

Top Quark Properties

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There is tremendous progress in the study of top quark properties from time when the top quark was discovered by CDF and D0 collaboration in 1995 at TEVATRON collider. In this review we try to summarize results and methods how these results have been achieved. Aim is to emphasize problems and how these problems have been solved. We stay in Standard Model framework and demonstrate what kind of support give experimental results to this framework.

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1 Introduction

Top quark was discovered in 1995 by two collaborations CDF and D0 [2] at TEVATRON collider at Fermi National Accelerator Laboratory (FNAL). From time of the discovery of existence of b-quark in 1977 (discovered also at FNAL) one of the most important tasks of HEP community was experimental discovery of partner of b-quark in weak isospin doublet - top quark. Eighteen years of intensive search for this particle was finished and era of experimental study of top quark properties started.

In this article we review current situation in a study of top quark properties, methods and results to determine top quark parameters. There are excellent review articles (e.g [3], [4]) which cover this topic in more details. Ambition of this paper is to update situation in experiments and emphasize some aspects from the author's point of view.

Until now (year 2008) only place to study experimentally top quark properties is FNAL and therefore all quoted experimental results come from two experiments CDF and D0 there.

The paper is organized following way:

In the Sec. 2 brief theoretical overview of the top quark properties from a point of view of the Standard Model is given.

In Sec. 3 after brief introduction of detector setup of the experiments CDF and D0 experimental results of the top quark parameter measurements are reviewed. There is always picked up specific published analysis which was from author's point of view interesting (for different reasons - novelty of approach, less model dependence etc.) and is followed in detail. We tried to simplify account and to emphasize important points of a specific analysis.

In concluding remarks improvements which can be expected based on integrated luminosity of the full TEVATRON run II are discussed and tasks which most likely will be left for future measurements at Large Hadron Collider (LHC) and International Linear Collider (ILC) are mentioned.

2 Theoretical framework

Standard model (SM) so far successfully passed all experimental tests and challenges (e.g. [1]). It describes weak, electromagnetic and strong interactions. Only gravitational interaction is left out of SM and we will ignore it in this paper. According to SM structure of basic building blocks of nature can be pictured as in Fig. 1.

In the first row of the picture one can see 3 families of leptons and 3 families of quarks. Quarks and leptons are spin 1/2 (spinor) particles. Quarks can interact with each other by all

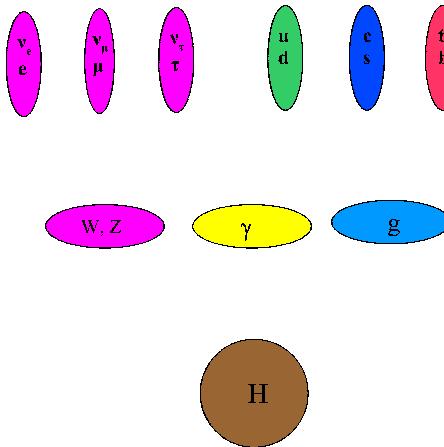


Fig. 1. Structure of fundamental blocks of matter.

three kinds of interactions. Each quark can be in one out of the three color states. Name of these states - color comes from an analogy with everyday life when white light can be decomposed into 3 constituent colors or vice versa. Also three quarks of different color can create white (colorless) object - strongly interacting particle - hadron (more precisely baryon). Also combination of (colored) quark and anti (colored) quark results in colorless hadron (more precisely meson). One can invent colorless combination also from more than three quarks but so far there was not found clear experimental evidence that such objects do exist. Strong interaction of quarks are described by Quantum chromo-dynamics (QCD) - theory which is integral part of Standard Model. QCD also explains that free quarks cannot be observed in nature - only composite colorless objects with quark constituents or fundamental colorless objects (e.g. leptons). Quarks and leptons are arranged in electroweak isospin doublets. Electric charge of quarks is 1/3 or 2/3 fraction of elementary charge. Upper partner of quark doublet posses +2/3 and lower partner -1/3 electric charge.

Leptons can interact only by electroweak interaction. Symmetry between quark and lepton families (same number of quark and lepton families) is very important from theoretical point of view. Lower partners in lepton doublet posses electric charge -1 (electron, muon and τ lepton) and upper components electric charge 0 (ν_e, ν_μ, ν_τ) neutrinos. Neutrinos interact through weak interaction only.

There are three kinds of spin 1 (vector) particles and their role in nature is to mediate interactions between particles. W^\pm and Z^0 -Bosons mediate (are carriers of) weak, photons (γ) electromagnetic and 8 gluons mediate strong interaction (as a colored objects they cannot be observed as a free particles in the nature).

Special role in this schema is reserved for last spin 0 (scalar) particle - Higgs particle. Up to now it is only particle from above structure which was not yet experimentally observed. This particle is responsible for symmetry breaking by which originally massless fundamental particles acquire mass. Main purpose of the new accelerator complex LHC (which will soon be operational) and experiments ATLAS, CMS [5] is discovery and study of the properties of Higgs particle.

2.1 The Standard Model predictions about top quark properties

Standard Model (SM) is frequently criticized because there are many independent parameters which need to be supplied to the model. Only after this supply does the Standard Model have a predictive power which can be experimentally verified. Masses of fundamental particles (including top quark) and values of the CKM [6] matrix cannot be determined from the first principles of the model. CKM matrix reflects observation that strong interaction quantum eigenstates of quarks are not the same as eigenstates for the weak interaction. Unitary CKM matrix transforms QCD eigenstates d, s, b quarks into weak interaction eigenstates d', s', b' .

$$\begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{bmatrix} \begin{bmatrix} d \\ s \\ b \end{bmatrix} = \begin{bmatrix} d' \\ s' \\ b' \end{bmatrix} \quad (1)$$

Matrix elements of CKM matrix represent probabilities for charged current transition of one quark to another. e.g. V_{tb} represents probability of charged current transition of top quark to b-quark.

A consequence of unitarity condition for three quark families CKM is

$$|V_{ub}|^2 + |V_{cb}|^2 + |V_{tb}|^2 = 1 \quad (2)$$

From experimentally determined V_{ub}, V_{cb} follows restriction on V_{tb}

$$0.9990 < |V_{tb}| < 0.9993 \quad (3)$$

Therefore top quark should dominantly decay into W and b-quark (but one should keep in mind that three quark family condition is crucial for this statement!).

Except for top mass - all other properties of top quark are predicted by Standard Model.

According to SM, t-quark as a weak isospin partner of b-quark should have charge +2/3 and spin 1/2. The weak interaction which is responsible for decay of e.g. muon is responsible also for decay of $t \rightarrow W^+ b$ and same formula applies. Top quark width is proportional to third power of top quark mass as can be seen from simplified formula below.

$$\Gamma_t \approx \frac{G_F m_t^3}{8\sqrt{2}\pi} \quad (4)$$

Where G_F is the Fermi coupling constant and m_t top quark mass. Top lifetime is inversely proportional to the top width. As one can see from above formula, for top quark mass 170 GeV/c² lifetime of t-quark is only $\sim 5 \times 10^{-25}$ seconds! Such short lifetime has a consequence. Typical time needed for fragmentation of quarks into hadrons is characterized by time scale

$1/\Lambda_{QCD} \sim 3 \times 10^{-24}$ sec. Because top quark lifetime is an order of magnitude smaller than time needed to create hadron from top quark we can draw a conclusion that top quark decays “immediately” into W and b-quark, therefore properties of top quark are directly transferred to decay products (and are not diluted by hadronization process). This property opens a unique opportunity to look at inner quark dynamics!

Dominant top quark production mechanism at hadron colliders is pair production of $t\bar{t}$. Lowest order Feynman diagrams for this process are drawn below.

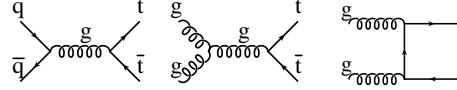


Fig. 2. Lowest order Feynman diagrams for $t\bar{t}$ pair production

Basic processes are quark anti quark annihilation and gluon - gluon fusion.

To calculate $t\bar{t}$ cross section for hadron-hadron reaction it is necessary to make convolution of elementary parton-parton cross section over parton distribution functions (PDF).

$$\sigma_{h_1 h_2}^{t\bar{t}}(\sqrt{s}, m_t) = \sum_{i,j=q,\bar{q},g} \int dx_i dx_j f_i^{h_1}(x_i, \mu^2) f_j^{h_2}(x_j, \mu^2) \times \hat{\sigma}^{ij \rightarrow t\bar{t}}(\rho, m_t^2, x_i, x_j, \alpha_s(\mu^2), \mu^2) \quad (5)$$

Where s is center of mass energy squared, m_t - top mass, x_i - fraction of longitudinal momentum of hadron carried by parton i , $f_i^h(x_i, \mu^2)$ is the PDF for parton i probed at energy scale of μ^2 in hadron h , $\alpha_s(\mu^2)$ - QCD strong running coupling constant at energy scale μ^2 , $\rho = 4m_t^2/\sqrt{s}$, effective center of mass energy squared for partonic process $\hat{s} = x_i x_j s$ and $\hat{\sigma}^{ij \rightarrow t\bar{t}}$ cross section for $t\bar{t}$ production by parton - parton interactions.

In [29] the theoretical cross section for $t\bar{t}$ production in $p\bar{p}$ collisions at 1.8 and 1.96 TeV in CMS (TEVATRON run I and II energy) was calculated (for selected top masses assumed) together with careful analysis of uncertainties. e.g. $\sigma_{p\bar{p}}^{t\bar{t}}(1.96, 175) = 6.70^{+0.71}_{-0.88}$ pb, $\sigma_{p\bar{p}}^{t\bar{t}}(1.96, 170) = 7.83^{+0.86}_{-1.04}$ pb. Cross section is dominated by by the $q\bar{q}$ production mechanism (80-90%), the gluon-gluon contribution was estimated to be between 10-20%.

At LHC ($p\bar{p}$ interaction at 14 TeV) fractions are reversed. The reason can be easily understood. Threshold \sqrt{s} for $t\bar{t}$ production is $2m_t$ and it corresponds to a typical fraction of longitudinal momentum $x \approx 2m_t/\sqrt{s}$. That means (for $m_t = 175$ GeV/ c^2) $x = 0.18$ at TEVATRON run II energy and $x = 0.025$ at LHC. As can be seen from Fig. 3 for $x = 0.025$ there is much larger probability for gluon than for u quark to be observed with given fraction of longitudinal momentum of proton. Lower threshold for x is also reason why cross section for $t\bar{t}$ production at LHC is two orders magnitudes higher than in case of TEVATRON while LHC energy is less than order of magnitude higher.

In Tab.1 a comparison of selected parameters of TEVATRON and LHC is shown. From this table one can see that cross section for $t\bar{t}$ production at LHC is two order of magnitude higher

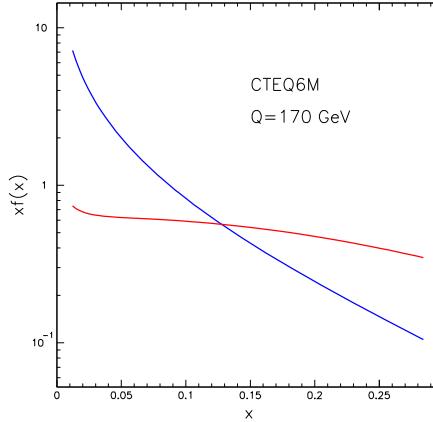


Fig. 3. Probability density to observe gluon (blue), up quark (red) with fraction of longitudinal momentum x in proton probed at energy scale 170 GeV. Curves are based on CTEQ6M parametrization [7], u-quark distribution dominates in a given x region quark distribution in proton.

Characteristic	Run 1	Run 2	LHC
interaction	$p\bar{p}$	$p\bar{p}$	pp
E_{CM} [TeV]	1.8	1.96	14
Luminosity [$\text{cm}^{-2}\text{s}^{-1}$]	$\approx 10^{31}$	$\approx 10^{32}$	$10^{33} - 10^{34}$
$\sigma_{t\bar{t}}(m_t = 175)$ [pb]	5.	6.7	830
q̄q fraction [%]	90	85	5
gg fraction [%]	10	15	95

Tab. 1. A comparison of the basic parameters for $t\bar{t}$ production at TEVATRON (run I, run II) and LHC

and luminosity at least order of magnitude higher than luminosity at TEVATRON one can expect at least 3 orders of magnitude larger statistics of produced top quarks at LHC in comparison with TEVATRON. LHC will be top factory and very detailed analysis will be possible there!

Standard Model also predicts possibility for “single top” production by electroweak process. Feynman diagrams for this production mechanism are shown in Fig. 4.

The first diagram represents s-channel, second t-channel and last one tW contributions to single top production. According to Standard Model in NLO approximation cross section (for top quark mass 175 GeV/c^2) for s-channel is $\sigma(p\bar{p} \rightarrow tb + X) = (0.88 \pm 0.11)|V_{tb}|^2$ pb and for t-channel $\sigma(p\bar{p} \rightarrow tqb + X) = (1.98 \pm 0.25)|V_{tb}|^2$ pb [9]. At TEVATRON tW channel contribution is expected to be negligible.

When considering t channel topology one needs to take into account there are no valence b-

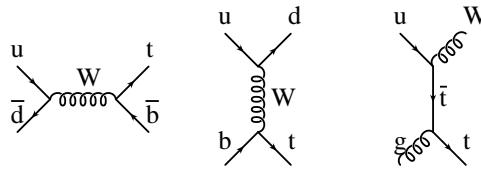


Fig. 4. Feynman diagrams for single top quark production

quarks in proton (or \bar{p}) structure. Therefore b-quark which enters t-channel diagram is always accompanied (compensated) by anti b-quark (and topology in final state is $dt\bar{b}$)!

Observation of single top production will be a very important piece of evidence for Standard Model. This process can be used for a direct measurement of CKM matrix element $|V_{tb}|$, it is a source of $\sim 100\%$ polarized top quarks, t-W-b vertex enters production level and therefore by cross section measurement of single top production one can measure top quark partial width $\Gamma(t \rightarrow Wb)$ and hence top lifetime! Specifically parton level cross section can be expressed as [10]

$$\hat{\sigma}(ub \rightarrow dt) = \sum_{\lambda=0,+,-} f_\lambda(x = \frac{m_t^2}{\hat{s}}) \left[\frac{16\pi^2 m_t^3}{\hat{s}(m_t^2 - M_W^2)^2} \right] \Gamma(t \rightarrow bW_\lambda^+) \quad (6)$$

where $f_\lambda(x)$ is flux of W - Bosons of a given helicity state $\lambda = 0, +, -$ [11], M_W is W-Boson mass, $\Gamma(t \rightarrow bW_\lambda^+)$ - partial width for decay of top quark into b and W of helicity λ .

Standard Model gives definite prediction about spin structure of top quark production and decay. Top quarks produced in pairs by strong interaction are unpolarized (more exactly polarization on level of a few percent is predicted by NLO QCD calculations [12]). Top quark decays through weak interaction $t \rightarrow W^+b$ and Standard Model predicts top mass dependent fractions of longitudinal and left-handed W polarization. In the limit of massless b-quark fraction of right handed W polarization should be (according to SM) 0. The topic is treated in more details in section 3.5. Even if in pair production top and anti-top quarks are unpolarized there is predicted correlation between spins of top and anti top quarks. Origin of correlation can be qualitatively understood the following way: At threshold $t\bar{t}$ system produced by $q\bar{q}$ annihilation mechanism is dominantly produced in 3S_1 angular momentum state [8]. From angular momentum conservation follows that spins of t and \bar{t} should be aligned (point in the same direction). Threshold production of $t\bar{t}$ by gluon-gluon fusion is dominantly produced in 1S_0 state and therefore spins of t and \bar{t} should be anti-aligned. Because at TEVATRON $t\bar{t}$ production dominates by $q\bar{q}$ mechanism one can expect to observe spin alignment of t and \bar{t} spins. The topic is treated in more details in section 3.6.

We close this section about Standard Model predictions about top quark properties with prediction of forward backward asymmetry in $t\bar{t}$ production. According to [13] as a consequence of interference of diagrams (a) and (b), (c) and (d) (see Fig. 5) there is forward-background asymmetry in case of $t\bar{t}$ production on level $\sim 5\%$. At this level top quark is more likely to follow direction of quark (from proton).

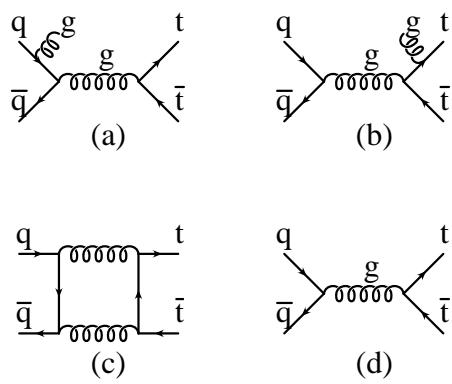


Fig. 5. Interference of diagrams (a) and (b), (c) and (d) has a consequence in forward backward asymmetry for top quark production

3 Experimental determination of top quark parameters

Experimental results we will discuss come from the two experiments at TEVATRON – CDF and D0.

TEVATRON is $p\bar{p}$ collider which during run I period (1992-1996) operated using 6 bunches of protons and anti-protons, with 3500 ns between bunch crossings and center of mass energy 1.8 TeV. Typical peak luminosity was $\approx 10^{31} \text{cm}^{-2}\text{s}^{-1}$. During this period each experiment recorded $\approx 120 \text{pb}^{-1}$ of data.

There was a major upgrade of TEVATRON and also for both of the experiments in years 1996 through 2000. Run II started in march 2001 and is supposed to continue until end of 2009 (2010?). It operates with 36 bunches of protons and anti-protons with bunch spacing of 396 ns and increased center of mass energy 1.96 TeV. Typical peak luminosity is order of magnitude higher than in run I and realistic integrated luminosity which will be achieved by end of run II is 6-8 fb^{-1} !

CDF and D0 are general purpose detector systems composed of following components:

- vertex detector
- tracking system
- electromagnetic calorimeters
- hadronic calorimeters
- muon system

Both detectors are azimuthally and forward backward symmetric. Both experiments use right-handed orthogonal coordinate systems (x, y, z) . Center of coordinate system coincides with center of detector, with z -axis along the proton beam direction. The y -axis is vertical and x -axis points to the center of accelerator ring. There are defined azimuthal angle ϕ , polar angle θ and frequently the pseudorapidity $\eta = -\ln(\tan(\theta/2))$ is used. Projection of momentum into transverse (x, y) plane is another frequently used variable p_T . This variable is directly related to a curvature of tracks in the magnetic field. In case of calorimetry there is used related variable $E_T = \sum E_i \sin(\theta_i)$. Where E_i, θ_i are energy of the cell i and polar angle of the cell i respectively. The summation is over all cells belonging to a given physical object (electromagnetic, hadronic shower).

Vertex detector and central tracker are placed inside solenoid magnet. Vertex detector is supposed to be efficient in identifying vertex of short lived particles. Specifically physical interest is directed to hadrons with b-quark in their structure - B-hadrons (e.g. B^\pm, B^0) or hadrons with c-quark in their structure - charmed hadrons (e.g. D^\pm, D^0). Typical mean lifetime of B-hadrons is $\tau \approx 1.6 \text{ ps}$ or in units $c\tau \approx 500 \mu\text{m}$. Charmed hadrons mean lifetime is shorter $c\tau \approx 100-300 \mu\text{m}$. Short lived particle decays into (relatively) long lived, charged (and/or neutral) particles like π, K or there is cascade of short lived particles but finally only long lived particles are left. Trajectory of charged particles in magnetic field has helix shape. It can be parametrized by 5 parameters. When charged particle passes through sensor it leaves trace - hit. From hits trajectory can be reconstructed. Long lived particles which come directly from interaction point have impact parameter (closest distance in transverse plane to interaction point) 0 and corresponding

coordinate along beam axes (z_0) also 0. Naturally within resolution provided by detector. Same quantities for particles which come from decay of e.g. B-hadrons will be non zero. Vertex detectors have different structure (for CDF and D0) but are based on the same microstrip silicon detector technology.

Detailed description of the CDF detector can be found in [14] and for D0 detector in [15]. Short descriptions can be found on web pages of above collaborations. In our description below we will concentrate just on final performance of subcomponents - resolution and coverage.

CDF detector

Schematic of CDF detector in elevation view is shown in Fig.6.

Silicon vertex detector has barrel structure and consists of three parts:

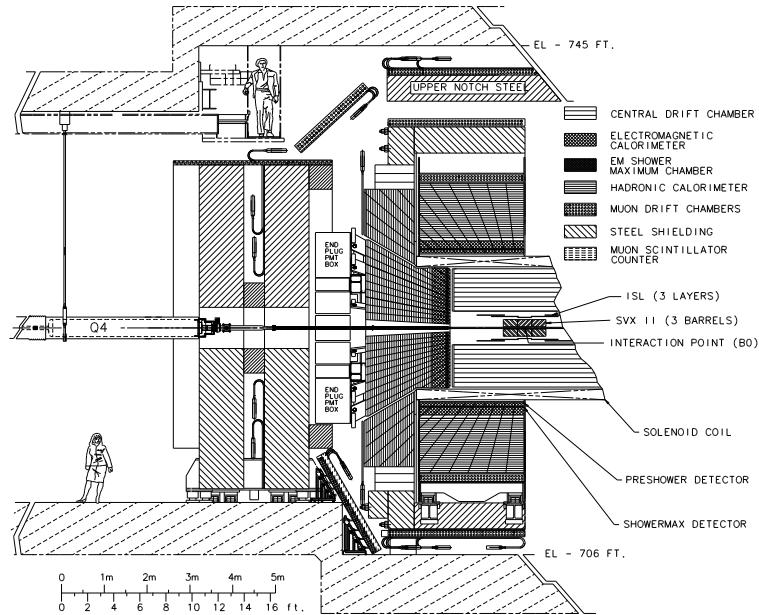


Fig. 6. CDF detector

- central SVXII detector consists of 5 layers of double sided silicon sensors
- outer ISL detector consists of two layers of double sided silicon sensors
- closest to beam pipe is Layer 00 - single sided silicon sensor

One of the important problems which needed to be solved is radiation hardness specially for Layer 00 which is closest to the beam pipe therefore most prone to radiation damage. Region covered by vertex detector in pseudorapidity $|\eta| < 2$ and impact parameter resolution

when SVXII and ISL can be combined is $40 \mu m$. But for $30 \mu m$ is responsible uncertainty in beamline. z_0 resolution is in optimal case (combination of SVXII and ISL) is $70 \mu m$. Device is capable of reconstructing tracks in 3D. This system is based on tremendous experience of CDF collaboration with silicon vertex detectors. Current system is the third generation of very successful sequence of silicon vertex detectors at CDF.

CDF Tracking Volume

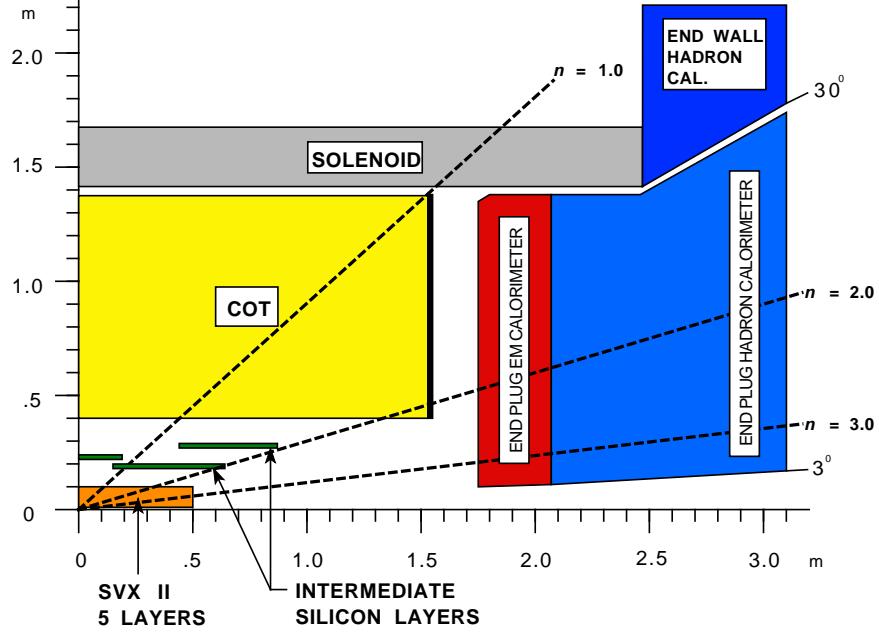


Fig. 7. Longitudinal view of CDF tracking system

Cylindrical drift chamber (COT) detector surrounds vertex detector and it is main tracking device for CDF collaboration. It is 3.1 m long, covers radial range from 40 to 137 cm and provides 96 measurement layers, organized into alternating axial and $\pm 2^\circ$ stereo superlayers. Provides the hit position resolution $\approx 140 \mu m$ and the momentum resolution

$$\frac{\sigma(p_T)}{p_T} = 0.0015 \times p_T \quad (7)$$

. The COT also provides information about energy loss of tracks (dE/dx) which can be used in particle identification. COT covers pseudorapidity $|\eta| \leq 1$. Schematic of CDF tracking system is shown in Fig. 7.

