



Proton decay in SUSY GUTs

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 We review proton decay in the SUSY SU(5) GUTs assuming the mini-split SUSY breaking model. In the mini-split SUSY breaking model, the squark and slepton masses are $O(10^{2-3})$ TeV while the gaugino masses are $O(1)$ TeV. As the result, even the minimal SUSY SU(5) GUT is still viable in the model. We present the motivation of the mini-split SUSY model and discuss the future prospects of proton decay searches in the SUSY SU(5) GUTs.

Subject Index B42

1. History of proton decay in GUTs

Proton decay is a window for the Grand Unified Theories (GUTs) [1,2] (see also Ref. [3] for a recent review of GUTs). GUTs have two aspects. The first one is the unification of three forces in the Standard Model (SM), and the other is the unification of quarks and leptons. As a result, baryon (B) and lepton (L) numbers are not conserved, and proton decay is predicted in GUTs. Since proton decay is mediated by the GUT particles, it has been argued that proton decay is a direct probe of the GUTs. The minimal gauge group of GUTs is SU(5). When assuming that quarks and leptons, including right-handed neutrinos, are unified into a GUT multiplet for each generation, the gauge group is SO(10) or larger.

Proton decay has been sought since Georgi and Glashow proposed the SU(5) GUTs and Georgi, Quinn, and Weinberg evaluated proton lifetime in the models in 1974 [1,2]. The X boson is introduced in the SU(5) GUT since the gauge group is extended from $SU(3)_C \times SU(2)_L \times U(1)_Y$ in the SM. Integration of the X boson generates baryon-number violating dimension-six operators for proton decay. The main decay mode is $p \rightarrow \pi^0 e^+$. The proton lifetime is proportional to the fourth power of the X boson mass (M_X). In the 1980s, large-volume detectors were constructed and operated to search for proton decay (see Ref. [4] for a review of experiments on proton decay in the 1980s). They included the Kamiokande (Kamioka Nucleon Decay Experiment) and IMB (Irvine–Michigan–Brookhaven) detectors. They are large water Čerenkov detectors of kiloton pure water with $O(10^3)$ PMTs. At that time, the GUT scale (M_{GUT}) in the minimal SU(5) GUT was expected to be around 10^{14} GeV from the unification of the three gauge coupling constants in the SM, though the uncertainties of the measured gauge

coupling constants were large [5]. The proton lifetime was predicted to be $\sim 10^{30}$ years. The experiments gave upper bounds on the partial lifetime¹ of $p \rightarrow \pi^0 e^+$ larger than $\sim 10^{32}$ years, and the minimal SU(5) GUT was considered to be excluded around 1986 (see Ref. [6] for a review of GUTs in 1986).

The minimal SU(5) GUT is the simplest model in the GUTs, though it has several problems. The gauge hierarchy problem is one of them [7–11]. The mass term of the Higgs boson in the SM suffers from quadratic divergence in the radiative correction. The gauge hierarchy between the weak and GUT scales is unstable even if we fine-tune parameters in the minimal SU(5) GUT at tree level. One of the solutions is the introduction of supersymmetry (SUSY) [12–15]. Superpartners are introduced for each particle in supersymmetric theories, and the radiative corrections to the scalar mass terms are at most logarithmically divergent. The gauge hierarchy is stable in the supersymmetric models. The supersymmetric extensions of the GUTs and SM are the SUSY GUTs and the SUSY SM, respectively (for reviews of SUSY SM and SUSY GUTs, see Refs. [16–18]). The SUSY breaking scale (m_{SUSY}) corresponds to the masses of superpartners in the SUSY SM. The SUSY breaking scale was expected to be below $O(1)$ TeV, since additional fine-tuning in the SUSY SM, the *little hierarchy* between the SUSY breaking and the weak scales, was not hopeful. It was pointed out that the GUT scale increased to $\sim 10^{16}$ GeV in the SUSY SU(5) GUTs, since the beta functions of the gauge coupling constants are changed due to the introduction of superpartners in the SUSY SM.

In the 1990s, the Weinberg angle was precisely measured in the LEP and SLC experiments. It was found that the three gauge coupling constants are successfully unified at $\sim 2 \times 10^{16}$ GeV when they are extrapolated to the higher energy in the SUSY SM [19–21]. This was the big step to considering that the SUSY GUTs would be more realistic beyond conceptual ideas. Since the test of gauge coupling unification was successful, the next test was proton decay. Since the GUT scale had increased to $\sim 2 \times 10^{16}$ GeV, the prediction of the partial proton lifetime of $p \rightarrow \pi^0 e^+$, induced by the X boson, was $10^{(35-36)}$ years [22]. It was safe from the bounds derived by the Kamiokande and IMB experiments. On the other hand, the SUSY GUTs have another source of proton decay.

The supersymmetric extensions of SM and GUTs introduce scalar fields with baryon or lepton numbers, squarks and sleptons, as superpartners of quarks and leptons, respectively. Thus, we can introduce baryon-number violating dimension-five operators [23,24]. In the minimal SUSY SU(5) GUT, the operators are generated by colored Higgs multiplets, which are SU(5) partners of the Higgs bosons and Higgsino in the SUSY SM. The operators are proportional to the inverse of the colored Higgs mass (M_{H_C}). The squarks and sleptons in the operators are heavier than proton mass, and then the dimension-six operators are generated by the exchange of gauginos or the Higgsino in the SUSY SM. Although only gaugino exchange diagrams were evaluated in the earlier papers [22,25,26], it was pointed out in Ref. [27] that the Higgsino exchange diagram may be comparable to the gaugino ones. The proton lifetime is proportional to $M_{H_C}^2 m_{\text{SUSY}}^2$. The dimension-five operators are more dangerous than the dimension-six ones. The proton decay rates are suppressed by the Yukawa coupling constants in the minimal SUSY SU(5) GUT. However, the predicted partial lifetime of $p \rightarrow K^+ \bar{\nu}$ is smaller than 10^{32} years in

¹The partial lifetime of the process is inverse of the event rate of the process, or the proton lifetime over the branching ratio.

the minimal SUSY SU(5) GUT, assuming $m_{\text{SUSY}} < O(1)$ TeV. The decay mode $p \rightarrow K^+ \bar{\nu}$ is the largest in the minimal SUSY SU(5) GUT unless it is accidentally suppressed.

