COULD LEPTONS BE COMPOSED FROM QUARKS OR ANTIQUARKS?

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Abstract

It is shown that within QCD extended by a scalar field theory with spontaneously broken scale invariance, the leptons could be composite bound states from three quarks (qqq) or antiquarks ($\bar{q}\bar{q}\bar{q}$). The matter-antimatter asymmetry of Universe, and some new lepton and hadron properties predicted in this picture are discussed.
Key-words: QCD, scale invariance, quark, lepton, antimatter.

1. Introduction

The primary goals of elementary particle physics are the discovery of the ultimate building blocks of nature and understanding the relationship between their forces.

In the Standard Model (SM) all known leptons (e, ν_e, μ, ν_μ, τ, ν_τ) and quarks (u, d, c, s, t, b) are grouped in three generations

$$\begin{pmatrix} e \\ \nu_e \end{pmatrix} \quad \begin{pmatrix} \mu \\ \nu_\mu \end{pmatrix} \quad \begin{pmatrix} \tau \\ \nu_\tau \end{pmatrix} \text{ - 6 leptons}$$

$$\begin{pmatrix} u \\ d \end{pmatrix} \quad \begin{pmatrix} c \\ s \end{pmatrix} \quad \begin{pmatrix} t \\ b \end{pmatrix} \text{ - 6 quarks}$$

and they are considered as fundamental (noncomposite) particles. Then all hadrons are composed from the quarks.

The remarkable agreement of the predictions of the SM with experimental observations shows the correctness of the spontaneously broken $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ gauge theory at low energies. SM cannot, however, be an ultimate fundamental theory by itself because of some major shortcomings. For example, (i) excessive multiplication of elementary particles without a principle restricting their choice, and (ii) a large number (=22) of arbitrary parameters (3 gauge couplings + 6 masses of the quarks + 3 mass of charged leptons + 3 neutrino masses + 4 parameters in the KobayashiMaskawa matrix + 1 strong charge-parity (CP) violating parameter + 2 Higgs potential parameters = 22). Nevertheless, within SM there is no clear solution to the particle-antiparticle (baryon-antibaryon, electron-positron) asymmetry of Universe, and to some other fundamental problems.

A few basic ideas for the possible resolutions of the SM shortcomings have been suggested – grand-unification [1], supersymmetry [2], supergravity [3], and one of them is the idea of composite particles [4]. The most compelling arguments in favor of a substructure of leptons and quarks in terms of more fundamental subunits are that there exist so many leptons and quarks, and that they seemingly form the pattern of three "generations". Mainly, there are two types of composite models: technicolour and preonic models.

The idea of the technicolour [5] presumes that the Higgs bosons are composite, but quarks, leptons, and techniquarks are elementary. This idea encounters several difficulties and is mostly excluded, because of the constraints from the flavour changing neutral current processes and the electroweak (EW) oblique corrections [6].

Preonic ideas propose that not only the Higgs bosons, but also the quarks and leptons are composites of a common set of constituents, generically called preons. A particular class of preon models in which the flavour and the colour attributes of quarks and leptons are carried by separate preonic constituents, so that quarks and leptons in their simplest forms may be viewed as fermion-boson composites, are initiated by Pati and Salam [7]. A similar idea that treats only quarks but not leptons as composites is considered by Greenberg [8]. The Greenberg's idea has been subsequently considered by Pati and Salam in a set of papers, and by many authors[1]. The approach developed in [19, 20, 21, 22, 23, 24] introduces a new phase in the preonic approach, when combined with local SUSY, provides a simple explanation for the protection of composite quark-lepton masses, the origin of diverse mass scales, family replication, and interfamily mass hierarchy. Nevertheless, the symmetry structure of the preon theory cannot strictly respect left-right, up-down and quark-lepton symmetries [25].

New idea suggested in this paper is that leptons could be composed from quarks (like the hadrons). This idea is based on the fact that there are more analogies between leptons and the lowest mass (spin^{Parity}, isospin - $J^P I = 1/2^+, 1/2$) baryons rather than between leptons and quarks: the leptons and baryons are colorless with entire electric charge in contrast to the color quarks with fractional electric charge. From this point of view quarks can be considered as fundamental particles for the leptons and hadrons: leptons, like hadrons in Quantum CromoDynamics (QCD), can be build from three quarks by an additional very short range Super Strong Dynamics (SSD).

$$
\begin{array}{lcccll}
 & \binom{e^+}{\bar{\nu}_e} & \binom{\mu^+}{\bar{\nu}_\mu} & \binom{\tau^+}{\bar{\nu}_\tau} & - & \text{6 leptons} \\
\text{SSD}\rightarrow & \uparrow & \uparrow & \uparrow & & \\
 & \binom{u}{d} & \binom{c}{s} & \binom{t}{b} & - & \text{6 quarks} \\
\text{QCD}\rightarrow & \downarrow & \downarrow & \downarrow & & \\
 & \binom{uud}{udd} & \binom{ccs}{css} & \binom{ttb}{tbb} & - & \text{6 lowest mass (JI=1/2,1/2) baryons} \\
 & \dots & \dots & \dots & - & \text{49 other ground states of baryons.}
\end{array}
$$

