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Quintessence background for $4D$ Einstein-Gauss-Bonnet black holes

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ABSTRACT

Our aim in this manuscript is to study $4D$ Einstein-Gauss-Bonnet (EGB) black hole (BH) surrounded by quintessence matter. We studied some thermodynamical properties of the model including Hawking temperature, entropy, Gibb's free energy and specific heat of the $4D$ EGB BH surrounded by quintessence matter. It is concluded that the Gibb's free energy is positive for all values of the horizon radius r_+. Therefore the BH is not stable globally. It is also noted that the specific heat for $r_+ < r^c$ the BH is locally stable and for $r_+ > r^c$ the BH is locally unstable.

1. Introduction

The recent astronomical observational data show that the universe is expanding with an accelerated rate [1–3]. It is believed that this expansion is due to a substantial negative pressure. There are two sources of negative pressure one is cosmological constant which acts is a repulsive force against the gravity and the other is the so called quintessence with equation of state which links the pressure p with the energy density ρ, so that $p = w\rho$, the parameter w of equation of state in the range $-1 \leq w \leq -\frac{1}{3}$ produces acceleration in the universe. The case $w = -1$ covers the cosmological constant. The earliest BH solution with quintessence matter was discussed by Kiselev [4]. This work was further elaborated to charged solution by Ainou [5]. Thermodynamics of the BH surrounded by quintessence matter was investigated in details in [6–10].

In the past few decades, a renewed interest has arisen in the investigation of higher order gravitational theories in higher dimensions due to M/superstring theory. The most widely studied model in the effective low energy action of superstring theory [11] is known as EGB gravitational theory [12]. Boulware and Deser [13] obtained the first BH solution in five dimensional EGB gravitational theory. The initiative work of [13], attracted many researchers [14–25] to investigate formation, thermodynamical properties and stability of the BH in the context of EGB gravitational theory. Recently, Ghosh et al. [26] studied Quintessence background for $5D$ EGB black holes (BHs).

It is widely known that the Gauss-Bonnet correction term L_{GB} to the Einstein-Hilbert action does not contribute to the gravitational dynamic for the spacetime having Dimensions $D < 5$. Therefore, in the 4-dimensional ($4D$) case the Euler-Gauss-Bonnet term becomes invariant and does not influence the gravitational dynamics. Glavan and Lin [27] formulated a novel $4D$ EGB gravitational theory by rescaling the Gauss-Bonnet term by $\alpha \to \frac{\alpha}{D-4}$. Motivated by the novel $4D$ EGB gravitational theory formulated by Glavan and Lin, a lot work related to the thermodynamics, formation and stability of the black holes has been done by many cosmologist and theoretical physicist. In modern physics this theory has a wide range of astrophysical and cosmological applications. Fernandes [28] has extended the work done by Glavan and Lin to charged BHs solutions. Ghosh and Maharaj [29] studied radiating BHs solutions in novel $4D$ EGB gravitational theory. Kumar and Ghosh [30] investigated Hayward regular BHs in $4D$ gravitational theory. Bardeen BHs was studied in [31]. Singh et al. [32] studied clouds of strings in $4D$ EGB BHs.

In this article, we investigate Black holes solutions in the novel $4D$ EGB gravitational theory surrounded by quintessence matter. The article is arranged as follows. In section 2, we formulated field equations and its solutions. We presented Thermodynamical properties of the $4D$ EGB BH surrounded by quintessence in section 3.

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2. Equations of motion in EGB gravitational theory

The EGB action can be written as

$$S = \frac{1}{16\pi}\int d^D x\sqrt{-g}[R + \alpha L_{GB}], \tag{1}$$

where R denotes the Ricci scalar and α is the Gauss-Bonnet constant. The Gauss-Bonnet Lagrangian L_{GB} is given by

$$L_{GB} = R^2 - 4R_{ab}R^{ab} + R_{abcd}R^{abcd}. \tag{2}$$

The variation of the above action given by (1) leads to the following equation of motion in EGB gravitational theory

$$G_{ab} + \alpha H_{ab} = T_{ab}, \tag{3}$$

where $G_{ab} = R_{ab} - \frac{1}{2}g_{ab}R$ is the Einstein tensor,

$$H_{ab} = 2\big[RR_{ab} - 2R_{ac}R^{c}_{b} - 2R^{cd}R_{acbd} + R^{cde}_{a}R_{bcde}\big] - \frac{1}{2}g_{ab}L_{GB}, \tag{4}$$

is the Lanczos tensor and T_{ab} is the stress energy momentum tensor. We consider a spherically symmetric spacetime defined by

$$ds^2 = -f(r)dt^2 + \frac{1}{f(r)}dr^2 + r^2 d\Sigma^2), \tag{5}$$

$$d\Sigma^2 = d\theta_1^2 + \sin^2\theta_1 d\theta_2^2 + ... + sin^2\theta_1 sin^2\theta_2 ..... \sin^2\theta_{D-3}d\theta^2_{D-2}. \tag{6}$$

