Vol. 29 (1998) *ACTA PHYSICA POLONICA B* No 5

SPIN EFFECTS IN EXCLUSIVE SEMILEPTONIC τ DECAYS*

J.H. Kühn

Institut für Theoretische Teilchenphysik, University of Karlsruhe
D–76128, Karlsruhe, Germany

(Received March 16, 1998)

Tau lepton production and decay to hadronic final states offer the unique possibility for studying low energy hadron dynamics and the weak couplings of the τ at the same time. Eventually one may even be sensitive to signals of physics beyond the Standard Model, like deviations from the $V-A$ structure of the charged currents, or even discover CP violation. Tau polarization and the polarization of the decay products may be crucial for these investigations.

PACS numbers: 11.30. Er, 11.15. -y, 13.35. Dx

1. Introduction

Three topics will be covered in detail during this talk:

(i) It will be demonstrated that the τ polarization can be recovered on an event by event basis, if the momenta of all decay products are measured and the relevant squared matrix element is known from theory [1].

(ii) The formalism of structure functions [2] will be reviewed. It allows to measure the τ polarization, at least its average value, even if the τ neutrino momentum is unknown. Structure functions help to separate the contributions of $0^+, 0^-, 1^+$ and 1^- states. They have been used to measure the sign of g_V^τ/g_A^τ and may also allow for a determiantion of g_V^q/g_A^q in τ decays.

(iii) The comparison of structure functions from the decays of τ^+ and τ^- allows for nontrivial tests of CP symmetry [3], even with unpolarized beams. Final states with $K\pi$ and $K\pi\pi$ from Cabbibo suppressed decays are particulary promising candidates for such an analysis.

* Presented at the Cracow Epiphany Conference on Spin Effects in Particle Physics and Tempus Workshop, Cracow, Poland, January 9–11, 1998.

2. Tau polarimetry with multi–meson states

The determination of the polarization of τ leptons produced in Z decays has lead to an important determination of the τ coupling to the Z boson, rivaling those from the forward-backward asymmetry of leptons or from the left-right asymmetry measured with longitudinally polarized beams. It is well known that the decay mode into a single pion leads to optimal analyzing power which expresses itself in an angular distribution of pions from decays of polarized τ 's of the form

$$dN \propto (1 + f \cos\theta)\,, \tag{1}$$

where $f = 1$. The angle between τ spin and pion momentum is denoted by θ. Decays into the ρ or a_1 meson or higher excitations exhibit reduced analyzing power, *e.g.* $f = (M^2 - 2Q^2)/(M^2 + 2Q^2)$ for a spin 1 final state of mass Q. In [3, 4] it has been argued that significant analyzing power can be recovered by exploiting information encoded in the momenta of the (pseudoscalar) mesons which are the actual decay products and are observed in the experiment. Detailed models have been used for the two- and three-meson channels to identify various angular distributions which enhance the sensitivity.

In [1] it was demonstrated explicitly that maximal sensitivity, corresponding to $f = 1$, can be recovered for *any* multi-meson final state, once the dynamics of the decay matrix element is known. Ingredients are the knowledge of all meson momenta and information about the τ rest frame. The latter is equivalent to reconstruction of the actual direction of flight of the τ and can be achieved in e^+e^- experiments with the help of vertex detectors [5].

The argument is based on the observation that the squared matrix element for semileptonic τ can always be written (in the τ restframe) in the form

$$|\mathcal{M}|^2 \propto 1 - \vec{h}\vec{s}. \tag{2}$$

The τ-spin direction is denoted by $\vec{s}$ and the polarimeter vector $\vec{h}$ is a function of all meson momenta. It has length $|\vec{h}| = 1$ and its direction therefore gives the (negative) spin direction of the original τ with unit probability. This holds true for meson final states only — the spin information is strongly diluted for leptonic τ decays as a consequence of averaging over the electron spin and neutrino momenta corresponding to a reduction of the length of $\vec{h}$. It is, however, retained in the direction of the $\bar{\nu}_e$.

In [1] it has been demonstrated that the norm of $\vec{h}$ is in fact equal to one for all semileptonic decays, which corresponds to maximal sensitivity.