Just outside COT with a few centimeters clearance is installed time-of-flight system (TOF) based on plastic scintillators and fine mesh photomultipliers. Resolution of TOF is ≈ 100 ps.

Superconducting solenoid magnet of length 4.8 m and radius 1.5 m surrounds tracking system and generates 1.4 T magnetic field parallel to the beam axis.

Segmented electromagnetic and hadronic calorimeters cover region $|\eta| < 3.64$. They measure the energy flow of interacting particles. Energy resolution for electromagnetic calorimeter in region $|\eta| < 1.1$ (CEM) is

$$\frac{\sigma(E_T)}{E_T} = \frac{0.135}{\sqrt{E_T}} + 0.02, \quad (8)$$

and in region $1.1 < \eta < 3.6$ (plug) is

$$\frac{\sigma(E)}{E} = \frac{0.16}{\sqrt{E}} + 0.001 \quad (9)$$

Corresponding resolution for hadronic calorimeter in these two separated regions is for central hadron calorimeter

$$\frac{\sigma(E_T)}{E_T} = \frac{0.75}{\sqrt{E_T}} + 0.03 \quad (10)$$

and for plug

$$\frac{\sigma(E)}{E} = \frac{0.74}{\sqrt{E}} + 0.04 \quad (11)$$

The muon system resides beyond the calorimetry. It covers region $|\eta| < 1.5$. There is minimum p_T for muon needed to pass through 5 absorption length of calorimeter (1.5 GeV/c) and region with additional instrumented iron (2 GeV/c). Matching of track in the muon system with track in COT identifies muon.

The beam luminosity is determined by gas Cherenkov counters in forward region $3.7 < |\eta| < 4.7$. They measure the average number of inelastic $p\bar{p}$ collisions per beam crossing.

Very important part of detector system is trigger system which should be able to select from huge number of interactions interesting events, and data acquisition system which should be able to cope with huge data flow. One should realize that there is 10 orders of magnitude difference between inelastic $p\bar{p}$ cross section and $t\bar{t}$ production cross section at TEVATRON!

There are three levels of trigger. First level limits rate of events to ≈ 18 kHz of selected events based on basic information from tracking system, calorimetry and muon system, level 2 trigger reduces rate to 300 Hz based on more refined information and level 3 trigger has access to the full information and reduces rate to ≈ 75 Hz which is written to permanent storage.

D0 detector

Schematic of D0 detector is shown in Fig. 8.

Most inner part of the detector is silicon vertex detector. Silicon vertex detector has hybrid barrel and disk structure. The central detector covers region $|z| < 32$ cm consists of 6 barrels with disks interspersed between them. Each barrel consists of 4 radial layers. Each disk module has 12 wedge shaped double sided detectors. The forward detectors consists of six disks of

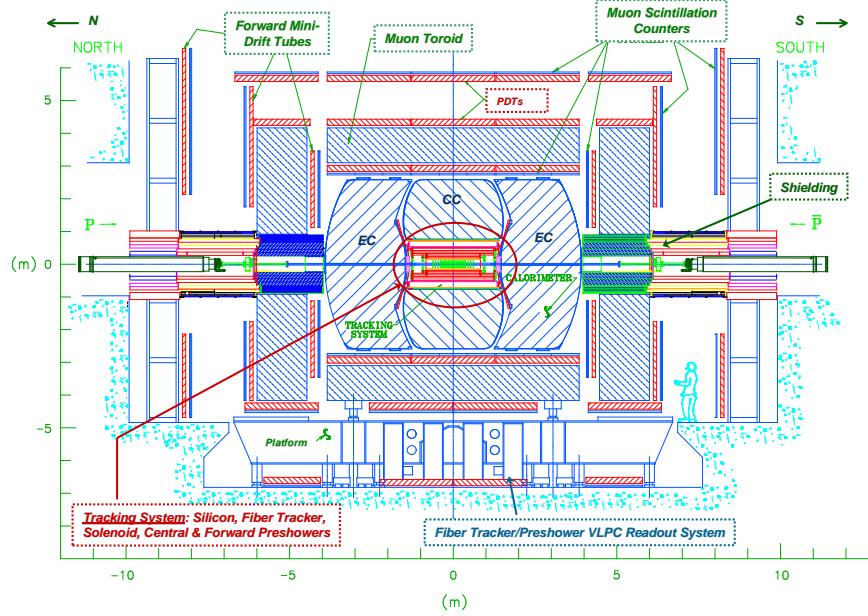


Fig. 8. D0 detector

similar design (see Fig. 9). Silicon vertex detector provides tracking information up to $|\eta| = 3$. It gives reconstructed vertex position resolution $15\text{--}40 \mu\text{m}$ in the transverse plane and $75\text{--}100 \mu\text{m}$ in z , depending on track multiplicity of the vertex.

The central fiber tracker is main tracking device for D0. It consists of 74000 scintillating fibers mounted on 8 concentric carbon fiber cylinders at radii 19.5 to 51.5 cm. Each cylinder supports four layers of fibers. The fibers are multi-clad and have diameter $830 \mu\text{m}$. Clear fiber waveguides carry the light for about 10 meters from scintillating fibers to the visible light photon counters! The combined silicon vertex detector and fiber tracker have excellent tracking performance. Full coverage of combined detector is for $|\eta| < 1.6$ and momentum resolution

$$\frac{\sigma(p_T)}{p_T} = 0.002 \times p_T \quad (12)$$

These detectors are located inside superconducting solenoid producing magnetic field 2 T in the beam direction.

Uranium liquid argon sampling with fine longitudinal and transverse segmentation provides full coverage $|\eta| < 4$ with energy resolution for electromagnetic showers

$$\frac{\sigma(E)}{E} \approx \frac{0.15}{\sqrt{E}} \quad (13)$$



Fig. 9. Disk/barrel structure of silicon vertex detector of D0 collaboration

and for hadronic jets

$$\frac{\sigma(E)}{E} \approx \frac{0.8}{\sqrt{E}} \quad (14)$$

The muon system is placed around calorimeters. The central part of muon system covers region $\eta < 1$, forward muon system extends coverage up to $\eta \approx 2$

Trigger system, as in case of the CDF, consists of the three levels. First level (L1) limits rate of events to ≈ 2 kHz based on basic information from detector components, level 2 (L2) reduces event rate by factor two by using hardware engine and microprocessors associated with specific sub-detectors to provide information for trigger decision tree and at level 3 (L3) candidates passed from L2 are sent to a farm of microprocessors where sophisticated algorithms are implemented and finally event rate is reduced to ≈ 50 Hz. These events are written to permanent storage.

Channels, particle and b-jet identification, variables

As was explained in the section 2.1 according to Standard Model pair $t\bar{t}$ production is dominant mechanism for top quark production at hadronic colliders. Each top (anti-top) quark very quickly decays into W-Boson and b-quark. As a result there are in an event W^+, W^-, b, \bar{b} . Decay of W-Bosons determines event topology. W-Boson can decay either into lepton and neutrino or into pair of quarks. If both W-Bosons decay into leptons - final state will consist of 2 leptons of opposite charge (e, μ) corresponding neutrino and anti-neutrino, b, \bar{b} . Such final state is called **“Dilepton channel”**. As a lepton could also be considered τ lepton but because τ lepton is from experimental point of view special (e.g. it can decay into hadrons in final state and ν_τ) it is treated separately.

Quarks cannot be observed as a free particles. They fragment by non perturbative QCD process into collimated jet of hadrons around original quark direction.

Therefore dilepton channel is characterized by 2 opposite charge energetic leptons (e or μ) 2 b-jets and 2 neutrinos.

In case when only one W-Boson decays into e or μ (and corresponding neutrino) and other decays into quarks final state is called **“Lepton + jets channel”**. It is characterized by energetic e

or μ lepton and corresponding neutrino, 2 b-jets and 2 (light) quark jets. In case when both W-Bosons decay harmonically (into quarks) final state is called “**All hadronic channel**”. It is characterized by 2 b-jets, 4 (light) quark jets.

In consideration of the optimal conditions for jet topology for a given channel one needs to take

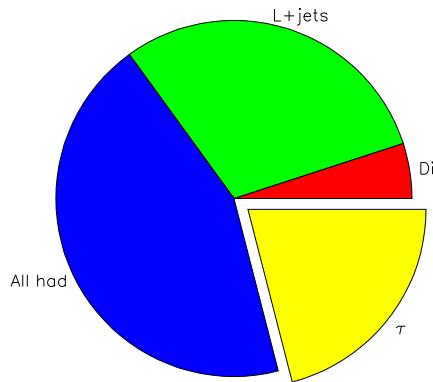


Fig. 10. Pie chart for fractions of different channels

into account that there can be created additional jets (e.g. by hard gluon radiation). There is also possibility to loose a jet because of jet reconstruction efficiency.

Branching ratios for different channels can be easily estimated. Coupling of W to different weak doublets is the same therefore (if we neglect mass of quarks in comparison with mass of W) also probability for decay is the same. There are 9 possible decays (3 - lepton doublets + 3(number of colors)x 2 quark doublets). Therefore probability for W decay into e or μ (and corresponding neutrino) is 2/9. Probability to decay into quarks is 2/3. Because decays of two W's are independent it follows that branching ratio for dilepton channel is 4/81, for lepton + jets channel 8/27 and for all hadronic channel 4/9. Rest is accounted by τ leptons in the final state. Graphically fractions of different channels are shown in Fig 10.

Identification of electrons is accomplished by a matching of track reconstructed in tracking system with electromagnetic shower in electromagnetic calorimeter. Details should be compatible with electron hypothesis. Ratio (E/p) of measured energy of electron in electromagnetic calorimeter to momentum of corresponding track measured by tracking system should be closed to 1.

Identification of muons is accomplished by matching of tracks in tracking and muon system. In corresponding cells in calorimeter deposited energy should be below muon compatibility threshold.

There are different methods of reconstructing jet kinematic characteristics. CDF collaboration prefers cone algorithm (details can be found in [22]). Raw reconstructed jet E_T is a sum of E_T of calorimetric towers in fixed cone $r = \sqrt{(\Delta\phi)^2 + (\Delta\eta)^2}$ around jet axis. D0 collaboration prefers more sophisticated version of the cone algorithm.

The raw jet energy is corrected by several kinds of corrections (see [23]) to get final energy which should correspond (within resolution) to energy of original quark which was materialized as a jet.

For identification of b-jets sophisticated b-tagging algorithms have been developed. In analysis quoted in this paper b-tagging algorithm is based on reconstruction of secondary vertexes of short lived particles inside jet reconstructed from tracks in silicon vertex detector. More details are in section 3.1.2.

Neutrinos, interacting with matter only by weak interaction, do not leave any trace in any part of the detector. But still at least transverse components of (vectorial) sum of transverse momenta of all neutrinos in the event can be estimated by imbalance of the total transverse momentum. Because initial state of $p\bar{p}$ interaction has $\vec{p}_T = 0$ from momentum conservation follows that also sum of \vec{p}_T of all particles created in interaction should be 0. All particles except neutrinos (or neutrino like) leave trace in detector and are accounted for. Therefore missing transverse energy \vec{E}_T

$$\vec{E}_T = - \sum_i \vec{E}_T^i - \sum_{jets} \vec{E}_T^{jets} - \sum_{\mu} \vec{p}_T^{\mu} \quad (15)$$

represents vectorial sum of transverse momenta of all neutrinos (or neutrino like objects) in the event. The notation \vec{E}_T^i means transverse energy deposited in cell i in calorimeter which was not attributed to any jet, \vec{E}_T^{jets} - transverse energy of jet, \vec{p}_T^{μ} - transverse momentum of isolated muon.

3.1 Cross section measurement

Cross section is a basic characteristic of a given reaction. This is the first quantity determined when there is a claim of production of a new particle or reaction. This quantity is also usually measured in a less model dependent way than other parameters. A comparison with theoretical prediction gives support or challenge for a theory.

Measurements of cross section in different channels represents statistically independent measurements and consistency of these measurements support idea of measurement of the same process. At small statistics in run I differences in measurement of cross section in different channels (even if there was no statistically significant difference) gave rise to theoretical speculations about two new particles (not just one top quark) close in mass.

In run II statistics is already more than order of magnitude larger. Speculations from run I can be given support or can be excluded.

Calculation of cross section according to a simple formula looks trivial

$$\sigma_{t\bar{t}} = \frac{N_{obs} - N_{bkg}}{\mathcal{A} \cdot \mathcal{L}} \quad (16)$$

Formula assumes that in observed number of events (N_{obs}) there is only a signal ($t\bar{t}$) and (known) background (N_{bkg}) which can be reliably estimated. In denominator there is a product of acceptance (\mathcal{A}) and integrated luminosity (\mathcal{L}). Acceptance is calculated by using generator for signal (e.g. [16], [17]), GEANT based model for detector simulation, selection and reconstruction procedure as used for the experimental data. One needs to assume specific top mass to calculate acceptance!

3.1.1 dilepton channel

A dilepton channel has an advantage of a simple topology. Signature for this channel are two energetic isolated leptons, (at least) two energetic jets and large missing transverse energy (\cancel{E}_T). By a proper kinematic selection it is possible to achieve reasonable signal to background ratio (S/B) without additional requirements (b-tagging or neural net selection)!

Description below follows analysis based on integrated luminosity 1.2 fb^{-1} collected with CDF detector [18]. Data are collected with inclusive electron or muon trigger which requires electron transverse energy $E_T > 18 \text{ GeV}$ or muon transverse momentum $P_T > 18 \text{ GeV}/c$. From this dataset are selected offline events with corresponding isolated leptons $E_T > 20 \text{ GeV}$ ($P_T > 20 \text{ GeV}/c$) and another lepton satisfying same E_T (P_T) condition but looser isolation requirement. Events with more than 2 leptons in a final state are rejected. To suppress Standard Model background couple more conditions are applied:

- $\cancel{E}_T > 25 \text{ GeV}$. If any lepton or jet is closer than 20° from the \cancel{E}_T direction, condition is strengthened to $\cancel{E}_T > 50 \text{ GeV}$.
- High \cancel{E}_T significance if ee or $\mu\mu$ invariant mass is in mass region of Z boson peak.
- At least 2 jets with $E_T > 15 \text{ GeV}$ (0 and 1 jet category belongs to the control sample)
- The summed transverse energy of leptons, jets and $\cancel{E}_T - H_T > 200 \text{ GeV}$ (the so called H_T cut)

- Two leptons should have opposite charge

Acceptance for candidate events was calculated by PYTHIA Monte Carlo program [16] simulating $t\bar{t}$ events with top mass $M_{top} = 175 \text{ GeV}/c^2$ combined with CDF detailed detector simulation and reconstruction package. Acceptance was found $\mathcal{A} = 0.808 \%$.

There are applied corrections for difference between Monte Carlo and data. These corrections are based on experimental data for Z boson to 2 lepton decay. Efficiency of inclusive lepton trigger is taken into account. It is measured in data samples selected by independent set of triggers. For above event selection there are expected two kinds of background. Background which has physically similar properties as $t\bar{t}$ signal. To this kind of background belongs diboson production (WW, WZ, ZZ) and $Z/\gamma^* \rightarrow \tau\tau$ accompanied by several jets. Another kind of background consist of physical process where some characteristic are mismeasured. To this background belongs $W\gamma$ production where γ is mismeasured as a (fake) lepton, $Z/\gamma^* \rightarrow ee, \mu\mu$ where \not{E}_T is mismeasured (fake \not{E}_T) and W accompanied by multiple jets where one jet is mismeasured as a (fake) lepton .

Amount of background from the first category is determined by Monte Carlo, second category

Source	Events per 1200 pb^{-1} vs Njet bins				
	0j	1j	$\geq 2j$	H_T	H_T, OS
Total background	139.34 ± 17.96	68.55 ± 10.74	44.87 ± 9.49	30.30 ± 5.89	25.56 ± 5.54
$t\bar{t}(\sigma = 6.7 \text{ pb})$	0.29 ± 0.03	7.49 ± 0.58	59.53 ± 4.53	57.41 ± 4.37	55.95 ± 4.26
Total SM expectation	139.63 ± 17.98	76.04 ± 11.10	104.40 ± 13.17	87.71 ± 8.85	81.52 ± 8.92
Data	143	84	114	88	77

Tab. 2. Summary table of background estimates, $t\bar{t}$ predictions and events in 1.2 fb^{-1} of data for each jet bin. For first three columns all cuts are applied except H_T cut and opposite charge conditions for leptons. The conditions are subsequently applied for the last two columns. The quoted uncertainties are summed (in quadrature) statistical and systematic uncertainties.

is estimated from analysis of data.

Acceptance for signal is based on MC and systematic errors come from several sources - MC

Source	Events per 1200 pb^{-1} after all cuts			
	ee	$\mu\mu$	$e\mu$	ll
Total background	6.86 ± 1.70	10.47 ± 2.06	8.23 ± 2.30	25.56 ± 5.54
$t\bar{t}(\sigma = 6.7 \text{ pb})$	12.18 ± 0.94	13.60 ± 1.04	30.17 ± 2.30	55.95 ± 4.26
Total SM expectation	19.04 ± 2.26	24.08 ± 2.68	38.40 ± 3.90	81.52 ± 8.92
Data	16	26	35	77

Tab. 3. Summary table by lepton flavor content of background estimates, $t\bar{t}$ predictions and final candidate events in 1.2 fb^{-1} of data. The quoted uncertainties are summed (in quadrature) statistical and systematic uncertainties.

generator (it is estimated by comparison of result of two different generators), uncertainty in tuned parameters for initial and final state radiation (ISR/FSR), choice of structure functions

(PDF) and most significant item is jet corrections. Purpose of jet corrections is to transform raw reconstructed jet kinematic characteristics to a kinematic characteristics of original quark. To estimate systematic error from this source corrections are changed by $\pm\sigma$ and shift in acceptance is a measure of systematic error. Total estimated systematic error from above sources is 4.2 %. For background there are also systematics related to fake leptons and cross section uncertainties. Results are summarized in two tables. In Tab. 2 the comparison of total SM expectation in control region (0 and 1 jet topology) demonstrates that amount of the background is well understood. Significant suppression of background is demonstrated in the last two columns by H_T cut and opposite charge condition for isolated leptons.

In the next table (Tab. 3) all selection cuts are applied but data are separated into different dilepton flavor categories. Cross section was calculated according to slightly modified eq.(16) to take more carefully into account contribution of different flavor categories.

The result is: $\sigma_{t\bar{t}} = 6.16 \pm 1.05_{stat} \pm 0.72_{syst} \pm 0.37_{lumi}$ pb, where the first uncertainty is statistical, the second is the convolution of the acceptance and background systematics and the third comes from the 6% uncertainty in the luminosity measurement.

3.1.2 lepton + jets channel

Lepton + jets channel is a “gold plated” channel for many top quark properties analysis-es. This channel played a crucial role in the top quark discovery.

Advantage of this channel is a large branching ratio (leading to this topology) combined with achievable good signal to background ratio.

Main background consists of the QCD production of $W + jets$. Trouble with this background is a large uncertainty in the theoretical estimate of cross section by perturbative QCD technique. Different (reasonable) choice of renormalization and factorization scale makes up to factor 2 difference. Fortunately this background (and therefore also uncertainty) can be reduced by requirement of at least one b-jet signature in the event. $t\bar{t}$ signal should have two b-jets therefore signal should pass very well b-jet signature requirement, background is dominated by light quark jets and it will be significantly reduced.

We follow below CDF analysis [19] based on integrated luminosity 1.12 fb^{-1} of data. The data were collected with an inclusive lepton trigger that requires electron $E_T > 18 \text{ GeV}/c$ (muon $P_T > 18 \text{ GeV}/c$). Offline selection required isolated electron $E_T > 20$ (muon $P_T > 20 \text{ GeV}/c$), $\cancel{E}_T > 30 \text{ GeV}$ and at least 3 jets with $E_T > 20 \text{ GeV}$. Scalar sum of all transverse energy in event (H_T cut) should exceed 250 GeV. At least one of the jets should be identified as b-jet.

For identification of b-jets (b-tagging) is used SecVtx algorithm [20]. This algorithm for identification of b-jets takes advantage of relatively long lifetime of hadrons with b-quark in their structure (B hadrons). Characteristic $c\tau$ for B hadrons is $\approx 500 \mu\text{m}$. Therefore (taking into account relativistic γ factor for observed mean lifetime in lab. frame) B-hadron travels typically several millimeters from interaction point where it was created until it decays. In critical region around beam pipe there is placed silicon vertex detector with sufficient resolution to find decay vertex close to primary interaction. b-tagging algorithm depends on many details where methodology and detector properties are interconnected but it can be viewed as a black box with two important parameters:

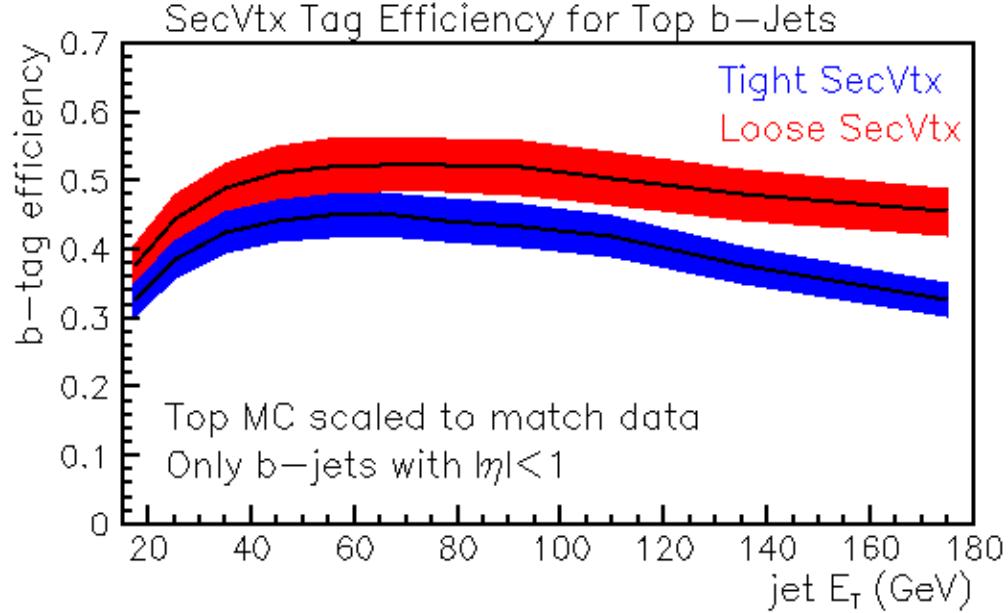


Fig. 11. Tagging efficiency for a “tight” and “loose” version of SecVtx algorithm as a function of E_T of b-jets

- efficiency to identify b-jet if a given jet really was a b-jet
- probability to identify as a b-jet jet which was in fact light quark (or gluon) jet. This property is also called miss-tag probability

As one can imagine these two parameters are correlated. By tuning procedure one can achieve smaller miss-tag probability but price will be smaller tagging efficiency. If efficiency is an issue one can use looser criteria for b-tagging procedure but price will be larger probability to misidentify non b-jet as a b-jet.

There are two versions of SecVtx algorithm at CDF - tight and loose. Properties of these two versions as a function transverse energy of jets (E_T) is shown in Fig.11,12. By a comparison of efficiency for SecVtx tagger on data and MC scale factor $SF = \epsilon_{DATA}/\epsilon_{MC}$ is determined. Scale factor for both cases (loose and tight) was found consistent with $SF \approx 0.95$.