For the above line element the EGB field equations (3), read as

$$\begin{aligned} T^0_0 = {} & \frac{D-2}{2}\left[\frac{f'}{r} + \frac{(D-3)f}{r^2} - \frac{(D-3)}{r^2}\right] - \frac{\alpha(D-2)(D-3)(D-4)}{2} \\ & \times\left[\frac{2ff'}{r^3} - \frac{2f'}{r^3} + \frac{(D-5)f^2}{r^4} - \frac{2(D-5)f}{r^4} + \frac{(D-5)}{r^4}\right], \end{aligned} \tag{7}$$

$$T^0_0 = T^1_1, \tag{8}$$

$$\begin{aligned} T^2_2 = {} & \frac{f''}{2} + (D-3)\left[\frac{f'}{r} + \frac{(D-4)f}{2r^2} - \frac{(D-4)}{2r^2}\right] - \frac{\alpha(D-3)(D-4)}{2} \\ & \times\left[\frac{2ff''}{r^2} - \frac{2f''}{r^2} + \frac{2f'^2}{r^2} + \frac{4(D-5)ff'}{r^3} - \frac{4(D-5)f'}{r^3}\right. \\ & \left. + \frac{(D-5)(D-6)f^2}{r^4} - \frac{2(D-5)(D-6)f}{r^4} + \frac{(D-5)(D-6)}{r^4}\right], \end{aligned} \tag{9}$$

$$T^2_2 = T^3_3 = ... = T^{(D-1)}_{(D-1)}. \tag{10}$$

The energy momentum tensor is given by [4]

$$T^0_0 = \rho(r), \tag{11}$$

$$T^b_a = \rho(r)\gamma\left[-(1+3B)\frac{r_a r^b}{r_n r^n} + B\delta^b_a\right], \tag{12}$$

where $B(r)$ denotes the quintessential parameter, we have

$$\langle T^b_a\rangle = -\rho(r)\frac{\gamma}{3}\delta^b_a = p(r)\delta^b_a, \tag{13}$$

and

$$\langle r_a r^b\rangle = \frac{1}{3}\delta^b_a r_n r^n. \tag{14}$$

Therefore we arrive at the following relation

$$p = w\rho, \quad w = \frac{1}{3}\gamma. \tag{15}$$

The quintessential state has $-1 \le w \le 0 \Rightarrow -3 \le \gamma \le 0$. In the present set up the free parameter B of the stress momentum tensor takes the form

$$B = -\frac{3w+1}{6w}. \tag{16}$$

For detail see [4] which sparks

$$T^0_0 = T^1_1 = \rho, \tag{17}$$

$$T^2_2 = T^3_3 = ... = T^{(D-1)}_{(D-1)} = -\frac{1}{2}\rho(3w+1). \tag{18}$$

In 4*D* case, the integral over the Gauss-Bonnet term is a topological invariant and does not influence the dynamics. However, according to [27], the coupling constant can be rescaled as

$$\alpha \to \frac{\alpha}{D-4}. \tag{19}$$

Now considering (19) with limit $D \to 4$ along with Eqs. (7), (9), (17) and (18) we get the following 4*D* EGB field equations

$$\rho = \frac{f'}{r} + \frac{f}{r^2} - \frac{1}{r^2} - \alpha\left[\frac{2ff'}{r^3} - \frac{2f'}{r^3} - \frac{f^2}{r^4} + \frac{2f}{r^4} - \frac{1}{r^4}\right], \tag{20}$$

$$-\frac{1}{2}\rho(3w+1) = \frac{f''}{2} + \frac{f'}{r} - \frac{\alpha}{2}\left[\frac{2ff''}{r^2} - \frac{2f''}{r^2} + \frac{2f'^2}{r^2} - \frac{4ff'}{r^3} + \frac{4f'}{r^3} + \frac{2f^2}{r^4} - \frac{4f}{r^4} + \frac{2}{r^4}\right]. \tag{21}$$

Now making use of (20) in (21), we get

$$\begin{aligned}&[r^2 f']' + (3w+1)[r(f-1)]' - \alpha\Bigg[2[(f-1)f']' - \left(\frac{f^2}{r}\right)' + 2\left(\frac{f}{r}\right)'\\ &+3w\left(\frac{f^2}{r}\right)' - 6w\left(\frac{f}{r}\right)' - \frac{3w-1}{r^2}\Bigg] = 0,\end{aligned} \tag{22}$$

the above equation has solution of the form given by

$$f(r)_{\pm} = 1 + \frac{r^2}{2\alpha}\left[1 \pm \sqrt{1 + \frac{8\alpha M}{r^3} + \frac{8\alpha q}{r^{3w+3}}}\right], \tag{23}$$

for $q = 0$ (23) reduces to the BH solution obtained in [27]. To investigate the general structure of the solution given by (23), we consider $M = q = 0$ or the limit $r \to \infty$ in Eq. (23), we get

$$\lim_{r\to\infty} f_+(r) = 1 + \frac{r^2}{\alpha}, \quad \lim_{r\to\infty} f_-(r) = 1. \tag{24}$$

In Eq. (23) the "±" sign represents two branches of solutions. For larger r Eq. (23) agrees with 4*D* Schwarzschild solution surrounded by the quintessence matter. Hence, the minus branch solution is well behaved. Therefore from here on we consider the negative branch solution in further investigations.