The matrix element for the semileptonic decay $\tau \to \nu_\tau + X$ can be written in the form

$$\mathcal{M} = \frac{G}{\sqrt{2}} \bar{u}(N)\gamma^\mu(1-\gamma_5)u(P)J_\mu, \tag{3}$$

where $J_\mu \equiv \langle X|V_\mu - A_\mu|0\rangle$ denotes the matrix element of the $V-A$ current relevant for the specific final state X. The vector J_μ depends in general on the momenta of all hadrons. The squared matrix element for the decay of a τ with spin s and mass M then reads

$$|\mathcal{M}|^2 = G^2(\omega + H_\mu s^\mu), \tag{4}$$

with

$$\omega = P^\mu(\Pi_\mu + \Pi^5_\mu), \qquad H_\mu = \frac{1}{M}(M^2 g_\mu{}^\nu - P_\mu P^\nu)(\Pi_\nu + \Pi^5_\nu) \tag{5}$$

and

$$\Pi_\mu = 2\left[(J^*\cdot N)J_\mu + (J\cdot N)J^*_\mu - (J^*\cdot J)N_\mu\right], \qquad \Pi^5_\mu = 2\,\mathrm{Im}\,\epsilon_\mu{}^{\nu\rho\sigma}J^*_\nu J_\rho N_\sigma. \tag{6}$$

This formula was derived in [4] and constitutes the basis for the simulation of spin effects in TAUOLA [6].

In the τ-rest frame, the function ω coincides with the time component of the four vector $\Pi_\mu + \Pi^5_\mu$ (multiplied with M) and the vector $\vec{H}$ with its space component (multiplied with M). The assertion that $|\vec{h}| = |\vec{H}/\omega| = 1$ is therefore equivalent to the statement that $\Pi_\mu + \Pi^5_\mu$ is null-vector. A simple calculation demonstrates that

$$\Pi^5_\mu \Pi^\mu = 0, \qquad \Pi^5_\mu \Pi^{5\mu} = -\Pi_\mu \Pi^\mu \tag{7}$$

and hence

$$(\Pi_\mu + \Pi^5_\mu)(\Pi^\mu + \Pi^{5\mu}) = 0 \tag{8}$$

which proves our assertion and gives, at the same time, a prescription of how to construct the direction of the spin on an event-by-event basis. Obviously, in order to perform this analysis, all hadron momenta must be measured and the dependence of the current J on these momenta known — either from theoretical considerations or from fits to experimentally measured distributions.

3. Structure functions

The determination of form factors in exclusive decays is not only required for a precise test of theoretical predictions. The separation of vector and axial vector amplitudes and their respective spin zero and one contributions is mandatory for a number of improved phenomenological studies like the determination of α_s and nonperturbative vacuum condensates based on moments of vector and axial spectral functions separately, or the unambigous separation of vector contributions (in particular in the $KK\pi$ channels) which allows in combination with CVC for an improved determination of $\alpha_{QED}(M_Z)$. Just like the search for CP violation (discussed below) or the measurement of g_V^τ/g_A^τ through parity violation in hadronic decays this can be performed with the help of a combined analysis of angular and energy distributions of the hadrons even without reconstruction of the τ-restframe. A particulary useful tool to disentangle the various contributions, to analyse the τ polarization and to test for new physics is the technique of structure functions to be discussed in this section [2].

The most general ansatz for the matrix element of the quark current in the three (two) meson case $J^\mu(q_1, q_2(, q_3)) = \langle h_1(q_1)h_2(q_2)(h_3(q_3))|V^\mu(0) - A^\mu(0)|0\rangle$ is characterized by four (two) complex form factors F_i, which are in general functions of $s_1 = (q_2+q_3)^2, s_2 = (q_1+q_3)^2$, $s_3 = (q_1+q_2)^2$ and Q^2 (which is conveniently chosen as an additional variable)

$$J^\mu(q_1,q_2) \;=\; T^{\mu\nu}\,(q_1-q_2)_\nu\,F + Q^\mu\,F_S\,, \tag{9}$$

$$\begin{aligned} J^\mu(q_1,q_2,q_3) \;=\; & T^{\mu\nu}\,[\,(q_1-q_3)_\nu\,F_1 \;+\; (q_2-q_3)_\nu\,F_2\,] \\ & + i\epsilon^{\mu\alpha\beta\gamma} q_{1\,\alpha} q_{2\,\beta} q_{3\,\gamma}\,F_3 + Q^\mu\,F_4\,, \end{aligned} \tag{10}$$

$T^{\mu\nu} = g^{\mu\nu} - (Q^\mu Q^\nu)/Q^2$ denotes a transverse projector. The form factors F_1 and F_2 in Eq. (10) originate from the $J^P = 1^+$ axial vector hadronic current, the form factor F_3 (F in Eq. (9)) from the $J^P = 1^-$ vector current, and correspond to a hadronic system in a spin one state, whereas F_4 (F_S) is due to the spin zero ($J = 0$) part of the axial-vector (vector) current matrix element. In specific cases, there are various simplifications. If the two mesons in Eq. (9) are two pions, $h_1h_2 = \pi^-\pi^0$, then the vector current is conserved and the scalar form factor vanishes, $F_S \equiv 0$ for $m_u = m_d$. In the three pion case, $h_1h_2h_3 = \pi^-\pi^-\pi^+$ or $\pi^0\pi^0\pi^-$, Bose symmetry relates F_2 to F_1, via $F_2(Q^2, s_1, s_2) = F_1(Q^2, s_2, s_1)$. G parity conservation requires $F_3 \equiv 0$ for $m_u = m_d$, and PCAC requires $F_4 \equiv 0$ for $m_u = m_d = 0$.