Another particular motivation for this idea is a possibility of clear explanation of electro-neutrality and baryon-electron (matter) dominance in Universe. Even if the total number of quarks is always exactly equal to the total number of antiquarks then after Big Bang and coalescence of the quarks and antiquarks to leptons and hadrons there are fluctuative differences in total numbers of leptons, antileptons, baryons and antibaryons. Then, after annihilation of all extra lepton and baryon pairs the remaining quarks in Universe are confined in protons, neutrons, and antineutrino[2], and, corresponding antiquarks are confined in electrons, and neutrinos.

There is an experimental test for this picture. Electroneutrality of Universe means the

[1]With W bosons treated as composites in some of them e.g. [9, 10, 11]. Some other composite models assume that quarks and leptons can be made most economically as bound states of either a boson and a fermion [12] or three fermions [13, 14, 15]. The supersymmetric version is considered in [16, 17, 18].

[2]For simplicity small amounts of antinucleons, positrons, antineutrons, and unstable mesons generated in reactions with cosmic rays are not listed and discussed here

equal numbers of protons and electrons. Then for equal numbers of quarks and antiquarks in Universe (zero total baryon charge concerned with quarks), one can relate the difference between the total numbers of neutrino (N_ν) and antineutrino ($N_{\bar\nu}$) to the total number of neutrons (N_n)

$$N_\nu - N_{\bar\nu} = N_n. \tag{1}$$

Unfortunately, the known estimation $N_\nu \sim N_{\bar\nu} \sim (10^8 - 10^9)N_n$ signifies extreme difficulties in measuring N_ν and $N_{\bar\nu}$ with the accuracy high enough to verify (1).

Nevertheless, this quark picture of lepton structure seems attractive enough to explore some of its consequences, despite the fact that it is rather unconventional.

In this way we need to choose reasonable SSD taking into account at least the following three points: (i) a clear dynamical mechanisms for vanishing of $\bar{u}\bar{d}\bar{d}$, $\bar{c}\bar{s}\bar{s}$, $\bar{t}\bar{b}\bar{b}$ neutrino masses and small nonzero masses of charged leptons $\bar{u}\bar{u}\bar{d}$, $\bar{c}\bar{c}\bar{s}$, $\bar{t}\bar{t}\bar{b}$, (ii) clear dynamical explanation for the absence of almost massless $q\bar{q}$ (meson-like) states in the lepton sector (in contrast to 36 meson ground states in QCD), and (iii) clear dynamical reasons for the large difference between numbers of the lepton (= 6) and baryon (= 55) ground states. This means that SSD has to be very different from QCD.

Another important piont is that various experimental constraints request that the leptons have a size [26]

$$\Lambda_L^{-1} < 10^{-17}cm \sim (10TeV)^{-1}. \tag{2}$$

Thus the bound-state dynamics of the quarks must be such that the masses of the qqq-leptons are extremely small compared to inverse sizes of the bound states. A possibility of keeping the masses of the tiny little size leptons small may be caused, in particular, by the scale invariance of the additional boson fields that critically strong interact with the quarks.

2. Spontaneously broken scale invariance and spinor-scalar solitons

Following the three points listed above lets introduce the additional (to QCD) self-interacting, color-singlet, odd G-parity scalar fields $\sigma_i(x)$ in the each generation i ($i = 1, 2, 3$) with a Lagrangian density of the most general renormalizable and scale invariant form, and with strong interaction between $\sigma_i(x)$ and color ($a = r, g, b$) quark fields $q_i^a(x)$ of the same generation[3]:

$$\mathcal{L}_{\sigma QCD} = \mathcal{L}_{QCD} + \sum_{i=1}^{3}\left(\frac{1}{2}\partial_\mu\sigma_i\partial^\mu\sigma_i - g\sigma_i\bar{q}_i^a q_i^a - \frac{1}{4}\lambda\sigma_i^4\right). \tag{3}$$

This model (lets call it σQCD) contains seven parameters well known from QCD lagrangian ($\mathcal{L}_{QCD}$) - Λ_{QCD} and quark masses[4]

$$\Lambda_{QCD}^{\overline{MS}} \approx 200MeV, \tag{4}$$

[3]Without any interaction between $\sigma_i(x)$ and quarks of other generations $j \neq i$

[4]Quark masses are given at renormalization point $\mu_{QCD} = 2GeV$

$$\begin{pmatrix} \hat{m}_u = (2-4)MeV \\ \hat{m}_d = (4-8)MeV \end{pmatrix}, \begin{pmatrix} \hat{m}_c = (1.15-1.35)GeV \\ \hat{m}_s = (0.08-0.13)GeV \end{pmatrix}, \begin{pmatrix} \hat{m}_t = (169 \pm 0.35)GeV \\ \hat{m}_b = (4.3 \pm 0.20)GeV \end{pmatrix}, \quad (5)$$

and two extra adjustable dimensionless parameters $g \sim 1$ and $\lambda \sim 1$. σQCD is scale invariant in the limit of zero quark mass and zero QCD condensates, and admits stable dynamical solutions for $\lambda > 0$.