Acceptance was calculated based on PYTHIA MC $t\bar{t}$ production with top mass set to $175 \text{ GeV}/c^2$. Naturally, detector simulation, reconstruction and channel selection procedure have been included in the calculation

Background is dominated by $W + \text{jets}$. This background can be divided into categories of light quark jets (they pass selection criteria due to mis-tag probability) and heavy flavor jets ($W + b\bar{b}$, $W + c\bar{c}$). There is also expected background from non W category (W signature is faked). Origin of this background is detector mismeasurement and it's amount is determined directly from the

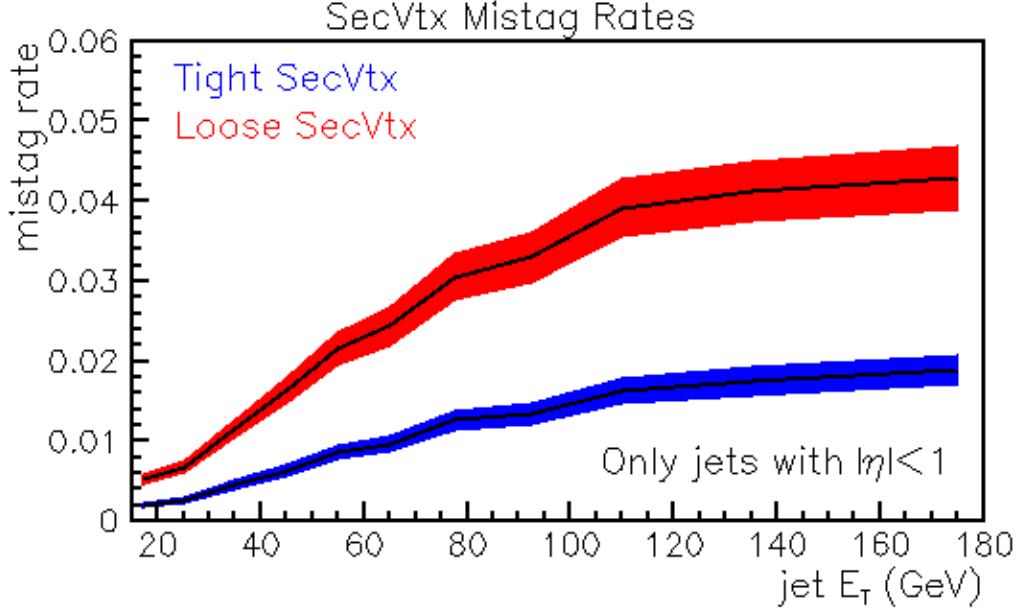


Fig. 12. Miss-tag probability for a “tight” and “loose”version of SecVtx algorithm as a function of E_T of b-jets

data. Last category of the background is di-Boson, single top and $Z \rightarrow \tau\tau$. Amount of this background was estimated by using theoretical cross sections. It was found to be negligible. Systematic errors have in addition to the same components as in the Dilepton channel (see previous section) uncertainties connected with b-tagging procedure. Systematic error is dominated by jet energy scale uncertainty and b-tagging scale factor uncertainty. Total estimated systematic error was determined to be 11.6 %.

Results are summarized in Tab.4. Category of one and two jet is background dominated and demonstrates consistency of background estimate with data.

Cross section determined by eq. (16) is $\sigma_{t\bar{t}} = 8.2 \pm 0.5(\text{stat}) \pm 0.9(\text{syst})$.

Source	Events per 1120 pb^{-1} vs Njet bins				
	1j	2j	3j	4j	$\geq 5j$
Total background	854.04 ± 225.17	427.52 ± 99.85	53.34 ± 13.75	16.69 ± 5.95	4.72 ± 1.74
$t\bar{t}(\sigma = 8.2 \text{ pb})$	9.15 ± 0.93	72.22 ± 7.35	133.68 ± 13.61	153.53 ± 15.63	53.58 ± 5.46
Data	1067	585	185	169	62

Tab. 4. Summary table of background estimates, $t\bar{t}$ predictions and events in 1.12 fb^{-1} of data for each jet bin.

3.1.3 All hadronic channel

Advantage of All hadronic channel is largest branching ratio in comparison of previous channels. This advantage is more than compensated by huge QCD background one needs to deal with. To get background under control and to achieve reasonable signal to background ratio one needs to implement all analysis artillery (kinematic selection, b-tagging). Below we follow analysis [21]. The All hadronic final state is characterized by the presence of at least 6 jets from the decay of top and anti top quarks. Special multi-jet trigger selects in real time candidates relying on calorimeter information. In the offline analysis jets are identified by grouping clusters of energy in calorimeter cells using fixed cone algorithm with a radius $r = \sqrt{(\Delta\phi)^2 + (\Delta\eta)^2} = 0.4$ [22]. Events satisfying the trigger requirements are reconstructed in terms of final state observables (tracks, vertexes, charged leptons, jets). Retained for further analysis are only events that are well contained in detector acceptance (primary vertex should lie inside luminous region $|z| < 60$ cm). Jet energies are corrected for detector response and multiple interaction. Each jet is required to have $E_T \geq 15$ GeV and $\eta \leq 2$. Events with identified isolated electrons or muons are removed from the sample and also events with $\cancel{E}_T / \sqrt{\sum E_T} \geq 3\sqrt{\text{GeV}}$ [24]. At this stage called pre-selection simulation shows that fraction of leptonic event is about 14% of the all accepted $t\bar{t}$ events. To avoid overlaps between jets requirement of minimal distance $\Delta r = \sqrt{(\Delta\phi)^2 + (\Delta\eta)^2} = 0.5$ between jets was applied. Background at this stage is expected to be 3 order of magnitude larger than signal! Finally topology is optimized for all hadronic $t\bar{t}$ signal by requirement of the number of selected jets $6 \leq N_{jet} \leq 8$. In the final sample expected signal is about 0.3% and fraction of leptonic $t\bar{t}$ amounts to 5% accepted $t\bar{t}$ events.

To improve signal to background ratio artificial neural network approach is implemented. Artificial Neural Network (ANN) analysis is a technique which resembles biological neural network decision making procedure. This technique has been successfully used in complex pattern recognition problems. Generally there is defined set of sensitive variables which characterizes object to be recognized, there is a procedure to train ANN to recognize certain type of objects and output of ANN gives a degree of recognition. Usually 1 means ANN for a sure recognized object and 0 means that for sure given object is of different kind than signal for which ANN was trained to recognize. Lot of software packages does exist for ANN analysis. Most popular in High Energy Physics are JETNET [25] and MLP [26].

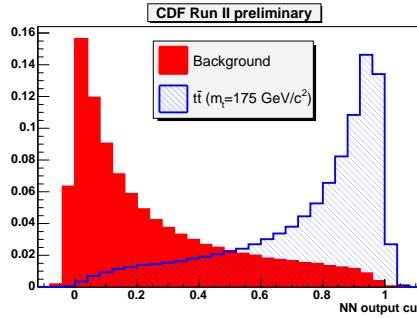
For a given task MLP package [26] as implemented by ROOT [27] was used. Sensitive variables selected for discrimination between signal and background by ANN are listed in Tab.5. The network was trained on the same-size samples of signal and background events with $6 \leq N_{jets} \leq 8$. Signal was modeled by PYTHIA v6.2 [16] followed by a simulation of the CDF II detector. The top quark mass for training was set to 175 GeV/ c^2 . The background is obtained from multi-jet data events themselves since the signal fraction at initial stage is expected to be very small.

Different structures of ANN have been investigated. Best signal to background ratio was obtained by ANN with 2 hidden layers with 20 and 10 hidden nodes and one output node. On Fig.13 one can see output node response distribution for $t\bar{t}$ signal and background. To optimize selection by ANN ratio of signal to standard deviation of signal + background was studied as a function of ANN output node cut. Optimal cut was found to be 0.94. Efficiency of this cut for $t\bar{t}$ signal is close to 5 % and S/B $\sim 1/12$.

Background for $t\bar{t}$ production in all hadronic channel is dominated by QCD heavy flavor pair ($b\bar{b}, c\bar{c}$) and (mistagged) light quark jet production. Theoretically, there are large uncertainties

Variable	Description
$\sum E_T$	Scalar sum of the transverse energies of all jets
$\sum_3 E_T$	As above except the two highest E_T jets
C	Centrality
A	Aplanarity
M_{2j}^{min}	Minimum dijet invariant mass
M_{2j}^{max}	Maximum dijet invariant mass
M_{3j}^{min}	Minimum trijet invariant mass
M_{3j}^{max}	Maximum trijet invariant mass
$E_T^{*,1}$	$E_T \sin^2 \theta^*$ for the highest- E_T jet
$E_T^{*,2}$	$E_T \sin^2 \theta^*$ for the next-to-highest- E_T jet
$\langle E_T^* \rangle$	Geometric mean over the remaining jets

Tab. 5. Input variables to the neural network

Fig. 13. Response of trained ANN to $t\bar{t}$ signal and background

in estimating this background. Background estimate is based on data in signal depleted region. Events with exactly 4 jets passing pre-selection procedure (without ANN) are expected to have $S/B \approx 1/3600$ and therefore satisfy criterion for signal depleted sample. Tag rate per jet is evaluated in this control sample and is parametrized in terms of variables sensitive to both the tagging efficiency for true heavy flavored objects and the rate of mistags (false tags). Based on this parametrization one can predict number of b-tagged jets in other samples (with higher jet multiplicity). Difference between prediction and direct result of b-tagging procedure on jets in a given category will be attributed to a $t\bar{t}$ source.

After kinematic and ANN selection there are left 1020 events with 1233 tags in category of $6 \leq N_{jets} \leq 8$. Estimated background amounts to 937 ± 30 tags. Because this estimate is based on all events passing selection before tagging contribution due to $t\bar{t}$ events should be subtracted. Average number of tags for $t\bar{t}$ events is expected $n_{tag}^{ave} = 0.95 \pm 0.07$. Excess tags are converted to a corresponding number of $t\bar{t}$ events and subtracted from the number of events before tagging and new estimate of background is obtained. After couple iterations the procedure converges. Final number of tags from background sources is reduced to 846 ± 37 tags. Results for different

jet topologies are in Tab.6. Cross section can be extracted by slightly modified eq. (16)

Source	Events per 1020 pb^{-1} vs Njet bins				
	4j	5j	6j	7j	8j
Background	16060 ± 575	2750 ± 92	536 ± 17	255 ± 8	146 ± 5
Corrected Background	15961 ± 677	2653 ± 112	481 ± 20	223 ± 10	142 ± 7
$t\bar{t}(\sigma = 8.3 \text{ pb})$	120 ± 20	266 ± 45	242 ± 41	101 ± 17	38 ± 7
Data	16555	3139	725	349	159

Tab. 6. Observed number of tags, expected background and signal after kinematic and ANN selection

$$\sigma_{t\bar{t}} = \frac{N_{obs} - N_{bkg}}{\epsilon_{kin} \cdot n_{tag}^{ave} \cdot \mathcal{L}_{int}} \quad (17)$$

where $N_{obs} = 1233$ and $N_{bkg} = 846 \pm 37$ are the number of total observed and background tags, respectively, in the signal region $6 \leq N_{jets} \leq 8$, $\epsilon_{kin} = 4.8 \pm 0.8\%$ is the signal kinematic selection efficiency, $n_{tag}^{ave} = 0.95 \pm 0.07$ is average number of tags in $t\bar{t}$ events and $\mathcal{L}_{int} = 1.02 \pm 0.06 \text{ fb}^{-1}$ is the integrated luminosity of the data sample. The $t\bar{t}$ cross section was measured to be $\sigma_{t\bar{t}} = 8.3 \pm 1.0(\text{stat})^{+2}_{-1.5}(\text{syst.}) \pm 0.5(\text{lumi.}) \text{ pb}$ for a top quark mass $175 \text{ GeV}/c^2$.

Relative systematic uncertainties affecting $t\bar{t}$ production cross section dominates jet energy scale uncertainty and uncertainty on average number of tags.

3.1.4 Summary

CDF collaboration cross section measurement in dilepton, lepton + jets and all hadronic channel was briefly introduced. There are more details in referenced papers. There are also other analysis produced by CDF collaboration as well as D0 collaboration [28]. CDF collaboration results are summarized in Fig.14 and D0's results in Fig.15. As one can see from above plots cross section measurements are consistent between each other for different channels and with theoretical NLO prediction for assumed top mass $175 \text{ GeV}/c^2$ [29]. There is also mutual consistency of results between CDF and D0 collaboration.

One may rise a question if these measurements exclude other interpretation of experimental findings than top quark as expected by Standard Model. Standard Model is considered a very successful theoretical concept but there are limitations. As a possible successor of Standard Model is so called Minimal Super-symmetric extension of Standard Model (MSSM). MSSM predicts existence of large number of new particles (to every particle in SM should exist supersymmetric partner). One possibility why so far there does not exist positive observation of any supersymmetric particle is that mass of supersymmetric particles is too large to be observed at TEVATRON. Other possibilities are that they buried in background because of low statistics (integrated luminosity), buried in $t\bar{t}$ candidate events if properties are close to properties of $t\bar{t}$ events or even interpretation of $t\bar{t}$ events can be challenged. At right conditions supersymmetric partners to top quark have properties similar to top. As was emphasized in [31] in framework of MSSM mass of scalar top \tilde{t}_1 from a theoretical point of view can be made arbitrary low. In this framework one can imagine pair production $\tilde{t}_1\tilde{t}_1$ and subsequent decay of $\tilde{t} \rightarrow \chi^+ + b$ and $\chi^+ \rightarrow W^+\chi^0$. In this case final state will be the same as in case of SM $t\bar{t}$ production. But

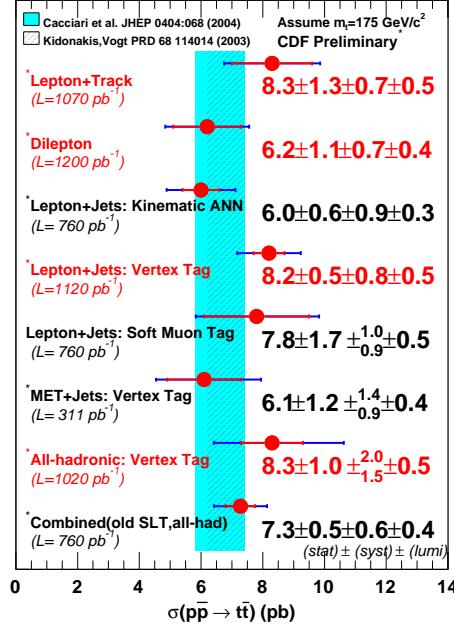


Fig. 14. Measurement of $t\bar{t}$ production cross section by CDF collaboration

theoretically cross section for $p\bar{p} \rightarrow \tilde{t}_1 \tilde{t}_1$ at 1.96 TeV is order of magnitude smaller than cross section for $t\bar{t}$ production for the same mass (e.g 175 GeV/c^2) of produced particle. Therefore measurement of $t\bar{t}$ cross section (consistent with SM expectation) indirectly rejects $\tilde{t}_1 \tilde{t}_1$ interpretation of observed events. Naturally question if in addition to $t\bar{t}$ signal there is also admixture of $\tilde{t}_1 \tilde{t}_1$ events can be answered only by detailed analysis of large statistic data.

There is also theoretical concept which claims that observed top is in fact particle with same properties as top except charge [32]. This particle decays $X \rightarrow W^- + b$. Because other properties are very close (same) just measurement of the $t\bar{t}$ cross section cannot exclude this possibility. Top charge measurement or single top cross section measurement can rule out (or support) this exotic hypothesis!

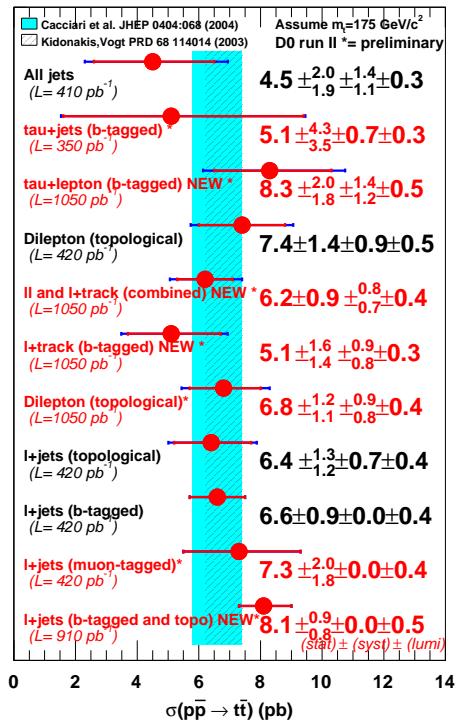


Fig. 15. Measurement of $t\bar{t}$ production cross section by D0 collaboration

3.2 Top quark mass measurements

A nontrivial question is even from a theoretical point of view definition of top quark mass we want to measure. The quarks have not been observed directly in the nature as a free particles, quark mass is not observable quantity therefore notion of quark mass relies on a theoretical construction.

There are (at least) two frequently used definitions - pole mass and $\overline{\text{MS}}$ mass.

Pole mass is defined as a position of a pole in quark propagator in perturbative QCD. To measure it one needs to be able to measure four momentum of the quark. Because quarks cannot be observed directly there is intrinsic limit on precision to which even in principle quark four momentum can be reconstructed. Light quarks hadronize into jets. Hadronization is nonperturbative procedure by which colored object quark is converted into jet of colorless hadrons. There is an inherent ambiguity of this procedure. Even in principle is not possible to reconstruct exactly kinematics characteristics of quarks from hadrons of jets. There is at least uncertainty on level of characteristic QCD scale (Λ_{QCD}). Point is that to produce colorless final object from colored object there should be picked up at least one quark unrelated to original quark to compensate it's color.

Other frequently quoted definition is $\overline{\text{MS}}$ mass.

QCD belongs to re-normalizable theories that means that divergent terms which appear at higher orders of perturbative expansion can be absorbed in such a way that physical (renormalized) quantities remain finite. There are several renormalization schemes which should be equivalent in limit of infinite expansion but in terms of finite perturbation order there can be advantage of using specific scheme because of faster convergence. One of them is $\overline{\text{MS}}$ scheme. Relationship between pole (M) and $\overline{\text{MS}}$ mass (\bar{m}) at a given energy scale \bar{m} is expressed by:

$$M = \bar{m}(\bar{m}) \left[1 + \frac{4}{3\pi} \bar{\alpha}_s(\bar{m}) + \dots \right] + O(\Lambda_{QCD})$$

where $\bar{\alpha}_s(\bar{m})$ is the strong interaction coupling constant in $\overline{\text{MS}}$ scheme at energy scale \bar{m} and $O(\Lambda_{QCD})$ reflects inherent ambiguity of a pole mass on the level of Λ_{QCD} . In paper [34] is claimed that $\overline{\text{MS}}$ mass is free of above ambiguity therefore this quantity can be more precisely determined!

All methods of the top quark mass determination used in TEVATRON experiments determine top quark pole mass. They generally rely on the three assumptions:

- signal is SM $t\bar{t}$ and it's amount was independently determined
- background is known QCD background and it's amount was independently determined
- there is nothing else in selected $t\bar{t}$ candidate samples except previous two items

Many methods have been developed and used for top quark determination in TEVATRON experiments. They can be divided in two categories:

- methods which rely on use of specific variable sensitive to top mass
- methods which rely on solution to kinematic equations for reconstruction of top mass

Methods based on single sensitive variables

As a consequence of a large mass of top quark and subsequent cascade decay one can find several variables sensitive to top quark mass. Most popular one are plotted in Fig. 16. Distributions in this figure have been produced by MC generator PYTHIA [16]. Reaction $p\bar{p} \rightarrow t\bar{t} + X$ at 1.96 TeV was simulated with subsequent decay $t \rightarrow Wb$. In this specific case W was forced to decay to electron or muon and corresponding neutrino. In simulation top quark mass was set to 160 (distributions in red color) or 180 GeV/c^2 (distributions in blue color). Range 160 - 180 GeV/c^2 represents very conservative range where according to present knowledge mass of top quark should be. One can see from Fig. 16 that P_T of leptons is less sensitive to top mass among

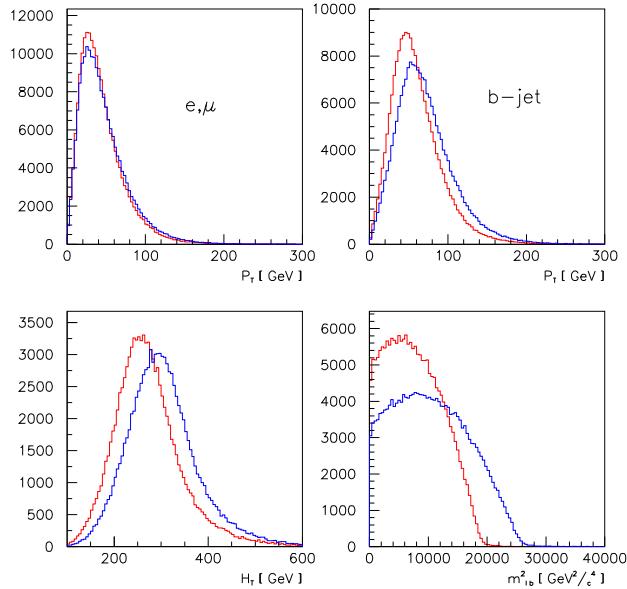


Fig. 16. Distribution of several variables sensitive to top mass. P_T of e, μ from top decay chain (W), P_T of b-jets, H_T and quadrature of effective mass of lepton and b-jet m_{lb}^2 . In red color distributions represent top mass $160 \text{ GeV}/c^2$, in blue top mass $180 \text{ GeV}/c^2$

all other variables displayed. But experimentally, isolated leptons from W decay are very well measured. Lower resolution can be compensated by a larger statistic and there can be advantage of lower systematic error by using this variable. But according to the first results this is not the case [35].

P_T distribution of b-jets shows much better sensitivity. But this distribution represents ideal case and great deal of sensitivity is lost when detector resolution is taken into account. Even better sensitivity is seen for H_T (already several times mentioned variable in this paper) which

is defined here as total transverse energy in $t\bar{t}$ production.

$$H_T = E_T^{b_1} + E_T^{b_2} + E_T^{l_1} + E_T^{l_2} + E_T^{\nu_1 + \nu_2} \quad (18)$$

The most sensitive variable is effective mass of lepton and b-jet squared (m_{lb}^2). Distribution of this variable in limit where we neglect lepton and b-jet masses can be expressed analytically.

$$\frac{dN}{dm_{lb}^2} = \frac{6(M_t^2 - M_w^2 - m_{lb}^2)(M_w^2 + m_{lb}^2)}{M_t^6 - 3M_t^2 M_w^4 + 2M_w^6} \quad (19)$$

This distribution is easily derived based on cascade decay kinematics and SM prediction of $\cos(\theta_l)$ distribution in W rest frame.

From this distribution one can derive relationship between top mass and average effective mass squared of lepton and b-jet ($\langle m_{lb}^2 \rangle$).

$$M_t = \sqrt{\langle m_{lb}^2 \rangle + \sqrt{\langle m_{lb}^2 \rangle^2 + 4 \langle m_{lb}^2 \rangle M_w^2 + M_w^4}} \quad (20)$$

Unfortunately all these relationships work for a “true” effective mass squared of lepton and b-jet where both particles come from decay of the same top quark. In reality in $t\bar{t}$ production in a most simple case there are two leptons and two b-jets in a final state and there is twofold ambiguity how to combine them. There are several options how to pick up correct pairs with reasonable probability. In more detail this problem will be discussed in top charge section of this article. Top mass determination by using m_{lb}^2 variable has at least in principle a potential for a method which does not rely on templates.

All above variables have been used for top mass determination in the past. Results and details can be found in [36], [37]. Advantage of methods based on above variables is relative simplicity, disadvantage is poorer resolution in comparison with more sophisticated methods which will be discussed later.