The hadronic decay into three (two) mesons is most easily analyzed in the hadronic rest frame $\vec{q}_1 + \vec{q}_2(+\vec{q}_3) = 0$ [2]. In experimental analyses, the four (two) complex form factors in Eq. (10) (Eq. (9)) appear as sixteen (four) real “structure functions” W_X, which are defined from the hadronic

TABLE I

The structure functions

	$J^\mu \longrightarrow$		
$J^{\nu\star}\downarrow$	$J^P=1^+$	$J^P=1^-$	$J=0$
$J^P=1^+$	$\mathbf{W_A}$ $W_C W_D W_E$		
$J^P=1^-$	$W_F W_G$ $W_H W_I$	$\mathbf{W_B}$	
$J=0$	$W_{SB} W_{SC}$ $W_{SD} W_{SG}$	$W_{SF} W_{SG}$	$\mathbf{W_{SA}}$
	$\underbrace{\qquad}_{h_1 h_2 h_3}$ (columns 1^+, 1^-, 0)	$\underbrace{\qquad}_{h_1 h_2}$ (columns 1^-, 0)	

tensor $H^{\mu\nu} = J^\mu J^{\nu\star}$ in the hadronic rest frame. For the precise definitions of W_X, we refer the reader to Ref. [2]. The contributions of the different F_i to W_X are summarized in Table I. After integration over the unobserved neutrino direction, the differential decay rate in the hadronic rest frame is given in the two-meson case by [2, 10]

$$d\Gamma(\tau^- \to 2h\nu_\tau) = \left\{\bar{L}_B W_B + \bar{L}_{SA} W_{SA} + \bar{L}_{SF} W_{SF} + \bar{L}_{SG} W_{SG}\right\}$$

$$\frac{G_F^2}{2m_\tau}\sin^2\theta_c \frac{1}{(4\pi)^3}\frac{(m_\tau^2 - Q^2)^2}{m_\tau^2}\ |\vec{q}_1|\ \frac{dQ^2}{\sqrt{Q^2}}\ \frac{d\cos\theta}{2}\ \frac{d\alpha}{2\pi}\ \frac{d\cos\beta}{2}\ . \tag{11}$$

The functions $\bar{L}_X$ depend on the angles α and β and energy of the hadronic system only. For the definition and discussion of the angles and leptonic coefficients $\bar{L}_X$ we refer the reader to Ref. [2, 10]. The hadronic structure functions W_X in the two meson case depend only on Q^2 and the form factors F and $\tilde{F}_S$ of the hadronic current and are given in Eq. (25) with $\tilde{F}_S \to F_S$. Similarly, the differential decay rate in the three meson case is given by [2]:

$$d\Gamma = \frac{G_F^2}{4m_\tau}\binom{\cos\theta_c^2}{\sin\theta_c^2}\sum_X \bar{L}_X W_X\ dPS^{(4)} \tag{12}$$

The sum in Eq. (12) runs (in general) over 16 hadronic structure functions W_X, which correspond to 16 density matrix elements for a hadronic system in a spin one and spin zero state (nine of them originate from a pure spin one state and the remaining originate from a pure spin zero state or from interference terms between spin zero and spin one, see Table I. These structure functions contain the dynamics of the hadronic decay and depend only on the form factors F_i. Note that W_A, W_B and W_{SA} alone determine $d\Gamma/dQ^2$ through

$$\frac{d\Gamma(\tau \to h_1 h_2 h_3 \nu_\tau)}{dQ^2} \propto \frac{(m_\tau^2 - Q^2)^2}{Q^4} \times \int ds_1\, ds_2 \left\{ \left(1 + \frac{Q^2}{m_\tau^2}\right) \frac{W_A + W_B}{6} + \frac{W_{SA}}{2} \right\} . \tag{13}$$

(Almost) all structure functions can be determined by studying angular distributions of the hadronic system, for details see [2]. This method allows to analyze separately the contributions from $J^P = 0^+, 0^-, 1^+$ and 1^- in a model independent way (see Table I).

