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THE INTERCEPT OF THE POMERON*

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ABSTRACT

We show that in Reggeon field theory the intercept of the Pomeron must be less than or equal to one. The mechanism responsible is an instability of the Reggeon vacuum when the bare intercept exceeds a critical value. At that critical value alone is the vacuum singularity at J=1 when t=0.

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Reggeon field theories[1] provide a constructive procedure for evaluating the importance of Reggeon cut corrections to a basic Regge pole exchange. When the intercept $\alpha(0)$ of the renormalized ($\underset{\sim}{P}$) singularity is precisely one, then multiple $\underset{\sim}{P}$ exchange modifies the large energy behavior of scattering amplitudes only by powers of log s; however, since all P exchanges are important, one must systematically sum all contributions. The use of the renormalization group for the Reggeon field theories is a powerful device for this.[2,3] Assuming that $\alpha(0)=1$, one finds scaling laws for multi-$\underset{\sim}{P}$ vertex functions. Total cross sections behave as

$$\sigma_T(s) \sim (\log s)^{-\gamma} \quad , \tag{1}$$

where γ, the anomalous dimension of the $\underset{\sim}{P}$ field $\phi(\vec{x}, \tau)$, has been estimated[2-5] to be $1/4 \leq -\gamma \leq 1/2$.

Unitarity in the t-channel is built into Reggeon field theory. It is an important matter to determine whether the constraints of s-channel unitarity are obeyed as well. Some s-channel requirements are rather simple to state: the Froissart bound, for example, requires $\alpha(0) \leq 1$, and $-\gamma \leq 2$ when $\alpha(0)=1$. For the case $\alpha(0)=1$, all numerical estimates of $-\gamma$ are less than two, and a general argument for this has been given by Cardy and Sugar in their study of lattice versions of the field theory.[6] The more stringent requirement $\alpha(0) \leq 1$ is not an obvious feature of the theory. One must inquire whether $\alpha(0) \leq 1$ for all values of the intercept α_0 of the basic or unrenormalized Pomeron. Clearly, if there are no Reggeon interactions, then when α_0 becomes larger than one, every $\underset{\sim}{P}$ and multi-$\underset{\sim}{P}$ exchange violates the Froissart bound. To restore it takes a special arrangement of the multi-$\underset{\sim}{P}$ couplings to particles as occurs, for example, in the eikonal formalism. In this paper we show that Reggeon interactions force

$\alpha(0) \leq 1$ whatever α_0 may be. Of course, more detailed tests of s-channel unitarity must be passed, but this is certainly crucial.

We study the Reggeon field theory appropriate to the very high energy behavior of processes involving $\underset{\sim}{P}$ exchange. The $\underset{\sim}{P}$ field $\phi(\vec{x}, \tau)$ is defined on a D dimensional impact parameter space, $\vec{x}$, and a one dimensional time (rapidity) space, τ. These variables are conjugate to the Reggeon momentum $\vec{q}$ (momentum transfer is $-\vec{q}^{\,2}$) and energy E=1-J. The action for the theory is:

$$A = \int d^D x \, d\tau \left\{ \frac{i}{2} \phi^+ \frac{\overleftrightarrow{\partial}}{\partial \tau} \phi - \alpha_0' \vec{\nabla} \phi^+ \cdot \vec{\nabla} \phi - \Delta_{0C} \phi^+ \phi - \eta_0 \phi^+ \phi \right.$$

$$\left. - \frac{i r_0}{2} \left[(\phi^+)^2 \phi + \phi^+ (\phi)^2 \right] \right\} , \qquad (2)$$

where $E = \alpha_0' \vec{q}^{\,2} + \Delta_0$, $(\Delta_0 = 1 - \alpha_0)$ is the bare trajectory, and r_0 is the (real) bare triple $\underset{\sim}{P}$ coupling; $\eta_0 = \Delta_0 - \Delta_{0C}$; Δ_{0C} is the value of Δ_0 at which $\alpha(0)=1$. The zero of the inverse $\underset{\sim}{P}$ propagator at $\vec{q}^{\,2}=0$ gives the renormalized intercept in terms of $\Delta = 1 - \alpha(0)$ as $i\Gamma^{(1,1)}(E=\Delta, \vec{q}=0) = 0$.

The quantity η_0 is like a mass term in conventional field theory. We are concerned with the behavior of this theory when η_0 becomes negative. As was pointed out by Abarbanel,[8] the passage through $\eta_0=0$ can bring about a situation where the field develops a non-zero vacuum expectation value and the "mass", Δ, remains positive. This is what occurs in Reggeon field theory as we now show.

