ELASTIC ELECTRON SCATTERING FROM TRITIUM AND HELIUM-3 *

H. Collard, R. Hofstadter, E. B. Hughes, A. Johansson, M. R. Yearian

Department of Physics and High-Energy Physics Laboratory, Stanford University, USA

Los Alamos Scientific Laboratory, USA

R. B. Day, R. T. Wagner

(Presented by R. HOFSTADTER)

INTRODUCTION

A study of the mirror nuclei tritium and helium-3 is of interest because the structures of these nuclei are not well known and because these nuclei provide the simplest systems in which evidence for the existence of a three-body nuclear force might be found. If such forces do not exist then the properties of these nuclei should be understandable in terms of the two-body nucleon-nucleon interaction. An attempt at such an understanding has been made by Blatt, Derrick, and Lyness [1] and by Blatt and Delves [2] who found the binding energy of the triton to be considerably underestimated when calculated with the help of the best known forms of the nucleon-nucleon interaction.

In the present experiment the electromagnetic structures of the nuclei of tritium and helium-3 have been investigated by the method of electron scattering. The elastic electron scattering cross sections have been measured from both nuclei during the same experiment to an accuracy of a few percent. The results should, therefore, be able to reveal quite small differences between the structures of the two nuclei. A preliminary report of this experiment together with an analysis of the results in terms of form factors describing the spatial distribution of individual nucleons in the nucleus has already been given, [3,4].

It appeared from the analysis of the preliminary experiments [5], using the original theory due to Schiff, that the spatial distribution of the pair of like nucleons in both nuclei was more extended than that of the odd nucleon. In the triton, for example, we refer to the neutrons as like nucleons and to the proton as the odd nucleon. Recently, Schiff [6] has shown that this feature can be explained by assuming a 4% admixture of a $^2S_{1/2}$ state of mixed symmetry into the nuclear ground state wave function which is primarily a completely symmetric $^2S_{1/2}$ state. This is, however, a considerably larger admixture than suggested by Blatt et al. [1] or by Blatt and Delves [2] and moreover it has recently been shown that such an explanation is not supported by the results of other experiments on the three-nucleon system. For instance, a 4% admixture of a state of mixed symmetry is not consistent with the electron-proton coincidence cross section measured by Johansson [7, 8] for the reaction $e + He^3 \rightarrow e + p + d$. Also Meister, Radha and Schiff [9]* have found it difficult to reconcile such a large admixture (of a $^2S_{1/2}$ state of mixed symmetry) with the known cross sections for the slow neutron capture reaction $n + d \rightarrow H^3 + \gamma$.

The preliminary theory and analysis of the early experimental data are unsatisfactory in the following respects:

a) the analysis yielded values for the neutron charge form factor which were smaller than zero and in disagreement with other measurements of this form factor [10, 11];

b) Levinger [12] has pointed out that in addition to the isovector exchange form factor which enters into the description of the magnetic form factor of three nucleon system there may also be a isoscalar exchange form factor;

c) it is not known to what extent Coulomb effects modify the wave function of the helium-3 nucleus. A simple estimate [13] of this effect suggests that the correction may well be of the

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* These authors have extended their calculations using the Gunn and the Irving-Gunn models with the same conclusion (private communication).

same order as the differences observed between the two nuclei.

In view of these difficulties and because the theory is presently being improved, we have confined ourselves in this paper to an analysis of the observed charge form factors of tritium and helium-3, assuming a knowledge of the charge form factor of the nucleons. This analysis, then, gives values for the form factors which describe the spatial distribution of the protons inside the nucleus.

