

# Structure of $\Lambda$ -Hypernuclei with Gogny type N-N and Y-N Interactions

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## Introduction

$\Lambda$ -hypernuclei constitutes the simplest of strange nuclei systems ( $S=-1$ ). For its theoretical description, both microscopic and phenomenological approaches have been employed [1]. In this work, we have studied the structure of  $\Lambda$ -hypernuclei through a phenomenological approach using self-consistent Hartree-Fock-Bogoliubov (HFB) calculations with Gogny force as an effective interactions in both  $N-N$  and  $\Lambda-N$  channels.

## Methodology

The Hamiltonian describing the  $\Lambda$ -hypernuclei can be written in second quantisation as,

$$\begin{aligned} \hat{H} = & \sum_{i,j} t_{ij}^N \hat{a}_i^\dagger \hat{a}_j + \sum_{i,j} t_{ij}^\Lambda \hat{c}_i^\dagger \hat{c}_j - \hat{T}_{c.m.} \\ & + \frac{1}{4} \sum_{ijkl} \bar{V}_{ijkl}^{NN} \hat{a}_i^\dagger \hat{a}_j^\dagger \hat{a}_l \hat{a}_k + \sum_{ijkl} V_{ijkl}^{\Lambda N} \hat{a}_i^\dagger \hat{c}_j^\dagger \hat{c}_l \hat{a}_k \end{aligned} \quad (1)$$

where creation (annihilation) operators  $\hat{a}_i^\dagger$  ( $\hat{a}_i$ ) for nucleons and  $\hat{c}_i^\dagger$  ( $\hat{c}_i$ ) for the  $\Lambda$  hyperon respectively,  $t_{ij}^N$  and  $t_{ij}^\Lambda$  are the nucleon and  $\Lambda$  hyperon kinetic energy matrix elements respectively,  $\bar{V}_{ijkl}^{NN}$  is the two body matrix element describing the anti-symmetrised  $N-N$  interaction,  $V_{ijkl}^{\Lambda N}$  is the two body matrix element describing the direct  $\Lambda-N$  interaction and  $\hat{T}_{c.m.}$  is the center of mass correction to total energy.

We have solved the Hamiltonian given in Eq. (1) in the framework of HFB method. Here, the minimisation of the total energy of  $\Lambda$  hypernuclei must also include the single particle energy of  $\Lambda$  hyperon in addition to the Hartree-Fock and pairing energies of nucleons. For the  $N-N$  interaction  $\bar{V}_{ijkl}^{NN}$ , the standard Gogny D1S parametrisation was used [2]. For the  $\Lambda-N$  interaction  $V_{ijkl}^{\Lambda N}$ , we used only the central term of Gogny force without also taking into consideration the isospin dependence. That is,

$$V(\vec{r}_1, \vec{r}_2) = \sum_{i=1,2} e^{-(r_{12}/\mu_i)^2} (W_i + B_i \hat{P}_\sigma) \quad (2)$$

This is justified because of weak spin-orbit part of  $\Lambda-N$  interaction and electrically neutral nature of  $\Lambda$  hyperon. The six unknown parameters ( $\mu_1$ ,  $\mu_2$ ,  $W_1$ ,  $B_1$ ,  $W_2$  and  $B_2$ ) in the Eq. (2) were determined through a  $\chi^2$  minimisation procedure called “Simulated Annealing method” (SAM) and the chosen observable for fitting is the experimental  $\Lambda$  single particle binding energies in various  $\Lambda$ -hypernuclei. The hypernuclear calculations performed in this work were done by incorporating the required modifications in *HFBaxial* code [3].

## Results and discussion

We have obtained various  $\Lambda-N$  Gogny parameter sets using SAM fitting protocol. In Fig. 1 we present our predicted binding energy (BE) results for  $1l$  ( $l = s, p, d, f$  and  $g$ )  $\Lambda$  states in  $\Lambda$ -hypernuclei with mass number  $A$

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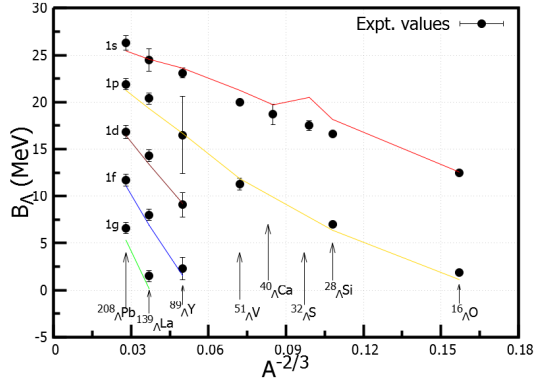


FIG. 1: Comparison of  $\Lambda$  single particle binding energy ( $B_\Lambda$ ) predictions with experimental values for  $\Lambda$ -hypernuclei with mass number  $A = 16$  to 208.

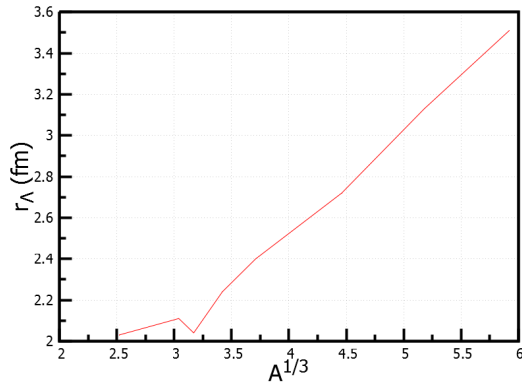


FIG. 2: Rms radii of ground state  $\Lambda$  orbitals ( $r_\Lambda$ ) in  $\Lambda$ -hypernuclei as a function of  $A^{1/3}$ .

$= 16$  to 208 using one such obtained  $\Lambda - N$  parameter set. The BE predictions with simplified  $\Lambda - N$  Gogny interaction are found to be in general reasonable agreement with the experimental data. In addition, we have observed a “kink” of 1s-shell  $\Lambda$  BE at  $^{32}_\Lambda\text{S}$  for all the obtained  $\Lambda - N$  parameter sets. This might be because HFB-Gogny quadrupole ( $\beta_2$ ) potential energy surface of  $^{32}_\Lambda\text{S}$  has a slightly broad energy minima extending from oblate to prolate region similar to the one observed for its ordinary nuclei counterpart  $^{32}\text{S}$  [4] and, the obtained BE result could have been improved

if these fluctuations in  $\beta_2$  degrees of freedom were taken into an account.

The results for root mean square (rms) radii of ground state  $\Lambda$  orbitals (1s-shell) are shown in Fig. 2. The values are in agreement with some hypernuclear mean field studies carried out using Skyrme forces [5]. Again, the “kink” in  $r_\Lambda$  was observed for  $^{32}_\Lambda\text{S}$  because of its correlation with the binding energy value. For the medium to heavy mass regions,  $r_\Lambda$  was found to be almost linearly increasing with the cubic root of the mass number  $A^{1/3}$  similar to the dependence of nuclei radii on  $A^{1/3}$ .

## Conclusions

The present work constitutes the first hypernuclear mean field studies performed using Gogny force in both  $N - N$  and  $\Lambda - N$  channels and the obtained BE predictions are in reasonable agreement with the experimental data. More refined calculations using density dependent term of Gogny force in  $\Lambda - N$  channel and the extension of the present study to multi-strange nuclei ( $S=-2$ ) will be performed in future.

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