In order to discuss the BH horizons surrounded by quintessence matter by definition of horizon, the largest value of the radius $r = r_+$ is called an event horizon if $f(r) = 0$. The BH hole admits three positive real horizons, namely: BH event horizon r_+, Cauchy horizon r_c and quintessential cosmological horizon r_q. In Fig. 1 and Fig. 2 the plot of the function $f(r)$ is shown in details for various values of the parameters, w, q and α. From Fig. 1, we see that for $w = -0.35$ there are two horizons i.e., quintessential cosmological horizon and BH event horizon. It is clear from Fig. 1 and Table 1 that by increasing the value of q the BH event horizon increases and the quintessential cosmological horizon decreases and with increasing α The BH event horizon decreases and quintessential cosmological horizon increases. From Fig. 1 and Table 2 we see that for $w = -0.45$ there are two horizons quintessential cosmological horizon and BH event horizon, by increasing the value of q the BH horizon disappears. From Fig. 2 and Table 3 we noted that there are three horizons for $w = -0.55$ and $q = 0.1$, with increasing q the BH horizon and Cauchy horizon disappears. From Fig. 2 and Table 4 we noted that for $w = -0.65$ there is only one horizon. Our results are consistent with the results given in Ref. [33]. In this work, we are mainly interested in BH thermodynamics, we have considered those values of parameters for which BH event horizon forms.

3. Thermodynamics

Next, we will investigate the thermodynamical quantities like Hawking temperature, entropy, Gibb's free energy and Specific heat related with the BH horizon r_+. In order to obtain the mass of the BH we set $f(r_+) = 0$, leads us to the following relation

$$M_+ = \frac{r_+}{2}\left(1 + \frac{\alpha}{r_+} - \frac{2q}{r_+^{3w+1}}\right), \tag{25}$$

for $q = 0$ it reduces to the result obtained by [28] for the 4*D* EGB BHs

$$M_+ = \frac{r_+}{2}\left(1 + \frac{\alpha}{r_+^2}\right), \tag{26}$$

and for vanishing coupling parameter it agrees with the mass of the Schwarzschild BHs [34,35]