One of the central problems in σQCD is understanding the nature of the vacuum part of the critically bounded solutions. In general, this vacuum can be characterized by simplest condensates – a σ-field condensate $\sigma_{vac}(x)$ and quark condensate $< vac|\bar{q}(x)q(x)|vac >$.

Because of scale invariance the energy U as a function of the σ-field strength has one minimum instead of two minima in the Friedberg-Lee model [27] which is more complicated and explicitly violates the scale invariance.

In the absence of quarks, the potential

$$U(\sigma) = \frac{1}{4}\lambda\sigma^4, \quad (6)$$

and therefore σ is massless field with the normal vacuum state at zero.

In presence of quarks strongly interacting with σ-field a localized bounded state with scale invariant and nonzero quark condensate

$$< vac|\bar{q}(x)q(x)|vac > = \frac{\gamma_{\bar{q}q}}{r^3} \quad (7)$$

may be formed, where the dimensionless constant $\gamma_{\bar{q}q}$ can be calculated self-consistently in σQCD or estimated from bag model[5].

For strongly bounded states the vacuum contribution (7) dominates on valence-quark contribution which vanishes when mass of the state tends to zero. In this case the potential U changes to

$$V(\sigma, r) = g\sigma < vac|\bar{q}(x)q(x)|vac > + \frac{1}{4}\lambda\sigma^4. \quad (9)$$

Therefore near the center of the localized solution (let assume the center in the origin) the σ-field finds a new deeper minimum

$$\sigma_{vac}(\mathbf{r}) = -\left(\frac{\lambda}{g} < vac|\bar{q}(\mathbf{r})q(\mathbf{r})|vac >\right)^{1/3} = -\left(\frac{\lambda}{g}\gamma_{\bar{q}q}\right)^{1/3} r^{-1} \quad (10)$$

at a large finite value $\sigma_{vac}(r)$ for every r (Fig.2): the interacting quarks form a nontopological spinor-scalar soliton.

In order to derive dynamical equations lets write for the scalar field

$$\sigma = \sigma_{vac} + \sigma_1, \quad (11)$$

[5]For MIT bag of radius R containing the fluctuating vacuum fields the quark condensate [28]

$$< vac|\bar{q}q|vac > \approx -\frac{0.15 \cdot 3 \cdot 2}{R^3} < 0 \quad (8)$$

where factors 3 and 2 correspond to three colors and two flavors in each generation). Outside the bag $< vac|\bar{q}q|vac > \longrightarrow 0$.

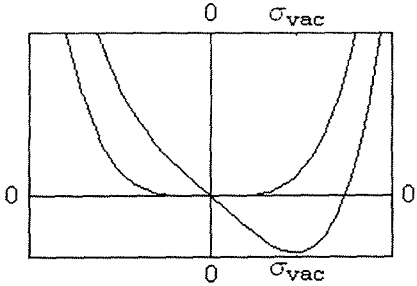

Figure 1: Typical form of the potential functions $U(\sigma)$, and $V(\sigma, r)$. Units on the vertical and horizontal axes are arbitrary

where σ_{vac} is a c-number field. It is convenient to work in the rest frame of the scalar field σ_{vac}. In this frame, $\sigma_{vac}(\mathbf{r})$ is time independent.

Then lets expand the operator $q(x)$ as follows:

$$q = \sum_k c_k q_k(r) \tag{12}$$

where $\{q_k\}$ is an arbitrary, complete orthonormal set of Dirac spinor functions and the c_k are fermion annihilation operators.

To lowest order in $\sigma_1(\mathbf{r})$ the $q_k(\mathbf{r})$ and $\sigma_{vac}(\mathbf{r})$ satisfy the coupled differential equations

$$(\vec{\alpha} \cdot \vec{p} + g\beta\sigma_{vac})q_k = \epsilon_k q_k \tag{13}$$

$$-\nabla^2\sigma_{vac} + \lambda\sigma_{vac}^3 = -g\sum_k q_k^\dagger \beta q_k \tag{14}$$

The sum in (14) is over all occupied quark states. Not only the "valence" quark states are needed in solving these equations self-consistently, but also the sea quarks. Eq. (13) defines a complete set of basis states $\{q_k\}$ in which the quark field operator $q(x)$ is expanded. Therefore for a static localized solution to (13,14) the sum contain vacuum and valence quark contribuions

$$\sum_k q_k^\dagger \beta q_k = < vac|\bar{q}(\mathbf{r})q(\mathbf{r})|vac > + < val|\bar{q}(\mathbf{r})q(\mathbf{r})|val >, \tag{15}$$

where the quark condensate can be used in form (7) with $\gamma_{\bar{q}q} = -0.15 \cdot 3 \cdot 2$ (see eq.(8)).

