varphi^2}{\varphi_0^2}\right)$ in our case, hence, $\Psi(x)$ would be real-valued for $|\varphi(x)| \leq \varphi_0$. In this case,

$$\Psi(x)=\frac{\varphi_0\arcsin\left(\frac{\varphi(x)}{\varphi_0}\right)+\varphi(x)\sqrt{1-\frac{\varphi^2(x)}{\varphi_0^2}}}{2}. \tag{3.24}$$

It can be checked easily that Ψ is a monotonic increasing function in φ, hence, the above relation is invertible. As a result, the function $\mathcal{K}(\Psi)$ is well-defined. Further, it is important to note that if $|\varphi(x)|>\varphi_0$ then, $f(\varphi(x))<0$ and as a result of it, the Hamiltonian density $\mathcal{H}(x)$ becomes unbounded from below. This causes instability, hence, $|\varphi(x)|\le\varphi_0$ must hold.

On the other hand, if the spin-tensor is given like the metric in the case of QFT in curved spacetime then, the action in (3.7) is the general form for free-field theory minimally coupled to curved spacetime with torsion. Further, if we consider $V(\varphi)=\frac{\lambda}{4!}\varphi^4$ in (3.7) with $\lambda>0$ and $\partial_\rho T^\rho<0$ holds then, the spontaneous symmetry breaking takes place.

3.3. Massless vector field theory minimally coupled to gravity including torsion

The minimally coupled action describing massless vector field theory is the following

$$S=-\frac{1}{4}\int\sqrt{-g}d^4x\, g^{\mu\alpha}g^{\beta\nu}F_{\mu\nu}F_{\alpha\beta}, \tag{3.25}$$

where because of torsion, we can write

$$F_{\mu\nu}=\tilde{\nabla}_\mu\mathcal{A}_\nu-\tilde{\nabla}_\nu\mathcal{A}_\mu=\partial_\mu\mathcal{A}_\nu-\partial_\nu\mathcal{A}_\mu-T^\rho_{\ \mu\nu}\mathcal{A}_\rho\equiv F^{(0)}_{\mu\nu}-T^\rho_{\ \mu\nu}\mathcal{A}_\rho. \tag{3.26}$$

It is important here to note that the presence of torsion breaks the invariance of action under the gauge transformation. Then, the above action can be written in the following way

$$S=-\frac{1}{4}\int\sqrt{-g}d^4x\, g^{\mu\alpha}g^{\beta\nu}\Big[F^{(0)}_{\mu\nu}F^{(0)}_{\alpha\beta}-2F^{(0)}_{\alpha\beta}T^\rho_{\ \mu\nu}\mathcal{A}_\rho+T^\rho_{\ \mu\nu}T^\sigma_{\ \alpha\beta}\mathcal{A}_\rho\mathcal{A}_\sigma\Big]. \tag{3.27}$$

Then, according to the definition

$$\Pi^{\mu\nu}_{\ \ \lambda}=\frac{1}{\sqrt{-g}}\frac{\delta S}{\delta T^\lambda_{\ \mu\nu}}=\frac{1}{2}g^{\mu\alpha}g^{\nu\beta}\Big[F^{(0)}_{\alpha\beta}\mathcal{A}_\lambda-T^\sigma_{\ \alpha\beta}\mathcal{A}_\sigma\mathcal{A}_\lambda\Big]\implies\Pi^{\mu\nu\lambda}=\frac{1}{2}g^{\mu\alpha}g^{\nu\beta}\Big[F^{(0)}_{\alpha\beta}\mathcal{A}^\lambda-T^\sigma_{\ \alpha\beta}\mathcal{A}_\sigma\mathcal{A}^\lambda\Big]. \tag{3.28}$$

According to (2.34), we can write

$$\begin{aligned}T^{\mu\lambda\nu}&=\Pi^{\lambda\mu\nu}-\Pi^{\nu\mu\lambda}=\frac{1}{2}g^{\mu\alpha}\left(F^{(0)}_{\alpha\beta}-T^\sigma_{\ \alpha\beta}\mathcal{A}_\sigma\right)(g^{\nu\beta}\mathcal{A}^\lambda-g^{\lambda\beta}\mathcal{A}^\nu)\\&=\frac{1}{2}g^{\mu\alpha}F^{(0)}_{\alpha\beta}(g^{\nu\beta}\mathcal{A}^\lambda-g^{\lambda\beta}\mathcal{A}^\nu)-\frac{1}{2}(T^{\sigma\mu\nu}\mathcal{A}_\sigma\mathcal{A}^\lambda-T^{\sigma\mu\lambda}\mathcal{A}_\sigma\mathcal{A}^\nu).\end{aligned} \tag{3.29}$$