First steps towards such analysis have been undertaken by various LEP experiments for the three pion final state. The measurements [9] are in fair agreement with the theoretical predictions [8] based on chiral Lagrangian providing the normalization at vanishing momentum transfer, and vector dominance to incorporate the proper resonance structure (Fig.1). The terms proportional to the structure function W_E are induced by parity violating

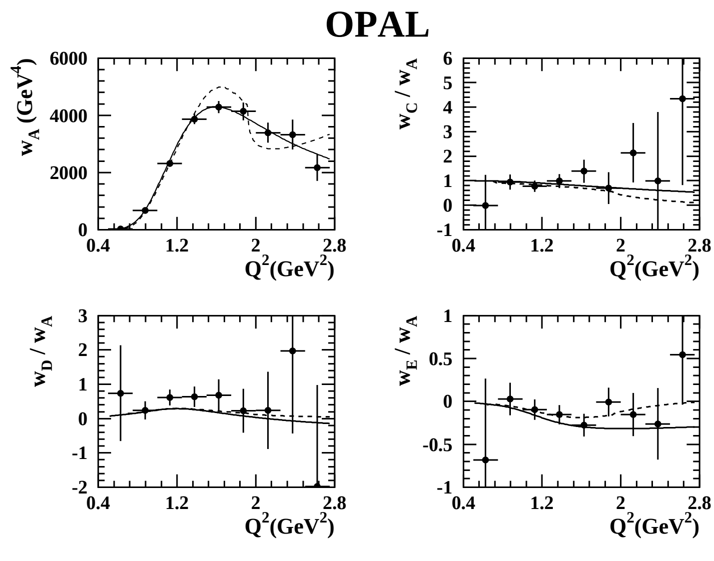

Fig. 1. Measured hadronic structure functions W_A, W_C, W_D and W_E (points) in comparison with the model predictions of the KS model (dashed line) and IMR model (dotted line) (from Ref. [9]).

couplings at the $\tau\nu W$ vertex. The analysis leads, therefore, to a determination of $\gamma_{VA} = 2g_\tau^V g_\tau^A/({g_\tau^V}^2 + {g_\tau^A}^2)$.

The result [9] $\gamma_{VA} = 1.29 \pm 0.26 \pm 0.11$ is consistent with the SM prediction $\gamma_{VA} = 1$ for lefthanded neutrinos. The model dependence of this analysis can, in principle, be removed through the experimental determination of the remaining functions W_A, W_C and W_D. Also, a potential scalar contribution to three pion channel has been searched for and an upper limit of 0.84 % has been obtained by this method.

4. CP violating in semileptonic τ decays with unpolarized beams

4.1. General considerations

CP violation has been experimentally observed only in the K meson system. The effect can be explained by a nontrivial complex phase in the CKM flavour mixing matrix. However, the fundamental origin of this CP violation is still unknown. In particular the CP properties of the third fermion family are largely unexplored. Production and decay of τ leptons might offer a particularly clean laboratory to study these effects. In this Section, we investigate the effects of possible non-Kobayashi-Maskawa-type of CP violation, *i.e.* CP violation effects beyond the Standard Model (SM) on semileptonic τ decays. Such effects could originate for example from multi Higgs boson models [7].

In [3] it has been shown that the structure function formalism [2] allows for a systematic analysis of possible CP violation effects in the two and three meson cases. Special emphasis is put on the $\Delta S = 1$ transition $\tau \to K\pi\nu_\tau$ where possible CP violating signals from multi Higgs boson models would be signalled by a nonvanishing difference between the structure functions $W_{SF}[\tau^- \to (K\pi)^-\nu_\tau]$ and $W_{SF}[\tau^+ \to (K\pi)^+\nu_\tau]$. Such a measurement is possible for unpolarized single τ's without reconstruction of the τ rest frame and without polarized incident e^+e^- beams. This difference is proportional to Im(hadronic phases) $\times$ Im(CP-violating phases), where the hadronic phases arise from the interference of complex Breit-Wigner propagators, whereas the CP violating phases could arise from an exotic charged Higgs boson. An additional independent test of CP violation in the two meson case would require the knowledge of the full kinematics and τ polarization.

The subsequent discussion is organized as follows: CP violating terms in the Hamiltonian for τ decays are discussed in a first step. It is shown that CP violating signals induced through the exchange of an exotic intermediate vector boson can only arise, if both vector and axial vector hadronic currents contribute to the same final state, *i.e.* for final state which are not eigenstates of G parity and involve three or more mesons. The kinematics and the relevant form factors and structure functions for tests of CP violation in the two meson case are presented, followed by a brief comment on CP violation effects in three meson final states.

