Horizon quantum mechanics and the inner side of black holes

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The Horizon Quantum Mechanics allows one to analyse the gravitational radius of spherically symmetric systems and compute the probability that a given quantum state is a black hole. We first review the global formalism and show that it reproduces a gravitationally inspired GUP relation but also leads to unacceptably large fluctuations in the horizon size of astrophysical black holes if one insists in describing them as (smeared) central singularities. On the other hand, if they are extended systems, like in the corpuscular models, no such issue arises and one can in fact extend the formalism to include asymptotic mass and angular momentum with the harmonic model of rotating corpuscular black holes. The Horizon Quantum Mechanics then shows that, in simple configurations, the appearance of the inner horizon is suppressed and extremal (macroscopic) geometries seem highly disfavoured.

Keywords: Black holes; Quantum physics

1. HQM for spherical sources

The world as we know it, is best described by quantum physics, and black holes should be represented as quantum objects as well. A first non-trivial question that follows is how much of the classical description of black holes given by general relativity we can still keep at the quantum level. One feature that should presumably live up into the quantum realm is that black holes are "gravitational bound states". Nonetheless, most of the existing literature simply analyses quantum effects on classical black hole space-times, and the proper quantum nature of the background itself is not fully accounted for. Finally, one might notice that a classical quantity that characterises such bound states (in the particular case when spherical symmetry is preserved) is the so called "gravitational radius",

$$R_{\rm H} = 2\, G_{\rm N}\, M \ , \tag{1}$$

where M here can be the local Misner-Sharp mass $M = M(r)$ or the total ADM mass of the system. A quantum treatment of the gravitational radius could therefore be a good starting point for developing a fully quantum theory of black holes, much like quantising the position of the electron proved to be a good starting point for a quantum description of the hydrogen atom.

The Horizon Quantum Mechanics (HQM)[1–13] was precisely proposed with the purpose of describing the gravitational radius of spherically symmetric compact sources and for determining the existence of a horizon in a quantum mechanical fashion. In a classical spherically symmetric system, the gravitational radius (1) uniquely determines the location of the trapping surfaces where the null geodesic expansion vanishes. The latter surfaces are proper horizons in a time-independent configuration, which is the case we shall consider here for simplicity. It is then straightforward to uplift this description of the causal structure of space-time to the quantum level by simply imposing the relation between the gravitational radius and the Misner-Sharp mass as a constraint to be satisfied by the physical states of the system[2], that is

$$0 = \left(\hat{M} - \frac{\hat{R}_{\rm H}}{2\,G_{\rm N}}\right) |\,\Psi\,\rangle = \sum_{\alpha,\beta} \left(E_\alpha - \frac{R_{{\rm H}\beta}}{2\,G_{\rm N}}\right) C(E_\alpha, R_{{\rm H}\beta})\,|\,E_\alpha\,\rangle|\,R_{{\rm H}\beta}\,\rangle \ , \tag{2}$$

where $|\,E_\alpha\,\rangle$ are eigenstates "Hamiltonian" operator $\hat{M}$, that is

$$\hat{M} = \sum_\alpha E_\alpha |\,E_\alpha\,\rangle\langle\,E_\alpha\,| \ , \tag{3}$$

and likewise $|\,R_{{\rm H}\alpha}\,\rangle$ are eigenstates of the gravitational radius operator,

$$\hat{R}_{\rm H} = \sum_\alpha R_{{\rm H}\alpha} |\,R_{{\rm H}\alpha}\,\rangle\langle\,R_{{\rm H}\alpha}\,| \ . \tag{4}$$

Solutions to the constraint (2) are given by

$$C(E_\alpha, R_{{\rm H}\beta}) = C(E_\alpha, 2\,G_{\rm N}\,E_\alpha)\,\delta_{\alpha\beta} \equiv C_{\rm S}(R_{{\rm H}\alpha}/2\,G_{\rm N})\,\delta_{\alpha\beta} \ , \tag{5}$$

and one can then define the horizon wave-function (HWF)[1] as

$$\psi_{\rm H}(R_{{\rm H}\alpha}) = \langle\,R_{{\rm H}\alpha}\,|\,\psi_{\rm H}\,\rangle = C_{\rm S}(R_{{\rm H}\alpha}/2\,G_{\rm N}) \ . \tag{6}$$

Once the HWF is introduced, one can define the probability that a given source with position wave-function $\psi_S = \psi_S(r)$ is a black hole as

$$P_{\rm BH} = \int_0^\infty \mathcal{P}_<(r < R_{\rm H})\,{\rm d}R_{\rm H} \ , \tag{7}$$

where $\mathcal{P}_<(r < R_{\rm H}) = P_{\rm S}(r < R_{\rm H})\,\mathcal{P}_{\rm H}(R_{\rm H})$ is the probability density that the source is found within its gravitational radius. The latter is in turn defined in terms of the probability

$$P_{\rm S}(r < R_{\rm H}) = 4\,\pi \int_0^{R_{\rm H}} |\psi_{\rm S}(r)|^2\,r^2\,{\rm d}r \tag{8}$$

that the source lies inside the radius $r = R_{\rm H}$ and the probability density

$$\mathcal{P}_{\rm H}(R_{\rm H}) = 4\,\pi\,R_{\rm H}^2\,|\psi_{\rm H}(R_{\rm H})|^2 \tag{9}$$

that $R_{\rm H}$ equals the gravitational radius.