In deriving equations for the critically bounded states the contribution from the vacuum polarization dominates over the vanishing valence quark contribution $< val|\bar{q}(\mathbf{r})q(\mathbf{r})|val >$ The last term in (15) can be omitted in calculations of $\sigma_{vac}(\mathbf{r})$. In this particular case equation (14) simplifies to

$$-\nabla^2\sigma_{vac} + \lambda\sigma_{vac}^3 = -g\frac{\gamma_{\bar{q}q}}{r^3}, \tag{16}$$

and has pure Coulomb solution (10) in respect to the scale symmetry of σQCD.

Inclusion of only σ_{vac}, the c-number part of the soliton field in (13) and (14), has led to what is essentially a mean-field approximation (MFA).

For the given $\sigma_{vac}(\mathbf{r})$ the total mass of the quark-scalar soliton can be calculated within MFA by use of the virial theorem [29]

$$\mathcal{E}_{soliton} = \int d^n x \left[\frac{n-3}{2} (\nabla \sigma_{vac})^2 + \frac{n+1}{4} \lambda \sigma_{vac}^4 + \left(\frac{m_q}{g} - \sigma_{vac} \right) \lambda \sigma_{vac}^3 \right], \tag{17}$$

The striking feature of Eq. (17) is the fact that for $n = 3$ (n is the number of spatial dimensions) the term with $\nabla \sigma_{vac} \sim r^{-4}$ vanishes, and the proportional to $\sigma_{vac}^4 \sim r^{-6}$ terms cancel out. This is an important feature, since σ_{vac}^4 is commonly held responsible for the stability of a theory with spontaneously broken scale symmetry. Thus in normal number of space dimensions ($n = 3$) the energy of the soliton diverges logarithmically

$$E_{soliton} = \sum_{quarks}^{valence} m_q \frac{\lambda}{g} \int d^3 x \, \sigma_{vac}^3 \approx -4\pi \sum_{quarks}^{valence} m_q \left(\frac{\lambda}{g} \right)^2 \gamma_{\bar{q}q} \ln \left(\frac{\Lambda_{\sigma QCD}}{\mu_{QCD}} \right). \tag{18}$$

The soliton energy (18) in MFA is proportional to sum of current masses of valence quarks (parameters of violation of scale symmetry), dimensionless constant $(\lambda/g)^2 \gamma_{\bar{q}q}$ ($\lambda \sim 1$ and $g \sim 1$, $\gamma_{\bar{q}q} \sim -1$), and a logarithmic factor. Therefore, taking into account (2) and (4) each valence quark contributes to the soliton mass

$$M_q \approx -4\pi m_q \left(\frac{\lambda}{g} \right)^2 \gamma_{\bar{q}q} \ln \left(\frac{\Lambda_{\sigma QCD}^2}{\Lambda_{QCD}^2} \right) > 116 \left(\frac{\lambda}{g} \right)^2 \gamma_{\bar{q}q} \, m_q \tag{19}$$

It should be noted that the spontaneous violation of scale symmetry in σQCD is the reason for the exact cancellation of the large $\sim \Lambda_{\sigma QCD}$ contributions to the mass of the system. Nevertheless, for $(\lambda/g)^2 \gamma_{\bar{q}q}$ assumed to be 1 and for known estimations of current quark masses (5) the quark contributions (19) to the soliton mass are still much greater than the desired lepton (especially neutrino) masses.

A dynamical reason for taking into account renormalization of quark masses and coupling constants in (18), and for further reducing of the soliton mass from the large effective quark masses (M_q) to the neutrino masses, is the quantum corrections due to the fluctuating field σ_1.

Although, the MFA already contains important nonlinear effects, deviations from this approximation are generated by σ_1. If effects due to σ_1 are not too great, the separation will be a useful one. Lets utilize the MFA to generate a representation in terms of which the corrections can be calculated. The Hamiltonian (without the terms due to vector gluons, and counterterms) can be written

$$\begin{aligned} \mathcal{H} &= \mathcal{E}_{soliton} + \sum_k \epsilon_k (b_k^\dagger b_k + d_k^\dagger d_k) \\ &+ \int d^3 x \left\{ \frac{1}{2} (\pi_1^2 + |\nabla \sigma_1|^2 + 3\lambda \sigma_{vac}^2 \sigma_1^2) + \frac{\lambda}{12} \sigma_{vac} \sigma_1^3 + \frac{\lambda}{24} \sigma_1^4 \right. \\ &+ \left. g \, (\bar{q}q - < vac|\bar{q}q|vac >) \sigma_1 \right\}, \end{aligned} \tag{20}$$

where π_1 is the momentum conjugated to σ_1, $b_k = c_k$ are the particle operators for $\epsilon_k > 0$ and $d_k = c_{\bar{k}}^\dagger$, where $k = (\kappa, m, \epsilon)$, $\bar{k} = (-\kappa, -m, -\epsilon)$ so that

$$q = \sum_{k \ (\epsilon_k > 0)} (b_k q_k + d_k^\dagger q_{\bar{k}}). \tag{21}$$