The Hamiltonian responsible for τ decays is decomposed into the conventional term of the SM, denoted by H_{SM}, a CP violating term of similar structure, induced *e.g.* by the exchange of a vector boson, $H_{\rm CP}^{(1)}$ and a CP violating term induced by scalar or pseudo scalar exchange $H_{\rm CP}^{(0)}$:

$$\begin{aligned} H_{SM} &= \cos\theta_c \frac{G}{\sqrt{2}}\, [\bar{\nu}\ \gamma_\alpha(g_V - g_A\gamma_5)\ \tau]\,[\bar{d}\ \gamma^\alpha(1-\gamma_5)\ u] + \text{h.c.} \\ H_{\rm CP}^{(1)} &= \cos\theta_c \frac{G}{\sqrt{2}}\, [\bar{\nu}\ \gamma_\alpha(g'_V - g'_A\gamma_5)\ \tau]\,[\bar{d}\ \gamma^\alpha(\chi_V^d - \chi_A^d\gamma_5)\ u] + h.c. \\ H_{\rm CP}^{(0)} &= \cos\theta_c \frac{G}{\sqrt{2}}\, [\bar{\nu}(g_S + g_P\gamma_5)\ \tau]\,[\bar{d}\ (\eta_S^d + \eta_P^d\gamma_5\,)\, u] + \text{h.c.} \end{aligned} \quad (14)$$

plus a similar term with the complex parameters η^d, χ^d replaced by η^s, χ^s for the $\Delta S = 1$ contribution.

The dominance of $(V-A)$ contributions to the leptonic current in leptonic and semileptonic τ-decays has been demonstrated experimentally under fairly mild theoretical assumptions. A pure $(V-A)$ structure of the leptonic current in H_{SM} will therefore be adopted for simplicity. A slight deviation from $(V-A)$ for the hadronic current can in principle be masked by the form factors. However, tight restrictions can be derived from the ratio $\Gamma(\tau\to\nu\pi)/\Gamma(\tau\to\nu\pi\pi)$ using as input f_π and the pion form factor from e^+e^- annihilation[1]. The study of CP violation will entirely rely on interference terms between H_{SM} and $H_{\rm CP}^{(1)}, H_{\rm CP}^{(0)}$. Interference terms between the dominant $(V-A)$ leptonic current in H_{SM} and a possible $V+A$ term in the leptonic current of $H_{\rm CP}^{(0,1)}$ are suppressed with the ratio between the mass of the τ neutrino and m_τ. Therefore only contributions from left–handed neutrinos will be included, *i.e.* $g_V = g_A = g'_V = g'_A = g_S = -g_P = 1$.

The hadronic matrix elements of the currents $\bar{d}\gamma^\alpha u$ (and similarly $\bar{d}\gamma^\alpha\gamma_5 u$) in H_{SM} and $H_{\rm CP}^{(1)}$ are of course identical. The spin zero part is closely related to the corresponding matrix element of the scalar current in $H_{\rm CP}^{(0)}$ through the equation of motion:

$$Q^\alpha\ \bar{d}\gamma_\alpha u = (m_u - m_d)\,\bar{d}u; \qquad -Q^\alpha\ \bar{d}\gamma_\alpha\gamma_5 u = (m_u + m_d)\,\bar{d}\gamma_5 u \quad (15)$$

[1] The relative sign between hadronic vector and axial vector current is not fixed through this consideration. It can be determined by interference measurements in the $K\pi\pi$ and $KK\pi$ channels, *e.g.* of the structure functions $W_{F,G,H,I}$ [2].

with $Q^\alpha = i\partial^\alpha$. The Hamiltonian in Eq. (14) can thus be written in the form

$$H = \cos\theta_c \frac{G}{\sqrt{2}} \left[\bar{\nu}\gamma_\alpha(1-\gamma_5)\tau\right] \left\{ \left((1+\chi_V^d) g^{\alpha\beta} + \frac{Q^\alpha Q^\beta}{m_\tau(m_u - m_d)} \eta_S^d \right) \bar{d}\gamma_\beta u \right.$$
$$\left. - \left((1+\chi_A^d) g^{\alpha\beta} + \frac{Q^\alpha Q^\beta}{m_\tau(m_u + m_d)} \eta_P^d \right) \bar{d}\,\gamma_\beta\gamma_5\, u \right\} + h.c. \quad (16)$$

and similarly for the Cabibbo suppressed mode. From this form it is evident that χ and η play a fairly different role. Effects from χ_V and/or χ_A can only arise if both vector and axial hadronic currents contribute to the same final state and hence only for final states which are not eigenstates of G parity and involve three or more mesons. Conversely, for all two meson decays and, adopting isospin symmetry, even all multipion states, CP violation cannot[2] arise from a complex χ. In contrast, CP violation can arise from a complex η, since $J=0$ and $J=1$ partial waves are affected differently in this case. For this reason contributions from nonvanishing χ will be ignored in the following.