There was recently proposed a very interesting new method [38] to determine top quark mass by a measurement of distribution of transverse distance between vertex from B hadron decay and primary vertex. This method exploits the fact that at TEVATRON t and \bar{t} are produced almost at rest. In this case boost of b - quark in top decay $\gamma_b \approx 0.4M_t/m_b$ is proportional to the top mass and therefore also lifetime of B-hadrons and distance traveled by them before they decay. Interesting feature of this method is that for a measurement of top mass is sufficient precise measurement of secondary vertexes by silicon vertex detector and there is not needed directly calorimetry (which introduces largest systematic error into measurement by an uncertainty in the jet energy scale). Sensitivity of this method can be judged from Fig. 17. Top mass determined by this method has been published in [39]

All the above methods are based on kinematics of top quark decay. Distributions of sensitive variables are compared with templates produced by MC generator combined with detector simulation and reconstruction package.

There is also a method of top mass determination based on dynamics of $t\bar{t}$ production. If Standard Model is correct one can translate cross section measurement to top mass measurement. There is an exponential dependence of $t\bar{t}$ cross section on top mass (every 20 GeV/ c^2 increase of mass of top means smaller cross section approximately by factor 2). Just by counting $t\bar{t}$ candidates we can measure top quark mass! More details in combination with kinematic reconstruction will be given later.

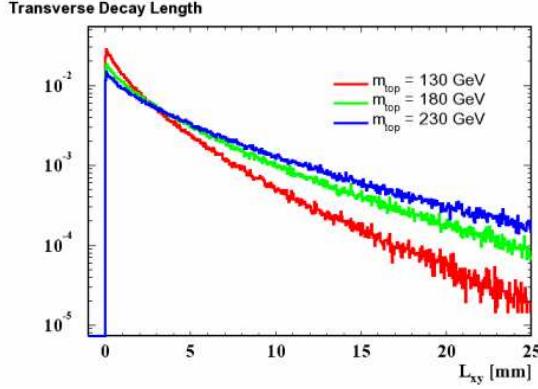


Fig. 17. Distribution of transverse decay length for top mass 130,180,230 GeV/c^2

Methods based on kinematic reconstruction

Classical way to reconstruct mass of unstable particle is through measurements of kinematic characteristics of all decay products and then to calculate effective mass which will be equal to mass of original unstable particle. In $t \rightarrow Wb$ decay there is at least one quark among decay products which cannot be directly measured. There are variety of methods developed for this category of top mass reconstruction. We have chosen 3 methods which will be described in more details. Selected methods have been developed primarily for top mass reconstruction in a specific channel.

3.2.1 Dilepton channel

Dilepton channel has one disadvantage in comparison with lept.+ jets or all hadronic channel. There is not enough kinematic constraints to reconstruct even in principle top mass on event by event basis. As one can easily check there are 22 independent variables constraint by 21 kinematic equations! Therefore kinematic reconstruction looks impossible. There is a way around. Assuming reaction $p\bar{p} \rightarrow t\bar{t} + X \rightarrow W^+ b W^- \bar{b} + X \rightarrow l^+ \nu b l^- \bar{\nu} \bar{b} + X$ following constraints can be used for a kinematics reconstruction.

$$\begin{aligned}
 M_t &= M_{\bar{t}} \\
 M_W &= 80.4 \\
 \vec{P}_b + \vec{P}_{W^+} &= \vec{P}_t \\
 \vec{P}_{\bar{b}} + \vec{P}_{W^-} &= \vec{P}_{\bar{t}} \\
 \vec{P}_{l^+} + \vec{P}_{\nu} &= \vec{P}_{W^+} \\
 \vec{P}_{l^-} + \vec{P}_{\bar{\nu}} &= \vec{P}_{W^-} \\
 P_{\nu_{1x}} + P_{\nu_{2x}} &= \not{P}_{T_x} \\
 P_{\nu_{1y}} + P_{\nu_{2y}} &= \not{P}_{T_y} \\
 P_{tz} + P_{\bar{t}z} &= P_{t\bar{t}z}
 \end{aligned}$$

Set of equations represents system with equal number of equations and independent variables. Therefore this set of equations can be solved and as a result top mass will be determined. One can measure kinematic characteristics of leptons, b-jets, x and y components of \cancel{E}_T . But one is not able to measure z component of \cancel{E}_T . Therefore one is not able from experiment supply right hand side of last equation. This equation is added there to make a system solvable but question how to get needed information is for a moment overlooked.

Another problem one needs to address is problem of measurement errors. Measured quantities are subjects to experimental errors. To take into account measurement errors it is adopted procedure when all measured quantities are smeared around measured values by amount of expected error. In this approach also last equation in a set can be inserted naturally. It is set $P_{t\bar{t}z} = 0$ with associated error corresponding to width of theoretical $P_{t\bar{t}z}$ distribution (to a very good accuracy Gaussian). Choice of $P_{t\bar{t}z}$ as a “theoretical” input is justified by very weak dependence of this distribution on top mass (in 0 order approximation it depends just on structure functions of quarks and gluons in proton (anti-proton)). Idea of this method was published in [40]. Application to data in [41].

Above set of equations can be reduced to a 4-th order polynomial equation. That means that

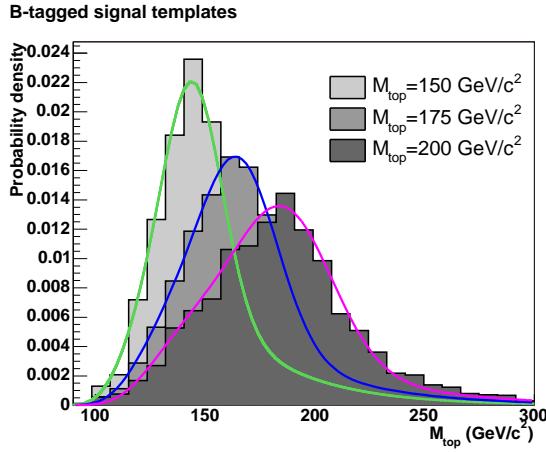


Fig. 18. Distribution of raw top mass for templates for b-tagged templates for top mass 150, 175 and 200 GeV/c^2

there can be up to 4 real solutions. Most probable situation are two solutions. There is also twofold ambiguity of pairing leptons and b-jets corresponding to top decay. Therefore there are up to 8 solutions. One can deal with multiple solutions by assigning appropriate weights. In [40] was taken other approach. There was set conditions applied and finally just one solution was selected. Still because of a smearing procedure for each event there will be a distribution of reconstructed (raw) top masses. As a representative top mass for event is picked up most probable value from distribution. There are produced MC templates for range of top mass parameters same way as was described distribution of most probable raw top masses. Distribution of reconstructed (raw) top masses from data is compared with templates and by likelihood fit is determined most prob-

able value of top mass parameter from MC which corresponds to data.

Most recent result by using this method was done for integrated luminosity 1.2 fb^{-1} [42]. Event selection is the same as in case of cross section analysis in dilepton channel (see section 3.1.1). Selected events have been divided into 2 sub-samples – events with at least one jet b-tagged and events where none of the jets have been b-tagged. b-tagged sample is practically background free. Fig. 18 shows distributions of raw top mass templates based on PYTHIA MC [16] for original top mass parameter set to 150, 175 and 200 GeV/c^2 . There are also superimposed curves from parametrization fit. These distributions represent b-tagged sample but for non b-tagged sample look very similar.

Fig. 19 shows the background distribution of raw top mass when the same procedure has been applied. When amount of signal and background is known (e.g. from cross section analysis)

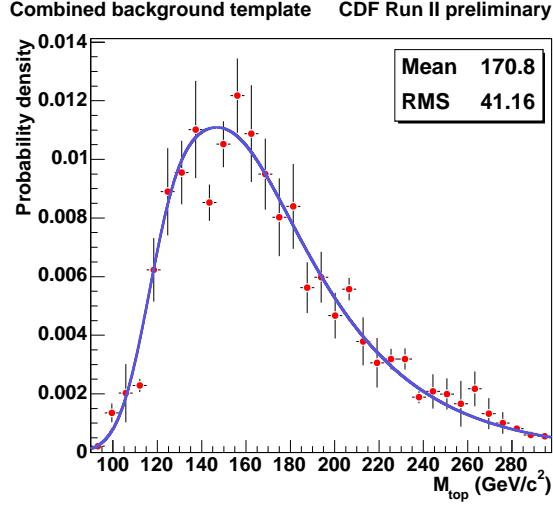


Fig. 19. Distribution of raw top mass for background treated as $t\bar{t}$ signal

and shape of raw top mass templates is parametrized (as well as background) top mass will be determined from data by likelihood fit. Definition of likelihood function is given below:

$$\mathcal{L} \equiv \mathcal{L}_{b\text{-tagged}} \times \mathcal{L}_{\text{non-tagged}} \quad (21)$$

$$\mathcal{L}_{\text{sub-sample}} \equiv \mathcal{L}_{\text{shape}} \times \mathcal{L}_{\text{nev}} \times \mathcal{L}_{\text{bg}} \quad (22)$$

$$\mathcal{L}_{\text{shape}} \equiv \prod_{i=1}^n \frac{n_s \times f_s(m_{t_i}^{\text{rec}}, m_t^{\text{orig}}) + n_b \times f_b(m_{t_i}^{\text{rec}})}{n_s + n_b} \quad (23)$$

$$\mathcal{L}_{\text{nev}} \equiv \frac{e^{-(n_s+n_b)} (n_s + n_b)^N}{N!} \quad (24)$$

$$-\ln(\mathcal{L}_{\text{bg}}) \equiv \frac{(n_b - n_b^{\text{exp}})^2}{2\sigma_{n_b}^2} \quad (25)$$

Where top mass m_t^{orig} , number of background events n_b and number of signal events n_s are

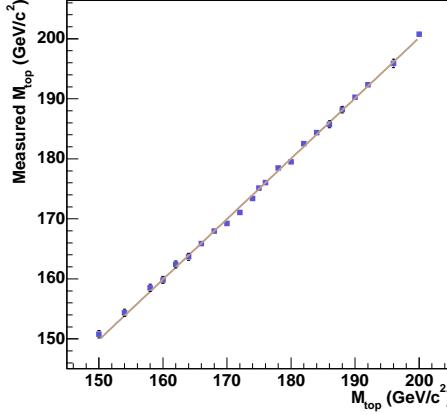


Fig. 20. Test of the top mass reconstruction procedure by pseudo-experiments.

fit parameters. The number of background events is constrained by Gaussian term \mathcal{L}_{bg} and total number of events by Poisson term \mathcal{L}_{new} , $f_s(m_{t_i}^{rec}, m_t^{orig})$ is a parametrization of signal as function of m_t^{orig} and $f_b(m_{t_i}^{rec})$ parametrization of background. Fit parameters are determined by maximum likelihood method.

Procedure was tested by pseudo-experiments where randomly picked up events from a pool of

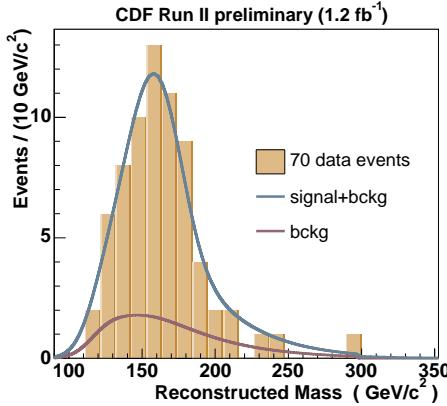


Fig. 21. Top mass fit to data

MC signal events for a given mass are combined with randomly picked up events from a pool of background and form set for which top mass is determined by a given procedure. As one can see

from Fig. 20 at least in the range of top masses between 160 - 200 GeV/c^2 original top mass and reconstructed one from pseudo-experiments (average) correspond well to each other.

Finally method is applied to data. In Fig. 21 distribution of raw top mass and fit is displayed. Top mass was determined $M_{\text{top}} = 169.7^{+5.2}_{-4.7}(\text{stat}) \text{ GeV}/c^2$.

Analysis of systematic errors determined as the most significant source uncertainty in jet energy scale contributing by 2.9 GeV/c^2 . Total systematic error was estimated 3.1 GeV/c^2 . Final result is $M_{\text{top}} = 169.7^{+5.2}_{-4.7}(\text{stat}) \pm 3.1(\text{syst})$

Combination with cross section mass dependence

By a small modification of likelihood function (22) also top mass dependence of cross section can be included in a fit and top mass resolution improved. The expected number of signal events can be expressed as:

$$n_s(M_{\text{top}}) = \sigma_{t\bar{t}}(M_{\text{top}}) \cdot a(M_{\text{top}}) \cdot \mathcal{L}^{\text{int}} \cdot p_{\text{mass}}^{\text{rec}} \quad (26)$$

where $\sigma_{t\bar{t}}(M_{\text{top}})$ is theoretical dependence of $t\bar{t}$ cross section as a function of top quark mass, $a(M_{\text{top}})$ is acceptance, \mathcal{L}^{int} is an integrated luminosity and $p_{\text{mass}}^{\text{rec}}$ probability to reconstruct raw top mass. Fitted parameters after this modification of likelihood function will be M_{top} and n_b . This way about 20 % top mass resolution improvement can be achieved.

Reconstruction of top quark mass by kinematic reconstruction (and templates) was done in previous section. One can also estimate top mass just by combination of theoretical prediction of top mass cross section dependence and cross section measurement [43] and compare these two measurements if they are compatible. In Fig. 22 extraction of top mass from top mass dependent theoretical cross section and from top mass dependent (acceptance) measured cross section is demonstrated. Result is $M_{\text{top}} = 178.3^{+10.1}_{-8.0}(\text{stat.})^{+4.0}_{-6.0}(\text{syst.}) \text{ GeV}/c^2$. It is important to note

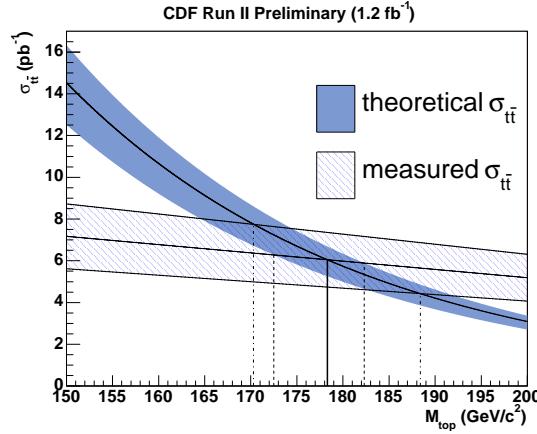


Fig. 22. Extraction of top mass from top mass dependence of theoretical cross section and top mass dependence of measured cross section

that two methodically very different approaches to top mass determination give a compatible result!

Results by using combination of top mass cross section dependence and kinematics based on modification of likelihood function (22) by (26) have been published in [44]. Reconstructed top mass was found (see Fig. 23) $M_{\text{top}} = 170.7^{+4.2}_{-3.9}(\text{stat}) \pm 2.6(\text{syst}) \pm 2.4(\text{theor})$. Improvement in statistical error was expected. Improvement in systematic error is because cross section constrained kinematic reconstruction of top mass is less sensitive to uncertainty in jet energy scale. It is still dominant source of systematic error but its estimated value is $1.8 \text{ GeV}/c^2$ (to be compared with $2.9 \text{ GeV}/c^2$ in case of traditional kinematics approach). Second most significant contribution to systematic error is uncertainty on estimate of integrated luminosity. Systematic uncertainty on top mass from this source was found to be $1.1 \text{ GeV}/c^2$.

Delicate question is treatment of uncertainty on calculated theoretical cross section. A conservative approach was adopted (after discussion with authors [29]) and systematic error from this source is written down separately.

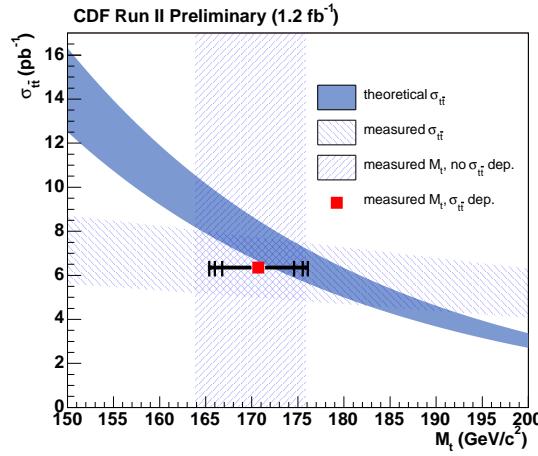


Fig. 23. Top mass measurements by combination of kinematics and cross section measurement. The blue solid bands shows theoretical $t\bar{t}$ cross section within its uncertainties. Hatched areas mark the cross section measurement as a function of top mass and independent top mass measurement based on kinematics and templates (see Fig. 21). Red square with error bars mark the measured top mass by combined cross section and kinematics method (see text).

3.2.2 Lepton + jets channel

As was already mentioned lepton + jets channel is a gold plated channel for most top properties analysis. In comparison with dilepton channel advantage for top mass analysis is not only higher statistics but also number of kinematic constrains is larger than number of independent variables. Therefore system is over-constrained. Specifically for 21 kinematic equations (constraints) there are 19 independent variables (if we use same conditions as in case of dilepton channel). Disad-

vantage is increased level of ambiguity for association of jets with correct decay chain item. In lepton + jets channel for top mass reconstruction there are selected events with isolated lepton and 4 jets (for cross section analysis are required at least 3 jets only). From these 4 jets there are 12 different combinations for assignment of these jets to specific items of top decay chain (b-jet combined with W leptonically or hadronically decaying, jets associated with hadronic decay of W). In case when one of the b-jets is b-tagged and its assignment will be strictly assigned to either hadronic or leptonic W (not to W decay) number of ambiguities is reduced to 6. In case when two b-jets are b-tagged, not tagged jets are assigned to W hadronic decay and there is still twofold ambiguity of assignment of b-tagged jets to leptonic or hadronic decaying W.

We follow CDF analysis published in [45] and more details can be found there. In this analysis two major improvements are implemented to improve resolution. For top mass reconstruction are used not only kinematic constraints but also top mass dependent dynamical information based on LO matrix element for $t\bar{t}$ production and for W + jets background. Second improvement concentrates on improving most significant systematic uncertainty - jet energy scale. W bosons which decays hadronically into two jets is used for “in situ” calibration of jet energy scale. Fitted parameters in this case will be M_{top} and jet energy scale.

Selection requires one isolated lepton with $E_T > 20$ GeV (e), $P_T > 20$ GeV/c (μ), exactly 4 jets with $E_T > 15$ GeV and pseudo-rapidity $|\eta| < 2.0$, at least one of the jets should be b-tagged, $E_T > 20$ GeV and for $\Delta\phi < 30$ GeV azimuthal angle between leading jet and E_T should be inside $0.5 < \Delta\phi < 2.5$. The last condition reduces non W background while retaining most of the signal. Fig. 24 shows dependence of acceptance as a function of top mass and jet energy scale (JES).

In Tab.7 expected contribution to the signal and background for 955 pb^{-1} sample is compared

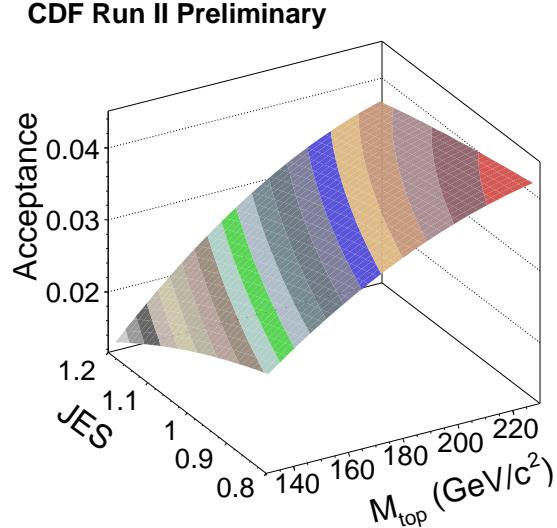


Fig. 24. Acceptance dependence as a function of JES and top mass

with data. Core of the method is definition of likelihood function. Likelihood function was de-

Source	Expected number of events
W + jets	14.5 ± 5.1
non-W	5.2 ± 2.6
EWK	2.2 ± 0.5
Total	22 ± 8.2
$t\bar{t}$ ($\sigma = 8.0 \text{ pb}$, $M_{\text{top}} = 170 \text{ GeV}/c^2$)	145.1 ± 16.5

Tab. 7. Signal, background and data passing selection criteria (integrated luminosity 955 pb^{-1})

fined:

$$\mathcal{L}(M_{\text{top}}, \text{JES}, C_s; \vec{x}) \propto \prod_{i=1}^N [C_s P_{t\bar{t}}(\vec{x}; M_{\text{top}}, \text{JES}) + (1 - C_s) P_{W+\text{jets}}(\vec{x}; \text{JES})] \quad (27)$$

Probability density for $t\bar{t}$ signal and dominant background (W+jets) is based on calculation of elementary processes cross section on parton level $\frac{d\sigma(y)}{dy}$ convoluted over parton distribution functions PDF's and detector resolution for observed variables x . It is claimed that dominant background adequately represents also non W+jets backgrounds.

$$P(\vec{x}) = \frac{1}{\sigma} \sum_{\text{perm jets}} \int \frac{d\sigma(\vec{y})}{dy} f(\tilde{q}_1) f(\tilde{q}_2) W(\vec{x}, \vec{y}) d\tilde{q}_1 d\tilde{q}_2 dy \quad (28)$$

$W(\vec{x}, \vec{y})$ describes detector resolution effects. It represents normalized probability of observing jet with E^{jet} when a parton with E^{parton} was produced. It is assumed that there is no correlation between different partons/jets. $W(\vec{x}, \vec{y})$ was determined by parameterizing the jet response in

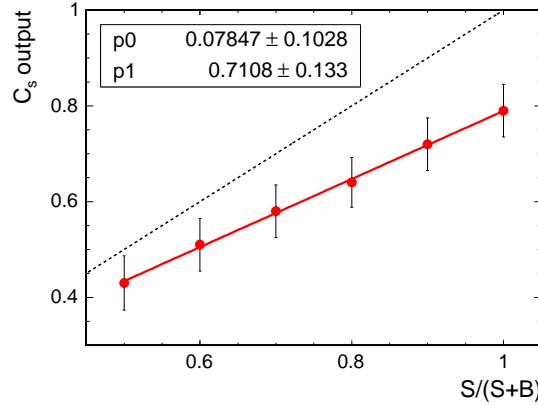


Fig. 25. Dependence of fitted output signal fraction C_s as a function of input signal fraction $S/(S+B)$ and a comparison with an ideal case

fully simulated $t\bar{t}$ created by MC [16] including detector simulation and resolution. PDFs $f(\tilde{q}_i)$ take into account the flavors of colliding quark and anti-quark and are given by CTEQ5L [48]. The final state described by $\frac{d\sigma(y)}{dy}$ contains 6 particles which introduces 20 integration variables. This number is reduced to 5 by imposing energy momentum conservation and also Dirac delta functions inside $W(\vec{x}, \vec{y})$.

Sum over all jet permutations means in case of one b-tagged jet 6 permutations of different jet assignments to proper items in top decay chain and in case of two b-tags (for two different jets) 2 permutations.

Jet energy scale is defined as ratio of observed jet energy to true jet energy $JES = E_{jet}^{obs}/E_{jet}$ and probability density $P(\vec{x})$ is evaluated as a function of assumed jet energy scale. It is also assumed that same jet energy scale factor is also valid not only for light quark jets but also for b-jets. Deviation from this assumption is treated as a systematic error.