The b_k and d_k are particle and antiparticle annihilation operators.

Lets also expand σ_1 in terms of an arbitrary, complete set of functions, e.g., $\{s_j\}$, as

$$\sigma_1 = \sum_j (2\omega)^{-1/2}(a_j^\dagger + a_j)s_j \tag{22}$$

$$\pi_1 = i\sum_j (2\omega)^{1/2}(a_j^\dagger - a_j)s_j \tag{23}$$

where the a_j and $a_j^\dagger$ are the usual Bose annihilation and creation operators. The index j is the collection of quantum numbers needed to describe the eigenstates s_j.

The s_j and ω_j can be fixed by requiring the s_j to satisfy the eigenvalue equation

$$(-\nabla^2 + 3\sigma_{vac}^2 - \omega_j^2)s_j(\mathbf{r}) = 0 \tag{24}$$

Now the Hamiltonian can be rewritten

$$\mathcal{H} = \mathcal{E}_{soliton} + \sum_k \epsilon_k(b_k^\dagger b_k + d_k^\dagger d_k) + \sum_j \omega_j(a_j^\dagger a_j + \frac{1}{2}) + \mathcal{H}', \tag{25}$$

where

$$\begin{aligned} \mathcal{H}' &= \int d^3x \Big[\quad g\ (\bar{q}q - <vac|\bar{q}q|vac>)\sigma_1 \\ &+ \frac{\lambda}{12}\sigma_{vac}\sigma_1^3 + \frac{\lambda}{24}\sigma_1^4 \Big] \end{aligned} \tag{26}$$

The $\{q_k\}$ and $\{s_j\}$ define a basis in terms of which corrections due to H' can be calculated. This is very analogous to the weak particle-surface coupling representation of the Bohr-Mottelson unified model [30]. The representation states and spectra are relatively easy to solve for once the self-consistent $\sigma_{vac}(\mathbf{r})$ has been obtained. Numerous approximation methods are available for handling H', such as perturbation theory or matrix diagonalization in a finite basis. Note that the nonlinear terms (σ_1^3 and σ_1^4) are not an essential complication. These terms additionally contribute to renormalization of quark masses m_q and coupling constants g and λ. In particular, due to quantum loop corrections, after the dimensional transmutation m_q, g, λ [31], and $\gamma_{\bar{q}q}$ [32] depend on the ratio of momentum transfer (p) to the characteristic scales Λ_{QCD}, and $\Lambda_{\sigma QCD}$:

$$m_q(p) = \hat{m}_q \left(\frac{\ln(\mu_{QCD}^2/\Lambda_{QCD}^2)}{\ln(p^2/\Lambda_{QCD}^2)} \right)^{16/11}, \tag{27}$$

$$g^2(p) \sim -\ln(\frac{p^2}{\Lambda_{\sigma QCD}^2})^{-1}, \tag{28}$$

$$\lambda(p) \sim -\ln(\frac{p^2}{\Lambda_{\sigma QCD}^2})^{-1}, \tag{29}$$

$$\gamma_{\bar{q}q}(p) \sim -\ln(\frac{p^2}{\Lambda_{\sigma QCD}^2}), \tag{30}$$

therefore the renormalized soliton energy

$$\mathcal{E}_{soliton} = \sum_{quarks}^{valence} \mathcal{M}_q, \tag{31}$$

where

$$\mathcal{M}_q(\Lambda_{\sigma QCD}) = -4\pi \left(\frac{\lambda}{g}\right)^2 \gamma_{\bar{q}q} \hat{m}_q \int_{\frac{\pi}{\Lambda_{\sigma QCD}}}^{\frac{\pi}{\mu_{QCD}}} \frac{dr}{r} \left(\frac{\ln(\mu_{QCD}^2/\Lambda_{QCD}^2)}{\ln(\pi^2/(r\Lambda_{QCD})^2)}\right)^{16/11} =$$

$$-\frac{22}{5}\pi\gamma_{\bar{q}q}\left(\frac{\lambda}{g}\right)^2 \hat{m}_q \ln\left(\frac{\mu_{QCD}^2}{\Lambda_{QCD}^2}\right)\left(1 - \frac{\ln\left(\frac{\mu_{QCD}^2}{\Lambda_{QCD}^2}\right)^{5/11}}{\ln\left(\frac{\Lambda_{\sigma QCD}^2}{\Lambda_{QCD}^2}\right)^{5/11}}\right). \tag{32}$$

It is ≈ 6 times smaller than (18) at $\Lambda_{\sigma QCD} = 10TeV$, and differs from (18) by the last logarithmic factor which is slightly decreasing with $\Lambda_{\sigma QCD}$.