4.2. Two meson decays: kinematics, form factors and structure functions

Transitions from the vacuum to two pseudoscalar mesons h_1 and h_2 are induced through vector and scalar currents only, where the latter can be related to the former with the help of Eq. (15). Expanding this hadronic matrix element along the set of independent momenta $(q_1-q_2)_\beta$ and $Q_\beta = (q_1+q_2)_\beta$ (see Eq.(9)) the general amplitude for the strangeness conserving decay

$$\tau^-(l,s) \to \nu(l',s') + h_1(q_1,m_1) + h_2(q_2,m_2)\,, \quad (17)$$

can be written as[3]

$$\mathcal{M} = \cos\theta_c \frac{G}{\sqrt{2}} \bar{u}(l',s')\gamma_\alpha(1-\gamma_5)u(l,s) \left(g^{\alpha\beta} + \frac{Q^\alpha Q^\beta}{m_\tau(m_u - m_d)} \eta_S \right)$$
$$\times \left[(q_1-q_2)^\delta\, T_{\delta\beta}\, F + Q_\beta\, F_S \right]$$
$$= \cos\theta_c \frac{G}{\sqrt{2}} \bar{u}(l',s')\,\gamma_\alpha(1-\gamma_5)\,u(l,s)\, \left[(q_1-q_2)_\beta\, T^{\alpha\beta}\, F + Q^\alpha\, \tilde{F}_S \right] \quad (18)$$

---

[2] In this aspect we disagree with Ref. [11] where it has been claimed that CP violation in the 2π channel can be induced through the exchange of an exotic intermediate vector boson.

[3] We suppress the superscript d (or s) in η_S in the following.

with

$$\tilde{F}_S = \left(1 + \frac{Q^2}{m_\tau(m_u - m_d)}\,\eta_S\right)\; F_S \tag{19}$$

and similarly for the $\Delta S = 1$ part. In Eq. (18) s denotes the polarization 4-vector of the τ lepton satisfying $l_\mu s^\mu = 0$ and $s_\mu s^\mu = -P^2$. P denotes the polarization of the τ in the τ rest frame with respect to its direction of flight in the laboratory frame. As stated before, terms proportioal to χ do not contribute to CP violation in the two meson case and have therefore been neglegted.

In the subsequent discussion we will not need to rely on the relation (19) between vector and scalar induced formfactors and thus define

$$\tilde{F}_s = F_s + \frac{\eta_s}{m_\tau} F_H. \tag{20}$$

The representation of the hadronic amplitude $\langle h_1 h_2 | \bar{u}\gamma_\beta d | 0\rangle = (q_1 - q_2)^\delta\, T_{\delta\beta}\; F \,+\, Q_\beta\, \tilde{F}_S$ corresponds to a decomposition into spin one and spin zero contributions, *e.g.* the vector form factor $F(Q^2)$ corresponds to the $J^P = 1^-$ component of the weak charged current, and the scalar form factor $F_S(Q^2)$ to the $J^P = 0^+$ component. Up to the small isospin breaking terms, induced for example by the small quark mass difference, CVC implies the vanishing of F_S for the two pion case. The small u and d quark masses enter presumably the couplings from charged Higgs exchange and thus cancel the apparent enhancement by the inverse power of $(m_u - m_d)$ in Eqs. (16), (18), (19). The perspectives are more promising for the $\Delta S = 1$ transition $\tau \to K\pi\nu$. The $J = 1$ form factor F is dominated by the $K^*(892)$ vector resonance contribution. However, in this case the scalar form factor F_S is expected to receive a sizable resonance contribution ($\sim 5\%$ to the decay rate) from the $K_0^*(1430)$ with $J^P = 0^+$ [10]. In the subsequent discussion we will include both $\pi\pi$ and $K\pi$ final states. The corresponding τ^+ decay is obtained from Eq. (18) through the substitutions

$$(1 - \gamma_5) \;\to\; (1 + \gamma_5), \qquad \eta_S \;\to\; \eta_S^*. \tag{21}$$

Reaction (17) is most easily analyzed in the hadronic rest frame $\vec{q}_1 + \vec{q}_2 = 0$. After integration over the unobserved neutrino direction, the differential decay rate in the rest frame of $h_1 + h_2$ is given by Eq. (11) [2,10] The hadronic structure functions W_X, $X \in \{B, SA, SF, SG\}$, which appear in Eq. (11) depend only on Q^2 and the form factors F and $\tilde{F}_S$ of the hadronic current. The dependence can be obtained from Eq. (34) in [2] with the replacements $x_4 \to 2\,\vec{q}_1$, $F_3 \to -iF$, $F_4 \to \tilde{F}_S$. One has:

$$W_B[\tau^-] \;=\; 4(\vec{q}_1)^2\,|F|^2\,, \tag{22}$$

$$W_{SA}[\tau^-] \;=\; Q^2\,|\tilde{F}_S|^2\,, \tag{23}$$

$$W_{SF}[\tau^-] \;=\; 4\sqrt{Q^2}|\vec{q}_1|\,\mathrm{Re}\left[F\tilde{F}_S^*\right]\,, \tag{24}$$

$$W_{SG}[\tau^-] \;=\; -4\sqrt{Q^2}|\vec{q}_1|\,\mathrm{Im}\left[F\tilde{F}_S^*\right]\,, \tag{25}$$

where $|\vec{q}_1| = q_1^z$ is the momentum of h_1 in the rest frame of the hadronic system:

$$\vec{q}_1^{\,z} = \frac{1}{2\sqrt{Q^2}}\left([Q^2 - m_1^2 - m_2^2]^2 - 4m_1^2m_2^2\right)^{1/2}. \tag{26}$$