ANALYSIS

Since the proton and the nuclei of tritium and helium-3 all have spin equal to 1/2 the elastic scattering cross section for these three nuclei can be described by the Rosenbluth formula (in units where $\hbar = c = 1$)

$$\sigma = \sigma_{NS} \frac{\left\{ F^2_{\text{ch}}(q^2) + F^2_{\text{mag}}(q^2)(1+K)^2 \times \frac{q^2}{4M^2}\left[1 + 2\left(1+\frac{q^2}{4M^2}\right)\text{tg}^2\frac{\theta}{2}\right]\right\}}{\left(1+\frac{q^2}{4M^2}\right)} \tag{1}$$

where

$$\sigma_{NS} = \left(\frac{Ze^2}{2E}\right)^2 \frac{\cos^2\frac{\theta}{2}}{\sin^4\frac{\theta}{2}} \frac{1}{1+\frac{2E}{M}\sin^2\frac{\theta}{2}},$$

q = the four momentum transfer; M = the mass of the scattering nucleus; Z = the charge of the scattering nucleus; $F_{\text{mag}}(q^2)$ = the magnetic form factor, normalized to unity at $q^2 = 0$, $F_{ch}(q^2)$ = the charge form factor, normalized to unity at $q^2 = 0$, K = the anomalous nuclear magnetic moment expressed in magneton units related to the scattering nucleus, i. e.

$K = 1.79$ for electron-proton scattering,

$K = 7.94$ for electron-triton scattering, and

$K = -4.20$ for electron-helium-3 scattering.

The data obtained for each particular value of q^2 were analyzed in such a way as to yield those charge and magnetic form factors for tritium and helium-3 which minimized the statistical function χ^2 defined as

$$\chi^2 = \sum_l \sum_i \frac{(\sigma_{l,i} - C_l X_{l,i})^2}{c_l^2 (\Delta X_{l,i})^2}$$

where the index l refers to the scattering angle and the index i to the target. The cross sections $\sigma_{l,i}$ were found from equation (1), using the proton form factors found by deVries et al [11], (combination b′), and trial form factors for tritium and helium-3. The quantities $X_{l,i}$ are the observed numbers of counts in the elastic peaks after the various corrections described above have been applied. The errors $\Delta X_{l,i}$ have been assumed to arise from counting statistics alone. The normalization constants C_l are also to be determined in such a way that χ^2 attains its minimum value. They can be eliminated from the expression for χ^2 by equating the derivatives of χ^2 with respect to C_l to zero.

Then with the definitions

$$T_{l,i} = \sigma_{l,i}/\Delta X_{l,i} \text{ and } E_{l,i} = X_{l,i}/\Delta X_{l,i}$$

we obtain

$$C_l = \sum_i T^2_{l,i} / \sum_i T_{l,i} E_{l,i}$$

and

$$\chi^2 = \sum_l \left\{ \frac{\sum_i \sum_{j>i} (T_{l,i}E_{l,j} - T_{l,j}E_{l,i})^2}{\sum_i T^2_{l,i}} \right\}$$

with

$$N = \left(\sum_l \sum_i 1\right) - \left(\sum_l 1\right) - 4 \text{ degrees of freedom.}$$

This expression for χ^2, at a particular value of q^2, depends only on the value of the charge and magnetic form factors of tritium and helium-3. These parameters are automatically adjusted, using an IBM 7090 computer, until a minimum value of χ^2 is found. The normalization constants C_l are computed at the same time, and hence the set of experimental cross sections which together with the final set of form factors generates the minimum value of the function χ^2.

RESULTS

The experimental cross sections are to be published in a more complete report of this work.

Table gives the charge and magnetic form factors of tritium and helium-3 which correspond to the minimum value of χ^2. The errors in these quantities were computed from the error matrix, which was also calculated by the computer. The quoted errors have been based on external or internal consistency, depending upon which gave the larger uncertainty. The final column in table shows the goodness of fit.

At all values of q^2, with the exception of $q^2 = 8$ fermi^{-2}, the value of χ^2/N is larger than unity, which is to be expected since only the rather small errors arising from counting statistics have been considered in this computation. The deviations from unity of χ^2/N are not sufficient to indicate a failure of the Rosenbluth formula.