These quantum corrections explicitly violate the scale invariance of σQCD and form the characteristic large momentum scale of this theory $\Lambda_{\sigma QCD} > 10TeV$. Therefore σQCD allows almost massless (relative to $\Lambda_{\sigma QCD}$) soliton solutions of very small radius $\sim \Lambda_{\sigma QCD}$. The scale symmetry of σQCD leads to dynamical reduction of the characteristic energy scale of $\Lambda_{\sigma QCD}$ to sum of relatively small effective masses of valence quarks (32).

The further reduction of the soliton mass is possible by taking into account the remaining interactions between different quarks via exchanges of quanta of σ_1 with quantum numbers chosen to be $I^G J^P = 0^- 0^+$. Mass of $m_{\sigma_1} = \frac{\sqrt{3}}{2}\Lambda_{\sigma QCD}$ is large enough to appear as critically strong very short-range interaction between the moderately massive effective quarks. In particular, these values of masses allow to avoid undesirable lowlying excitations which are absent in the lepton's data.

The choice of the odd G-parity σ_1-meson allows to avoid undesirable lowlying meson-like $q\bar{q}$ states. Because the source of this interaction is meson exchange it is related by crossing symmetry to the $q\bar{q}$ interaction which can be deduced directly by simply changing the sign of the odd G-parity exchange terms [33]. The most significant feature is that the short-range attraction in the qq system which is produced by the exchange of $I^G = 0^-$ σ_1-meson becomes a strongly repulsive short-range force in $q\bar{q}$ systems. In addition, in $q\bar{q}$ system there is annihilation interaction which is always repulsive. Therefore in σQCD lowlying $q\bar{q}$ mesons don't exist.

3. Critically bounded states

A specific feature for generation of the critically bounded (and therefore ultrarelativistic) states by the critically strong short-range interaction is that MFA is not enough even for rough estimations (because the strong two-particle or three-particle correlations play important role in such systems).

For example a relativistic three body approach for three equal fermions of mass m interacting via scalar zero-range forces [34] (in contrast to MFA) manifests the critically strong binding of three-particle system (zero total mass) when mass of the two-particle system $M_2 = M_c = 1.35m$ (Fig.3).

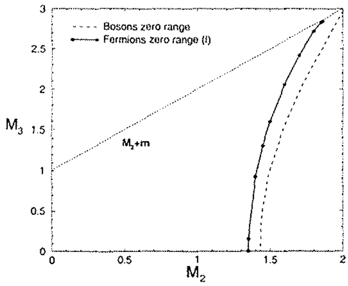

Figure 2: Three-fermion bound state mass M_3 versus two-fermion mass M_2 (solid line) in comparison to the three-boson bound state mass (dash line) calculated in the framework of the Light-Front Dynamics for zero-range scalar interaction [34]. The mass M_3 of the relativistic three-body bound state exists only when M_2 is greater than a critical value M_c ($\approx 1.43\,m$ for bosons and $\approx 1.35\,m$ for fermions, m is the constituent mass). For $M_2 = M_c$ the mass M_3 turns into zero

For $M_2 < M_c$ there are no three-particle solutions with real value of M_3, what means from the physical point of view that three-body state no longer dominate in the system and configurations with quark-antiquark pairs became essential for the critically bounded states. This means that in σQCD there is a critical value for coupling constant $g_{crit}(m_{q_1}, m_{q_2}, m_{q_3}, \lambda, \alpha_{QED}, Q) \approx 1$ so that for $g \geq g_{crit}$ masses of elecrtically neutral ($Q = 0$) states with tree valence quarks (udd, css, tbb) became extremely small and these states can be interpreted as antineutrinos[6].

In order to estimate masses of the charged leptons lets assume

$$(\mathcal{M}_{q_1} + \mathcal{M}_{q_2} + \mathcal{M}_{q_3})\sqrt{1 - g^2/g^2_{crit}}$$

falloff of the 3-body mass versus g to the critical point $g_{crit} \approx 1.1$ (Fig.3).