The hadronic structure functions $W_X[\tau^+]$ are obtained by the replacement $\eta_S \to \eta_S^*$ in $\tilde{F}_S$ in Eqs. (23)–(25), (19). CP conservation implies that all four structure functions are identical for τ^+ and τ^-. With the ansatz for the form factors formulated in Eq. (18) CP violation can be present in W_{SF} and W_{SG} only and requires complex η_S. As will be shown in the subsequent discussion CP violation in W_{SG} is maximal for fixed η_S in the absence of hadronic phases whereas W_{SF} in contrast requires complex η_S and hadronic phases simultaneously.

In [3] it has been demonstrated that W_{SF} can be measured in e^+e^- annihilation experiments in the study of single unpolarized τ decays even if the τ rest frame cannot be reconstructed. In this respect the result differ from earlier studies of the two meson modes where either polarized beams and reconstruction of the full kinematics [11] or correlated fully reconstructed τ^- and τ^+ decays were required [12]. The determination of W_{SG}, however, requires τ polarization and the knowledge of the full τ kinematics.

The differential rate (Eq. (11)) for the CP conjugated process can be obtained from the previous results by reversing all momenta $\vec{p} \to -\vec{p}$, the τ spin vector $\vec{s} \to -\vec{s}$, the polarization $P \to -P$, and the transition $\gamma_{VA} \to -\gamma_{VA}$. CP therefore relates the differential decay rates for τ^+ and τ^- as follows:

$$d\Gamma[\tau^-](\gamma_{VA}, P, W_X[\tau^-]) \to d\Gamma[\tau^+](-\gamma_{VA}, -P, W_X[\tau^+])\,. \tag{27}$$

Note that the coefficients $\bar{L}_X$ contain the full γ_{VA} and P dependence. From the interference between the spin-zero spin-one terms (denoted by the subscript $X = SF, SG$) one can construct the following CP-violating quantities:

$$\Delta_X \;=\; \frac{1}{2}\left(\bar{L}_X(\gamma_{VA}, P)\,W_X[\tau^-] - \bar{L}_X(-\gamma_{VA}, -P)\,W_X[\tau^+]\right) \tag{28}$$

$$\;=\; \bar{L}_X(\gamma_{VA}, P)\,\frac{1}{2}\left(W_X[\tau^-] - W_X[\tau^+]\right) \tag{29}$$

$$\;\equiv\; \bar{L}_X \Delta W_X\,. \tag{30}$$

As mentioned before the hadronic structure functions $W_X[\tau^-]$ and $W_X[\tau^+]$ differ only in the complex parameter η_S in

$$\tilde{F}_S[\tau^-] = F_s + \frac{\eta_s}{m_\tau} F_H \tag{31}$$

and

$$\tilde{F}_S[\tau^+] = F_s + \frac{\eta_s^*}{m_\tau} F_H \tag{32}$$

and one obtains for the only nonvanishing spin-zero spin-one interference term $L_{SF} W_{SF}$

$$\Delta W_{SF} \;=\; 4\sqrt{Q^2}|\vec{q}_1| \; \frac{1}{m_\tau} \; \mathrm{Im}\,(F F_H^*) \;\; \mathrm{Im}\,(\,\eta_S) \;\; . \tag{33}$$

In essence this measurement analyses the difference in the correlated energy distribution of the mesons h_1 and h_2 from τ^+ and τ^- decay in the laboratory. As already mentioned, ΔW_{SF} is observable for single τ^+ and τ^- decays without knowledge of the τ rest frame. Any nonvanishing experimental result for ΔW_{SF} would be a clear signal of CP violation. Note that a nonvanishing ΔW_{SF} requires nontrivial hadronic phases (in addition to the CP violating phases η_S) in the form factors F and F_S. Such hadronic phases in F (F_S) originate in the $K\pi\nu_\tau$ decay mode from complex Breit Wigner propagators for the K^* (K_0^*) resonance. Sizable effects of these hadronic phases are expected in this decay mode [10].

Once the τ rest frame is known and a preferred direction of polarization exists both angles α and β are well defined and one may proceed further, determine with the help of $\sin\alpha$ also W_{SG} and thus perform a second independent test for CP violation.

