

## Ground State Spin-Parity Assignments of Deformed Odd-A Nuclei

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### Introduction

The spherical shell models have been successful in ground state spin-parity assignments to spherical or near-spherical nuclei but fail in case of well deformed nuclei having large intrinsic quadrupole moment. In such cases, we have to resort to the models having non-spherical nuclear potential. In present work, we performed Nilsson model calculations [1] and predicted the ground states (i.e. single particle states with appropriate spin-parity assignments) of odd-A nuclei in rare-earth mass region ( $151 < A < 191$ ) and also compared the results with compared the available experimental data [2].

### Methodology

In the present article, we considered the Modified Harmonic Oscillator (MHO) potential with strong spin-orbit coupling and appropriate correction ( $l^2 - \langle l^2 \rangle$ ). The explicit form MHO Hamiltonian is given as [1]:

$$H_{sp} = -\frac{\hbar}{2M} \nabla^2 + \frac{1}{2} M \left[ \omega_{\perp}^2 (x^2 + y^2) + \omega_z^2 \right] \quad (1)$$

where,  $\omega_{\varepsilon} = \omega_0 \left[ 1 - \frac{2}{3} \varepsilon_2 \right]$  and  $\omega_{\perp} = \omega_0 \left[ 1 + \frac{1}{3} \varepsilon_2 \right]$

are oscillator frequencies and  $\varepsilon_2$  is the quadrupole deformation.

The explicit form of Nilsson model potential containing spin-orbit interaction and correction terms is given in below listed equation (2) [1]. In this equation,  $P_3$ ,  $P_4$ ,  $P_6$  and  $\varepsilon_3$ ,  $\varepsilon_4$ ,  $\varepsilon_6$  represent the Legendre polynomials, octupole, hexadecapole and hexinda-tesserapole deformations, respectively. The  $k$  &  $\mu$  are potential strength parameters.

$$H = -\frac{1}{2} \hbar \omega_0 \Lambda + \frac{1}{2} \hbar \omega r^2 \left[ 1 - \frac{2}{3} \varepsilon_2 \left[ \frac{4\pi}{5} \right]^{1/2} \times \left[ Y_{20} \cos \gamma - (Y_{22} + Y_{2-2}) \frac{\sin \gamma}{\sqrt{2}} \right] + 2\varepsilon_3 P_3 + 2\varepsilon_4 P_4 + 2\varepsilon_6 P_6 \right] - k \hbar \omega_0 \left[ 2\vec{l}_i \cdot \vec{s} + \mu (l_i^2 - \langle l_i^2 \rangle) \right] \quad (2)$$

The Nilsson's state is characterized by quantum number  $\Omega^{\pi} [N n_z \Lambda] \Sigma$  where,  $\Omega$  is the projection of total angular momenta on symmetric axis,  $N$  is the principal quantum number,  $n_z$  is the number nodes in wavefunction along z-axis and  $N = n_{\perp} + n_z$ , where  $n_{\perp} = n_x + n_y$  is the sum of oscillator quanta along the  $x$  and  $y$  axis. The quantum number  $\Lambda$  and  $\Sigma$  are the projection of orbital angular momenta and spin angular momenta single particle on symmetric axis. In present formulation, the  $\Omega = \Lambda + \Sigma$ , which represents the projection of the particle total angular momenta ( $j$ ) on the symmetric axis is the only good quantum number.

### Results and discussion

In present work, we performed Nilsson model calculations to predict the ground state spin-parity and Nilsson state of odd-A nuclei in rare-earth mass region ( $151 < A < 191$ ). The model predictions are compared with the experimental data extracted from Evaluated Nuclear Structure Data Files [2]. We considered only the prolate nuclei having significant quadrupole deformation ( $\varepsilon_2 > 0.15$ ) [3] and extracted 123 odd-Z nuclides lying in the rare-earth mass region ( $151 < A < 191$ ). Among these 123 nuclides, the experimental data of ground state spin and parity are available only for 100 nuclei. On the basis of present Nilsson

model calculations, we successfully reproduced the experimentally observed ground state spin-parity of 57 nuclides and suggested appropriate Nilsson states for the cases where experimental data is not available (Table-1)

Similarly, for odd-N nuclei, we identified total 123 systems and experimental data (ground state spin and parity) of 102 cases are available [3]. On the basis of present calculations, we successfully reproduced the ground state spin and parity of 43 nuclides and predicted for 13 nuclei where experimental data is not available (Table-2). Complete testing of present model calculations and identification of possible reasons for violation are in progress.

Table 1: Predicted Nilsson states in odd-Z nuclei

S.No.	Nuclide	Nilsson State $\Omega^\pi [Nn_z\Lambda]\Sigma$
1.	<sup>151</sup> Cs	1/2 <sup>+</sup> [420] ↑
2.	<sup>153</sup> La	1/2 <sup>-</sup> [550] ↑
3.	<sup>155</sup> La	1/2 <sup>-</sup> [550] ↑
4.	<sup>157</sup> La	1/2 <sup>-</sup> [550] ↑
5.	<sup>153</sup> Pr	3/2 <sup>-</sup> [541] ↑
6.	<sup>155</sup> Pr	3/2 <sup>-</sup> [541] ↑
7.	<sup>161</sup> Pr	3/2 <sup>-</sup> [541] ↑
8.	<sup>163</sup> Eu	5/2 <sup>+</sup> [413] ↓
9.	<sup>165</sup> Eu	5/2 <sup>+</sup> [413] ↓
10.	<sup>171</sup> Tb	3/2 <sup>+</sup> [411] ↑

Table 2: Predicted Nilsson states in odd-N nuclei

S.No.	Nuclide	Nilsson State $\Omega^\pi [Nn_z\Lambda]\Sigma$
1.	<sup>153</sup> Ba	5/2 <sup>-</sup> [523] ↓
2.	<sup>153</sup> Ce	5/2 <sup>+</sup> [642] ↑
3.	<sup>155</sup> Ce	5/2 <sup>-</sup> [523] ↓
4.	<sup>163</sup> Nd	5/2 <sup>-</sup> [512] ↑
5.	<sup>161</sup> Sm	1/2 <sup>-</sup> [521] ↓
6.	<sup>167</sup> Sm	7/2 <sup>-</sup> [514] ↓
7.	<sup>165</sup> Gd	7/2 <sup>+</sup> [633] ↑
8.	<sup>171</sup> Gd	9/2 <sup>+</sup> [624] ↑
9.	<sup>171</sup> Dy	7/2 <sup>-</sup> [514] ↓
10.	<sup>175</sup> Dy	1/2 <sup>+</sup> [651] ↓
11.	<sup>185</sup> Yb	9/2 <sup>-</sup> [505] ↑
12.	<sup>185</sup> Hf	3/2 <sup>-</sup> [512] ↓
13.	<sup>167</sup> W	3/2 <sup>+</sup> [651] ↑

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## References

- [1] S.G. Nilsson *et al.*, Nucl. Phys. A, 131, 1 (1969).
- [2] Evaluated Nuclear Structure Data File (ENSDF-Sept.-2022) available at [www.nndc.bnl.gov](http://www.nndc.bnl.gov)
- [3] P. Moller *et al.*, Atomic Data and Nuclear Data Tables, 109, 1 (2016).