$$M_+ = \frac{r_+}{2}. \tag{27}$$

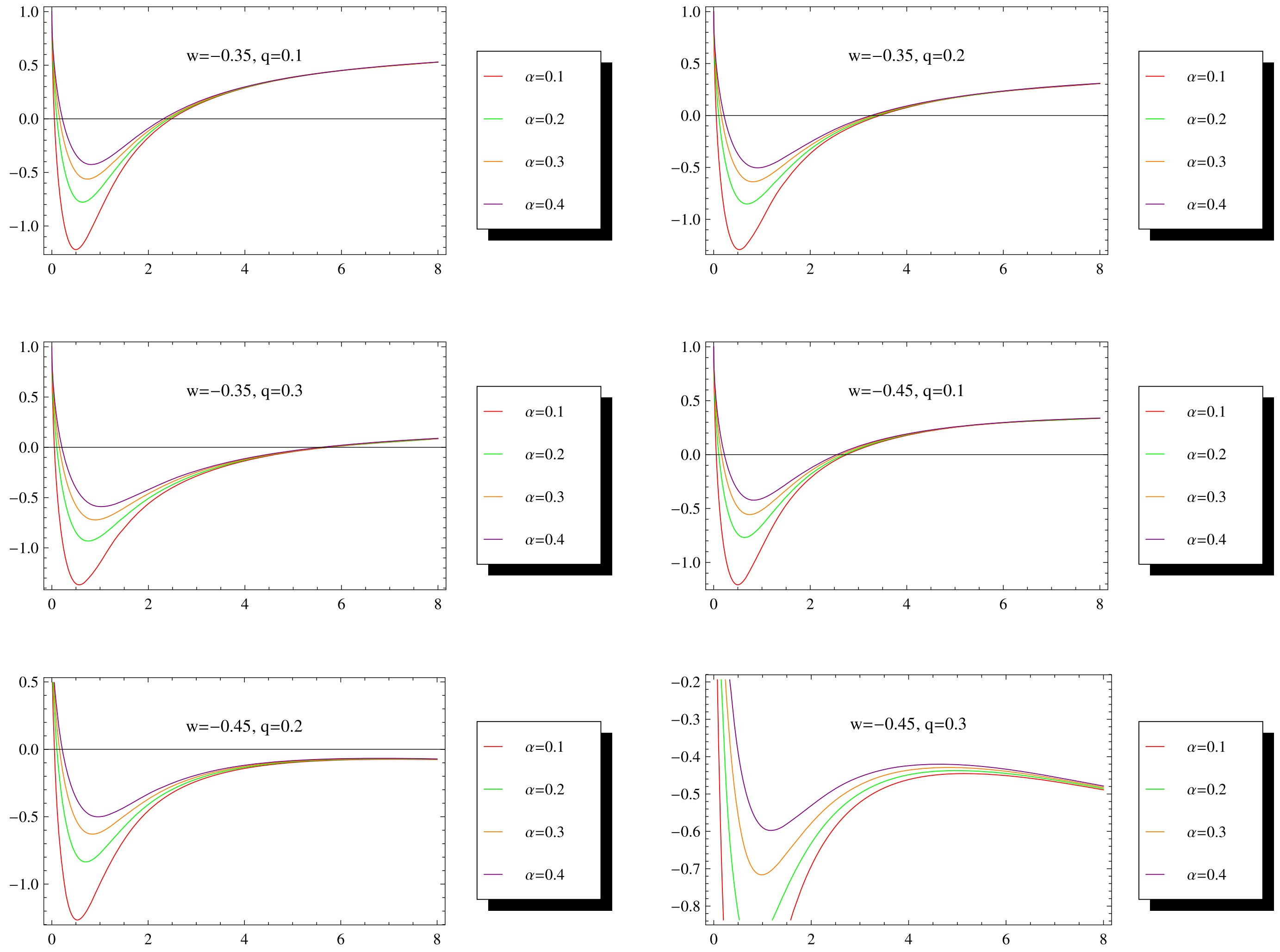

Fig. 1. Performance of metric function versus radius r for various values of parameters.

Table 1
The above table demonstrates Cosmological horizon r_q and Black hole horizon r_+ for $M = 1$ and $w = -0.35$.

$q = 0.1$			$q = 0.2$			$q = 0.3$		
α	r_q	r_+	α	r_q	r_+	α	r_q	r_+
0.1	0.0510797	2.478330	0.1	0.0508472	3.43010	0.1	0.506190	5.74332
0.2	0.1448300	2.424332	0.2	0.1034390	3.37574	0.2	0.102438	5.68652
0.3	0.1605340	2.367680	0.3	0.1579210	3.31953	0.3	0.155479	5.62859
0.4	0.2196510	2.308000	0.4	0.2144820	3.26130	0.4	0.209795	5.56948

Table 2
The above table demonstrates Cosmological horizon r_q and Black hole horizon r_+ for $M = 1$ and $w = -0.45$. NH stand for no horizon.

$q = 0.1$			$q = 0.2$			$q = 0.3$		
α	r_q	r_+	α	r_q	r_+	α	r_q	r_+
0.1	0.051219	2.74551	0.1	0.511221	NH	0.1	0.051024	NH
0.2	0.105013	2.68377	0.2	0.104467	NH	0.2	0.103934	NH
0.3	0.161690	2.61925	0.3	0.161180	NH	0.3	0.158617	NH
0.4	0.221668	2.55151	0.4	0.218220	NH	0.4	0.215017	NH

The Hawking temperature can be calculated by the formula $T = \frac{k}{2\pi}$, where k stands for surface gravity defined by [36–38]

$$k^2 = -\frac{1}{4} g^{tt} g^{ab} g_{tt,a} g_{tt,b}. \tag{28}$$

Hence we get the following relation for the $4D$ EGB BH surrounded by quintessence matter

$$T_+ = \frac{r_+^{3w+1}(r_+^2 - \alpha) + 6wqr_+^2}{4\pi r_+^{3w+2}(r_+^2 + 2\alpha)}. \tag{29}$$

