# COSMIC BALLS OF TRAPPED NEUTRINOS

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Fermions trapped inside a closed domain wall may cool to degeneracy and form a long-lived structure. In the context of spontaneous left-right symmetry breaking, we show that trapped right-handed neutrinos cool due to annihilations to electron-positron pairs if the initial temperature is less than $0.21m_e$. The surface tension of the wall must be less than $(1.93\ \mathrm{TeV})^3$. The lifetime of the neutrino-ball (NB) is determined by neutrino annihilation to three photons and may be comparable to the age of the universe. These NBs are in the $10^4$-$10^7$ solar mass range and radiate $\gamma$-rays in the few hundred KeV range at a rate of $10^{40}$-$10^{43}$ ergs/sec. NBs die in a $10^{56}$-$10^{59}$ erg electron-positron burst.

Domain walls, like monopoles and cosmic strings, may form during phase transitions as the early universe cools. In fact they must arise when there is a spontaneous breakdown of a discrete symmetry. But if walls form there must be at least one wall per horizon volume, by causality. This leads to a cosmological disaster, since this wall will eventually dominate the total energy contained in the horizon volume. Therefore if walls form there must be a mechanism to remove the "infinite walls", the walls which stretch across horizon volumes. Two ways have been discussed (see [1] for references). One involves the presence of suitable cosmic strings. Loops of string puncture the wall and the whole system of holey walls and strings bounding walls can decay away before wall domination. Another possibility involves a slight explicit breaking of the original discrete symmetry. Further discussion of the wall removal mechanism is not required here.

Does this imply that there are no remnants of a possible wall era in the early universe? One remnant occurs if walls in some region happen to collapse into a black hole. We may estimate the mass of such a black hole by considering a wall in the form of a spherical bubble. If the bubble is too large it becomes a black hole. If $\sigma$ is the mass/area of the wall then the mass of such a black hole is

$$M \approx (16\pi G^2\sigma)^{-1} \approx 10^{8-9}/\sigma[\mathrm{TeV}^3] \text{ solar masses.} \qquad (1)$$

This result relates two interesting mass scales. $10^{8-9}$ solar masses is at the upper end of the mass range of supermassive black holes discussed in connection with quasars and galactic nuclei. And one TeV is a mass scale in particle physics at which new physics is expected.

Here we shall discuss another possible remnant of a wall era. This occurs when the bubble traps particles, and the result is analogous to a balloon. To see when this occurs consider two real fields $\varphi_1$ and $\varphi_2$ with a potential having the discrete symmetry $\varphi_1 \leftrightarrow \varphi_2$;

$$V(\varphi_1,\varphi_2) = \lambda(\varphi_1^2 - \varphi_0^2)^2 + \lambda(\varphi_2^2 - \varphi_0^2)^2 + \tilde{\lambda}\varphi_1^2\varphi_2^2. \qquad (2)$$

For a range of couplings there are two degenerate vacua, either $\langle\varphi_1^2\rangle = 0$, $\langle\varphi_2^2\rangle = \varphi_0^2$ or $\langle\varphi_1^2\rangle = \varphi_0^2$, $\langle\varphi_2^2\rangle = 0$. In either vacuum the masses of the $\varphi_1$ and $\varphi_2$ particles are different, and these masses are interchanged in the other vacuum. Thus a discrete symmety which interchanges fields has yielded particle masses which change across a wall.

Left-right symmetric particle theories have a parity symmetry under interchange of left- and right-handed fermions. To describe the real world this symmetry must be spontaneously broken. Then the theory will have two degenerate vacua, as in the previous example, which may be distinguished by large values for one or the other of $\langle\psi_L\psi_L\rangle$ or $\langle\psi_R\psi_R\rangle$ for some fermion $\psi$. On one side of a wall $\psi_L$ has a large Majorana mass and $\psi_R$ is light, while on the other side the roles are reversed. On the first side if a $\psi_R$ particle has insufficient energy, it cannot penetrate the wall.