The SuperKamiokande experiment started in 1996. The detector is a large water Čerenkov detector of 50 kilotons pure water with $O(10^4)$ PMTs. The sensitivities to proton decay were quite enhanced. The experiment updated the upper bounds on partial lifetimes of various proton decay modes. The current bounds on the partial lifetimes $p \rightarrow \pi^0 e^+$ and $p \rightarrow K^+ \bar{\nu}$ are 2.4×10^{34} years [28] and 6.6×10^{33} years [29], respectively. When the Higgsino exchange diagrams were included in the analysis of the dimension-five operator proton decay in 1999, it was found that accidental suppression of the proton decay did not happen and that the minimal SUSY SU(5) GUT was also excluded [27].

The minimal SUSY SU(5) GUT was excluded, though the success of the gauge coupling unification in the SUSY GUTs was still quite beautiful. People believed that some mechanism would suppress the dimension-five operator proton decay. The introduction of some symmetries, such as the Peccei–Quinn symmetry [23,30] or the R symmetry in the extra-dimensional models [31], was proposed to suppress it.

In 2008, the LHC experiments started. The superpartners in the SUSY SM and the Higgs boson in the SM were expected to be discovered at the experiments. People guessed that the squark might be discovered earlier than the Higgs boson. However, the opposite was true. The Higgs boson was discovered in 2012 [32,33], while the superpartners have not yet been discovered (for the latest results of the SUSY searches, see Refs. [34,35]). In addition, the observed Higgs boson mass is around 126 GeV. It is too heavy to be predicted in the SUSY SM if the stops, the superpartners of top quarks, are lighter than a few TeV [36–40]. This is consistent with the null result of superpartner searches at LHC.

The superpartner masses might be much heavier than $O(1)$ TeV even if they exist. However, since people had considered that the SUSY breaking scale is below $O(1)$ TeV from the viewpoint of the little hierarchy problem, this fact had a big impact on studies of physics beyond the SM (BSM). One choice is to abandon SUSY. Another choice is to accept the possibility that the squarks and sleptons might have masses heavier than the weak scale [41–47]. Some people are considering the non-SUSY GUTs again. However, in this review, we follow the second choice, called the mini-split SUSY model, assuming that the squark and slepton masses are around $O(10^{2-3})$ TeV [48–51]. While we have to abandon SUSY as a solution to the little hierarchy problem, we reserve the other advantages of the SUSY GUTs, such as gauge coupling unification, WIMP dark matter, and the gauge hierarchy problem. In addition, the various issues in the SUSY SM are solved in the model. They include the dimension-five operator proton decay. The dimension-five operator proton decay is suppressed by increasing the squark and slepton masses, and even the minimal SUSY SU(5) GUT is revived.

Now new experiments for proton decay are being prepared for. The main target of the HyperKamiokande, DUNE, and JUNO experiments is to study the neutrino oscillation, though they have higher sensitivities to proton decay searches. Proton decay may be discovered there.

In 2002, a meeting of theorists and experimentalists in particle physics was held at the International Institute of Advanced Studies, Japan with the title “Origin of Matter” [52]. Prof. Koshihara gave a summary talk at the meeting. He said, “I have to give candid advice to you, especially theorists. Why are you still studying GUTs and SUSY, which have already been excluded?” It is assumed that he might consider that GUTs and SUSY were excluded by the Kamiokande and

SuperKamiokande experiments. At that time and now, theorists still have reasons to consider SUSY and GUTs and furthermore hope that proton decay will be sought.

In the next section we give a general review of baryon-number violating nucleon decay, the current experimental bounds on proton decay, and the future prospects. In Section 3, we discuss dimension-six and five operator proton decay in the mini-split SUSY model after introducing it. Section 4 is devoted to a summary of this review. In this review, we assume SUSY GUTs in 4D spacetime, but not non-SUSY GUTs and GUTs in extra-dimensional space since our space and time are limited.

2. Baryon-number violating nucleon decay processes and future prospects

Rare processes are sensitive to BSM, for which the energy frontier experiments are difficult to access. Among them, the baryon-number violating nucleon decays are the most sensitive to physics beyond the SM at an extremely high energy scale. Assuming that there are no lighter fermions than protons except for leptons, the processes are classified as follows.

$$\Delta(B + L) = 2 \text{ nucleon decay: } p \rightarrow \pi^0 e^+, n \rightarrow \pi^0 \bar{\nu}, \dots$$

The lowest effective operators are dimension-six in the SM. A list of the operators is given in Refs. [53,54]. Some of them are predicted in various GUT models. The various modes are being updated by the SuperKamiokande experiment [55]. The future prospects are presented below.