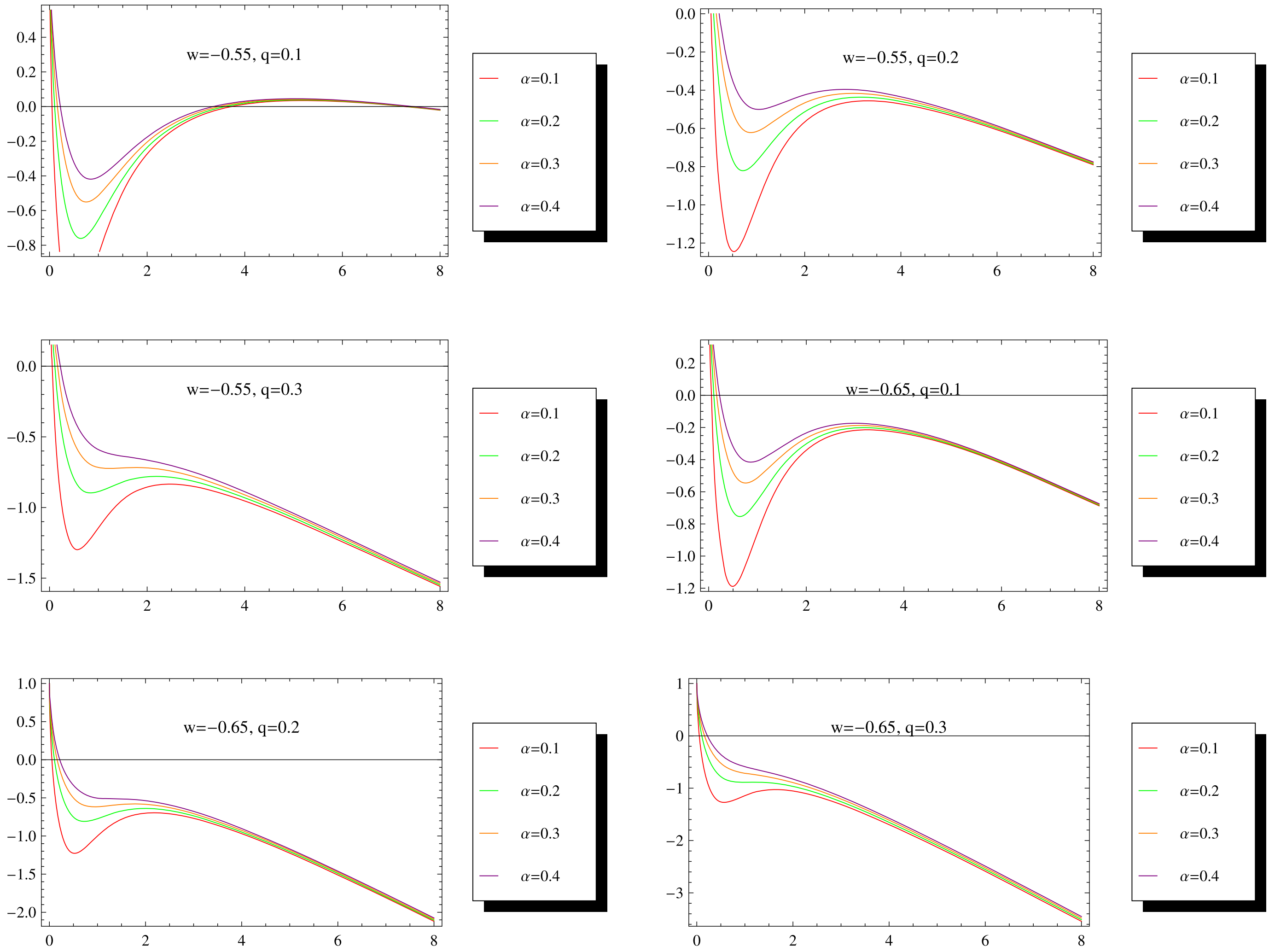

Fig. 2. Performance of metric function versus radius r for various values of parameters.

Table 3
The above table demonstrates Cosmological horizon r_q, Cauchy horizon r_c and Black hole horizon r_+ for $M = 1$ and $w = -0.55$.

$q = 0.1$				$q = 0.2$			$q = 0.3$		
α	r_q	r_c	r_+	α	r_q	r_+	α	r_q	r_+
0.1	0.2229870	3.35300	7.48170	0.1	0.0512365	NH	0.1	0.0511966	NH
0.2	0.1623740	3.47670	7.42175	0.2	0.1050030	NH	0.2	0.1047250	NH
0.3	0.1052860	3.59975	7.35898	0.3	0.1614380	NH	0.3	0.1605310	NH
0.4	0.0512765	3.72300	7.29302	0.4	0.2207070	NH	0.4	0.2185900	NH

Table 4
The above table demonstrates Cosmological horizon r_q and Black hole horizon r_+ for $M = 1$ and $w = -0.65$.

$q = 0.1$			$q = 0.2$			$q = 0.3$		
α	r_q	r_+	α	r_q	r_+	α	r_q	r_+
0.1	0.0513002	NH	0.1	0.0512837	NH	0.1	0.0512673	NH
0.2	0.1054260	NH	0.2	0.1052810	NH	0.2	0.1051370	NH
0.3	0.1627760	NH	0.3	0.1622230	NH	0.3	0.1616820	NH
0.4	0.2238440	NH	0.4	0.2223490	NH	0.4	0.2209130	NH

For $q = 0$ the above relation for the temperature reduces to the $4D$ EGB BH [28]

$$T_+ = \frac{1}{4\pi r_+}\left(\frac{r_+^2 - \alpha}{r_+^2 + 2\alpha}\right), \tag{30}$$

and in limit $\alpha \to 0$ we get the temperature of the Schwarzschild BH [34,35]