Tritium and helium-3 Form Factors

q^2, fermi-2	F_{ch} (H^3)	F_{mag} (He^3)	F_{ch} (He^3)	F_{mag} (H^3)	χ^2/N
1.0	0.622 ± .007	0.653 ± .022	0.567 ± .004	0.676 ± .075	1.09
1.5	0.503 ± .007	0.475 ± .015	0.431 ± .004	0.479 ± .046	1.54
2.0	0.387 ± .007	0.379 ± .012	0.329 ± .004	0.385 ± .031	2.42
2.5	0.ç12 ± .006	0.312 ± .008	0.258 ± .003	0.291 ± .020	2.16
3.0	0.267 ± .005	0.242 ± .006	0.209 ± .002	0.203 ± .014	2.19
3.5	0.215 ± .004	0.199 ± .005	0.1614± .0017	0.167 ± .010	1.59
4.0	0.175 ± .004	0.167 ± .004	0.1326± .0015	0.128 ± .009	1.64
4.5	0.137 ± .003	0.139 ± .003	0.1013± .0010	0.118 ± .005	1.10
5.0	0.118 ± .004	0.109 ± .005	0.0813± .0012	0.093 ± .008	2.27
6.0	0.0758± .0041	0.0792± .0032	0.0548± .0015	0.0566± .0056	3.45
8.0	0.0295± .0039	0.0416± .0018	0.0173± .0010	0.0318± .0026	0.63

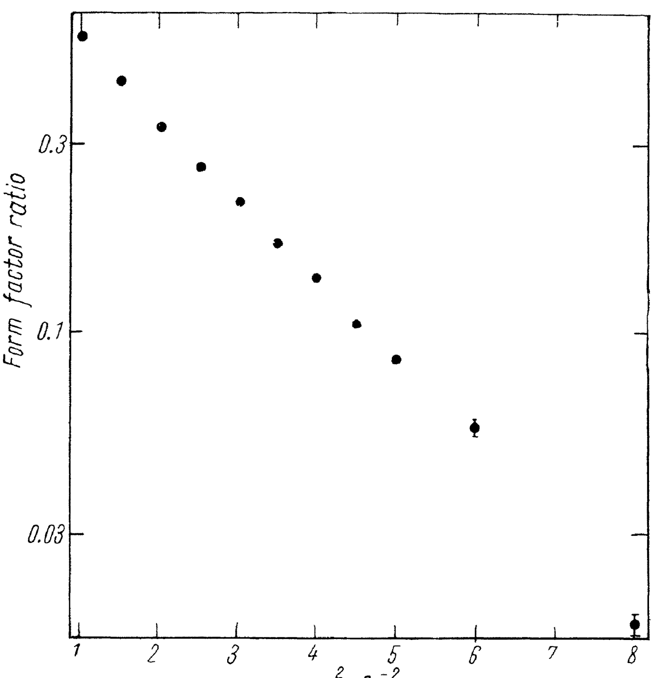

Fig. 1. The charge form factor of helium-3 function of four-momentum transfer squared.

In Fig. 1 we show the variation of the charge form factor of helium-3 with the four-momentum transfer q^2. For the sake of comparison the other form factors are given in Fig. 2 in the form of ratios of form factors. Several conclusions can be drawn from these ratios:

a) the charge and magnetic form factors of tritium seem to be very similar, with the exception of the point at $q^2 = 8$ fermi^{-2};

b) the charge form factor of helium-3 is significantly smaller than the magnetic form factor, although the effect is not quite as marked as was originally suggested by the data of Collard and Hofstadter [14];

c) both the charge and magnetic form factors of tritium seem to decrease less rapidly with increasing q^2 than the corresponding form factor in He^3.

The description of the magnetic form factors of tritium and helium-3 given by Schiff [6] cannot be used without modification to extract information about the three-body wave function unless the contributions due to meson exchange are calculated. However, with the assumptions that the nuclei of tritium and helium-3 can be described by three-nucleon wave functions and that the charge form factors of these nucleons are given by the free nucleon form factors, the charge form factors of tritium and helium-3 can be written quite generally in the form

$$F_{ch}(\mathrm{He}^3) = F_{ch}(p)\, F_L(\mathrm{He}^3) + \frac{1}{2} F_{ch}(n)\, F_0(\mathrm{He}^3); \qquad (2)$$

$$F_{ch}(\mathrm{H}^3) = F_{ch}(p)\, F_0(\mathrm{H}^3) + 2F_{ch}(n)\, F_L(\mathrm{H}^3) \qquad (3)$$

where the form factors F_L (H^3) and F_L (He^3), describing the spatial distribution of like nucleons, and F_0 (H^3) and F_0 (He^3), describing the spatial distribution of the odd nucleon, are all different due to the existence of a Coulomb repulsion in helium-3.