The differential rate for the CP conjugated process can be obtained as before, with the additional substitution $\sin\alpha \to -\sin\alpha$.

$$d\Gamma[\tau^-](\sin\alpha, \gamma_{VA}, P, W_X[\tau^-]) \to d\Gamma[\tau^+](-\sin\alpha, -\gamma_{VA}, -P, W_X[\tau^+])\,. \tag{34}$$

From the interference between the spin-zero spin-one terms one can construct the following CP-violating quantities:

$$\begin{aligned} \Delta_X \;&=\; \frac{1}{2}\left(\bar{L}_X(\sin\alpha, \gamma_{VA}, P)\, W_X[\tau^-] - \bar{L}_X(-\sin\alpha, -\gamma_{VA}, -P)\, W_X[\tau^+]\right) \\ &=\; \bar{L}_X(\sin\alpha, \gamma_{VA}, P)\, \frac{1}{2}\left(W_X[\tau^-] - W_X[\tau^+]\right) \qquad (35) \\ &\equiv\; \bar{L}_X \Delta W_X\,, \qquad (36) \end{aligned}$$

$$\Delta W_{SF} \;=\; 4\sqrt{Q^2}|\vec{q}_1| \; \frac{1}{m_\tau} \; \mathrm{Im}\,(F F_H^*) \;\; \mathrm{Im}\,(\eta_S) \;\; . \tag{37}$$

$$\Delta W_{SG} \;=\; 4\sqrt{Q^2}|\vec{q}_1| \;\frac{1}{m_\tau}\; \mathrm{Re}\,(FF_H^*)\;\; \mathrm{Im}\,(\eta_S)\;\; . \tag{38}$$

Any observed nonzero value of these quantities would again signal a true CP violation. Eqs.(37) and (38) show that the sensitivity to CP violating effects in ΔW_{SF} and ΔW_{SG} can be fairly different depending on the hadronic phases. Whereas ΔW_{SF} requires nontrivial hadronic phases ΔW_{SG} is maximal for fixed η_S in the absence of hadronic phases.

4.3. Three meson decays

The structure function formalism [2] allow also for a systematic analysis of possible CP violation effects in the three meson case. Some of these effects have already been briefly discussed in [13]. The $K\pi\pi$ and $KK\pi$ decay modes with nonvanishing vector and axial vector current are of particular importance for the detection of possible CP violation originating from exotic intermediate vector bosons. This would be signalled by a nonvanishing difference between the structure functions $W_X(\tau^-)$ and $W_X(\tau^+)$ with $X \in \{F,G,H,I\}$. A difference in the structure functions with $X \in \{SB,SC,SD,SE,SF,SG\}$ can again be induced through a CP violating scalar exchange.

REFERENCES

[1] J.H. Kühn, *Phys. Rev.* **D52**, 3128 (1995).

[2] J.H. Kühn, E.Mirkes, *Phys. Lett.* **B286**, 381 (1992); *Z. Phys.* **C56**, 661 (1992); *Z. Phys.* **C67**, 364 (1995).

[3] J.H. Kühn, E.Mirkes, *Phys. Lett.* **B398**, 407 (1997).

[4] J.H. Kühn, F. Wagner, *Nucl. Phys.* **B236**, 16 (1984); K. Hagiwara, A.D. Martin, D. Zeppenfeld, *Phys. Lett.* **B235**, 198 (1989); A. Rouǵe, *Z. Phys.* **C48**, 77 (1990).

[5] J.H. Kühn, *Phys. Lett.* **B313**, 458 (1993).

[6] S. Jadach, J.H. Kühn, Z. Wąs, *Comput. Phys. Commun.* **64**, 275 (1991); M. Jeżabek, Z. Wąs, S. Jadach, J.H. Kühn, *Comput. Phys. Commun.* **70**, 69 (1992); S. Jadach, Z. Wąs, R. Decker, J.H. Kühn, *Comput. Phys. Commun.* **76**, 361 (1993).

[7] Y. Grossman, *Nucl. Phys.* **B426**, 355 (1994) and references therein.

[8] J.H. Kühn, A. Santamaria, *Z. Phys.* **C48**, 445 (1990).

[9] OPAL-Coll., K. Ackerstaff *et. al.*, *Z. Phys.* **C75**, 593 (1997); OPAL-Coll., P.Abreu *et. al.*, CERN-EP/98-14.

[10] M. Finkemeier, E. Mirkes, *Z. Phys.* **C72**, 619 (1996).

[11] Y.S. Tsai, *Phys. Rev.* **D51**, 3172 (1995).

[12] C.A. Nelson *et al.*, *Phys. Rev.* **D50**, 4544 (1994) and references therein.

[13] M. Finkemeier, E. Mirkes, Proceedings of the Workshop on the Tau/Charm Factory, Argonne, IL, (1995), [hep-ph/9508312].