C_s represents fraction of signal. One can use eq.(27) to fit C_s as a function of input signal fraction (S/(S+B)). From result in Fig. 25 for top mass 172.5 and JES=1 one can see that even for case of 100% signal fraction from fit is close to 0.8. This apparent discrepancy is explained by fraction of jets in signal which cannot be matched to partons. In this case given procedure treats them as background. On subsample of signal when all jets are matched to partons fitted signal fraction is 1. Linearity check of reconstructed (output) top mass as a function of input top

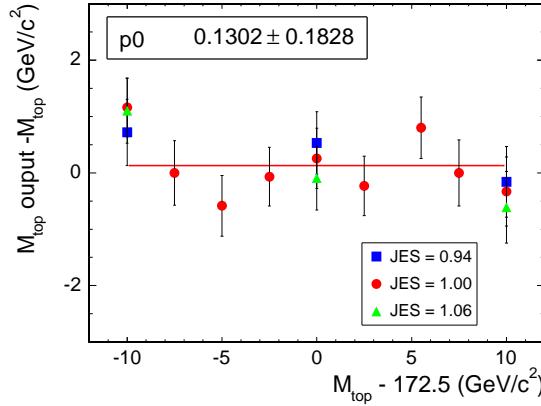


Fig. 26. Linearity check of reconstructed top mass as a function of input top mass for 83% of signal and 17% of background. Input JES was fixed at 0.94, 1.0 and 1.06 values

mass for fixed input jet energy scale from 0.94 to 1.06 is presented in Fig. 26 for 83% signal and 17% background. Within statistical errors reconstructed top mass in a given range is unbiased. When given procedure is applied to data result is $M_{top} = 170.8 \pm 2.2(\text{stat.}) \pm 1.4(\text{syst.}) \text{ GeV}/c^2$. Here statistical uncertainty includes uncertainty $1.5 \text{ GeV}/c^2$ due to jet energy scale. Systematic errors are dominated by uncertainty from initial and final state radiation which was estimated to be $1.1 \text{ GeV}/c^2$.

3.2.3 All hadronic channel

From the point of view of ratio of number of kinematic constraints (19) to number of independent variables (16) all hadronic channel looks suited best for top mass reconstruction. But large background and also large level of ambiguity of association of the jets with correct decay chain items have risen a question if at all can be top quark mass reconstructed in this channel. For 6 jets in this channel there is 90-fold ambiguity for assignment of these jets to proper items in top decay chain, in case when one jet is b-tagged 30-fold ambiguity remains and with 2 b-tagged jets still 6-fold ambiguity persists.

We follow method published first in [49] where results on statistic of 310 pb^{-1} from CDF experiment are presented. The method is based on event by event likelihood and uses all 90 possible jet permutations. By likelihood fit are determined two parameters - top mass M_{top} and signal fraction (called also sample purity) \mathcal{P} .

Event selection is based on multi-jet trigger which relies solely on calorimetry. It requires at least 4 clusters with transverse energy $E_T > 15 \text{ GeV}$ and scalar sum of transverse jet energies exceeds 125 GeV . Offline there are imposed kinematic requirements based on scalar sum of transverse jet energies and event shape observables. Full description is given in [56]. For the final top quark mass measurement are considered only events with exactly six jets, each with $E_T > 15 \text{ GeV}$ and pseudo-rapidity $|\eta| \leq 2$. Jets are identified as clusters of energy deposits in the calorimeter segments (towers) which fall within a cone radius $r = \sqrt{\Delta\phi^2 + \Delta\eta^2} = 0.4$. The jet energies are calibrated based on instrumental calibration analysis of the data control sample [57]. There are applied additional jet-pattern corrections, specific to $t\bar{t}$ which are parametrized independently for b-quark jets and light quark jets. In order to reduce background it is imposed requirement that at least one jet should be b-tagged. But this information is not used to reduce ambiguity of the jet assignments. Instead weight factor for each jet is defined using CDF jet probability algorithm [63] which takes into account probability that jet originates from the b-quark.

By applying energy momentum conservation momenta of t and \bar{t} are derived, the two top quark masses $m_i^{1,2}$ and their estimated uncertainties $\sigma_i^{1,2}$ determined. In kinematic fit constraint on W mass $80.4 \text{ GeV}/c^2$ and natural width are implemented. All 90 possible permutations of jets assignments are applied, equality of $m_i^{1,2}$ is not imposed. Weight factor is defined:

$$w_i = \exp\left(-\frac{1}{2}\chi^2\right) \prod_{j=1}^2 p_j^b \prod_{j=3}^6 p_j^q \quad (29)$$

where exponential term is a measure of compatibility of a given jet combination with $t\bar{t}$ kinematics, second factor determines probability that b-jet assignment is correct and the last term probability that 4 jets assigned to W decay into light quarks are consistent with this assignment. Likelihood consists of two terms. Signal likelihood is a convolution of two Breit-Wigner distributions $F_{BW}(m'_j|M_{\text{top}})$ with two Gaussians $G(m'_j|m_i^j, \sigma_i^j)$ which describes the experimental resolution. $\sigma_i^{1,2}$ is uncertainty in reconstructed top quark masses $m_i^{1,2}$.

$$\mathcal{L}_i^{sig}(M_{\text{top}}) = \prod_{j=1}^2 \int G(m'_j|m_i^j, \sigma_i^j) F_{BW}(m'_j|M_{\text{top}}) dm'_j \quad (30)$$

The M_{top} independent likelihood term \mathcal{L}_i^{bg} corresponds to the two-dimensional posteriori probability density function of $m_i^{1,2}$ obtained from ALPGEN [47] MC multi-jet QCD background. All MC events pass through detector simulation, reconstruction and event selection procedure same as data.

Likelihood for a given event j and total likelihood are defined:

$$\mathcal{L}^j(M_{top}, \mathcal{P}) = \sum_{i=1}^{90} w_i [\mathcal{P} \mathcal{L}_i^{sig}(M_{top}) + (1 - \mathcal{P}) \mathcal{L}_i^{bg}] \quad (31)$$

$$\mathcal{L}_{tot}(M_{top}, \mathcal{P}) = \prod_{j=1}^{N_{ev}} \mathcal{L}^j(M_{top}, \mathcal{P}) \quad (32)$$

Top mass extracted from likelihood fit will be biased due to presence of wrong jet combinations, background events, jets assigned from initial and final state radiation. In Fig. 27 a comparison of fitted mass for pseudo-experiment samples as a function of input mass and background fraction is presented. Calibration curves are used for a final determination of top mass. $Pull = \frac{M_{top} - m_{gen}}{\sigma_{res}}$

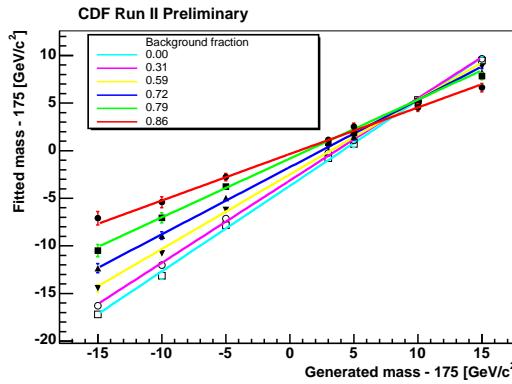


Fig. 27. Calibration curves for dependence of fitted top mass as a function of input top mass for different background fractions

is very sensitive variable to correct top mass reconstruction. If procedure is correct, mean of pull distribution should be equal to 0 (if reconstructed top mass M_{top} is unbiased) and have width 1 (if σ_{res} correctly represents uncertainty on reconstructed top mass). These properties should be constant as a function of input top mass (m_{gen}). As can be seen from Fig. 28 constancy condition is honored. But width is wider by about 17% and therefore for final determination of uncertainty on top mass estimated uncertainty is appropriately inflated. When 290 events of data have been analyzed by above procedure purity was found $\mathcal{P} = 0.21 \pm 0.07$ and top mass $M_{top} = 177.1 \pm 4.9(stat)$ for this sample. From systematic errors the most significant is jet energy scale uncertainty. It amounts $4.4 \text{ GeV}/c^2$. The total systematic error was estimated to be $4.7 \text{ GeV}/c^2$

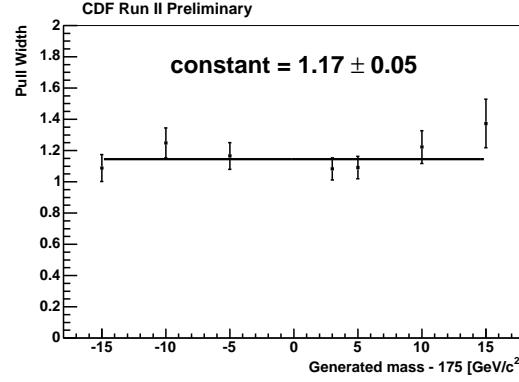


Fig. 28. Pull width dependence as a function of input top mass

3.2.4 Summary

All methods for top mass reconstruction rely on an assumption of known background and $t\bar{t}$ signal. These assumptions are tested in details and there was not found any discrepancy which could undermine this assumption.

Statistical error $\sigma_{M_{top}}$ can be roughly estimated as

$$\sigma_{M_{top}} \approx \frac{\sigma_{res}}{\sqrt{N_{sig}}}$$

where σ_{res} is intrinsic top mass resolution of the method and N_{sig} is number of signal events. Rough estimate $\sigma_{res} \approx 30$ gives a quick guess what top mass resolution can be achieved for a given number of signal events.

Presented examples of methods for top mass reconstruction have features which are specific for a given channel but other can be (with a slight modification) applied to other channels too. A combination of cross section and kinematics can be used in lept. + jets channel and all hadronic channel too. In situ calibration of jet energy scale can be used also in All hadronic channel. This is a very important way how to reduce systematic error of uncertainty on jet energy scale. And this uncertainty is reduced with increasing statistics!

Already at present statistical and systematic errors are very close. Statistical errors are reduced with increasing statistics however same does not work for systematic errors. One of the most significant systematic errors after jet energy scale uncertainty is uncertainty from initial and final state radiation. Systematic errors will present main challenge in future precise measurement of top mass.

In Fig.29 one can see prediction for dependence of the total error (systematic and statistical errors added in quadrature) as a function of the statistics. Statistical error is expected to be inversely proportional to the square root of statistics. If systematic error will be fixed expected behavior will follow upper curve. If systematic error will go down same way as statistical one dependence will follow lower curve. In situ energy calibration is an example how systematic error can be

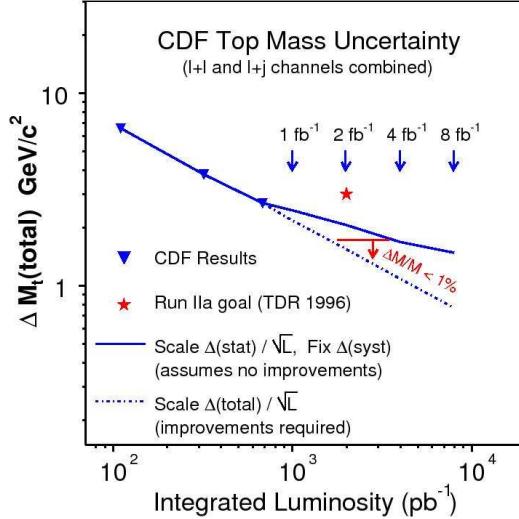


Fig. 29. Expected total top mass error (systematic and statistical added in quadrature) dependence as a function of accumulated statistics (for more detail see text)

improved with increased statistics. But then there will dominate other components of systematic errors. Most likely outcome is dependence between two curves. It should be also mentioned that goal in the CDF TDR proposal for top mass measurement was total precision of $3 \text{ GeV}/c^2$ which was already achieved.

Both experiments CDF and D0 used variety of methods to reconstruct top quark mass in the three discussed channels. Summary of CDF results is in Fig. 30. For D0 summary results are in Fig. 31. From these results one can draw a conclusion that within experimental uncertainties measurement of top quark mass in different channels executed by different methods are consistent with the statement that mass of the same physical object was measured - Standard Model top quark!

In case when we have a confidence that in different decay channels same physical object was measured one can combine different measurements (even from both experiments CDF and D0) to improve precision of top quark mass measurement. Proper procedure which takes into account correlations between different measurements and contributing items to measurements was

developed [50] and last updated result (spring 2008) of combination of CDF and D0 experiment is $M_t = 172.6 \pm 0.8(\text{stat}) \pm 1.1(\text{syst})$.

Most attention to a top mass measurement is attracted due to a possibility to estimate mass of Higgs particle by precise measurement of mass of W boson and top quark mass. Constraints on SM Higgs or MSSM Higgs given by LEP2 and TEVATRON measurements of top mass $M_t = 172.6 \pm 1.4$ and $M_W = 80.398 \pm 0.025 \text{GeV}/c^2$ are in Fig. 32 produced by authors of [51]. Precision top quark and W boson mass measurements (blue circle) limits region of Higgs mass (if Standard Model is correct) or possible masses of SUSY particles if MSSM theoretical description is correct.

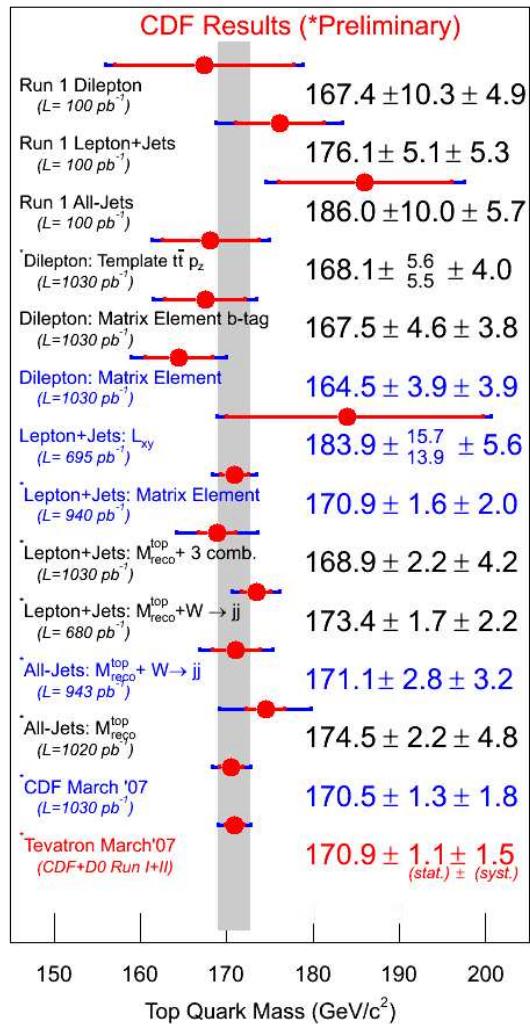


Fig. 30. Summary of top mass measurements by the collaboration CDF

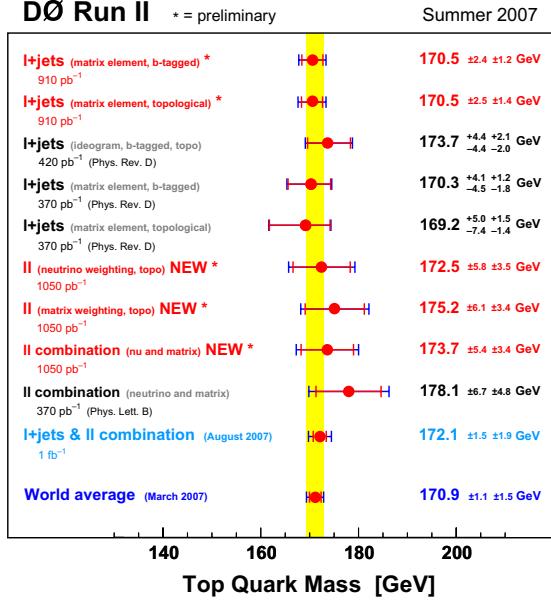
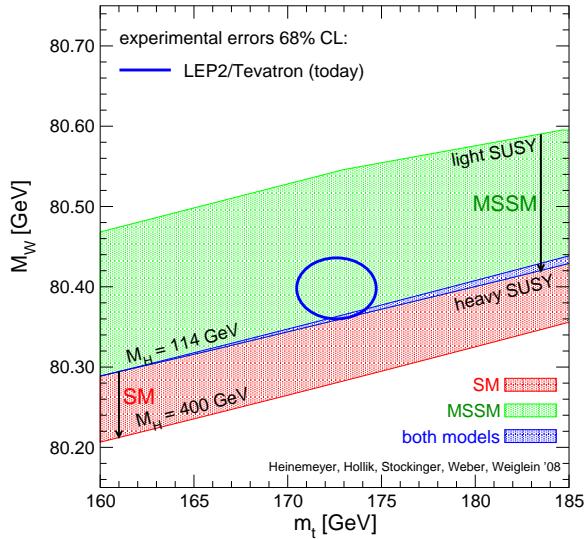


Fig. 31. Summary of top mass measurements by the collaboration D0

Fig. 32. Allowed region for range of M_t and M_W in case when Higgs mass is varied from $114 \text{ GeV}/c^2$ and SM assumed (red band) and allowed region determined by a scan over possible SUSY particle masses in case of MSSM (green band).

3.3 Single top production

Pair production of t and \bar{t} quarks by strong interaction can be considered theoretically and experimentally (as can be seen from previous sections) as well established. Theory predicts existence also another mechanism of single t (\bar{t}) quark production by electroweak interaction. Theoretical cross section for single top production is expected to be about 40% of pair production cross section however signal is so deeply buried in background that it presented challenge to find evidence for single top production. Experimental evidence for this channel was found twelve years after top quark discovery was announced! Evidence was established by D0 collaboration [52] and latter supported by CDF [55]. This analysis is a milestone in analysis in high energy physics. It completely relies on advanced methods of statistical treatment. As was demonstrated in previous sections advanced treatments like Artificial Neural Network based methods can be very useful and improve precision of a given measurement. But it was never only way how to get result up to now.

Conditions to find a signal in single top channel are following: signal to background ratio 1:15-20, number of events after final selection is even lower than what is expected from background only (uncertainty on background level is much larger than expected signal) and signal and background in terms of a single sensitive variables look indistinguishable. Can the existence of a signal under these conditions be established? It is clear that classic counting approach fails. Below we follow procedure proposed by D0 collaboration [52] by which the first evidence for this channel was claimed.

Data have been analyzed at the integrated luminosity 0.9 fb^{-1} , triggers required a jet and electron or muon. Search focuses on top quark in lepton channel mode - isolated lepton, high \cancel{E}_T and b-jet which is accompanied by b-jet in s-channel production and light quark and b-quark in t-channel production (it is rarely reconstructed because it is produced mainly in forward direction and low E_T). Main background consists of W boson in association with jets, pair production of $t\bar{t}$ and cases when jet was mismeasured as lepton (fake leptons) - multi-jet, non W background.

Specifically, jets are reconstructed by cone algorithm with radius $r = \sqrt{(\Delta\eta)^2 + (\Delta\phi)^2} = 0.5$ to cluster energy deposits in the calorimeter. The leading jet was required to have $E_T > 25 \text{ GeV}$ and pseudo-rapidity $|\eta| < 2.5$, the second leading jet $E_T > 20 \text{ GeV}$ and $|\eta| < 3.4$ and subsequent jets $E_T > 15 \text{ GeV}$ and $|\eta| < 3.4$. Requirement for events was exactly one isolated electron with $P_T > 15 \text{ GeV}/c$ and $|\eta| < 1.1$ or one isolated muon with $P_T > 18 \text{ GeV}/c$ and $|\eta| < 2.0$. Only events satisfying condition $15 < \cancel{E}_T < 200 \text{ GeV}$ and \cancel{E}_T should not be aligned or anti aligned in azimuth with lepton or selected jets. One or two of the jets have been required to be identified as originating from long lived b hadrons by neural network b-jet tagging algorithm.

For simulation of signal SINGLETOP NLO MC event generator was used, it was checked that it reproduces s-channel and t-channel kinematics according to direct NLO theoretical calculations [9]. Interface to Pythia [16] is used for hadronization of generated partons.

To simulate $t\bar{t}$ and W + jets backgrounds ALPGEN leading order MC package [47] was used and interfaced with PYTHIA for hadronization. Normalization of $t\bar{t}$ was based on integrated luminosity and predicted cross section [30]. The multi-jet background was modeled using data that contain non-isolated lepton but otherwise resemble the lepton + jets data set. W + jets background combined with multi-jet background is normalized to lepton + jets data set separately for each channel (defined by lepton flavor and jet multiplicity) before b-jet tagging. In W + jets

simulation $Wb\bar{b}$ and $Wc\bar{c}$ components are scaled by 1.5 ± 0.45 factor. MC events are input to GEANT [53] simulation of D0 detector. Differences between data and simulations are corrected by proper weights.

Selection procedure passed 1398 b-tagged lepton + jets data events. Expected amount from single top signal there is 62 ± 13 . Results for expected signal, background and data are summarized in Tab.8. There is no signal over expected background visible. Next step is to calculate multivari-

Source	Events per 900 pb^{-1} after all cuts		
	2j	3j	4j
tb	16 ± 3	8 ± 2	2 ± 1
tqb	20 ± 4	12 ± 3	4 ± 1
Total background	686 ± 41	460 ± 39	253 ± 38
Data	697	455	246

Tab. 8. Number of expected and observed events in 0.9 fb^{-1} for e and mu, 1 b-tag and 2 b-tag channels combined as a function of 2-4 jets

ate discriminants that separate signal from background and thus enhance probability to observe single top quarks. Decision tree technique (see specific references in [52]) is used to create these discriminants. A decision tree is a machine-learning technique that applies cuts iteratively to

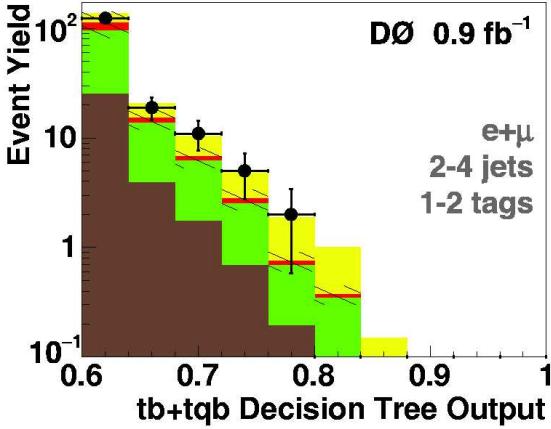


Fig. 33. Distribution of output from boosted decision tree for sum of all 12 channels for tb+tqb search. Different colors represent different contribution: green - $W+jets$, brown - multi-jets, red - $t\bar{t}$ and yellow - single top scaled according to measured cross section

classify events. Discrimination power is claimed to be further improved by averaging over many decision trees constructed using the adaptive boosting algorithm AdaBoost. Based on analysis of signal and background 49 sensitive variables have been identified. The variables may be clas-

sified into three categories: individual object kinematics, global event kinematics and variables based on angular correlations. Those with most discrimination power include the invariant mass of all the jets in the event, the invariant mass of the reconstructed W boson and the highest E_T b-tagged jet, the angle between the highest E_T b-tagged jet and lepton in the rest frame of the reconstructed top quark, and lepton charge times the pseudorapidity of the untagged jet. It is claimed that reducing number of variables always reduces sensitivity of analysis.

Events are divided into 12 channels based on lepton flavor (e or μ), jet multiplicity (2,3 or 4) and number of identified b jets (1 or 2). Boosted decision tree (DT) is applied to each of the 12 channels for three searches: tb+tqb, tqb, and tb. A boosted decision tree produces a quasi-continuous output distribution O_{DT} ranging from 0 to 1. Closer to 0 means more background like event, closer to 1 more signal like. Distribution of O_{DT} in signal enhanced region is displayed in Fig. 33. There is seen excess of single top signal over background!

To convert excess of signal into measured cross section Bayesian approach was applied. Binned

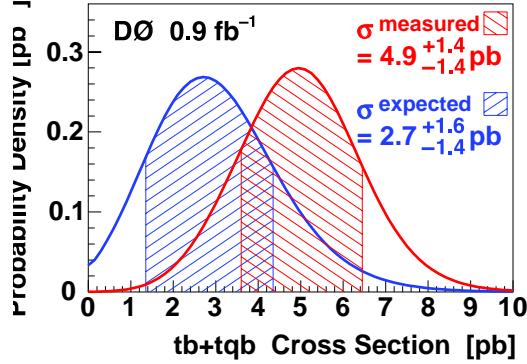


Fig. 34. Expected SM and measured Bayesian posterior probability densities for the tb + tqb cross section. Shaded lines indicate 1 standard deviation above and below peak positions.

likelihood was formed as a product over all binned channels (lepton flavor, jet multiplicity, tag multiplicity) of the decision tree discriminant, separately for tb+tqb, tqb, and tb analysis. Poisson distribution for observed counts was assumed and flat nonnegative probabilities for the signal cross section. Systematic uncertainties have been taken into account by assuming their Gaussian prior. The final posterior probability density is computed as a function of the production cross section. As a measured cross section is considered peak position in posterior probability density and symmetric 68% region determines uncertainty of measurement. A posterior probability density distribution as a function of tb + tqb cross section is shown in Fig. 34. There is also included expected distribution based on Standard Model. By above procedure was measured single top cross section $\sigma(p\bar{p} \rightarrow tb + X, tqb + X) = 4.9 \pm 1.4$ pb which is consistent with expectation from Standard Model. Same procedure was applied separately for tb, tqb channels. Result $\sigma(p\bar{p} \rightarrow tb + X) = 4.2^{+1.8}_{-1.4}$ pb and $\sigma(p\bar{p} \rightarrow tqb + X) = 1.0 \pm 0.9$ pb. The uncertainties include statistical and systematic components combined.