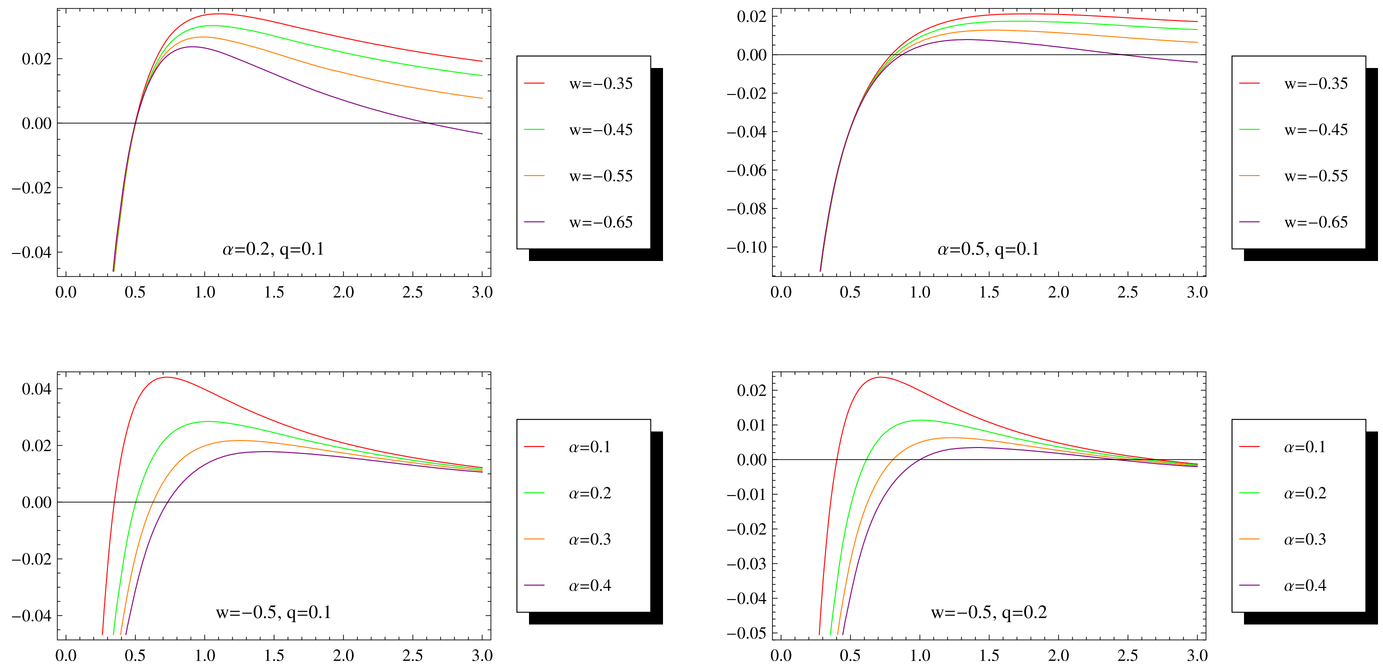

Fig. 3. Performance of Hawking temperature T_+ versus horizon radius r_+ for various values of parameters.

$$T_+ = \frac{1}{4\pi r_+}, \tag{31}$$

The behavior of temperature T_+ versus the horizon radius r_+ for various values of w, α and q is plotted in Fig. 3 which sparks that near the center of the BH the Hawking temperature T_+ is negative. The temperature T_+ increases very fastly with increasing the horizon radius r_+ and after gaining a maximum value it decreases smoothly with increasing r_+. It is also depicted that for decreasing w the T_+ decreasing fastly and by increasing the q the Hawking temperature again becomes negative.

Now using the first law of thermodynamics $dM_+ = T_+ dS_+$ we obtained the following expression for the BH entropy [39–41]

$$S_+ = \frac{A}{4}\left[1 + \frac{4}{r^2}\alpha \log(r_+)\right], \tag{32}$$

which does not lead to the usual entropy area law $S_+ = \frac{A}{4}$ [37], here $A = 4\pi r_+^2$ represents area of the BH horizon. This result consistent with the result obtained by Kumar and Ghosh [30]. It is worthwhile to note that the quintessence matter has no effects on the entropy of the $4D$ EGB BH. Entropy formula we borrowed is quite same as the one consistent with the AH of an FRW solution given in Ref. [42], which carried out by taking the relation between modified field equation and first law of thermodynamics [43]. Generally, it is considered that the said entropy formula associated with AH of an FRW Universe is the same as one of BH horizon in the same gravity theory [44]. Secondly, a logarithmic term is universally present as a sub-leading correction to the Bekenstein-Hawking area entropy, in the microscopic statistical interpretation of BH entropy such as in string theory, loop quantum gravity, thermal and/or quantum fluctuations in a fixed BH background (see references cited in [43]). For more detail about Eq. (32) can be found in Ref. [45].

For $\alpha \to 0$ it reduces to the entropy of Schwarzschild BH

$$S_+ = \pi r_+^2. \tag{33}$$

In order to discuss the global stability of the BH we use the Gibb's free energy defined as $F_+ = M_+ - T_+ S_+$ [34]. A thermodynamical system is globally stable if the Gibb's free energy for the system is negative, otherwise it will be globally unstable [34]. For our model the Gibb's free energy yields the relation given by

$$F_+ = \frac{r_+}{2} + \frac{\alpha}{2r_+} - \frac{r_+(r_+^2 - \alpha)}{4(r_+^2 + 2\alpha)} - \frac{\alpha \log(r_+)}{r_+(r_+^2 + 2\alpha)} - \frac{q}{r_+^{3w}} - \frac{3wq}{2r_+^{3w-2}(r_+^2 + 2\alpha)} - \frac{6\alpha q \log(r_+)}{r_+^{3w}(r_+^2 + 2\alpha)}. \tag{34}$$

For $q = 0$ it reduces to the $4D$ EGB BH given by

$$F_+ = \frac{r_+}{2} + \frac{\alpha}{2r_+} - \frac{r_+(r_+^2 - \alpha)}{4(r_+^2 + 2\alpha)} - \frac{\alpha \log(r_+)}{r_+(r_+^2 + 2\alpha)}, \tag{35}$$

the above equation in the limit $\alpha \to 0$ reduces to the Gibb's free energy for the Schwarzschild Black holes [34]

$$F_+ = \frac{r_+}{4}, \tag{36}$$

The Gibb's free energy F_+ versus the horizons radius r_+ is plotted in Fig. 4 which shows that the $4D$ EGB BH is globally unstable.