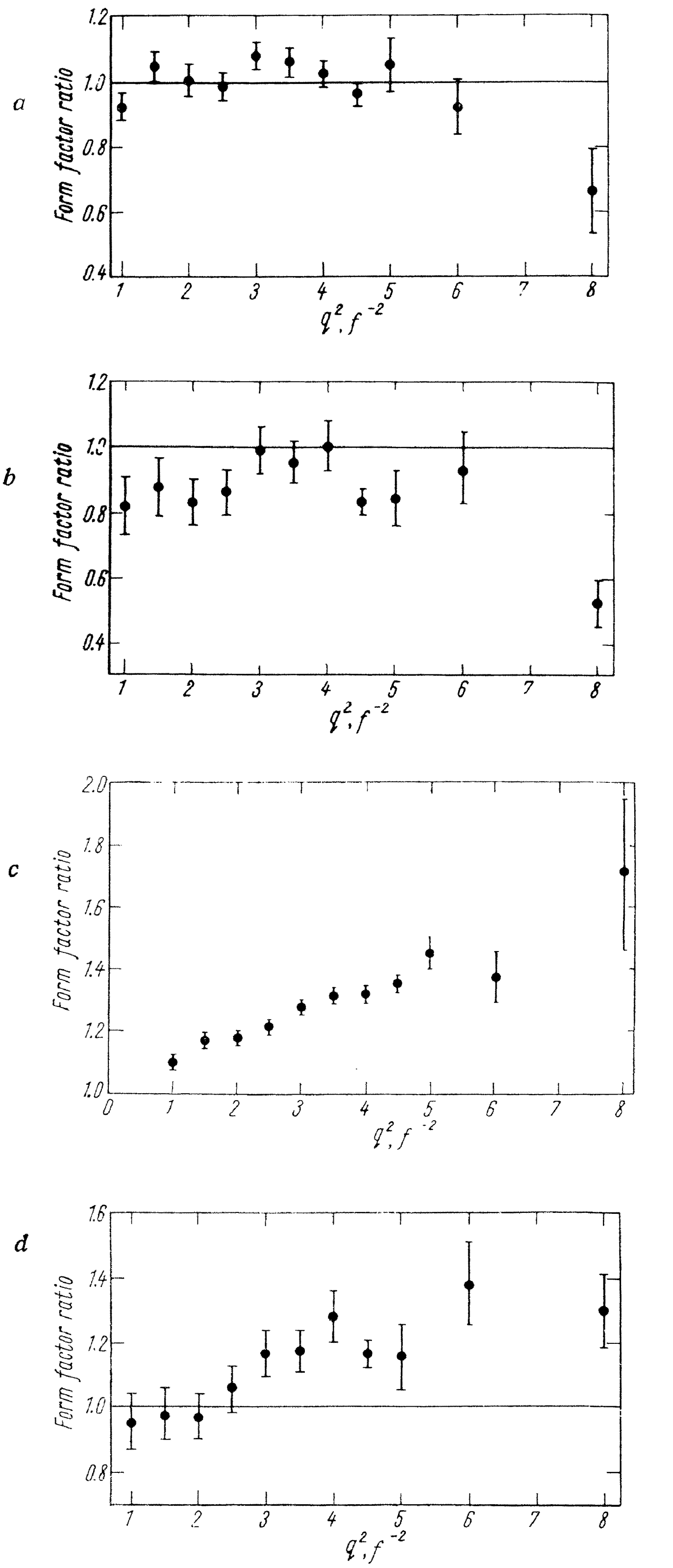

Fig. 2. The form factor ratios:
a — F_{ch} (H^3)/F_{mag} (He^3); b — F_{ch} (He^3)/F_{mag} (He^3); c — F_{ch} (H^3)/F_{ch} (He^3); d — F_{mag} (H^3)/F_{mag} (He^3).