$$\Delta(B - L) = 2 \text{ nucleon decay: } n \rightarrow \pi^+ e^-, \dots$$

These are induced by the dimension-seven effective operators in the SM, and they include the SM Higgs boson or derivative. A list of the operators is given in Refs. [56,57]. They are predicted in some SO(10) GUTs with intermediate scales. They might be linked to baryogenesis, since the $B-L$ number generated in the early universe is not be washed out by the sphaleron process [58,59]. The charge discrimination is required to identify the $\Delta(B - L) = 2$ modes. The bounds on them have not been updated since the 1990s.

$$\Delta B = 2 \text{ dinucleon decay: } pp \rightarrow \pi^+ \pi^+, \dots$$

These processes are possible inside a nucleus. They are induced by dimension-nine effective operators in the SM. It is pointed out that some SO(10) GUTs with intermediate scales predict them, and that they may be linked to Majorana neutrino masses. The lower limit of lifetimes for neutrons bound in ^{16}O from the SuperKamiokande experiment gives the bound on the $n-\bar{n}$ oscillation as $\tau(n-\bar{n}) > 4.7 \times 10^8 \text{ s}$ [60]. This is better than the latest bound from the free neutron experiment $\tau(n-\bar{n}) > 8.6 \times 10^7 \text{ s}$ [61].

The SuperKamiokande experiment gives upper bounds on the various decay modes mentioned above. The $\Delta(B - L) = 2$ or $\Delta B = 2$ processes might be interesting since they are predicted in some SO(10) GUTs with intermediate scales. However, the successful gauge coupling unification suggests SUSY GUTs without intermediate scales. Thus, we concentrate on the $\Delta(B + L) = 2$ nucleon decay, $p \rightarrow \pi^0 e^+$ and $p \rightarrow K^+ \bar{\nu}$. They are induced by the dimension-six and five operators in the SUSY SU(5) GUTs. The baryon-number violating neutron decay processes are also being sought, though the bounds are comparable to or weaker than the proton decay processes of the counterparts [55].

The current bounds on the processes are derived by the SuperKamiokande experiments as $\tau(p \rightarrow \pi^0 e^+) > 2.4 \times 10^{34} \text{ years}$ [28] and $\tau(p \rightarrow K^+ \bar{\nu}) > 6.6 \times 10^{33} \text{ years}$ [29]. The $\pi^0 e^+$ mode can be fully constructed by the water Čerenkov detector, and the backgrounds are suppressed. The

experimental sensitivities are determined by the fiducial volume of the detector. On the other hand, the $K^+\bar{\nu}$ mode has a missing momentum, and it suffers from the atmospheric neutrino background. It is necessary to understand atmospheric neutrinos in order to study the $K^+\bar{\nu}$ mode.

The HyperKamiokande detector [62] is now being constructed. It is also a water Čerenkov detector of 187 kilotons pure water. Due to the larger volume, it is the best prospect for $p \rightarrow \pi^0 e^+$. However, due to the larger statistics, more dedicated studies are needed to suppress the atmospheric neutrino background. It was found by a simulation that the 90% CL limit may reach $7.9 (13) \times 10^{34}$ years for a 10 (20) year run, assuming that a second tank is installed after 6 years.

For the $K^+\bar{\nu}$ mode, the JUNO [63] and DUNE experiments [64,65] could compete with the HyperKamiokande experiments [62] even though their fiducial values are smaller. These are liquid scintillator detectors so they have better energy resolution to reject the atmospheric neutrino backgrounds. The prospects of the 90% CL limit of the JUNO, DUNE, and HyperKamiokande experiments are $1.9(4.0) \times 10^{34}$ years, $3.3(6.5) \times 10^{33}$ years, and $3.2(5.0) \times 10^{34}$ years for a 10 (20) year run, respectively.

3. Proton decay in SUSY SU(5) GUTs

Now we discuss proton decay in SUSY GUTs. However, we first review the mini-split SUSY model, which is consistent with the current experimental data, and discuss the GUT-scale mass spectrum expected from the gauge coupling unification in the model.

3.1 Mini-split SUSY model

The SUSY SM is motivated by (i) the gauge hierarchy problem between the weak and GUT scales, (ii) the successful gauge coupling unification, compatible with SUSY GUTs, and (iii) the stable lightest SUSY particle (LSP) due to the R parity, which is a candidate for the dark matter in the universe. However, it is known that the SUSY SM with $m_{\text{SUSY}} < O(1)$ TeV has encountered some problems since the supersymmetric extension of the SM was introduced. They are listed as follows.

(1) The FCNC and CP problems:

While flavor-violating and/or CP-violating SUSY breaking terms are allowed in the Lagrangian, they are constrained by FCNC and CP-violating processes, such as $K^0-\bar{K}^0$ mixing and electric dipole moments [66]. If the off-diagonal terms are not suppressed in the squark mass matrices compared with the diagonal ones, the squark masses are larger than $O(10^{(2-3)})$ TeV.

(2) The gravitino problem:

The gravitino is the superpartner of the gravitino with spin 3/2. It only has gravitational interactions suppressed by the Planck scale. If it is unstable, the lifetime is given by $\tau_{3/2} \sim 10 \text{ s} \times (10 \text{ TeV}/m_{3/2})^3$ ($m_{3/2}$ is the gravitino mass). If the gravitino mass is $O(1)$ TeV, successful nucleosynthesis may be spoiled [67].

(3) The dimension-five operator proton decay problem:

Baryon-number violating dimension-five operators are induced in the SUSY GUTs as mentioned above. In addition, the operators suppressed by the Planck scale may be generically present, though they are constrained by the proton decay searches. They may be

suppressed by some global symmetries, while global symmetries are argued to be incompatible with quantum gravity.