In Fig. 35 is addressed a question what is a probability that observed result can be obtained from

background only hypothesis. From ensemble of 68150 background-only pseudo-datasets, with all systematic uncertainties included, background fluctuates to give SM cross section 2.9 pb or higher 2.9% cases which correspond to 2.1 standard deviations for a Gaussian distribution. The probability that background fluctuates up to produce the measured cross section 4.9 pb or greater is 0.035% corresponding to 3.4 standard deviations in Gaussian model. The same exercise was

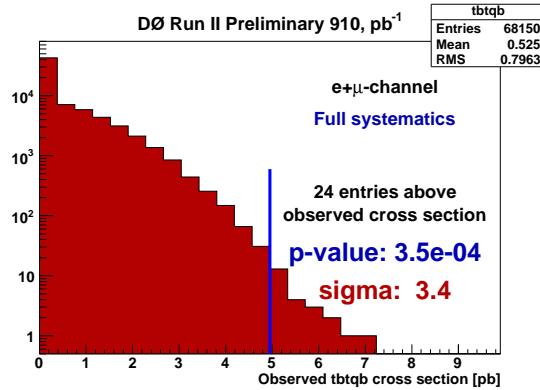


Fig. 35. Test of procedure of measurement cross section on background-only pseudo-datasets

repeated with pseudo-datasets where single top signal was included on level expected from SM. Probability to measure cross section 4.9 pb or larger was found to be 11%.

From above discussion follows that significance of the result (3.4σ) was a result of upward fluctuation of signal in D0 experiment. This can also explain why CDF had problems to find evidence for single top production (lack of upward fluctuation) but finally both experiments CDF and D0 claim to see evidence for single top production in their experiments. By using different multivariate statistical approaches both experiments find consistent results. Summary of D0 and CDF results are in Fig. 36. As one can see from a comparison that both results (D0 and CDF) are consistent with Standard Model expectations however cross section measured by D0 is factor 2 larger than cross section measured by CDF. As was already several times mentioned upward fluctuation can be a reason. In high energy physics to state that new process was observed there needs to be reached significance better than 5 standard deviations from a contrary hypothesis that given process was not observed. There is a good chance that with integrated luminosity about 4 fb^{-1} both experiments CDF and D0 will be able to claim observation of this process!

Task of detailed studies of single top production will be probably left for next generation of HEP experiments at LHC which should start at the end of this year (2008).

Importance of experimental evidence for single top production by experiments D0 and CDF is not only in finding another evidence supporting standard model picture of nature. For experimental high energy physics is of at least the same importance successful exploration of new methods of analysis which are able to isolate tiny signal inside huge background. These methods will find even more challenging application in search for Higgs boson. Not to mention that single top production will be for Higgs search a background which needs to be well understood and under control.

CDF and DØ tb+tqb Cross Section

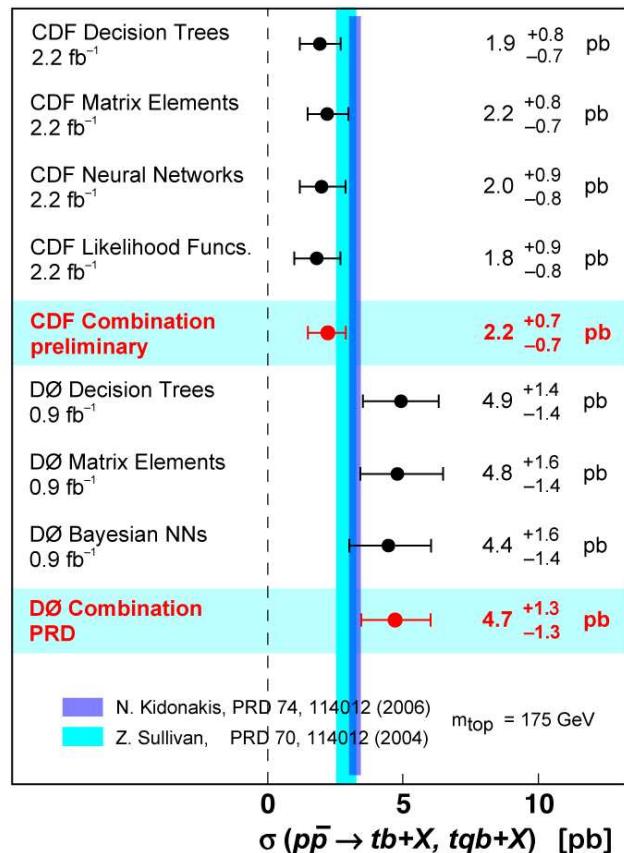


Fig. 36. Summary of measurements of single top cross section by D0 and CDF collaboration

3.4 Top Charge

There does exist alternative model [32] which claims even better agreement with electroweak data than Standard Model. There is large number models which claim that in low energy limit they give the same or better description as Standard Model. Some of them are considered (e.g MSSM) to be a possible successors of the SM.

What makes this model [32] special is that in this model is postulated existence of exotic particle with all properties same as top quark except charge. Charge of exotic particle is 4/3 (in comparison with 2/3 for top). According to this theory what was discovered as top quark is in fact exotic quark. True top quark mass (according to [32]) is much larger ($\approx 270 \text{ GeV}/c^2$). Therefore only by measurement of top charge one can distinguish between SM and exotic model. More exactly there are also other verifiable differences - let's call exotic particle XM top.

XM top lifetime is longer by factor ≈ 18 . Therefore does not hold usual argument of SM that hadronization cannot occur. In case of XM top it can. But consequences could escape experimentalists attention.

Cross section for single XM top production is about factor 18 smaller [33]. Therefore if single top production gains discovery level XM top will be in trouble.

In any case, independent of existence or nonexistence of alternative models, task for experimentalist is to measure top charge and make clear underlying assumptions which led to a given measurement.

Basically there are two approaches to measure top charge:

- based on electromagnetic coupling (measurement of cross section $t\bar{t}\gamma$). This measurement will wait for LHC.
- based on reconstruction of charges of decay products - b and W

At TEVATRON one needs to consider limited statistics. That can be at some level compensated by an appriory information.

Reconstruction of top charge in $t\bar{t}$ production from decay to b and W can be simplified to a reconstruction of sign of b-jet charge, W charge and decision to what is correct pairing of Ws to b-jets (there are two W and two b-jets in final state). We consider only 2 channels – dilepton channel or lept. + jets channel.

b-jet charge can be determined by identification of decay products (e.g. B^\pm, D^\pm , lepton) associated with b-jet.

There is also well known procedure to determine jet charge by weighted sum of charges of the tracks associated with jet. Specifically, following formula is known to work:

$$Q_{jet} = \frac{\sum |\vec{p}_i \cdot \vec{P}_{jet}|^{0.5} Q_i}{\sum |\vec{p}_i \cdot \vec{P}_{jet}|^{0.5}} \quad (33)$$

Where \vec{p}_i, Q_i is momentum and charge of a track associated with a jet and \vec{P}_{jet}, Q_{jet} momentum and charge of the jet. Methods based on identification of specific particles associated with b-jet suffer from relatively low identification efficiency. Therefore approach based on eq. (33) was preferred.

Reconstructed b-jet charge distribution by eq. (33) $P_b(x)$ is generally a function of x in the range $<-1,1>$. Because of CP symmetry of $p\bar{p}$ reaction it should hold relation between b and \bar{b}

reconstructed charge distribution $P_b(x) = P_{\bar{b}}(-x)$. This relation can serve also as a consistency check.

One more step can be taken to simplify problem. If reconstructed b-jet charge is smaller than 0 it is considered -1/3 if it is greater than 0 it is considered +1/3. Probability to obtain this way correct charge (P_b^c) or wrong charge (P_b^w) is following:

$$P_b^c = \int_{-1}^0 P_b(x) dx. \quad (34)$$

$$P_b^w = \int_0^1 P_b(x) dx. \quad (35)$$

There hold the following trivial relations:

$$P_b^w = 1 - P_b^c, P_b^c = P_{\bar{b}}^c, P_b^w = P_{\bar{b}}^w$$

Let's denote P^{++} probability to measure +2/3 top charge, charge given top hypothesis true and P^{--} is probability to measure -4/3 given exotic hypothesis is true. P^{++} can be determined based on probability of correct pairing and probability of correct b-jet charge reconstruction:

$$P^{++} = P_b^c P_{lb} + (1 - P_b^c)(1 - P_{lb}) \quad (36)$$

P_{lb} is a probability for a correct pair combination of lepton and b-jet from the same top decay. It is also called Purity.

In case when only difference in top and exotic particle hypothesis is charge formulas for P^{++} and P^{--} will be the same. Also assumption that these two hypotheses are mutually exclusive seems plausible. Therefore we have binomial problem (there are only two possible and exclusive outcomes) to solve. What is probability to observe N^{++} or less pairs out of N pairs of lepton and b-jet if top hypothesis is true?

$$P(N^{++}) = \sum_{i=0}^{N^{++}} \binom{N}{i} (P^{++})^i (1 - P^{++})^{N-i} \quad (37)$$

In case of exotic hypothesis considered true one needs to replace ++ to -- in above formula. Relation eq. (37) in fact represents formula for P-value calculation (standard statistics) given true corresponding hypothesis (top).

Task of reconstructing top (exotic) charge can be reformulated following way: Let's have binomial distribution of known individual probability p. There is an ambiguity if p holds for (++) or (--) combination. H_0 hypothesis can be formulated for either option. Rejecting one hypothesis means support for the other.

Background treatment

Background is supposed to contribute with equal probability (0.5) to both charge combinations (top and exotic). This statement should be cross-checked in both MC and Data! Probability to measure +2/3 charge when top hypothesis is true taking into account correct $t\bar{t}$ and background fractions is:

$$P_{comb}^{++} = P^{++} P_F^{t\bar{t}} + 0.5 P_F^{Bkg} \quad (38)$$

Where $P_F^{t\bar{t}}$ and P_F^{Bkg} are $t\bar{t}$ and background fractions respectively. For P-value calculation P^{++} in formulas (37) should be replaced by P_{comb}^{++} . Treatment of exotic hypothesis is straightforward.

3.4.1 P-value distribution

Standard statistical treatment is to set α level (usually 0.05 or 0.01) and calculate P-value. P-value was defined by e.g (Pnpp:). (I) t is a random number and probability to observe value smaller than α when hypothesis under consideration is true is α . This is true for continuous probability distribution functions but not exactly true for discontinuous ones. Let's calculate probability distribution of P-value defined by eq. (37).

$$n(P_l, N) = \binom{N}{P_l^{-1}(N^{++})} (P^{++})^{P_l^{-1}(N^{++})} (1 - P^{++})^{N - P_l^{-1}(N^{++})} \quad (39)$$

Where $P_l^{-1}(N^{++})$ is an inverse function to P_l (value of N^{++} corresponding to value of P_l). P_l can acquire only discreet values depending on N . Probability to observe $P_l \leq \alpha$ is

$$P_{\alpha, N} = \sum_{P_l \leq \alpha} n(P_l, N) \quad (40)$$

eq. (40) represents probability to reject true top hypothesis by set α level.

What is probability to reject the exotic hypothesis (based on same α level) when top hypothesis is true? In this case P-value probability distribution has a form:

$$n^{ex}(P_l, N) = \binom{N}{P_l^{-1}(N - N^{++})} (P^{++})^{P_l^{-1}(N - N^{++})} (1 - P^{++})^{N - P_l^{-1}(N - N^{++})} \quad (41)$$

Probability to observe $P_l \leq \alpha$ is

$$P_{\alpha, N}^{ex} = \sum_{P_l \leq \alpha} n^{ex}(P_l, N) \quad (42)$$

In Fig. 37 a two dimensional distribution of probability (based on eq. (42)) to reject “wrong” hypothesis at 95% CL as a function of purity and number of measurements is shown. It is quantitatively expressed rather trivial message that for large purity is sufficient modest number of measurements to reject wrong hypothesis but for poor purity very large number of measurements is necessary.

3.4.2 Treatment of uncertainty of individual probability

In case of individual probability (P^{++}) is known with some uncertainty what is probability to reject true hypothesis based on α level cut and what is probability to reject wrong hypothesis based on the same cut? Probability to reject true hypothesis based on α level cut is in this case a convolution of formula 40 over distribution of individual probability function. Formula (37) is modified:

$$P(N^{++}) = \sum_{i=0}^{N^{++}} \binom{N}{i} \int f(P^{++}) (P^{++})^i (1 - P^{++})^{N - i} dP^{++} \quad (43)$$

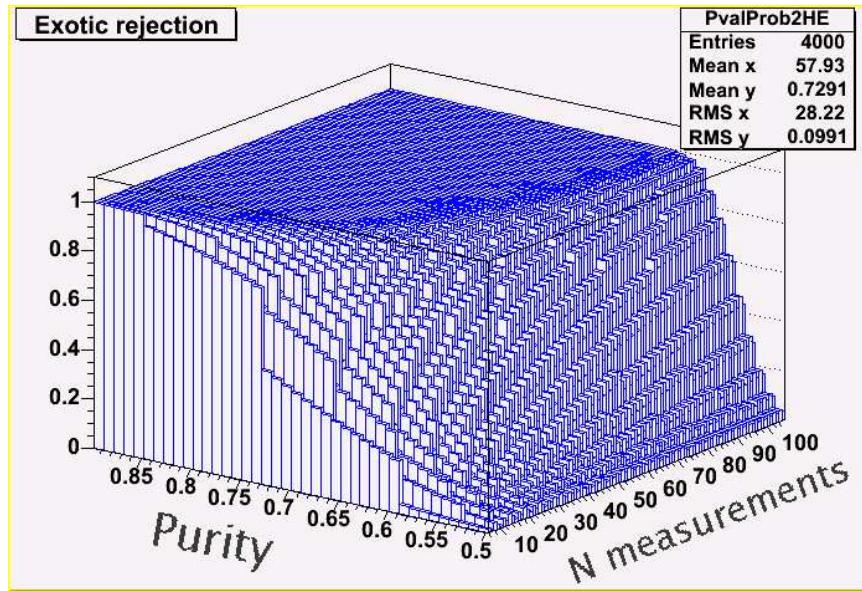


Fig. 37. Two dimensional distribution of probability to reject “wrong” hypothesis at 95% CL as a function of purity and number of measurements

where $f(P^{++})$ is distribution function of P^{++} . All previous formulas have the same form only relation (37) is replaced by (43).

3.4.3 Bayes factor

Statistical treatment based on rejection of hypothesis with observed P-value lower than preset α -cut can lead to a situation e.g. when both hypotheses are accepted or rejected. This kind of drawback is eliminated by Bayes Factor approach.

In statistics Bayes Factor (BF) is a standard tool for a comparison of two hypotheses. Odds of one hypothesis over other. Convention is:

2 ln(BF)	evidence
0-2	Not worth mentioning
2-6	positive
6-10	strong
> 10	very strong

In our case definition of Bayes factor is following:

$$BF = \frac{\int prob^{SM}(N^{++}, N, p) \cdot f(p) dp}{\int prob^{Ex}(N^{++}, N, p) \cdot f(p) dp} \quad (44)$$

Where $prob^{SM}(N^{++}, N, p)$ - probability to observe N^{++} pairs out of N for a given purity p , $f(p)$ - we assume Gaussian distribution of purity where mean is estimated purity and σ estimated uncertainty on purity.

Statistical analysis

Now we have all tools ready for statistical analysis. Strategy in statistical analysis was decided following [58]:

- Standard Model hypothesis is considered true. Standard Model hypothesis will be rejected only if P-value under assumption this hypothesis is true is found smaller than 0.01.
- Bayes factor will be calculated
- probability to reject SM hypothesis (by 1% cut) if exotic hypothesis is true will be calculated

3.4.4 Monte Carlo Study

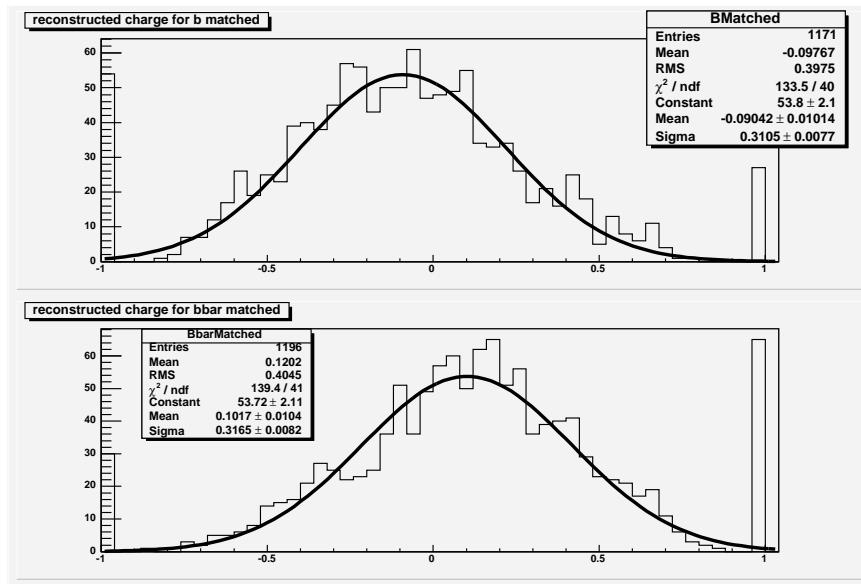


Fig. 38. Charge distribution for matched b-jets (upper plot) and \bar{b} -jets

Bellow analysis follows CDF results [58]. For top charge study two channels have been used: lepton + jets channel and dilepton channel. There are similarities in treatment in both channels

but there are also differences. There is needed to establish procedure for determination of b-jet charge and procedure for assignment of b-jet and lepton from the same top decay. In Fig. 38 distribution of b-jet charge calculated according to eq. (33) is shown. Tracks used in calculation have been selected around cone $r = \sqrt{(\Delta\phi)^2 + (\Delta\eta)^2} = 0.4$ around jet axis. To get rid of charge asymmetry (excess of positive charge because of interaction with detector material) $P_T \geq 1.5\text{GeV}$ and $D_0 < 0.03\text{ cm}$ cuts have been implemented. If calculated charge by eq. (33) is smaller than 0 jet is considered b-jet (charge -1/3) if calculated charge is greater than 0 jet is considered \bar{b} -jet (charge +1/3). Probability that this way decision taken is right is close to 60%. Next problem one needs to solve is a right combination of lepton and b-jet from the same top decay. This problem is approached differently in dilepton channel and lept. + jets channel. In dilepton channel two leptons and two b-jets represent two fold ambiguity how leptons and b-jets can be combined. There are four possible combinations of effective mass squared between lepton and b-jet (M_{lb}^2). In Monte Carlo simulation one “knows” which combination is “correct”, which combination is “wrong”. In Fig. 39 there are two superimposed distributions - maximal

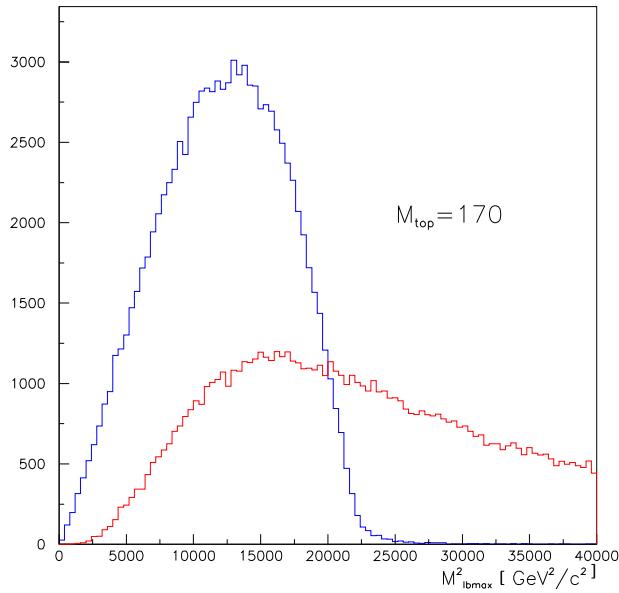


Fig. 39. Distribution of max M_{lb}^2 for correct pairing (blue) and wrong pairing (red)

value of M_{lb}^2 for “correct” combination and for “wrong” one. One can see that for top mass 170 GeV/c^2 contribution of “correct” combination in region of $M_{lb}^2 > 21000\text{GeV}/c^2$ is negligible. Therefore if in event is a combination with $M_{lb}^2 > 21000\text{GeV}/c^2$ choice of complementary combination is with very high probability right choice. Probability for correct assignment this way is close to 95% and efficiency (probability to observe in event $M_{lb}^2 > 21000\text{GeV}/c^2$) close to

40%.

In lept.+ jets channel correct pairing is estimated by kinematic reconstruction of the event. Probability of correct pairing this way is close to 83%.

As a result one obtains certain number of pairs of lepton and b-jet which support top hypothesis (charge 2/3) and certain number of pairs which support exotic hypothesis (charge 4/3). Under conditions of CDF experiment and a given analysis based on MC simulation it was expected in 1.5 fb^{-1} integrated luminosity to observe 171.07 ± 25.66 pairs from “signal” and 8.23 ± 3.55 pairs from background (combined lept.+jets and dilepton channels).

Scale factor between Data and simulation

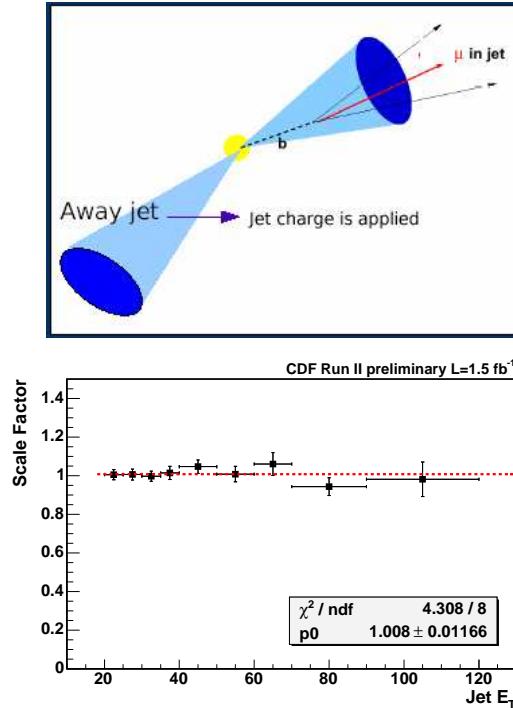


Fig. 40. Idea for determination b-jet charge purity from enhanced b-dijet data (upper picture) and dependence of scale factor as a function of b-jet E_T

Detector simulation combined with physical event generator is tuned after many tests and comparisons with real experiment to reproduce most of features of experiment very well. Still question how reliable is e.g. probability of b-jet charge determination based on MC needs to be answered. It can be answered by determination of “scale factor” which converts probability of correct b-jet charge determination in MC to corresponding measurement in real experimental

data. Roughly idea is simply to select b-jets from trigger lepton enhanced dijet sample. Lepton associated with jet enhances probability that a given jet is a b-jet. Leptons from b-hadrons semileptonic decay have a characteristic transverse momentum distribution (details can be found in e.g. [64]) and one can statistically determine fraction of true b-jet in a sample. Charge of lepton determines charge of b-jet. Second jet in a dijet sample (so called “away jet”) is considered to have opposite sign to semileptonic jet (see Fig. 40). This jet is used for determination of b-jet charge by formula 33. Scale factor determined by above procedure was found within statistical and systematic errors equal to 1.