The logical candidate for $\psi$ is the neutrino. $\nu_R$ is a singlet under the known gauge interactions SU(3)×SU(2)×U(1) and is therefore allowed to have a Majorana mass larger than the scale of weak interactions. $\langle\nu_R\nu_R\rangle \neq 0$ may therefore be the order parameter which spontaneously breaks a left-right symmetric theory down to SU(3)×SU(2)×U(1). In other regions of the early universe it may instead be $\nu_L$ which is very massive, and domain walls will separate the two vacua. The very massive neutrinos quickly decay away in a theory of this sort and we are left with light neutrinos of opposite chirality on opposite sides of a wall. The wall is opaque for these neutrinos whereas it is transparent for all other light particle species. A closed domain wall which traps $\nu_R$'s on the inside will be referred to as a neutrino-ball (NB).

The NBs of interest will form near the end of the wall era, before the walls start to dominate. We assume that a small number density of NBs escape the infinite wall removal mechanism.

We consider static spherical NBs since background neutrinos damp oscillations. The trapped relativistic neutrino gas has a pressure $^1/_3\rho$ which balances the pressure $2\sigma/r$ due to the surface tension $\sigma$, giving $\rho = 6\sigma/r$. The total mass of the NB is then $M = 4\pi r^2\sigma + {}^4/_3\pi r^3\rho = 12\pi r^2\sigma$. This also shows that ${}^2/_3$ of the total mass is neutrino energy and ${}^1/_3$ is wall energy. The initial energy density and number density of neutrinos, $\rho_i$ and $n_i$, are determined by the initial temperature $T_i$. For a given $\sigma$, $T_i$ determines all NB properties.

Neutrinos may annihilate into other particles which may then escape. They annihilate weakly, $\bar{\nu}_R\nu_R \Rightarrow e^-e^+$, with standard weak cross-section $\sigma \approx G_F{}^2m_e{}^2$. (The weak interactions are standard, except that they are parity flipped.) They may also annihilate into photons via a one-loop graph obtained by closing the electron lines and attaching photons. The two photon amplitude is suppressed by the neutrino mass.[1] The three photon mode goes via a dimension 10 effective low energy operator, and this yields the estimate $\sigma \approx G_F{}^2\alpha^3E_\nu{}^{10}/m_e{}^8$. This small one loop quantum effect turns out to determine the NB lifetime.

We will discuss three phases of NB evolution. The first is a cooling phase due to the $e^-e^+$ annihilation mode. The second is a long-lived radiating phase causing a slow shrinkage in size. The third is an explosive death occurring at a critical size when the $e^-e^+$ annihilation mode turns back on.

But the initial cooling requires that $T_i$ be sufficiently below $m_e$. Then only the neutrinos on the high energy tail annihilate, and the average energy, $\rho/n$, of the remaining neutrinos falls. This produces a nonzero chemical potential $\mu$, the same for $\nu_R$ and $\bar{\nu}_R$. (We consider equal numbers of $\nu_R$'s and $\bar{\nu}_R$'s.) The annihilation rate slows and the neutrinos slowly approach degeneracy.

We wish to derive the critical $T_i$ below which this happens. This will translate into a lower bound on the mass for a long-lived NB. Most neutrinos which annihilate will have energies close to $m_e$. This allows us to relate the change in the total mass of the NB to the change in the total number of trapped neutrinos, $dM/M = {}^2/_3(m_e n/\rho)dN/N$. This in turn implies the following relation between change in number density and energy density,
$dn/n = \{-(3\rho/m_e n) + 3\}d\rho/\rho$. This relation may be solved to yield

$$w(x) = xy+x^3(1-y) \tag{3}$$

where $x \equiv \rho/\rho_i$, $w \equiv n/n_i$, $y \equiv T_i/T_C$, and $T_C \equiv 0.21155m_e$.

On the other hand for complete neutrino degeneracy, $n_d = 2\mu_d^3/3\pi^2$ and $\rho_d = \mu_d^4/2\pi^2$, and this gives $w_d(x) = 1.9244x^{3/4}$. Thus the NB approaches degeneracy only if there exists an $x_{ac}$ (after cooling) such that $w(x_{ac}) = w_d(x_{ac})$. This occurs if $y \leq 1.000231$.

This sets a lower bound on the initial NB mass since $M_i = M_C y^{-8}$ where $M_C \approx 0.5\times10^5(\sigma[\mathrm{TeV}^3])^3$ solar masses. On the other hand the initial mass must satisfy $2GM_i \lesssim r_i$ or else the NB is a black hole. This gives an upper bound $M_i \lesssim 1.3\times10^8/\sigma[\mathrm{TeV}^3]$ solar masses. These two bounds on $M_i$ are only consistent if $\sigma^{1/3} \lesssim 1.93$ TeV. (We have assumed two neutrino flavors.)