It was considered that these problems would be solved by some SUSY breaking mechanisms or some cosmological scenarios. For example, people constructed many models of SUSY breaking to generate universal SUSY breaking scalar mass terms with state-of-the-art technology of quantum field theories. Now a new problem has appeared, i.e., the observed Higgs boson mass. In the minimal SUSY SM, the Higgs boson mass is smaller than the Z boson mass at tree level, and the radiative correction from the top quark and stop loops raises it, but not to 126 GeV if the stops are lighter than $O(1)$ TeV [36–40].

Now we introduce the mini-split SUSY model [48–51]. In this model, the SUSY breaking terms are generated by the gravity mediation, and the gravitino mass is the order parameter in the SUSY breaking in the supergravity (for reviews of SUSY SM and SUSY GUTs, see Refs. [16–18]). The gravitino mass is $O(10^{(2-3)})$ TeV, and the SUSY breaking scalar masses are also comparable to the gravitino mass. Then, the problems mentioned above are automatically solved in this setup. The gaugino masses are generated by the anomaly mediation mechanism [68,69] so that they are suppressed by one-loop factors compared with the gravitino mass. The gaugino masses are proportional to the beta functions of gauge coupling constants. Then, the $SU(2)_L$ gaugino, called the wino, is the lightest of the gauginos. The Higgsino mass would be comparable to the gravitino mass. However, it could be lighter if the Higgsino mass is suppressed by some symmetry.

The wino or the Higgsino is the LSP in the mini-split SUSY model, and they are dark matter candidates. If they are coupled with the thermal bath in the early universe and they explain the observed dark matter abundance, the wino mass is about 3 TeV [70] and the Higgsino mass is 1 TeV [71]. Since they have electroweak interactions, their masses are $O(1)$ TeV, not $O(100)$ GeV.

The difference between the beta functions of tree gauge coupling constants in the SUSY SM comes from the gauginos and the Higgsino in addition to the gauge and Higgs bosons. Since the gauginos are $O(1)$ TeV, the successful unification of the gauge coupling constants is not spoiled. Furthermore, if the Higgsino mass is $O(10^{(2-3)})$ TeV, the gauge coupling unification is improved, as discussed below [72].

The little hierarchy between the weak and SUSY breaking scales would be accidental in the mini-split SUSY model. The parameters in the Higgs potential must be finely tuned so that the Higgs vacuum expectation value is much smaller than the SUSY breaking scale. However, the big hierarchy between the SUSY breaking and GUT scales is still stabilized by supersymmetry.

Last, we have an additional advantage of the mini-split SUSY model. The model building of SUSY breaking becomes much easier due to the anomaly mediation, which is built in supergravity. What we need is only the introduction of the hidden sector decoupled from the SUSY SM, which is responsible for spontaneous SUSY breaking [48–51]. The supergravity generates the hierarchical SUSY breaking masses for scalar bosons and gauginos. This is the reason why “mini-split SUSY” is called “pure-gravity mediation” in Refs. [48,49].

The possible experimental tests of the mini-split SUSY model are limited since squarks and sleptons are quite heavy. The LSP, wino or Higgsino, is a window to the model. The future prospects of the direct [73–75] and indirect dark matter searches [76,77] may reach the prediction of wino and Higgsino dark matter.

Another window is the violation of fundamental symmetries. The electric dipole moment of an electron is induced by Barr–Zee two-loop diagrams in the Higgsino–wino system, and the prediction may be within the future prospects of the experiments [78]. Proton decay might be another one. The minimal SUSY SU(5) GUT is revived in the mini-split SUSY model. The prediction may be reached by future experiments.

3.2 GUT-particle mass spectrum from the gauge coupling unification

The gauge coupling unification gives constraints on the GUT-particle mass spectrum [79]. First, let us assume the minimal SUSY SU(5) GUT for simplicity². In this case, the GUT-particle mass spectrum includes the X boson mass (M_X), the GUT breaking Higgs mass (M_Σ), and the colored Higgs mass (M_{H_c}). The GUT gauge symmetry is broken by the vacuum expectation value of the SU(5) adjoint Higgs multiplet. The three gauge coupling constants in the SM are related to the SU(5) one via the renormalization-group equations. Then, we get two relations among the three gauge coupling constants after removing the SU(5) gauge coupling constant:

$$\begin{aligned} \frac{3}{\alpha_2(m_Z)} - \frac{2}{\alpha_3(m_Z)} - \frac{1}{\alpha_1(m_Z)} &= \frac{1}{2\pi} \left[\frac{12}{5} \ln \left(\frac{M_{H_c}}{m_Z} \right) - \frac{2}{5} \ln \left(\frac{M_{\tilde{H}}^4 M_H}{m_Z^5} \right) + 4 \ln \left(\frac{m_3}{m_2} \right) \right], \\ \frac{5}{\alpha_1(m_Z)} - \frac{3}{\alpha_2(m_Z)} - \frac{2}{\alpha_3(m_Z)} &= \frac{1}{2\pi} \left[12 \ln \left(\frac{M_X^2 M_\Sigma}{m_Z^3} \right) - 2 \ln \left(\frac{m_2}{m_Z} \right) + 4 \ln \left(\frac{m_3}{m_Z} \right) \right]. \end{aligned} \quad (1)$$

Here, $\alpha_i(m_Z)$ ($i = 1-3$) are the gauge coupling constants in the SM at the Z boson mass (m_Z). The U(1) gauge coupling constant is given by $\alpha_1 = 5\alpha_Y/3$. The factor $5/3$ in α_1 comes from the GUT normalization. $m_{\tilde{H}}$, m_H , M_3 , and M_2 are the masses of the Higgsino, the second Higgs boson, the gluino, and the wino, respectively, in the SUSY SM. We take a common mass for squarks and sleptons for simplicity so that their dependences disappear in the above equations. The first equation comes mainly from the splitting between the masses of the Higgs bosons and the Higgsino in the SUSY SM and the colored Higgs mass, which are embedded into the SU(5) $\mathbf{5}$ and $\bar{\mathbf{5}}$ multiplets; the second one is from the mass splitting in the gauge and GUT breaking Higgs multiplets. It is found that the colored Higgs mass is constrained from the gauge coupling constant unification. On the other hand, the X boson mass is not determined since it is correlated with the GUT breaking Higgs mass.