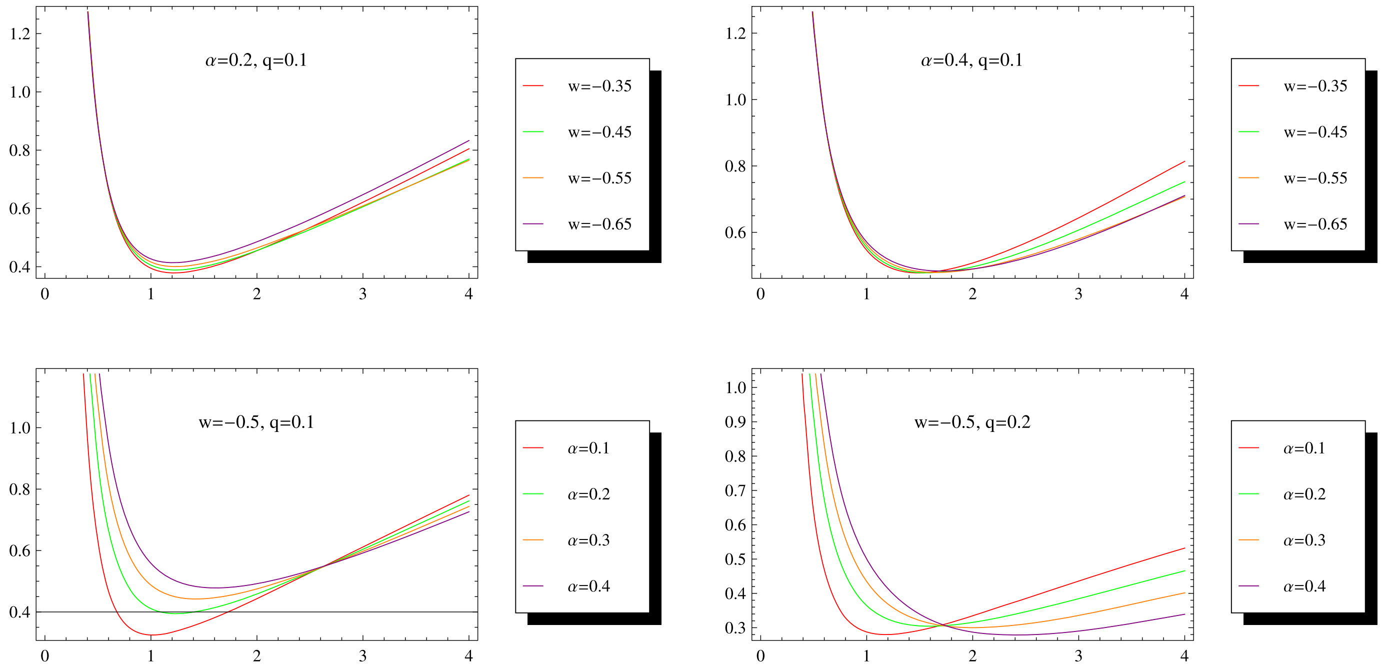

Fig. 4. Performance of Gibb's free energy F_+ versus horizon radius r_+ for various values of parameters.

The specific heat defined as $C_+ = \frac{dM_+}{dT_+} = \frac{\partial M_+}{\partial r_+} \partial r_+ \partial T_+$ [40,41] gives us information about the local thermodynamical stability of the BH. A thermodynamical system is locally stable if $C_+ > 0$ and unstable if $C_+ < 0$. The specific heat for our model takes the following form

$$C_+ = -\frac{2\pi \left(r_+^{3w+1} - \alpha r_+^{3w-1} + 6wq\right)\left(r_+^2 + 2\alpha\right)^2}{r_+^{3w+3} - 5\alpha r_+^{3w+1} + 12wqr_+^2 - 2\alpha^2 r_+^{3w-1} + 18w^2\alpha r_+^2 + 36\alpha w^2 q}. \tag{37}$$

For $q = 0$ we get the specific heat of the $4D$ EGB BHs

$$C_+ = -\frac{2\pi r_+^2 \left(1 - \frac{\alpha}{r_+^2}\right)\left(r_+^2 + 2\alpha\right)^2}{r_+^4 - \alpha\left(5r_+^2 + 2\alpha\right)}. \tag{38}$$

In the limit $\alpha \to 0$ we obtained the specific heat for the Schwarzschild BH [34,35]

$$C_+ = -2\pi r_+^2, \tag{39}$$

In Fig. 5 we graphically demonstrated the behavior of specific heat for various values of w and α. From the graphical representation of the specific heat it is clear that the sign of C_+ changes around the critical parameter r^c and at $r_+ = r^c$, C_+ is discontinuous. The specific heat $C_+ > 0$ for $r_+ < r^c$. Therefore the BH is locally stable for $r_+ < r^c$ and the specific heat $C_+ < 0$ for $r_+ > r^c$, which suggest that the BH is locally unstable. Thus, the specific heat of a $4D$ EGB BH, for various values of w and α, is negative for $r_+ > r^c$ and is positive for $r_+ < r^c$. The phase transition happens from a lower mass BH having negative specific heat to higher mass BH having positive specific heat.