Final result

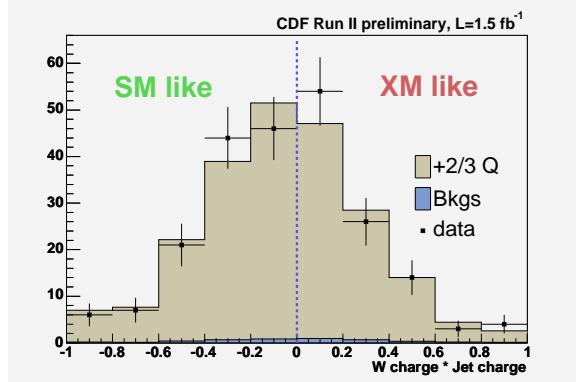


Fig. 41.

Above procedure has been applied to 1.5 fb^{-1} CDF data. Probability to determine correctly top charge (taking into account also scale factor) for signal was estimated to be $P_s = 0.572 \pm 0.003(\text{stat}) \pm 0.008(\text{syst})$ for background $P_b = 0.505 \pm 0.005$ (background is expected to contribute equally to top and to exotic hypothesis. The result is consistent with this statement). There have been found 124 pairs supporting SM hypothesis and 101 pairs supporting exotic one (combined lept. + jets and dilepton channel result).

P-value (under assumption of SM hypothesis true) was found 0.31 - consistent with the assumption and calculated Bayes factor 12.01 means very strongly support for SM (top charge 2/3) in comparison with exotic hypothesis (charge 4/3). Probability to reject SM hypothesis (P-value < 0.01 under assumption of SM true) in case when exotic hypothesis is in fact true was found 87%. Therefore exotic hypothesis is claimed to be rejected with 87 % probability.

In Fig. 41 one can see distribution quantity which is product of W charge and b-jet charge calculated according to eq. (33) paired by pairing procedure to W. This product would be in case of absolute precision and SM true equal exactly to $-1/3$.

Another easy way how to view result is through Fig. 42. In Fig. 42 there are superimposed two curves and marked two regions. Blue line, border of blue region represents P-value under assumption of SM top hypothesis for N^{++} number of pairs supporting this hypothesis out of

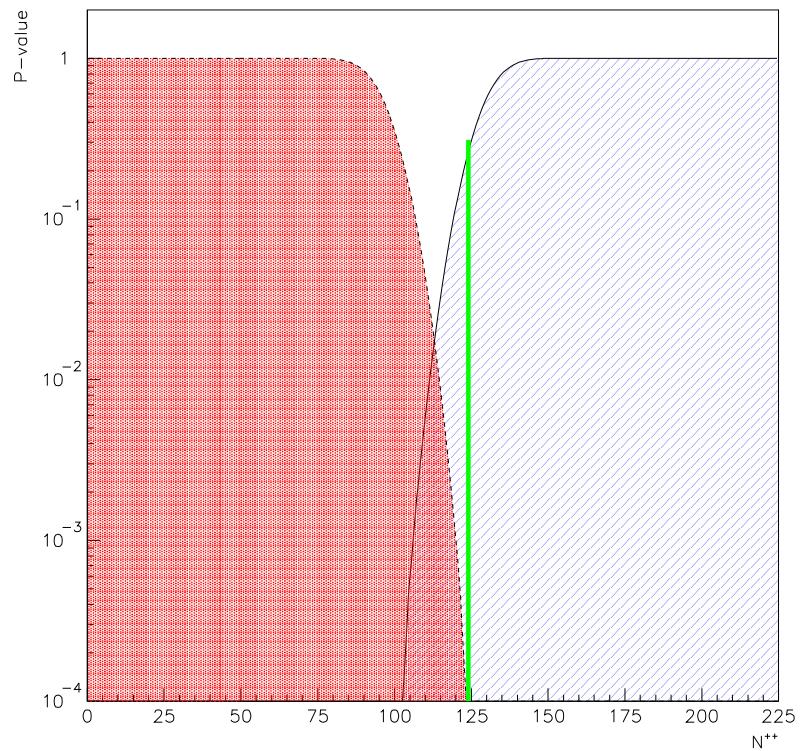


Fig. 42. Probability to observe N^{++} pairs or less supporting SM top if SM hypothesis is true - blue curve (region) and complementary case when exotic hypothesis (XM) is correct. Probability to correctly determine top charge was set to $P_s = 0.572$

225 pairs. Red border has same meaning for exotic hypothesis. Probability for correct determination of top charge (purity) was set to value 0.572. One can immediately judge from figure P-value in case SM hypothesis assumed true or XM hypothesis assumed true. Green line represents experimental result $N^{++} = 124$ and one can see that it corresponds to P-value ≈ 0.3 under assumption SM hypothesis true or $\approx 10^{-4}$ under assumption of XM true. This is just good order of magnitude estimate because e.g. uncertainty on purity was not taken into account.

3.5 Measurement of W-Boson helicity in top decays

According to the Standard Model the top quark decays into W and b-quark with almost 100% probability and very fast (see 2.1). Short lifetime prevents hadronization of top quark therefore its properties are transferred directly to decay products without modification caused by hadronization! This is a unique property of the top quark! Standard Model makes specific prediction about W polarization in case of $t \rightarrow Wb$ decay. Precise measurement of W polarization can reveal new physics beyond the Standard Model (if it takes place).

A short description of W helicity expectations in top rest frame predicted by Standard Model is given below.

W-Boson as a vector (spin 1) particle can have projections of spin on the direction of motion (helicity) +1,0,-1. We talk about right-handed, 0 (longitudinal) and left-handed helicity states. In the b-quark massless limit in top decay (because V-A charge current weak interaction responsible for decay) b-quark can be only left handed and \bar{b} -quark (in \bar{t} decay) only right-handed. Top quark spin is $\frac{1}{2}$ and only options for W^+ helicity states therefor are left-handed (W_-) and longitudinal (W_0). In case of \bar{t} decay – W^- helicity states can be right-handed (W_+) and longitudinal (W_0). These are only options for W how to combine with left-handed b-quark to make combined spin projection $\pm 1/2$. In Fig. 43 there is the graphical explanation.

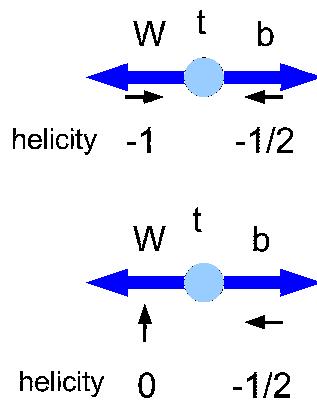


Fig. 43. Decay of top quark into W and b in top rest frame. Upper part represents top spin projection $+1/2$, W helicity -1 and b-quark helicity $-1/2$ (both left-handed), Lower part represents top spin projection $-1/2$, W helicity 0 (longitudinal) and b-quark helicity $-1/2$ (left-handed)

If what we call top quark is in fact exotic quark as predicted in [32] which decays to W^+ and \bar{b} then naively one would expect that W helicity can be only 0 and right-handed! But as it turns out [33] in this exotic model prediction about W helicity is the same as in case of SM! But for

considered exotic model SM assumption about fast decay preventing hadronization or spin flip is not valid. Because that specific model also predicts lifetime of exotic top 18x longer than SM top! Therefore really conclusive possibility to rule out exotic option is to measure top charge! Standard Model gives specific prediction about fraction of cases when top quark decays into definite helicity states of W-Boson. e.g. for longitudinal fraction F_0 SM in the tree approximation predicts:

$$F_0 = \frac{\Gamma(t \rightarrow W_0 b)}{\Gamma(t \rightarrow W_0 b) + \Gamma(t \rightarrow W_+ b) + \Gamma(t \rightarrow W_- b)} = \frac{M_t^2}{2M_W^2 + M_t^2} \quad (45)$$

In Standard Model $\Gamma(t \rightarrow W_+ b)$ is (close to) 0. Naturally there holds equality $F_+ + F_- + F_0 = 1$. Therefore for top mass $175 \text{ GeV}/c^2$ SM predicts $F_0 = 0.7$, $F_- = 0.3$ and $F_+ = 0$.

The W-Boson polarization is reflected in angular distribution of leptons from $W \rightarrow l\nu$ decay. Generally $\cos(\theta^*)$ distribution is expressed by:

$$\frac{dN}{dcos(\theta^*)} \propto F_- \cdot \frac{3}{8}(1 - \cos(\theta^*))^2 + F_0 \cdot \frac{3}{4}(1 - \cos^2(\theta^*)) + F_+ \cdot \frac{3}{8}(1 + \cos(\theta^*))^2 \quad (46)$$

where θ^* is the angle between momentum of the charged lepton in the W rest frame and the momentum of the W-Boson in the top quark rest frame. In Fig. 44 are shown $\cos(\theta^*)$ distributions for different helicity states of W and for SM expectation.

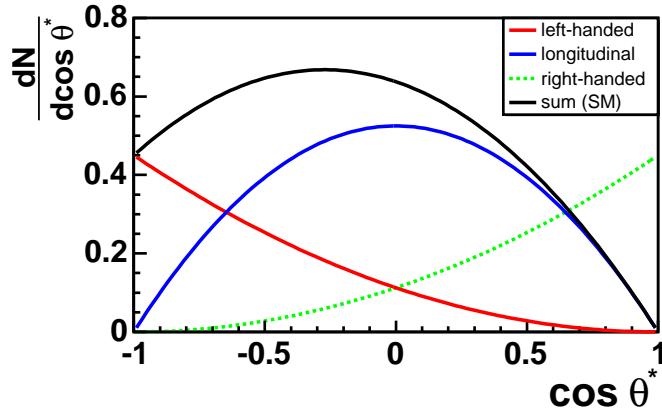


Fig. 44. Expected angular distribution of leptons in W rest frame in case W_-, W_0, W_+ helicity states compared to SM expectation

Below we follow procedure and results published by the CDF collaboration [59]. The analysis is done using lept. + jets channel. Selection requirements are close to one used for cross section measurement (see section 3.1.2). There is an extra requirement that event is removed if lepton and another object form effective mass in Z-Boson mass window. In the data sample corresponding to integrated luminosity 1.9 fb^{-1} 484 $t\bar{t}$ candidate events have been selected.

Background was estimated to be 86.53 events. It's composition was discussed in detail in section 3.1.2.

It is necessary to fully reconstruct four momenta of t and \bar{t} . One can think that it would be sufficient to reconstruct top quark associated with W which undergoes semileptonic $W \rightarrow l\nu$. But to find out which jet in the event is most probably associated with W from top quark decay one needs to try different jet associations and to reconstruct t and \bar{t} four momenta.

Measured momenta of jets and \cancel{E}_T are corrected to parton level. \cancel{E}_T corresponds to transverse momentum of neutrino from leptonic decay of W -Boson. Longitudinal component of neutrino can be reconstructed when constrain on W mass in W -Boson semileptonic decay is applied. This way longitudinal component of neutrino will be obtained as a solution of a quadratic equation. In 70% cases there are two real solutions and both have to be taken into account. In 30% cases solutions become complex. In this case x and y components of \cancel{E}_T are varied (starting from the measured values) until imaginary part of the solution vanishes. In this case there is single solution.

All jets in the event (not only 4 leading jets) are considered for assignment to the two b -quarks and the two light quarks (from hadronic W decay). Therefore number of hypotheses considered for complete kinematic reconstruction is quite large - $N_\nu \cdot N_j \cdot (N_j - 1) \cdot (N_j - 2) \cdot (N_j - 3)/2$. Where N_ν is number of neutrino solutions and N_j number of jets. For each hypothesis is calculated quantity ψ which gives quantitative estimate how well given hypothesis matches the $t\bar{t}$ configuration.

$$\psi = P_\nu \cdot P_{b-light} \cdot \chi^2$$

where P_ν is the weighting factor for the calculated longitudinal component of neutrino, $P_{b-light}$ is a measure of likeness that for a given jet assignment true light quarks are assigned as the b -quarks, χ^2 is a measure how well within expected uncertainties reconstructed W mass from assigned jets corresponds to M_W , M_{top} leptonic to M_{top} hadronic and fraction of transverse energy from two top quarks in event to total transverse energy.

ψ is calculated for each hypothesis in the event and hypothesis with smallest ψ is chosen to be used for a calculation of $\cos(\theta^*)$ distribution.

Extraction of helicity fraction F_+ and F_0 from measured $\cos(\theta^*)$ distribution was performed by binned likelihood fit. The likelihood function $L(a)$ (where "a" stands for the free parameters - helicity fractions).

$$L(a, \beta) = e^{-\frac{(1-\beta)^2}{\sigma_\beta^2}} \cdot \prod_{k=1}^{N_{bins}} \frac{\mu_k^{exp}(a, \beta)^{n_k} e^{-\mu_k^{exp}(a, \beta)}}{n_k!} \quad (47)$$

$\mu_k^{exp}(a, \beta)$ denotes the number of events expected to be observed in bin k of the reconstructed $\cos(\theta_{rec}^*)$ and n_k actually measured number of events in the same bin. $\mu_k^{exp}(a, \beta)$ is a sum of the expected number of events from signal $\mu_k^{sig,exp}$ and background $\mu_k^{BG,exp}$:

$$\mu_k^{exp}(a, \beta) = N_{sig} \cdot \mu_k^{sig,exp}(a) + N_{BG} \cdot \mu_k^{BG,exp} \cdot \beta$$

and $\mu_k^{sig,exp}$ is calculated via:

$$\mu_k^{sig,exp} \propto \sum_i \mu_i^{sig}(F_0, F_+) \cdot \epsilon_i \cdot S(i, k)$$

The starting point is the theoretically predicted number of signal events in each bin $\mu_i^{sig}(F_0, F_+)$ (dependence on F_- is eliminated by constrain $F_0 + F_+ + F_- = 1$). Event selection acceptance depends on $\cos(\theta^*)$ therefore for each bin is applied different event selection efficiency ϵ_i . The migration matrix S takes migration effects due to the finite resolution of detector and the reconstruction method into account. The matrix element $S(i, k)$ represents probability for an event with true $\cos(\theta^*)$ in bin i to contribute to bin k. Both migration matrix and selection efficiency ϵ_i have been checked to be independent of W helicity fractions.

$\mu_k^{BG,exp}$ is the normalized background estimate derived from the background templates.

$N_{BG} \cdot \beta$ is the total number of background events. Gaussian term in likelihood (47) constrains number of background events to estimated expected value within estimated error.

Number of the signal events N_{sig} is determined from number of observed events N_{obs} and number of background events as $N_{sig} = N_{obs} - N_{BG} \cdot \beta$.

Data are separated into exactly one b-tag and more than one b-tag categories. Likelihood function is separately calculated for each category and product of these two likelihoods is the combined likelihood used to perform a fit of W-Boson helicity fractions. Fit was executed under three separate conditions:

- Fit to determine F_0 when F_+ is fixed to 0 (as expected from SM)
- Fit to determine F_+ when $F_0 = 0.7$ (as expected from SM)
- A two dimensional fit to determine simultaneously F_0, F_+ .

Systematic uncertainty was studied by means of pseudo-experiments (PE). The systematic uncertainty is given by the difference between mean fit result for PE with systematic affected sample and the mean fit result for default sample. Three most relevant sources of systematic errors have been identified - jet energy scale uncertainty, final state radiation uncertainty and background shape. Nominal value of top quark mass in systematic studies was $175 \text{ GeV}/c^2$. Uncertainty on top mass was not included in systematic uncertainties but there was performed a check in $\pm 5 \text{ GeV}/c^2$ region around top mass $175 \text{ GeV}/c^2$ and it was demonstrated that fit reproduces well W helicity fractions (which are top mass dependent).

Results:

Performing the fit with fixed $F_+ = 0$ the fraction of helicity 0 W-Bosons was determined to be $F_0 = 0.66 \pm 0.10(stat) \pm 0.06(syst)$.

When F_0 was fixed to $F_0 = 0.7$ right-handed fraction of W-Bosons F_+ was found $F_+ = 0.01 \pm 0.05(stat) \pm 0.03(syst)$. Background parameter β was determined $\beta = 1.02 \pm 0.20$.

Assuming $F_0 = 0.7$ the 95% CL for F_+ was determined to be $F_+ < 0.12$.

The simultaneous fit of F_0 and F_+ yielded the results $F_0 = 0.38 \pm 0.21(stat) \pm 0.07(syst)$, $F_+ = 0.15 \pm 0.10(stat) \pm 0.05(syst)$ and $\beta = 1.0 \pm 0.2$.

The corresponding distribution of $-\Delta \ln L$ is shown in Fig. 45.

Both results are consistent with SM prediction for W helicity fractions.

Consistent results have been obtained by several other independent analysis at CDF [60], [61] and D0 [62]. As can be seen from Fig. 46 different methods point to the same conclusion - support for SM prediction. Consistency with Standard Model prediction for W helicity fractions means also indirect support for other prediction of SM which implicitly assumed - very short lifetime of SM top quark which prevents hadronization or spin flip for top quark to happen.

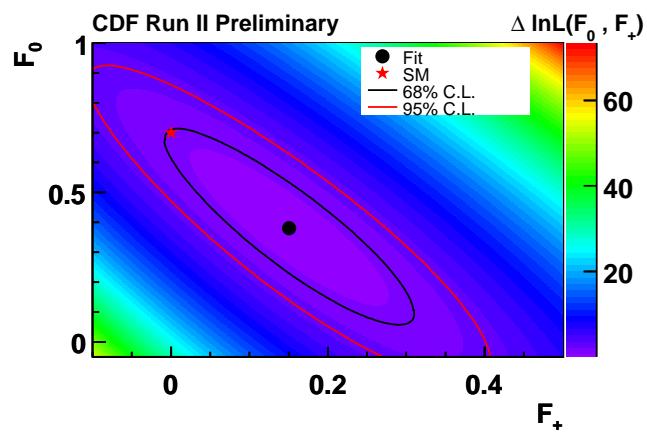


Fig. 45. $\Delta \ln L$ dependence as a function of F_+, F_0 around fitted value (black point). Position of SM predicted value for F_+, F_0 is marked by red star. The black line indicate 68% CL contour, red one 95% CL.

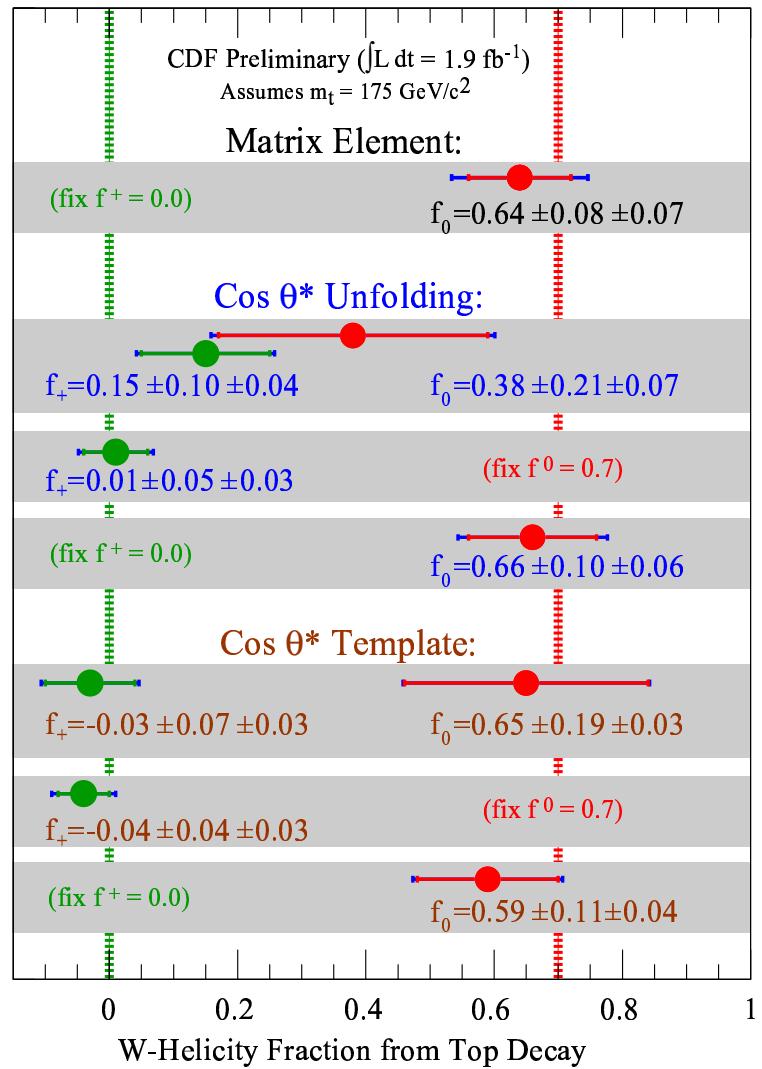


Fig. 46. Summary of the results of CDF collaboration on W helicity fraction measurement

3.6 Spin - Spin correlations

Correlations between spins of t and \bar{t} in $t\bar{t}$ production have been extensively theoretically studied in the past [65]. Interest in these studies was provoked by unique feature of the top quark - very fast decay, mean lifetime order of magnitude smaller than time needed for hadronization or spin flip. Therefore spin properties of top quark are directly transferred to the decay products. These correlations are very sensitive to a new physics, deviations from the Standard Model. The large correlations in an appropriate reference frame have been predicted in Standard Model framework.

So far there are no (statistically significant) experimental results about t and \bar{t} spin correlations. Therefore in this section we will try to analyze situation from an experimentalist point of view how these correlations can be approached and what kind of problems can be faced.

It is well known [71] that best analyzer of top spin is charged lepton from the decay chain $t \rightarrow Wb \rightarrow l\nu b$. Therefore we concentrate on dilepton channel.

A two-dimensional distribution (in an appropriate reference frame) of $\cos(\theta_+)$, $\cos(\theta_-)$ (for lepton with charge + and - respectively) has a simple form (see e.g. [71]):

$$\frac{1}{\sigma} \frac{d\sigma^2(p\bar{p} \rightarrow t\bar{t} \rightarrow l^+l^-X)}{dcos\theta_+ dcos\theta_-} = \frac{1}{4}(1 - C \cdot \cos(\theta_+) \cos(\theta_-)) \quad (48)$$

Where C is spin-spin correlation coefficient in a given reference frame. One can define variable

$$z = \cos(\theta_+) \cos(\theta_-) \quad (49)$$

and derive from the 2 dimensional distribution eq. (48) one dimensional distribution

$$\frac{1}{\sigma} \frac{d\sigma(p\bar{p} \rightarrow t\bar{t} \rightarrow l^+l^-X)}{dz} = -1/2 \cdot (1 - C \cdot z) \cdot \ln(|z|) \quad (50)$$

Very attractive feature has distribution:

$$\frac{-1}{\ln(|z|)} \frac{1}{\sigma} \frac{d\sigma(p\bar{p} \rightarrow t\bar{t} \rightarrow l^+l^-X)}{dz} = 1/2 \cdot (1 - C \cdot z) \quad (51)$$

Correlation coefficient is in this case proportional to a slope of a linear distribution. No information is lost in going from the two-dimensional eq. (48) to the one dimensional eq. (51). Dealing with one dimensional distribution, specially in case of low statistics should be an advantage.

Standard Model prediction in NLO approximation [67] for TEVATRON and LHC is summarized

Reference frame	TEVATRON	LHC
C_{beam}	0.806	-0.072
C_{hel}	-0.389	0.311

Tab. 9. A comparison of the NLO Standard Model prediction for spin-spin correlation coefficient in beam and helicity frames in case of TEVATRON and LHC

in Tab. 9. As one can see from above comparison Standard Model predicts larger correlations in

the case of TEVATRON. But this drawback in case of LHC will be more than compensated by three orders of magnitude larger expected statistics.

There are several options how to determine correlation coefficient C .