The gaugino masses in the SUSY SM are generated by the anomaly mediation as [68,69]

$$\begin{aligned} M_1 &= \frac{33}{3} \frac{\alpha_1}{4\pi} \left(m_{3/2} + \frac{1}{11} L \right), \\ M_2 &= \frac{\alpha_2}{4\pi} (m_{3/2} + L), \\ M_3 &= -3 \frac{\alpha_3}{4\pi} m_{3/2}, \end{aligned} \quad (2)$$

where L comes from the Higgsino and the Higgs boson one-loop diagrams,

$$L = m_{\tilde{H}} \sin 2\beta \frac{m_H^2}{|m_{\tilde{H}}|^2 - m_H^2} \log \frac{|m_{\tilde{H}}|^2}{m_H^2},$$

with $\tan \beta$ the vacuum expectation value ratio of two Higgs bosons in the SUSY SM. L is not negligible if the Higgsino mass is $O(10^{(2-3)})$ TeV. In the mini-split SUSY model, m_H is $O(10^{(2-3)})$ TeV. We take the ratio of the gaugino masses as a free parameter.

In Fig. 1 we show the colored Higgs mass M_{H_c} , evaluated from the gauge coupling unification,

²The particle contents and Lagrangian of the minimal SUSY SU(5) GUT are given in Ref. [22].

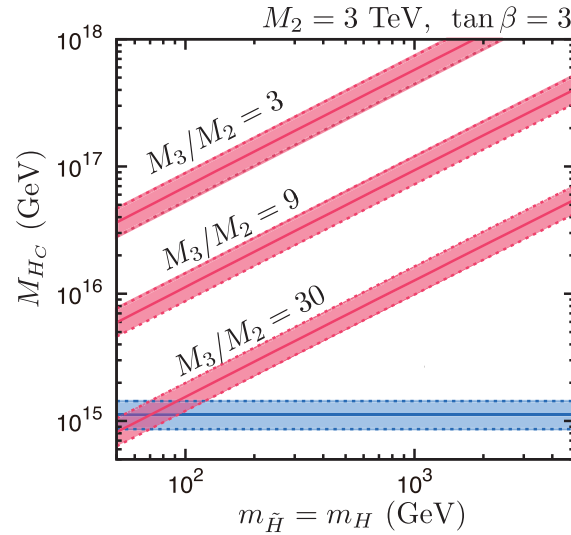


Fig. 1. Colored Higgs mass, M_{H_C} , evaluated from gauge coupling unification, as a function of $M_{\tilde{H}} = M_H$ (red regions). Here, the wino mass is 3 TeV and $\tan \beta = 3$. The gluino and wino mass ratios are taken to be 3, 9, 30. The blue region is the prediction of SUSY SM with $m_{\text{SUSY}} < O(1)$ TeV. This figure is originally from Ref. [72].

as a function of $M_{\tilde{H}} = M_H$ (red regions). This figure is originally from Ref. [72]. Here, the wino mass is 3 TeV, and $\tan \beta = 3$. The width of the region comes mainly from the uncertainty of $\alpha_3(m_Z)$. The gluino and wino mass ratios are taken to be 3, 9, 30. The blue region is the prediction of the SUSY SM with $m_{\text{SUSY}} < O(1)$ TeV. It is pointed out in Ref. [79] that the colored Higgs mass is lower than the GUT scale ($\sim 10^{16}$ GeV) in the SUSY SM with $m_{\text{SUSY}} < O(1)$ TeV. In the mini-split SUSY model, the colored Higgs mass could be predicted around the GUT scale. This implies that gauge coupling unification could be more precisely realized in the model.

Next, we define the GUT scale as $M_{\text{GUT}} \equiv (M_X^2 M_\Sigma)^{1/3}$, and we show M_{GUT} as a function of the gluino mass M_3 in Fig. 2. This figure is originally from Ref. [72]. Here, we take $M_{\tilde{H}} = M_H = 10^3$ TeV, $\tan \beta = 3$, and $M_2 = 300$ GeV and 3 TeV. The blue region is the prediction of the SUSY SM with $m_{\text{SUSY}} < O(1)$ TeV. The uncertainty of the prediction from $\alpha_3(m_Z)$ is much smaller than that of the colored Higgs mass.

When the gaugino masses are greater, M_{GUT} is lower. We cannot determine the X boson mass from the gauge coupling unification, since the X boson mass is correlated with the GUT breaking Higgs mass in M_{GUT} . However, the lower GUT scale might suggest the enhancement of the dimension-six proton decay.

Now we discuss the model dependence of the prediction of the GUT-particle mass spectrum. In the minimal SUSY SU(5) GUT, the SU(5) gauge symmetry is broken by the SU(5) adjoint Higgs multiplet. The large splitting between the colored Higgs mass and the masses of the Higgsino and Higgs bosons in the SUSY SM is given by hand. This tree level fine-tuning is not avoidable in the model, though the fine-tuning is not spoiled by the radiative correction due to supersymmetry. This problem is called the doublet–triplet splitting problem. The missing partner model [80,81] is introduced as the solution to the problem³. In the model, the SU(5)

³The particle contents and Lagrangians of the missing partner model and the Peccei–Quinn symmetric extension are given in Ref. [30].