It is possible to obtain the function $x_{ac}(y) \equiv \rho_{ac}/\rho_i = r_i/r_{ac}$ (see [1] for a plot). But for $y$ not too close to unity, say 0.95 or less, we find that $x_{ac}(y)$ increases very slowly with $y$, with $x_{ac}(0.9) \approx 2$. We also find that the chemical potential after cooling $\mu_{ac}(y)$ increases very slowly with $y$, with $\mu_{ac}(0.9) \approx {}^1/_2 m_e$. Above $y \approx 0.95$, $\mu_{ac}(y)$ rapidly approaches $m_e$.

We may estimate the time scale for cooling, $\tau_C$. We set $1/\tau_C \approx (dN/dt)/N = (dn/dt|_{ann})/n$ where $dn/dt|_{ann}$ is due to annihilations and not to the changing volume. This is calculated using the weak cross section and using the number density of neutrinos with energy greater than $m_e$. We find $\tau_c \approx 2\times10^3 y\exp(9.45/y)$ sec $\gtrsim$ 1 year.

As the neutrinos cool and approach degeneracy annihilations to electrons become less and less frequent. Eventually annihilations to photons becomes the dominant energy loss mechanism. Annihilations to electrons still occur, but only occur to the extent needed to keep the neutrinos close to degeneracy. We may find the fraction $p(t)$ of annihilations which produce photons. The relation, $dn/n = (-(3\rho/\bar{E}n) + 3)d\rho/\rho$, is derived as before where $\bar{E}$ is the average energy of an annihilating neutrino. But since the NB remains close to degeneracy we have $dn/n = {}^3/_4 d\rho/\rho$, which then requires $\bar{E} = \mu(t)$. We may write

$\bar{E} = \eta\mu(t)p(t) + m_e(1-p(t))$ where $\eta\mu(t)$ is the average energy of an annihilating neutrino when photons result. ($\eta$ is some number like 0.9.) In this way we obtain $p(t) = (m_e - \mu(t))/(m_e - \eta\mu(t))$. We thus find that $p(t) \approx 1$ as long as $\mu(t)$ is sufficiently less than $m_e$. Only when $\mu(t) \rightarrow m_e$ do annihilations to electrons again become important.

The lifetime $\tau$ of a NB is the time it takes the NB to shrink and heat up to the point where $\mu(t) \rightarrow m_e$. We estimate this time to be $\tau \approx (dN/dt)/N \approx (n/4)\sigma(\bar{\nu}\nu \rightarrow \gamma\gamma\gamma) \approx (m_e/\mu_{ac})^{13}10^{14-15}$ seconds. The factor $(m_e/\mu_{ac})^{13}$ is $\approx 10^3$ for $y = 0.95$ and $\approx 10^5$ for $y = 0.75$. We thus find a lifetime which is of order the age of the universe. It is of interest that this estimate of the lifetime is obtained from known physics and is independent of $\sigma$, the unknown mass density of the wall.

A typical NB has a radius of 10 - 100 light seconds, a mass of $10^{4-7}$ solar masses, and radiates few hundred KeV $\gamma$-rays at a rate of $10^{40-43}$ ergs/sec. In the final burst the NB converts into $10^{56-59}$ ergs of $e^+e^-$ pairs. Note that the last three numbers quoted vary as $\sigma^3$; we used $\sigma^{1/3} \approx 1 - 2$ TeV.

We have not discussed NB production. But we do know that it must be very inefficient; more than one NB for every $\approx 10^6$ horizon volumes at the time of formation would lead to NB domination. We also expect that most long-lived NBs are of similar size close to the minimum size; very much larger ones would be extremely suppressed.

Various astrophysical questions remain. The NB mass range makes them candidate seeds for galaxy formation. Then perhaps NBs are located in (active?) galactic nuclei. And note that NBs accrete matter, possibily resulting in supermassive black holes. But even if most or all NBs expired in the past, their imprint could still be left on the X-ray and $\gamma$-ray background.

1) B. Holdom, Phys. Rev. D36 (1987) 1000.