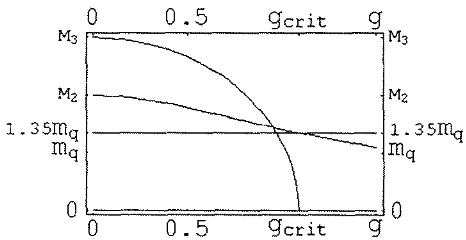

Figure 3: Typical behavior of a critically bounded $3q$, and strongly bounded $2q$ systems of quarks with equal masses (M_q) vs coupling constant g

Then small deviation from the critical point caused by difference in electromagnetic interactions of electrically neutral ($Q = 0$) qqq states (neutrinos) and charged ($Q = \pm 1$)

[6]Corresponding almost massless states with tree valence antiquarks ($\bar{u}\bar{d}\bar{d}$, $\bar{c}\bar{s}\bar{s}$, $\bar{t}\bar{b}\bar{b}$) can be interpreted as neutrinos.

states can be written in the following form[7]

$$m_{charged} = \sqrt{\frac{2\alpha_{QED}}{g_{crit}}} \left(2\mathcal{M}_q^{up}(\Lambda_{\sigma QCD}) + \mathcal{M}_q^{down}(\Lambda_{\sigma QCD})\right), \qquad (33)$$

where $\alpha_{QED} = 1/137$. Therefore, for a given value of still arbitrary $\Lambda_{\sigma QCD}$ and λ, σQCD allows estimate the masses of e, μ, τ leptons.

4. Proton mean life time and lepton scale

One of the most fundamental predictions of the quark structure of leptons is that proton is the first radial excitation of positron, and therefore proton must be unstable with decay, in particular, to $e^+\gamma$ via M1 transition. The proton mean life time

$$\tau(p \to e^+\gamma) \sim \left(\frac{4\alpha_{QED}}{9}\Lambda_{\sigma QCD}\left(\frac{m_p}{2\Lambda_{\sigma QCD}}\right)^3\left(\frac{m_p}{4\Lambda_{\sigma QCD}}\right)^9\right)^{-1} \qquad (34)$$

must satisfy the experimental upper limit $\tau(p \to e^+\gamma) > 4.6\ 10^{32}$ years. This take place if $\Lambda_{\sigma QCD} > 1.2\ 10^6 GeV$.

5. Masses of charged leptons in σQCD

Taking into account that for $\Lambda_{\sigma QCD} > 1.2\ 10^6 GeV$ $\mathcal{M}_q$ is almost independent on $\Lambda_{\sigma QCD}$, and chosing $\lambda = 0.1$ σQCD predict the following values for masses of the charged leptons

$$m_e = 0.48(0.51)MeV, \quad m_\mu = 105(106)MeV, \quad m_\tau = 13.7(1.8)GeV, \qquad (35)$$

where experimental values are given in parenthesis. The too large difference between the calculated masses and the data for m_τ may be caused by too large differences between masses of t and b quarks.

6. On EW properties of the composed leptons

In the simplest approach one expects parity to be conserved. Of course, this contradicts observation. Therefore one must attribute the observed parity violation to details of the σQCD dynamics which may be related to the fact that the observed fermions are much lighter than $\Lambda_{\sigma QCD}$. One way to accommodate the parity violation would be to assume the lepton structure only for the lefthanded fermions, and to construct the right-handed fermions differently (e.g. by interpreting them as elementary objects). Another way arises if there is additional force which is magnetic in origin. However, in this case one expects both P- and CP-violation, and it is not understood why the observed CP-violation is small. The problem of parity violation persists in all substructure models.

Nevertheless, some electroweak properties of the composed leptons (in particular, the axial coupling constant of the composed leptons $g_A = 1$ in contrast to $g_A \approx 1.25$ for nucleons) can be explained and estimated in σQCD.

[7] $\sqrt{1-(g_{crit}-\alpha)^2/g_{crit}^2} \approx \sqrt{2\alpha/(g_{crit})}$ for $\alpha \ll 1$

7. Conclusion

It is shown that leptons could be composite bound states from quarks or antiquarks ($e^- = |\bar{u}\bar{u}\bar{d}>$, $\nu_e = |\bar{u}\bar{d}\bar{d}>$, $e^+ = |uud>$, $\bar{\nu}_e = |udd>$, $\mu^- = |\bar{c}\bar{c}\bar{s}>$, $\nu_\mu = |\bar{c}\bar{s}\bar{s}>$, $\mu^+ = |ccs>$, $\bar{\nu}_\mu = |css>$, $\tau^- = |\bar{t}\bar{t}\bar{b}>$, $\nu_\tau = |\bar{t}\bar{b}\bar{b}>$, $\tau^+ = |ttb>$, $\bar{\nu}_\tau = |tbb>$) within QCD extended by addititional scalar fields with sufficiently strong coupling constant and large renormalization scale $\Lambda_{\sigma QCD} > 10^6 GeV$: ground states of this theory are almost massless and localized at small radii $\sim \Lambda^{-1}_{\sigma QCD}$. They can be interpreted as the leptons. Baryons of $SU(3)$ multiplets in this picture are radial excitations of the leptons. At low energies, the small wave function overlap between the lepton and the hadron states then naturally leads to the large enough proton life time, the lepton number conservation, and do not affect the electroweak interactions. The lepton number symmetry is explicitly broken on the $\Lambda_{\sigma QCD}$ scale, but electrical charge and quark baryon number are exactly conserved. This theory leads to natural solution for the antimatter problem: the quarks are hidden in nucleons and $\bar{\nu}_e$, and the antiquarks are hidden in e^- and ν_e. In particular, this means that usual matter consists of equal numbers of quarks and antiquarks and can annihilate to photons and, correspondingly, can be created from photons in processes concerned with gravitational singularities.