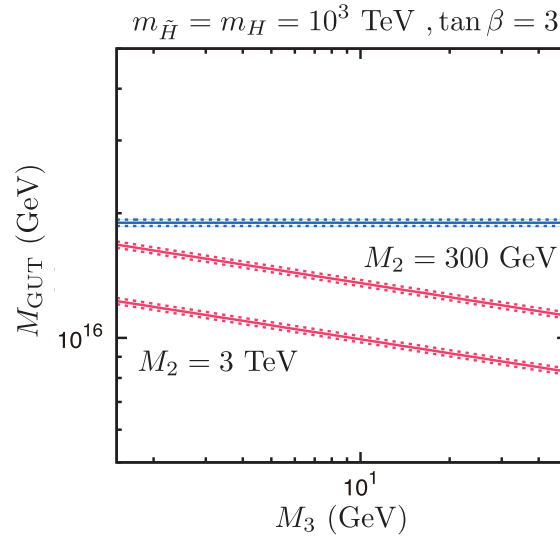


Fig. 2. $M_{\text{GUT}} \equiv (M_X^2 M_\Sigma)^{1/3}$, evaluated from gauge coupling unification, as a function of gluino mass, M_3 . Here, we take $M_{\tilde{H}} = m_H = 10^3$ TeV, $\tan \beta = 3$, and $M_2 = 300$ GeV and 3 TeV. The blue region is the prediction of SUSY SM with $m_{\text{SUSY}} < O(1)$ TeV. This figure is originally from Ref. [72].

gauge symmetry is broken by the **75D** Higgs multiplet, and **50** and $\bar{\mathbf{50}}$ are also introduced for the mechanism to work. In this model, the mass splitting among the components in the **75D** Higgs multiplet contributes to the difference between the three gauge coupling constants in the SM. As the result, the *effective* colored Higgs mass, which appears in the dimension-five operators, is 2×10^4 times larger than the prediction of the minimal SUSY SU(5) GUT [82]. The GUT scale M_{GUT} is lower than that of the minimal model by only 1.4.

The **75**, **50**, and $\bar{\mathbf{50}}$ D Higgs multiplets make big contributions to the beta function of the SU(5) gauge coupling constant. In particular, the SU(5) gauge coupling constant increases above the mass of **50** and $\bar{\mathbf{50}}$. It is suggested from the gauge coupling unification that the effective colored Higgs mass is much larger than the GUT scale, though the perturbative picture is broken in the minimal model. If the Peccei–Quinn symmetry is introduced and pairs of **50** and $\bar{\mathbf{50}}$ and **5** and $\bar{\mathbf{5}}$ Higgs multiplets are introduced, the perturbative picture is not broken and the dimension-five operator proton decay is suppressed [30].

The minimal SUSY SU(5) GUT and the missing partner model are simple models. However, unknown particles with GUT scale masses may be introduced and mass splitting in the GUT multiplets may be generated due to their coupling to the GUT breaking Higgs field. In this case, they may contribute to the differences between the gauge coupling unification, and the above results of the GUT-particle mass spectrum may be merely qualitative expectations from the gauge coupling unification.

3.3 Dimension-six operator proton decay

The baryon-number violating dimension-six operators are induced by the X boson in the SUSY SU(5) GUTs (Fig. 3). The proton lifetime is given as

$$\tau(p \rightarrow \pi^0 e^+) \simeq 1 \times 10^{35} \times \left(\frac{M_X}{10^{16} \text{ GeV}} \right)^4 \text{ years}, \quad (3)$$

where M_X is the X boson mass. Thus, if $M_X < 10^{16}$ GeV, HyperKamiokande may discover X boson proton decay. The GUT scale might be smaller than 10^{16} GeV in the mini-split SUSY

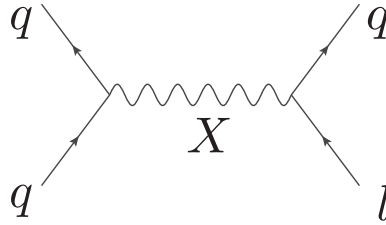


Fig. 3. Dimension-six operators induced by the X boson.

model as mentioned above, though the gauge coupling constant unification cannot determine the X boson mass itself.

The prediction for the proton lifetime of the X boson can be precisely evaluated now if explicit models of the SUSY GUT are assumed. The procedure for the evaluation is as follows. (1) The effective dimension-six operators are derived by integrating the X boson. The matching conditions are derived at the one-loop level [83]. (2) The Wilson coefficients of the operators at the hadronic scale are evaluated with renormalization group equations at the two-loop level [84,85]. Threshold corrections to them at the SUSY breaking scale may be included, though they are negligible [83]. (3) Using the hadronic matrix element evaluated by lattice QCD, the proton decay rate is evaluated. The latest lattice QCD results for the matrix elements of proton decay are given by Ref. [86], in which the matrix elements are evaluated at the physical pion mass. The proton decay of the X boson is less sensitive to the details of the GUT model if the X boson mass is assumed. However, the model dependence appears through the matching condition of the operators at the one-loop level. The correction to the proton decay rate from one-loop matching in the minimal SUSY SU(5) GUT is about 5% [83]. In the missing partner model with Peccei–Quinn symmetry, the correction is $O(10)\%$ [87]. This is because the model has larger SU(5) multiplets, such as **75**.