Beside formal developments, we have applied the HQM to specific states of the harmonic black hole model[14], which can be considered as a working realisation of the corpuscular black holes proposed by Dvali and Gomez[15]. If the black hole contains only one quantum constituent, with energy $M \simeq \hbar/\lambda$, where λ is the Compton length of the constituent, the results is a black hole only provided $\lambda \lesssim \ell_{\rm p}$ (the Planck length), corresponding to a mass $M \gtrsim m_{\rm p}$ (the Planck mass). For this case one also recovers a Generalised Uncertainty Principle[3]

$$\Delta r \sim \frac{\hbar}{\Delta p} + \Delta R_{\rm H} \sim \frac{\hbar}{\Delta p} + \Delta p\ , \tag{10}$$

and a quantum hoop conjecture[4]. However, the fluctuations in the horizon size are given by

$$\langle\,\Delta \hat{R}_{\rm H}^2\,\rangle \sim \lambda^{-2} \sim \Delta p^2\ , \tag{11}$$

which becomes extremely large for macroscopic black holes with $M \simeq \hbar/\lambda \gg m_{\rm p}$. This result supports the idea that a black hole must instead be made by a large number N of constituents of energy $\varepsilon \simeq \hbar/\lambda$ (such that $M \simeq N\,\varepsilon \sim \sqrt{N}\,m_{\rm p}$ and $\lambda \sim R_{\rm H} \sim \sqrt{N}\,\ell_{\rm p}$). For the latter case, horizon fluctuations can be very small, of order $1/N$, and the quantum harmonic model is simple enough that one can determine explicitly the probability that the chosen states are indeed black holes[16].

2. HQM for rotating sources

In a space-time generated by an axially-symmetric rotating source, we exploit the fact that the asymptotic behaviour of the system is described by the Kerr metric. We can then uplift to a quantum condition for the physical states the classical relation that determines the two horizon radii

$$R_\pm = M \pm \sqrt{M^2 - a^2} \tag{12}$$

from the mass M and angular momentum $J = a^2\,M^2$ of the source which holds in the (asymptotic) Kerr metric[10]. The formalism described in the previous section now doubles with the introduction of two gravitational radius operators, $\hat{R}_\pm$, and corresponding HWF's we can denote as $\psi_\pm = \psi_{\rm H}(R_\pm)$. The probability $P_{\rm BH} = P_+$ that the whole source is a black hole is then obtained from ψ_+, and the probability P_- that the inner horizon is also realised is likewise determined from ψ_-.

The harmonic black hole model[14] also allows one to compute the above two probabilities for states of a large number N of gravitons with non-vanishing total angular momentum[10]. The main result is that, for extremal configurations with $M^2 \simeq a^2$, one finds

$$P_+ \simeq P_{1+}^N \ll 1\ , \tag{13}$$

where $P_{1+} \lesssim 1$ is the probability that each constituent is inside R_+. For instance, we typically obtain $P_1 \simeq 0.7$ and, since $N \sim M^2/m_{\rm p}^2 \gg 1$ for astrophysical sources, it appears that such configurations are very unlikely to have a horizon and be black holes. Moreover, the inner horizon R_- also has a small probability to exist for large sources, to wit

$$P_- \simeq P_{1-}^N \ll 1 \; , \tag{14}$$

where now $P_{1-} \ll 1$ is the probability that each constituent is inside the inner horizon R_-. A typical value is $P_{1-} \simeq 0.05$ and $N \gg 1$.

The overall conclusion is that quantum fluctuations for rotating geometries, although negligibly small for the horizon of non extremal configurations, appear to be large enough to spoil the causal structure of extremal configurations and eliminate the presence of inner horizons. We recall that the latter are in fact rather problematic in the semiclassical description, because of the effect called mass inflation[17]. We remark that the HQM seems to provide a solution to this problem in general, since the inner horizon is removed also for electrically charged sources[5].

Acknowledgments

R.C. and A.G. are partially supported by the INFN grant FLAG. The work of R.C. and A.G. has also been carried out in the framework of activities of the National Group of Mathematical Physics (GNFM, INdAM). O.M. was supported by the grant LAPLAS 4.

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