If we put all the torsion tensor components in *left-hand side*, we obtain the following equation

$$\begin{aligned}&T^{\alpha\beta\gamma}\Big[\delta^\mu_\alpha\delta^\nu_\beta\delta^\lambda_\gamma+\frac{1}{2}\mathcal{A}_\alpha\delta^\mu_\beta\left(\delta^\lambda_\gamma\mathcal{A}^\nu-\delta^\nu_\gamma\mathcal{A}^\lambda\right)\Big]=\frac{1}{2}g^{\mu\alpha}F^{(0)}_{\alpha\beta}(g^{\lambda\beta}\mathcal{A}^\nu-g^{\nu\beta}\mathcal{A}^\lambda)\\&\implies T^{\alpha\beta\gamma}\mathcal{S}^{\mu\nu\lambda}_{\ \ \ \ \alpha\beta\gamma}(\mathcal{A})=\frac{1}{2}g^{\mu\alpha}F^{(0)}_{\alpha\beta}(g^{\lambda\beta}\mathcal{A}^\nu-g^{\nu\beta}\mathcal{A}^\lambda)\\&\implies T^{\alpha\beta\gamma}=[\mathcal{S}^{-1}(\mathcal{A})]^{\alpha\beta\gamma}_{\ \ \ \ \mu\nu\lambda}\frac{1}{2}g^{\mu\rho}F^{(0)}_{\rho\sigma}(g^{\lambda\sigma}\mathcal{A}^\nu-g^{\nu\sigma}\mathcal{A}^\lambda)=[\mathcal{S}^{-1}(\mathcal{A})]^{\alpha\beta\gamma}_{\ \ \ \ \rho[\nu\sigma]}\mathcal{A}^\nu F^{(0)\rho\sigma}.\end{aligned} \tag{3.30}$$

In principle, one can calculate the components of $\mathcal{S}^{-1}(\mathcal{A})$ in terms of vector field components $\mathcal{A}_\mu$s. Hence, the matter action can be written by substituting the last line in the previous equation

$$\begin{aligned}S_{\mathcal{A}}&=-\frac{1}{4}\int\sqrt{-g}\,d^4x\,g^{\mu\alpha}g^{\nu\beta}\Big[F^{(0)}_{\mu\nu}F^{(0)}_{\alpha\beta}-2F^{(0)}_{\mu\nu}[\mathcal{S}^{-1}(\mathcal{A})]^\rho_{\ \alpha\beta\lambda[\nu\sigma]}\mathcal{A}^\nu F^{(0)\lambda\sigma}\mathcal{A}_\rho\\&+[\mathcal{S}^{-1}(\mathcal{A})]^\rho_{\ \mu\nu\lambda_1[\lambda_2\lambda_3]}[\mathcal{S}^{-1}(\mathcal{A})]^\sigma_{\ \alpha\beta\lambda_4[\lambda_5\lambda_6]}F^{(0)\lambda_1\lambda_3}F^{(0)\lambda_4\lambda_6}\mathcal{A}^{\lambda_2}\mathcal{A}^{\lambda_5}\mathcal{A}_\rho\mathcal{A}_\sigma\Big]\\&=-\frac{1}{4}\int\sqrt{-g}\,d^4x\,g^{\mu\alpha}g^{\nu\beta}F^{(0)}_{\kappa_1\kappa_2}F^{(0)}_{\kappa_3\kappa_4}\Big[\delta^{\kappa_1}_\alpha\delta^{\kappa_2}_\beta\delta^{\kappa_3}_\mu\delta^{\kappa_4}_\nu-2\delta^{\kappa_3}_\mu\delta^{\kappa_4}_\nu[\mathcal{S}^{-1}(\mathcal{A})]^{\rho\ \ \kappa_1[\nu\kappa_2]}_{\ \alpha\beta}\mathcal{A}_\nu\mathcal{A}_\rho\\&+[\mathcal{S}^{-1}(\mathcal{A})]^{\rho\ \ \kappa_1[\lambda_2\kappa_2]}_{\ \mu\nu}[\mathcal{S}^{-1}(\mathcal{A})]^{\sigma\ \ \kappa_3[\lambda_5\kappa_4]}_{\ \alpha\beta}\mathcal{A}_{\lambda_2}\mathcal{A}_{\lambda_5}\mathcal{A}_\rho\mathcal{A}_\sigma\Big].\end{aligned} \tag{3.31}$$

Note that in this case also torsion modifies the coefficient of the quadratic term in derivative and makes it dynamical in terms of matter field variables. The solution of the following equation

$$\Big[\delta^{\kappa_1}_\alpha\delta^{\kappa_2}_\beta\delta^{\kappa_3}_\mu\delta^{\kappa_4}_\nu-2\delta^{\kappa_3}_\mu\delta^{\kappa_4}_\nu[\mathcal{S}^{-1}(\mathcal{A})]^{\rho\ \ \kappa_1[\nu\kappa_2]}_{\ \alpha\beta}\mathcal{A}_\nu\mathcal{A}_\rho+[\mathcal{S}^{-1}(\mathcal{A})]^{\rho\ \ \kappa_1[\lambda_2\kappa_2]}_{\ \mu\nu}[\mathcal{S}^{-1}(\mathcal{A})]^{\sigma\ \ \kappa_3[\lambda_5\kappa_4]}_{\ \alpha\beta}\mathcal{A}_{\lambda_2}\mathcal{A}_{\lambda_5}\mathcal{A}_\rho\mathcal{A}_\sigma\Big]=0, \tag{3.32}$$