Another example of model dependence comes from the low-energy model below the GUT scale. We assume that the low-energy model is the SUSY SM with three generations. However, the model might have additional matter. For example, E_6 GUTs or string theories may introduce extra matter families. If they form SU(5) multiplets, the gauge coupling unification is not spoiled. The introduction of additional matter increases the SU(5) gauge coupling constant, and the proton decay rate is enhanced [88].

In Fig. 4, we show the ratio of the proton lifetime with and without the extra matter as a function of the extra matter mass. Here, we introduce SU(5) $\mathbf{10} + \bar{\mathbf{10}}$ multiplets. We introduce 1, 2, and 3 pairs of $\mathbf{10} + \bar{\mathbf{10}}$ in the lines from top to bottom, respectively, in the figure. It is found that the proton lifetime becomes much shorter when the extra matter is introduced.

In this review the SUSY SU(5) GUT is assumed. It is motivated by the successful gauge coupling unification. If proton decay is discovered and the branching ratios of proton decay are measured, we could get more information on the model behind the discovery. In Ref. [89] the branching ratios in the dimension-six proton decays in the SUSY SU(5), SO(10), and E_6 GUT models are evaluated. In Ref. [90] they compare the prediction of flipped and unflipped SU(5) GUTs.

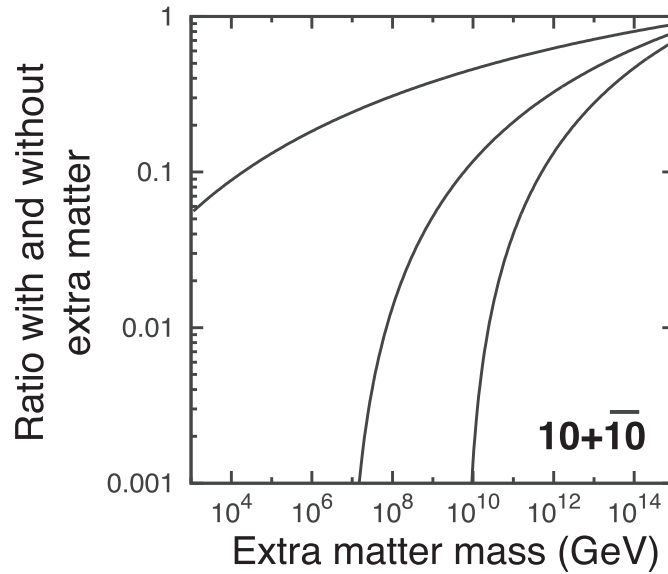


Fig. 4. Ratio of proton lifetime with and without extra matter as a function of the extra matter mass of $(\mathbf{10} + \overline{\mathbf{10}})$. Here, proton decay is induced by the X boson. 1, 2, and 3 pairs of $\mathbf{10} + \overline{\mathbf{10}}$ are introduced in the lines from top to bottom, respectively. This figure is originally from Ref. [88].

3.4 Dimension-five operator proton decay

Integrating out the colored Higgs multiplets, we get the baryon-number violating dimension-five operators. The effective superpotential for the operators is given as [22]

$$W = \frac{1}{2M_{H_C}} f_{u_i} f_{d_l} V_{kl}^* e^{i\varphi_i} Q_i Q_j Q_k L_l + \frac{1}{M_{H_C}} f_{u_i} f_{d_l} V_{kl}^* e^{i\varphi_i} \overline{U}_i \overline{E}_j \overline{U}_k \overline{D}_l \quad (4)$$

where i, j, k, l stand for generations. Q and L are the chiral superfields of the $SU(2)_L$ doublet quark and lepton, respectively, and $\overline{U}, \overline{D}, \overline{E}$ are those of the $SU(2)_L$ singlet quarks and lepton, respectively. The phase factor $e^{i\varphi_i}$ ($\varphi_1 + \varphi_2 + \varphi_3 = 0$) is an extra generic phase factor in the SUSY $SU(5)$ GUT. The Yukawa coupling constants f_{u_i} and f_{d_i} are related to the quark masses m_{u_i} and m_{d_i} as $m_{u_i} = f_{u_i} v \sin \beta$ and $m_{d_i} = f_{d_i} v \cos \beta$ (v is the vacuum expectation value of the SM Higgs field). The Yukawa coupling constants and the Kobayashi–Maskawa matrix V in the above effective superpotential are given at the GUT scale.

Due to the antisymmetric nature of $SU(3)_C$ in Eq. (4), we get $i \neq k$ in Eq. (4). With the hierarchical structure of the Yukawa coupling constants, it favors the final states of proton decay to include the strange quarks. Thus, the main mode is $p \rightarrow K^+ \bar{\nu}$.

The dimension-five operators in Eq. (4) include squarks and/or sleptons, which are heavier than protons. An extra superpartner exchange is required for protons to decay. The gluino exchange is suppressed when the squark masses degenerate. The wino and Higgsino exchanges are dominant for the first and second operators in Eq. (4), respectively (Fig. 5).