Our tool is the effective action Γ which generates the one Reggeon irreducible, proper vertex functions $\Gamma^{(n,m)}$. It is a functional of the c-number fields $\psi(\vec{x},\tau)$, $\psi^+(\vec{x},\tau)$ and has the expansion:[9]

$$\Gamma\left[\psi,\psi^+\right] = \sum_{m,n=1}^{\infty} \frac{1}{n!m!} \int d^Dx_1\, d\tau_{x_1} \ldots d^Dy_m\, d\tau_{y_m}$$

$$\times\, i\tilde{\Gamma}^{(n,m)}(\vec{x}_1,\tau_{x_1},\ldots,\vec{y}_m,\tau_{y_m})\, \psi(\vec{x}_1,\tau_{x_1})\ldots\psi^+(\vec{y}_m,\tau_{y_m}) \quad . \tag{3}$$

The vacuum expectation values of ϕ and ϕ^+ are denoted by v and w, respectively. They satisfy the equations

$$\left.\frac{\delta\Gamma}{\delta\psi}\right|_{\psi=v,\,\psi^+=w} = 0 \ , \qquad \left.\frac{\delta\Gamma}{\delta\psi^+}\right|_{\psi=v,\,\psi^+=w} = 0 \quad . \tag{4}$$

We wish to construct Γ, as well as we can, and search for v and w. From translational invariance we know that v and w are independent of $\vec{x}$ and τ, so we take ψ and ψ^+ to be constants. Factoring out a delta function, the effective action becomes

$$\Gamma\left[\psi,\psi^+\right] = \sum_{m,n=1}^{\infty} \frac{1}{n!m!}\, i\Gamma^{(n,m)}(0,0)(\psi-v)^n(\psi^+-w)^m \ . \tag{5}$$

The $\Gamma^{(n,m)}$ are vertex functions for the displaced field operators ϕ-v, ϕ^+-w, evaluated at $E_i=\vec{q}_i=0$.

The heart of our argument resides in the zero loop (or classical) approximation to Γ.

$$\Gamma\left[\psi,\psi^+\right] = -\eta_0\, \psi^+\psi - \frac{ir_0}{2}\, \psi^+\psi(\psi^+ + \psi) \quad . \tag{6}$$

The vacuum expectation values v and w are determined by (4) as solutions to

$$\eta_0 v - \frac{ir_0}{2}\,(2vw + v^2) = 0 \quad , \tag{7}$$

and

$$\eta_0 w - \frac{ir_0}{2}(2vw + w^2) = 0 \quad . \tag{8}$$

These have the solutions

$$\begin{array}{llll} \text{I.} & v=w=0 \ , & \text{II.} & v=w=-2\eta_0/3ir_0 \ , \\ \text{IIIa.} & v=0 \ , \quad w=-2\eta_0/ir_0 \ , & \text{IIIb.} & v=-2\eta_0/ir_0 \ , \quad w=0 \ . \end{array} \tag{9}$$

For $\eta_0>0$ only the solution with v=w=0 is stable when quantum fluctuations are taken into account. For $\eta_0<0$ the stable solutions are IIIa and IIIb. The instability of the other solutions can most easily be seen by studying the perturbation expansion for the $\Gamma^{(n,m)}$. One finds that the functional integrals which give the individual terms in the perturbation series are not well defined and the Feynman diagrams have unphysical singularities. In the zero loop approximation

$$\Delta = -\left.\frac{\partial^2\Gamma}{\partial\psi^+\partial\psi}\right|_{\psi=v,\,\psi^+=w} = \eta_0 + ir_0(v+w) = \begin{cases} \eta_0 (\eta_0>0) \\ -\eta_0 (\eta_0<0) \end{cases} \tag{10}$$

so Δ always remains ≥ 0 or $\alpha(0)\leq 1$.

There are a host of quantum corrections to the zero loop approximation for Γ, and we must discuss their effect. We can construct Γ explicitly near D=4 using the renormalization group and an expansion in ϵ=4-D. After some algebra we find the scaling form for the renormalized Γ [10,11]

$$\Gamma \underset{|\eta|\to 0}{\longrightarrow} E_N\left(\frac{E_N}{\alpha'}\right)^{2-\epsilon/2}\left|\frac{\eta}{E_N}\right|^{3-\epsilon/6}\Big\{-dxy\left[1+\frac{\epsilon}{6}(\ell n\, 2-1)\right]$$

$$-\frac{1}{2} ig_1\, xy(x+y) - xy\,\frac{\epsilon}{6}\left[d+ig_1(x+y)\right] \ell n\left[d+ig_1(x+y)\right] + 0\left(\epsilon^2 \text{ or } (xy)^2\right)\Big\}. \tag{11}$$