gives a non-trivial vacuum configuration. Like the earlier case, here also we can write the action $S_{\mathcal{A}}$ in (3.31) in the following form

$$S_{\mathcal{A}} = -\frac{1}{4}\int \sqrt{-g}d^4x\, \mathcal{O}^{\kappa_1\kappa_2\kappa_3\kappa_4}_{\mu\nu\alpha\beta}(\mathcal{A}) g^{\mu\alpha} g^{\nu\beta} F^{(0)}_{\kappa_1\kappa_2} F^{(0)}_{\kappa_3\kappa_4}$$
$$\mathcal{O}^{\kappa_1\kappa_2\kappa_3\kappa_4}_{\mu\nu\alpha\beta} = \Big[\delta^{\kappa_1}_{\alpha}\delta^{\kappa_2}_{\beta}\delta^{\kappa_3}_{\mu}\delta^{\kappa_4}_{\nu} - 2\delta^{\kappa_3}_{\mu}\delta^{\kappa_4}_{\nu}[\mathcal{S}^{-1}(\mathcal{A})]^{\rho\ \ \kappa_1[\nu\kappa_2]}_{\alpha\beta}\mathcal{A}_\nu\mathcal{A}_\rho \tag{3.33}$$
$$+ [\mathcal{S}^{-1}(\mathcal{A})]^{\rho\ \ \kappa_1[\lambda_2\kappa_2]}_{\mu\nu}[\mathcal{S}^{-1}(\mathcal{A})]^{\sigma\ \ \kappa_3[\lambda_5\kappa_4]}_{\alpha\beta}\mathcal{A}_{\lambda_2}\mathcal{A}_{\lambda_5}\mathcal{A}_\rho\mathcal{A}_\sigma\Big].$$

Finding the field equation from the above action is not that complicated. However, we want to point out that the field equation corresponding to the above action leads to a non-linear electrodynamics which has no gauge invariance because of the torsion.

4. Conclusion

Since the massless radiation fields have long-range behavior [75–77], therefore, probing certain physical observables in terms of them would be easier compared to massive fields because of their short-range nature (exponential decay after $\frac{1}{mass}$ length scale). Since the observed Standard Model fermion particles are massive particles (even neutrino oscillation phenomena shows the mass property of them), hence, it is difficult to probe the nature of the torsion of spacetime manifolds from their dynamics although they have a natural tendency to couple with torsion. On the other hand, the gauge bosons like Photons [78], scalar massless bosons like Higgs field [79] and critical bosonic excitations [80] are accessible as a tool to probe Torsion from various sources like near stars in the form of radiations. In this article, we have mentioned two non-trivial features of these fields, one of which is non-trivial vacuum configurations and break down of gauge invariance due to coupling with Torsion. From the Higgs mechanism [81–83] in Standard Model physics, it has been proved that these two features can be observed after a certain high energy scale, found in different collider experiments. This observation suggests to us that the strength of torsion coupling with radiation fields might be so weak in nature that to probe them, we need to go beyond energy scales that current colliders have achieved so far [84–94].

Furthermore, through our examples, we have shown that the non-trivial vacuum configuration leads to the mass gap of the system after which only the coupling of torsion can be probed since torsion itself depends on the dynamics of excitations about the mass-gap. This is actually huge as the mass-gap scale is inversely related to Newton's gravitation constant. These non-trivial vacuum configurations can also be understood as non-trivial saddle-points of on-shell action in which it is extremized. As a consequence, these vacuum configurations contribute maximally in the Feynman's path integral or functional integral formalism [95,96] in order to find generating functional or partition function.

Another interesting feature that we have shown in this article is that $U(1)$ gauge invariance of the photon which is an internal symmetry of the field is broken by torsion tensor, not from the non-trivial vacuum configuration. It is broken by a completely different spacetime feature which is nothing to do with diffeomorphism invariance, an internal symmetry of GR without the inclusion of Torsion. One of the important consequences of gauge symmetry breaking is the presence of longitudinal polarization of vector field which is recently observed in a quantum optics experiment from quenching the vacuum [97] and in [98,99].

Within a general and mathematically straightforward manner, the inclusion of torsion in GR has been explicitly shown in which modified terms that actually appear in the weak field limit of the Einstein equation lead to the break-down of infinitesimal diffeomorphism or gauge invariance. These terms appear as mass terms comparing with the Fierz-Pauli action. This suggests a new and completely different effect in dynamics and appearance of new degrees of freedom or more correctly new set of polarizations of gravitation waves or radiations other than the 2-polarizations in GR due to the torsion of spacetime [32,100] which is independent of the metric tensor of spacetime.

Showing these features in the on-shell action for radiation fields, we suggest a new way to check the correctness of the assumption of the torsionless connection in GR both through high energy physics experiments on radiation fields and from the observation of extra polarizations of gravitational waves, and their dynamics in short-wavelength scales where the effect of coupling with Torsion plays an important role.

Declaration of competing interest

The author declares that there is no conflict of interest.

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