To evaluate F_0 (H^3) and F_L (He^3) we need to know the charge form factors of the neutron and proton and to make an estimate of the values of the form factors F_L (H^3) and F_0 (He^3). Although there is at present some disagreement

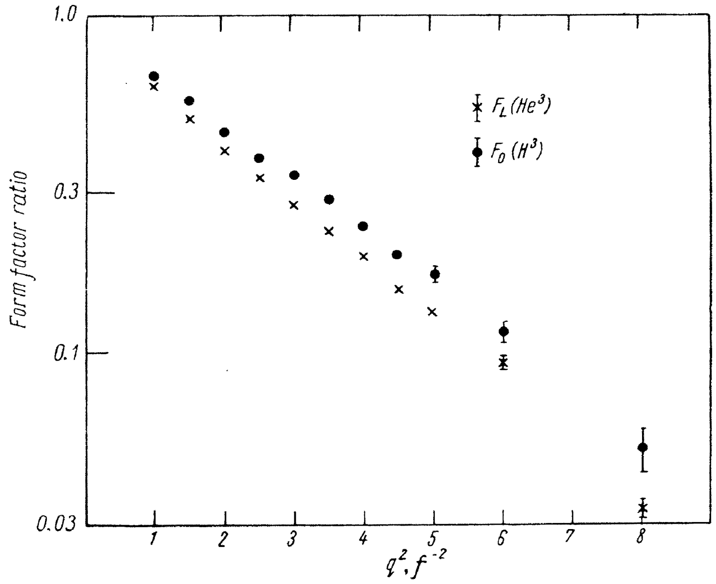

Fig. 3. Variation of the form factors F_0 (H^3) and F_L (He^3) with four-momentum transfer squared.

between various measurements of the charge form factor of the neutron, it is generally accepted that the value of F_{ch} (n) is small (of the order of 0.1) in the range of q^2 of interest to the present experiment. Since the contribution to F_0 (H^3) and F_L (He^3) of the terms involving $F_{ch}(n)$ is therefore small, we have decided to use the values of E_{ch} (n) given by deVries et al. Also we have made the approximations that

$$F_0(He^3) = F_L(H^3) = \frac{1}{2}(F_L(He^3) + F_0(H^3)),$$

which are acceptable because the form factors F_0 (He^3) and F_L (H^3) enter only into the terms involving E_{ch} (n).

With these assumptions we have solved the equations (2) and (3) for $F_0(H^3)$ and $F_L(He^3)$. The results are given in Fig. 3 which shows that the form factor describing the spatial distribution of the protons in helium-3 decreases much more rapidly with increasing q^2 than the form factor describing the spatial distribution of the proton in tritium. This result is of course expected, but at this time it is not possible to say to what extent the difference should be attributed to Coulomb repulsion or to a weaker nuclear force between the like particles. Also, in view of the inadequate state of the theory we do not think it meaningful to attempt to use these body form factors as a means of determining the ground state wave functions of the three body nuclei.

NUCLEAR RADII

Model independent determinations of the r m s radii for the charge and magnetic moment distributions of tritium and helium-3 were made by plotting $1 - F_{ch}(q^2)$ and $1 - F_{mag}(q^2)$ as a function of q^2 and finding the slope at the origin of a curve fitted by the method of least squares to these points. Since the form factor can be expressed as $F(q^2) = 1 - \frac{q^2a^2}{6} +$ higher terms, the slope of $1 - F(q^2)$ at the origin is

$$\left(\frac{d\,[1 - F(q^2)]}{dq^2}\right)_{q^2=0} = \frac{a^2}{6}$$

from which the r. m. s radius «a» can be determined.

The charge and magnetic form factors for both tritium and helium-3 were fitted with a third order curve and the values obtained for the r m s radii, (in units of fermis) were as follows:

$a_{ch}(H^3) = 1.67 \pm 0.05 \quad a_{ch}(He^3) = 1.82 \pm 0.06$

$a_{mag}(H^3) = 1.65 \pm 0.06 \quad a_{mag}(He^3) = 1.61 \pm 0.10$

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