Here E_N is a normalization point for the renormalized theory, α' and η are the renormalized α'_0 and η_0, $d=\frac{\eta}{|\eta|}$ and $g_1=(8\pi)^{D/4}(\epsilon/6)^{1/2}$. The scaling variables x and y are

$$x = \psi \frac{\alpha'}{|\eta|}\left(\frac{\alpha'}{E_N}\right)^{-\epsilon/4} \left|\frac{\eta}{E_N}\right|^{\epsilon/6} , \qquad y = \psi^+ \frac{\alpha'}{|\eta|}\left(\frac{\alpha'}{E_N}\right)^{-\epsilon/4} \left|\frac{\eta}{E_N}\right|^{\epsilon/6} . \tag{12}$$

The exponents in Eqs. (11) and (12) will be modified by terms of $0(\epsilon^2)$ when higher quantum corrections are added. For $\eta>0$ the stable solution to Eq. (4) is still v=w=0. For $\eta<0$ there are again two stable solutions. The one which arises from taking into account quantum corrections to Solution IIIa of Eq. (9) is

$$v=0 , \qquad w \underset{\eta\to 0-}{\longrightarrow} \left(\frac{E_N}{\alpha'}\right)^{D/4} \left|\frac{\eta}{E_N}\right|^{1-\epsilon/6} \left(-2ig_1^{-1}\right) \times \left[1+\frac{\epsilon}{6}(\ln 2-1)\right] , \tag{13}$$

and the one which follows from Solution IIIb is obtained by interchanging the values of v and w.

In order to study the behavior of the $\underset{\sim}{P}$ trajectory function it is necessary to write down a renormalization group equation for $\Gamma^{(1,1)}(E,k^2,\eta)$ directly, since Eq. (11) will only give us information about $\Gamma^{(1,1)}(0,0,\eta)$. After some additional algebra we find[10, 11]

$$\Delta = 1-\alpha(0) \underset{|\eta|\to 0}{\longrightarrow} E_N \left|\frac{\eta}{E_N}\right|^{1+\epsilon/12} \left\{1-\frac{\epsilon}{6}\left[\frac{1}{2}+(d-1)\ln 2\right]+0(\epsilon^2)\right\} \tag{14}$$

and

$$\alpha'_R \underset{|\eta|\to 0}{\longrightarrow} \alpha' \left|\frac{\eta}{E_N}\right|^{-\epsilon/24} \left[1-\frac{\epsilon}{6}\left(\frac{5}{4}-2\ln 2\right)(d-1)+0(\epsilon^2)\right] . \tag{15}$$

Here α'_R is the slope parameter of the renormalized $\underset{\sim}{P}$. Again $\alpha(0)\leq 1$ for either sign of η. Notice that α'_R diverges as $|\eta|\to 0$ with just the power one would expect from the direct calculations with $\eta=0$.[2, 3]

Although it may well be that the ϵ-expansion is not rapidly convergent in two-dimensions, we believe that the qualitative features of our calculation will hold there. First, as long as the theory continues to have an infra-red stable fixed point, then it is possible to derive scaling laws which give the small η behavior in terms of critical exponents which are independent of the coupling constant. Moreover, as long as the renormalization group functions have the analyticity properties indicated by perturbation theory, then it is possible to show that the partial wave amplitude will have no singularities to the right of J=1 for $\vec{q}^{\,2} \geq 0$ $(t \leq 0)$.[10]

We are now in a position to enumerate the possible high energy behaviors which can be obtained in Reggeon field theory. The two quantities which set the energy scale are η_0 and r_0^2/α_0' (for D=2). For $\eta_0=0$ and $r_0^2/\alpha_0' \,\ell n\, s \gtrsim 1$ we will see the scaling behavior of Refs. 2 and 3 and the total cross section will be given by Eq. (1). For $r_0^2/\alpha_0' > |\eta_0| > 0$ we will see approximate scaling for $(r_0^2/\alpha_0')\,\ell n\, s \geq 1 > |\eta_0|\,\ell n\, s$, but at energies such that $|\eta_0|\,\ell n\, s \geq 1$ the high energy behavior will be dominated by the renormalized $\underset{\sim}{P}$ pole which will be below one. Finally for $|\eta_0| > r_0^2/\alpha_0'$ we will not see the scaling behavior at all. For $|\eta_0|\,\ell n\, s \geq 1$ the renormalized pole will again dominate. At "low" energies $(|\eta_0|\,\ell n\, s,\ (r_0^2/\alpha_0')\,\ell n\, s < 1)$ the behavior of the scattering amplitudes will not be controlled by the infra-red stable fixed point. It will depend on the strengths of the couplings of the $\underset{\sim}{P}$'s to each other and to the external particles. Fortunately, perturbation theory is applicable in this domain, so one may hope to describe the data in terms of a finite number of parameters.

Since total cross sections are approximately constant at high energies and r_0 is small $|\eta_0|$ must also be small. The question of why nature chooses $\Delta_0 \approx \Delta_{0C}$ is outside the scope of Reggeon field theory. It is of course a crucial problem for a complete understanding of high energy diffraction scattering.

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