The dimension-six operators are generated by the wino/Higgsino exchange, and the Wilson coefficients are suppressed by $1/M_{H_C} m_{\text{SUSY}}$. Exactly speaking, m_{SUSY} is a function of the superpartner mass spectrum. When the squark and slepton masses (m_S) are comparable to M_2 and $m_{\tilde{H}}$, $m_{\text{SUSY}} \simeq 2m_S$. On the other hand, when m_S is much heavier than M_2 or $m_{\tilde{H}}$ as in the mini-split SUSY model, m_{SUSY} is given by m_S^2/M_2 or $m_S^2/m_{\tilde{H}}$. This comes from the chirality flip of the wino or Higgsino. Thus, the dimension-five operator proton decay is more efficiently suppressed in the mini-split SUSY model. When $m_{\tilde{H}} \gg M_2$, the Higgsino exchange contribution

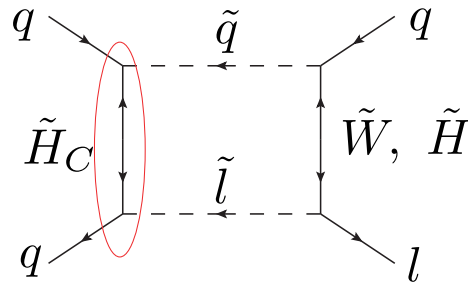


Fig. 5. Proton decay induced by colored Higgs exchange. The red circle corresponds to a dimension-five operator induced by colored Higgs exchange. An extra wino (\tilde{W}) or Higgsino (\tilde{H}) exchange is required for proton decay since squarks (\tilde{q}) and sleptons (\tilde{l}) are heavier than protons.

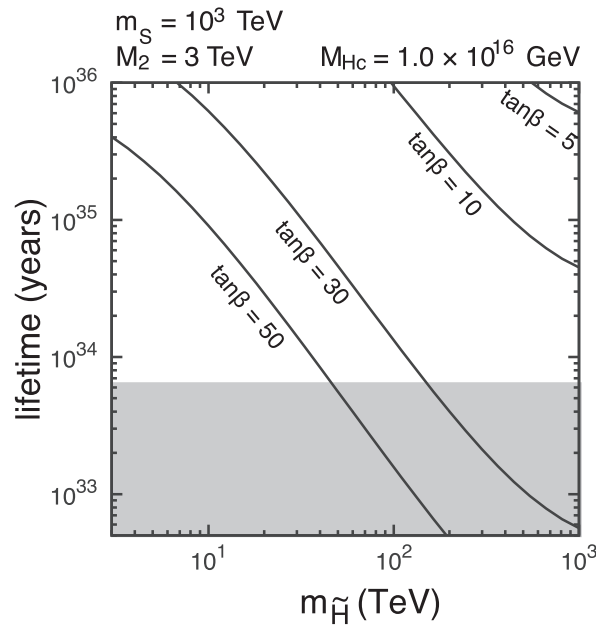


Fig. 6. The partial lifetime of $p \rightarrow K^+ \bar{\nu}$ as a function of the Higgsino mass in the minimal SUSY SU(5) GUT. The wino mass is assumed to be 3 TeV, the squark and slepton masses are 10^3 TeV, and $M_{H_C} = 1.0 \times 10^{16}$ GeV. Solid lines are for $\tan \beta = 5, 10, 30,$ and 50 from top right to bottom left, respectively. The shaded region is excluded by the current bound $\tau(p \rightarrow K^+ \bar{\nu}) > 6.6 \times 10^{33}$ years. This figure is originally from Ref. [91].

is dominant in the amplitude. In such a case, the partial lifetime of $p \rightarrow K^+ \bar{\nu}$ is approximately given as

$$\tau(p \rightarrow K^+ \bar{\nu}) = 4 \times 10^{35} \times \sin^4 2\beta \left(\frac{0.1}{\bar{A}_R} \right)^2 \left(\frac{M_S}{10^2 \text{ TeV}} \right)^2 \left(\frac{M_{H_C}}{10^{16} \text{ GeV}} \right)^2 \text{ years.} \quad (5)$$

Here, we take $m_S = m_{\tilde{H}}$. The symbol \bar{A}_R is for the radiative correction, and the definition is given in Ref. [91]. The factor $\sin^4 2\beta$ comes from both the colored Higgs and Higgsino exchanges.

In Fig. 6 the partial lifetime of $p \rightarrow K^+ \bar{\nu}$ is shown as a function of the Higgsino mass. Here, $M_S = 10^3$ TeV, $M_2 = 3$ TeV, $M_{H_C} = 1.0 \times 10^{16}$ GeV. Solid lines are for $\tan \beta = 5, 10, 30,$ and 50 from top right to bottom left, respectively. The shaded region is excluded by the current bound, $\tau(p \rightarrow K^+ \bar{\nu}) > 6.6 \times 10^{33}$ years. As expected, the larger Higgsino enhances the proton decay

rate. Notice that the lifetime is proportional to the fourth power of m_S since the amplitude is proportional to $m_{\tilde{H}}/m_S^2$ for $m_{\tilde{H}} \ll m_S$. It is found that the minimal SUSY SU(5) GUT is still viable, and that future experiments may find dimension-five operator proton decay in the model.

In the above study of dimension-five operator proton decay, we assume that the off-diagonal terms in the squark and slepton mass matrices are suppressed for simplicity. They may be present in the mini-split SUSY model. The rates and branching fractions of proton decay can be changed if they are introduced [92].

4. Summary of this review

We review proton decay in the SUSY SU(5) GUTs assuming the mini-split SUSY breaking model. In the mini-split SUSY breaking model, the squark and slepton masses are $O(10^{(2-3)})$ TeV while the gaugino masses are $O(1)$ TeV. As a result, even the minimal SUSY SU(5) GUT is still viable in the model. We present the motivation of the mini-split SUSY model and discuss the future prospects of proton decay searches in the SUSY SU(5) GUTs.

Some serious issues for GUTs have appeared since this was proposed. However, new ideas have appeared to deal with these issues. The concept of GUTs is still fascinating to researchers in particle physics. We hope that more clues will appear to us. Is one of those proton decay?

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