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References

[1] H. Georgi and S. Glashow, Phys. Rev. Lett. **32**, 438 (1974).

[2] P. Nath and R. Arnowitt, CERN-TH. Preprint 7227/94 Nub-TH-3089/94, CTP TAMU 28/94.

[3] E. Witten, Phys. Lett. B **155**, 151 (1985).

[4] L. Lyons, Prog. In Part. and Nucl. Phys. **10**, 227 (Pergamon Press, NY, 1983).

[5] For a review, see E. Farhi and L. Susskind, Phys. Rev. **74** 277 (1981) and references therein.

[6] D. C. Kenndy and B. W. Lynn, Nucl. Phys. B **322**, 1 (1989); D. C. Kennedy, Nucl. Phys. B **351**, 81 (1991); M. E. Peskin and T. Takeuchi, Phys. Rev. Lett. **65**, 964 (1990); G. Altarelli and R. Barbieri, Phys. Lett. B **253**, 161 (1991); G. Altarelli , R. Barbieri and S. Jadach, Nucl. Phys. B **369**, 3 (1992); and references therein.

[7] J. C. Pati and A. Salam Phys. Rev. D 10 275 1974 (footnote 7).

[8] O. W. Greenberg, Private communication with J C Pati (1974).

[9] H. Terezawa, Prog. Theor. Phys. **64**, 1763 (1980).

[10] H. Fritzsch and G. Mandelbaum, Phys. Lett. B **102**, 113 (1981).

[11] O. W. Greenberg and J. Sucher, Phys. Lett. B **99**, 339 (1981).

[12] H. Fritzsch and G. Mandelbaum, Phys. Lett. B **102**, 319 (1981).

[13] H. Harari and N. Seiberg, Phys. Lett. B **100**, 41 (1981).

[14] H. Harari and N. Seiberg, Phys. Lett. B **98** 269 (1981).

[15] H. Harari and N. Seiberg Phys. Lett. B **102**, 263 (1981).

[16] J. C. Pati and A. Salam, Nucl. Phys. B **214**, 109 (1983).

[17] J. C. Pati and A. Salam, Nucl. Phys. B **234**, 223 (1984).

[18] R. Barbieri, Phys. Lett. B **121**, 43 (1983).

[19] J. C. Pati, M. Cvetic and H. Sharachandra, Phys. Rev. Lett. **58**, 851 (1987).

[20] J. C. Pati, Phys. Lett. B **228**, 228 (1989).

[21] K. S. Babu, J. C. Pati and H. Stremnitzer, Phys. Lett. B **256**, 206 (1991).

[22] K. S. Babu, J. C. Pati and H. Stremnitzer, Phys. Rev. Lett. **67**, 1688 (1991).

[23] J. C. Pati, Nucl. Phys. B Proc. Suppl. A **22**, 115 (1991) and references therein.

[24] K. S. Babu and J. C. Pati, Phys. Rev. D **48**, 1921 (1993).

[25] J. Kimj, J. Phys. G: Nucl. Part. Phys. **24**, 1881 (1998).

[26] O.W. Greenberg, and C.A. Nelson, Phys. Rev. D**10** 2567 (1974).

[27] R. Friedberg and T. D. Lee, Phys. Rev. D**15**, 1694 (1977); **16**, 1096 (1977); **18**, 2623 (1978); T. D. Lee, Particle Physics and Introduction to Field Theory (Harwood Academic, New York, 1981); K. Huang and D. R. Stump, Phys. Rev. D **14**, 223 (1976).

[28] K. A. Milton, Phys. Lett. B **104**, 49 (1981).

[29] J. Rafelski, Phys. Rev. D **16**, 1890 (1977).

[30] A. Bohr, and B. R. Mottelson, Nuclear Structure (Benjamin, New York, 1969, 1973), Vols. I and II.

[31] A. Hasenfratz and P. Hasenfratz, Nucl. Phys. B **270**, 687 (1986).

[32] C. Alexandrou, E. Follana, H. Panagopoulos, E. Vicari, Nucl. Phys. B **580**, 394 (2000).

[33] J. S. Ball, A. Scotti, and D. Y. Wong, Phys. Rev. **142**, 1000 (1966).

[34] V. A. Karmanov, and J. Carbonell, Few-Body Systems, **0**, 1 (2004)