Furthermore, it is also worthwhile to mention that the critical radius r^c varies drastically due to the presence of quintessence matter, it is concluded that the quintessence matter strongly affects the thermodynamical stability of the BH. The value of r^c increases by increasing the value of w for a fixed value of the Gauss-Bonnet coupling constant α. The critical radius r^c is also increases by increasing α for fixed w.

4. Conclusion

In the present paper, we have investigated thermodynamical aspects of the $4D$ EGB BH surrounded by quintessence matter. We have solved the $4D$ EGB field equation with quintessence matter. It is found that the horizons of the BH strongly depend on the quintessence parameter w, there are two horizons, event horizon and quintessence horizon. The BH horizon disappears by decreasing the value of the quintessence parameter w.

We also studied the behavior of the Hawking temperature versus the horizon radius r_+ for various values of w, α and q is plotted in Fig. 3 which sparks that near the center of the BH the Hawking temperature T_+ is negative and with increasing r_+ it increases very fastly and becomes positive. After gaining a maximum value it decreases smoothly. The stability of the $4D$ EGB BH surrounded by quintessence is also analyzed, using the definition of Gibb's free energy and the specific heat. It is noted that the Gibb's free energy is positive in the entire domain which sparks that the BH is globally unstable. The specific heat $C_+ > 0$ for $r_+ < r^c$. Therefore the BH is locally stable for $r_+ < r^c$ and the specific heat $C_+ < 0$ for $r_+ > r^c$, which suggest that the BH is locally unstable.

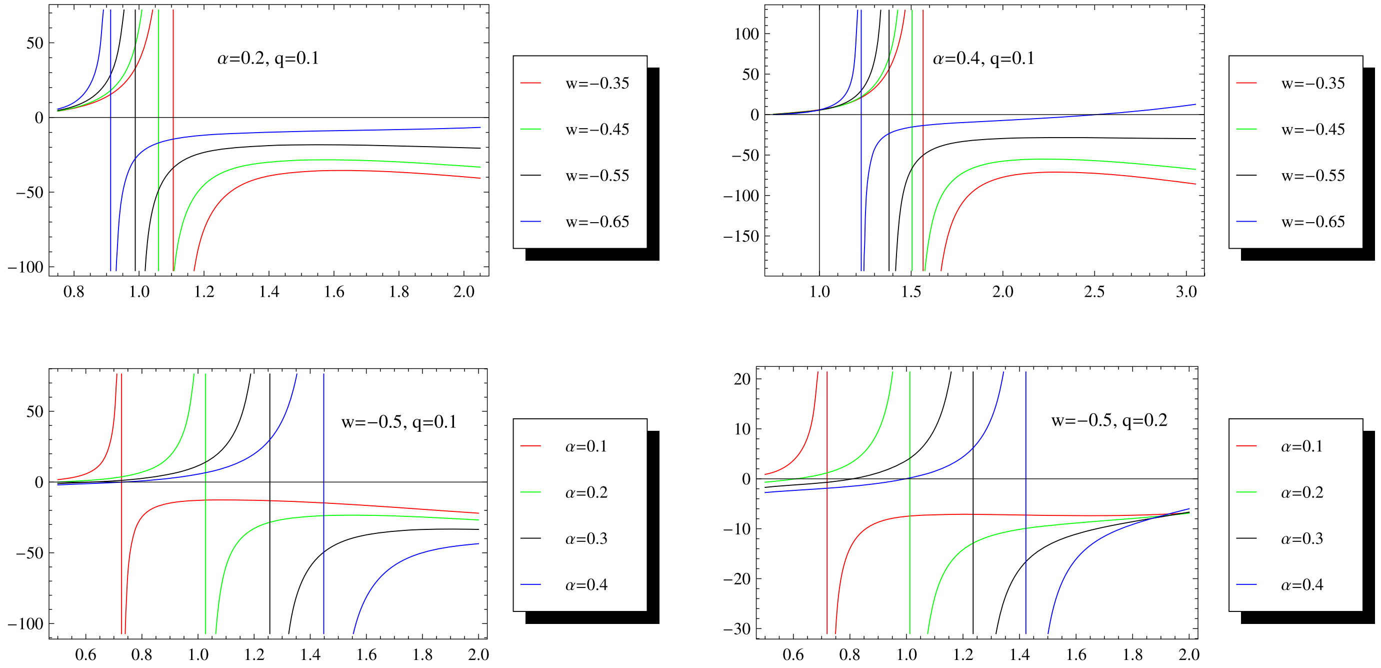

Fig. 5. Performance of specific heat C_+ versus horizon radius r_+ for various values of parameters.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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