- by a fit to a two-dimensional distribution described by eq. (48)
- from eq. (48) follows:

$$C = -9 < \cos\theta_+ \cos\theta_- > \quad (52)$$

- from endpoints of distribution eq. (48)
for points (1,-1) and (-1,-1) follows:

$$\begin{aligned} (-1, -1) &\rightarrow 1/4(1-C) = N_- \\ (1, -1) &\rightarrow 1/4(1+C) = N_+ \end{aligned}$$

then

$$C = \frac{N_+ - N_-}{N_+ + N_-} \quad (53)$$

- from asymmetry of “same side” and “opposite side” pairs.
Let’s define same side

$$N_s = \int_{-1}^0 \int_{-1}^0 \frac{d\sigma^2}{d\cos\theta_+ d\cos\theta_-} d\cos\theta_+ d\cos\theta_- + \int_0^1 \int_0^1 \frac{d\sigma^2}{d\cos\theta_+ d\cos\theta_-} d\cos\theta_+ d\cos\theta_-$$

and opposite side

$$N_{op} = \int_{-1}^0 \int_0^1 \frac{d\sigma^2}{d\cos\theta_+ d\cos\theta_-} d\cos\theta_+ d\cos\theta_- + \int_0^1 \int_{-1}^0 \frac{d\sigma^2}{d\cos\theta_+ d\cos\theta_-} d\cos\theta_+ d\cos\theta_-$$

one can easily see that

$$C = 4 \cdot \frac{N_{op} - N_s}{N_{op} + N_s} \quad (54)$$

- from a fit to z distribution eq. (50)
- according to eq. (51) z distribution has linear dependence. Fit to this dependence determines correlation coefficient.

In a limit of the infinite statistics all these determinations should be equivalent. In case of a modest statistics, selection and reconstruction bias, there can be advantage of the specific determination.

Reference frames

Because results crucially depend on a choice of reference frame we decided to devote special paragraph to this topic. We also realized that there are at least two definitions of the beam frame defined by theorists who calculated correlation coefficient in these frames. Ambiguity in definition of above frames in the literature was resolved with help of original authors [68, 69] We consider three reference frames:

- Beam reference frame I
- Beam reference frame II
- Helicity reference frame

Beam frame I [68]

There are defined four vector $QL \equiv (0, 0, 1, 1)$ and $QR \equiv (0, 0, -1, 1)$ in lab frame (in case of TEVATRON means in fact CMS frame). QR is transformed by Lorentz transformation into top rest frame. Cosine of angle between transformed QR and lepton from top decay in top rest-frame is $\cos(\theta_+)$. $\cos(\theta_-)$ is defined by using QL and \bar{t} rest frame same way.

Beam frame II [69]

$QL \equiv (0, 0, 1, 1)$ in lab frame is transformed to “Zero Momentum Frame” (frame where $t\bar{t}$ system has 0 momentum). Lepton four momenta are transformed to ZMF first and then to corresponding top (for + charged lepton) and \bar{t} (for negative charge lepton). $\cos(\theta_+)$ is defined by direction of QL in ZMF and charge + lepton in top rest frame. $\cos(\theta_-)$ is defined by direction of QL in ZMF and negative charge lepton in \bar{t} rest frame.

Helicity frame

There is a general agreement on definition of helicity frame. What may be unusual for novice is importance of ZMF stage.

Top, anti top and leptons four momenta are transformed to ZMF first and then from this frame to top (anti top) rest frame. Direction from top (anti top) rest frame to ZMF frame is the helicity axis. Angle between this direction and direction of lepton in a given rest frame is the wanted angle in the helicity frame.

3.6.1 Spin-spin correlation coefficient determination from different MC generators

For a test of discussed procedures for determination of spin correlation coefficient we employed two event generators – Pythia [16] and TopRex [70]. In fact TopRex does use interface to Pythia. By means of these generators $p\bar{p} \rightarrow t\bar{t} + X$ reaction at 1.96 TeV is generated and t (\bar{t}) is forced to decay in leptonic mode (dilepton channel). Top mass was set to $170 \text{ GeV}/c^2$ for purpose of these tests.

Standard Pythia - ideal case

In the standard Pythia (at least up to version 6.x) spin correlations in $t\bar{t}$ production are not included. However spin content of individual t (\bar{t}) is correctly propagated along t (\bar{t}) decay chain. Just spin of t and \bar{t} are not correlated. In the ideal case, "true" - generator level top four vectors are used in the transformations between appropriate frames.

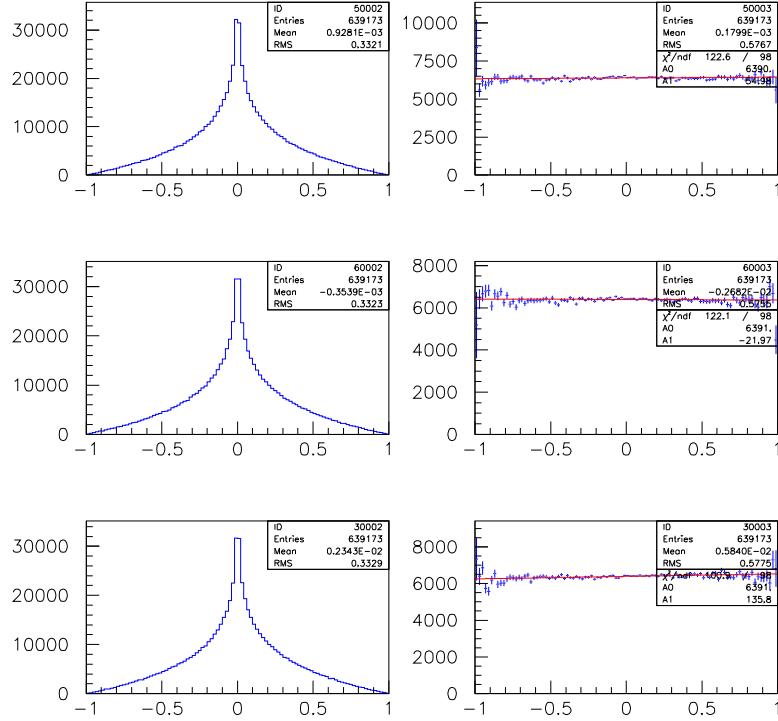


Fig. 47. Distribution of variable z in beam frame I,II (upper two rows) and helicity frame - standard Pythia

In Fig. 47 distribution of variable z (see eq. (49)) in beam frames I,II and helicity frame is shown (left column). Right column displays z distribution weighted by $-1/\ln(|z|)$. Absence of spin-spin correlation is easily identified by flat shape of this distribution.

Modified Pythia - TopRex - ideal case

In TopRex $t\bar{t}$ spin-spin correlations are correctly included. We use version 4.11 where LO matrix element is implemented. One can compare spin correlations in two dimensions for beam frame

I and II in Fig. 48 and helicity frame in Fig. 49. The same z distribution as in Fig. 47 (standard Pythia) is presented in Fig. 50 (TopRex). One can clearly see difference between previous case. More precisely, relevant conditions for spin correlations generated by TopRex are $M_t = 170$ GeV/c 2 and PDF structure function CTEQ5L.

In Tab. 10 is result of determination of spin-spin correlation coefficient by different approaches. The first two items in the table are published results of theoretical calculations using leading

Method	C_{beamI}	C_{beamII}	C_{helicity}
LO (CTEQ5) [71]	–	0.910	-0.456
LO (CTEQ6L) [72]	–	0.928	-0.471
$-9 < z >$	-0.711 ± 0.004	0.9 ± 0.004	-0.468 ± 0.004
Fit (eq. 51)	-0.711 ± 0.004	0.904 ± 0.004	-0.467 ± 0.004
Asymmetry (SS,OS)	-0.711 ± 0.005	0.905 ± 0.005	-0.466 ± 0.005
End Points 2D	-0.6 ± 0.03	0.858 ± 0.02	-0.5 ± 0.03

Tab. 10. The $t\bar{t}$ spin-spin correlation parameter determined in different ways.

order matrix element but different PDF structure functions. Other items are different estimates of correlation coefficient based on analysis of TopRex generated events.

There is a good agreement between different ways of determination of correlation coefficient and also between LO theoretical prediction and above determinations. As one would expect at high statistics different approaches give at high statistic very close results.

One can make a very rough estimate of the needed statistics to distinguish between correlation coefficient as expected from Standard Model and no correlations at 3σ accuracy. To achieve this correlation coefficient should be estimated with precision better than 0.3 (prediction for corr. coef. from SM in LO approximation in beam frame is ≈ 0.9). As one can see from Fig. 50 rms of (unweighted) distribution in variable z is for all frames ≈ 0.3 . If we use estimate of correlation coefficient based on eq. (52) estimate of error is:

$$\sigma(C) = \frac{9\text{rms}(z)}{\sqrt{N}} \quad (55)$$

Where N is number of events. Result is ≈ 80 events. This estimate should be considered as a lower limit because all input data are taken from ideal condition set up. One can expect that in reality there will be needed to fight with selection and reconstruction bias.

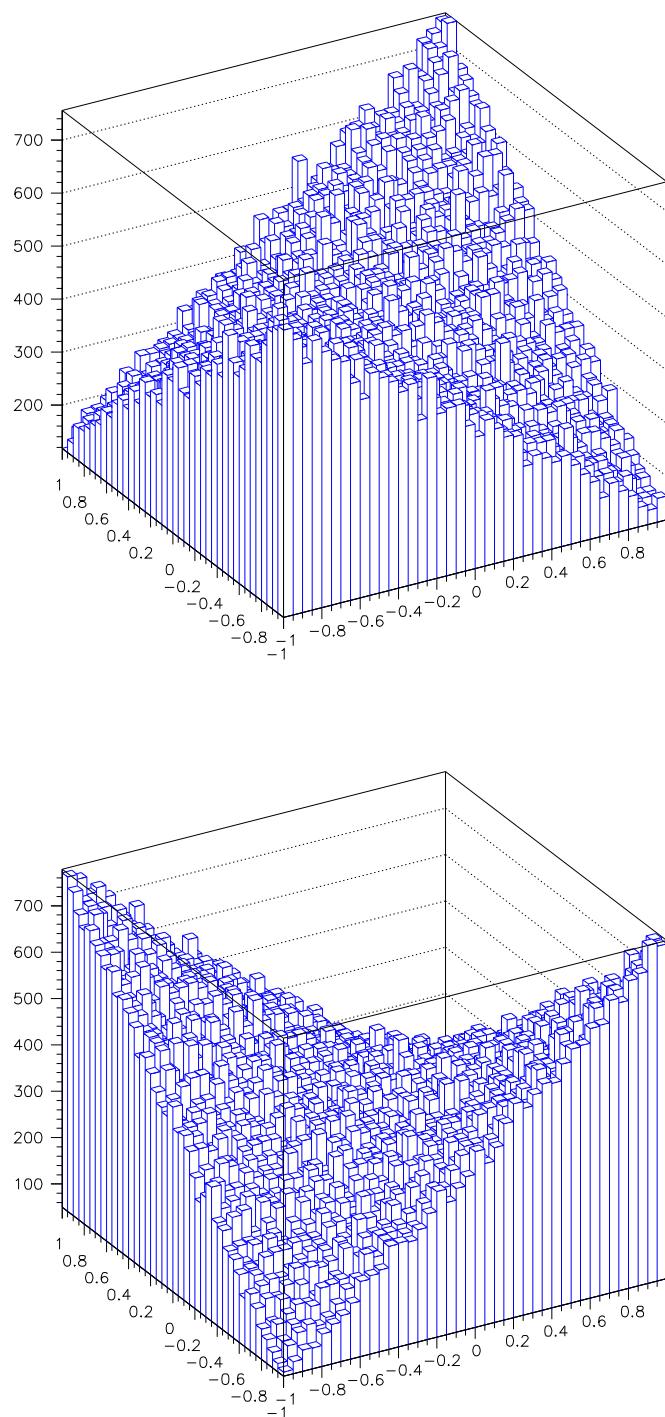


Fig. 48. A two dimensional distribution of $\cos(\theta_1) \times \cos(\theta_2)$ in beam frame I (upper plot) and beam frame II (lower plot) - TopRex

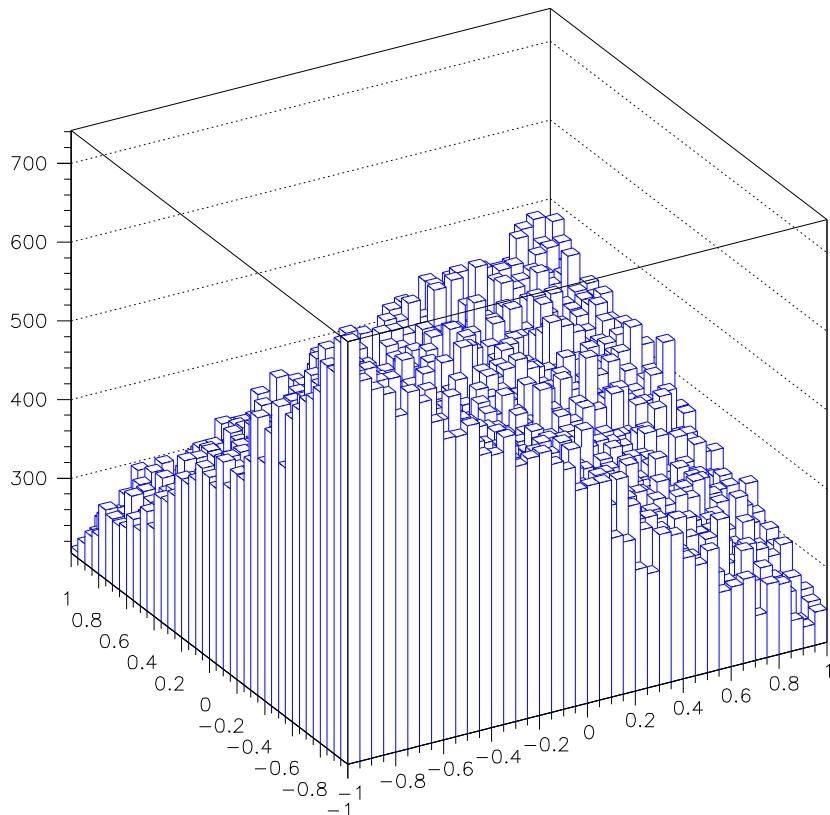


Fig. 49. A two dimensional distribution of $\cos(\theta_1) \times \cos(\theta_2)$ in helicity frame - TopRex

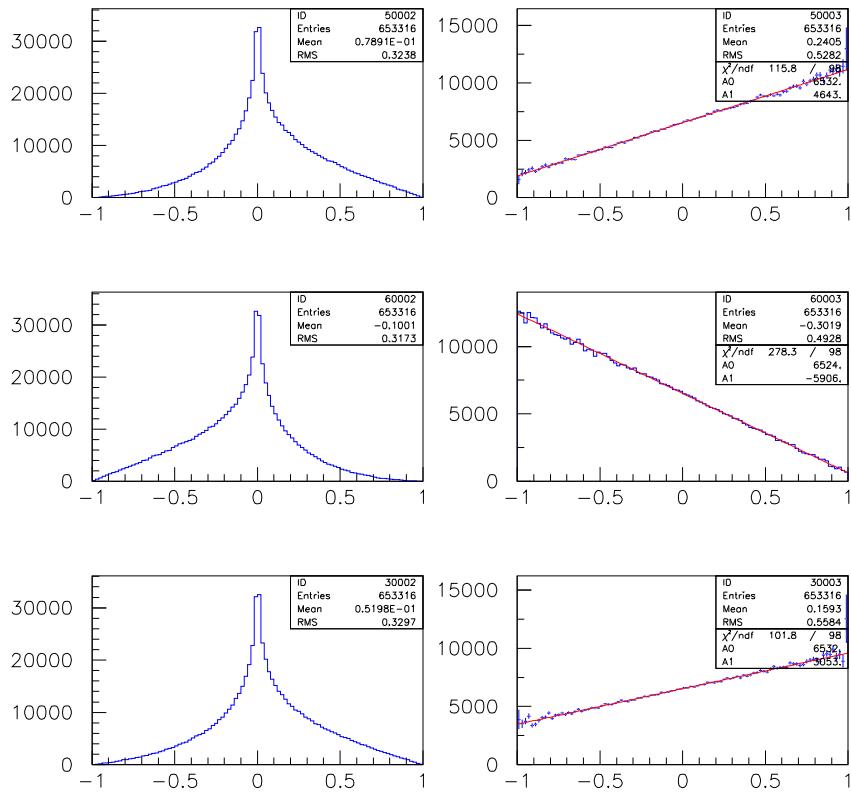


Fig. 50. Distribution of variable z in beam I,II frames and helicity frame (see text)

3.7 Measurement of top quark width and lifetime

Top quark lifetime and width are related through Heisenberg's uncertainty relation. Therefore if one is able experimentally measure one quantity it implicitly measures also other.

Lifetime can be measured directly if it is long enough that resolution of detector can distinguish it from 0. Otherwise measurement will result in the upper limit. It can be measured indirectly if there is relationship between observed specific feature and top lifetime and if some calibration is possible. e.g. one can imagine that amount of radiated gluons directly from top quark is proportional to the top quark lifetime.

Top quark width can be measured directly by top mass measurement. It can be measured indirectly e.g. by single top cross section measurement (see 2.1).

Two of the above mentioned options have been already realized and details are discussed below.

Direct measurement of top quark lifetime

A direct measurement of top quark lifetime was addressed by CDF study [73]. This study had no chance to measure top quark lifetime if Standard Model is correct. But it is important to make measurement even in such case (otherwise if we strictly believe to Standard Model any measurement is not necessary).

Idea behind this measurement is to measure with best precision detector allows distance between interaction point of $p\bar{p}$ scattering and decay vertex of W^\pm from $t \rightarrow Wb$ decay.

Study [73] is based on statistics 318 pb^{-1} collected by CDF collaboration between March 2002 and September 2004. Selection of enhanced $t\bar{t}$ candidate sample is close to one explained in section 3.1.2 - cross section measurement based on lept. + jets channel. Basically, isolated lepton, at least 3 high E_T jets (at least one of them b-tagged), $\cancel{E}_T > 20 \text{ GeV}$ are required. Requirements are strengthened on quality of reconstructed track corresponding to lepton (e or μ) in silicon vertex detector - at least three measurements in $r - \phi$ layers (transverse plane) for each track are required. Rejected are also events when effective mass of isolated lepton based on "tight" criteria and track passing loose lepton criteria requirements have effective mass between 76 and $106 \text{ GeV}/c^2$.

Finally, 97 electron tracks and 60 muon tracks for measurement of top quark lifetime have been selected.

As sensitive variable for a measurement of top lifetime is selected impact parameter d_0 of lepton from W-Boson decay. Impact parameter is defined as smallest distance between transverse projection of track and collision point. More precisely, implemented definition is: $d_0 = Q(\sqrt{x_0^2 + y_0^2} - \rho)$, where x_0, y_0 are x,y coordinates of center of circle (track of charged particle in magnetic field forms helix in space or circle in transverse plane), ρ is radius of the circle and Q sign of charge of the track.

Impact parameter resolution is measured by using leptons from Z^0 -Boson dilepton decay (Drell-Yan sample). Pair of leptons of the same flavor, opposite charge, satisfying tight lepton criteria with effective mass in range between 83 and $106 \text{ GeV}/c^2$ have been used for impact parameter resolution study. In Fig. 51 impact parameter distribution for electron and muon tracks is shown.

This distributions represents intrinsic detector resolution which includes uncertainty of the beam position ($\approx 25 \mu\text{m}$) as well as intrinsic resolution of the single track helix parametrization.

Background to $t\bar{t}$ production is dominated by "prompt" leptons from W-Boson decay. In W-

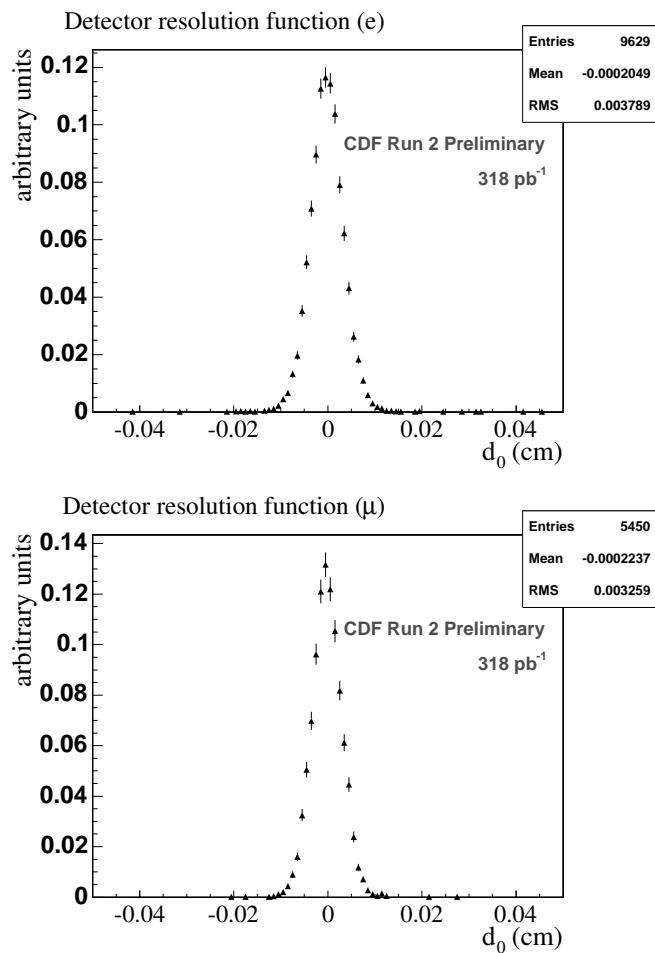


Fig. 51. impact parameter resolution for isolated electrons and muons from Z^0 decay

Boson decay there is the τ decay fraction present and τ 's mean lifetime is relatively large ($c\tau \approx 90\mu m$). Another component which broadens impact parameter distribution is QCD component of background (non W background) where leptons come from heavy flavor (b,c jets). Electrons from electron positron conversion which somehow pass through selection procedure are also source of fake “long lifetime” component. And also muons from cosmic which passed cosmic ray filter. Sum of these “long lifetime” contributions and “prompt component” for electron and muon tracks are shown in Fig. 52. Based on experimentally determined impact parameter resolution by convolution of resolution with exponential decay of preset lifetime of top templates for $c\tau$ in range of 0-500 μm have been produced. See Fig. 53. Binned likelihood method is used to fit combination of background and signal templates to data. Result of a comparison of data and the best fit is shown in Fig. 54.

Fit converges to 0 lifetime. Statistical Feldman-Cousins (FC) [75] approach is used to estimate correspondence between measured and true lifetime at 95 % confidence level (CL). Result (which takes also into account systematic errors) is presented in Fig. 55.

From FC analysis follows that at 95 % CL mean lifetime $c\tau$ of top quark is smaller than 52.5 μm or $1.75 \cdot 10^{-13}$ s!

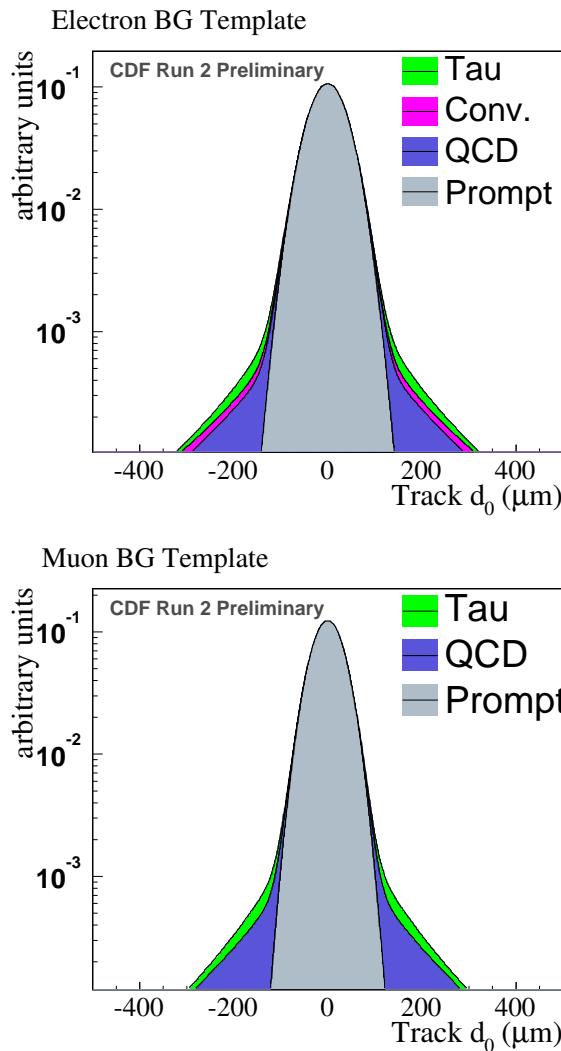


Fig. 52. Influence of different kind of background on impact parameter distribution for electron and muon tracks

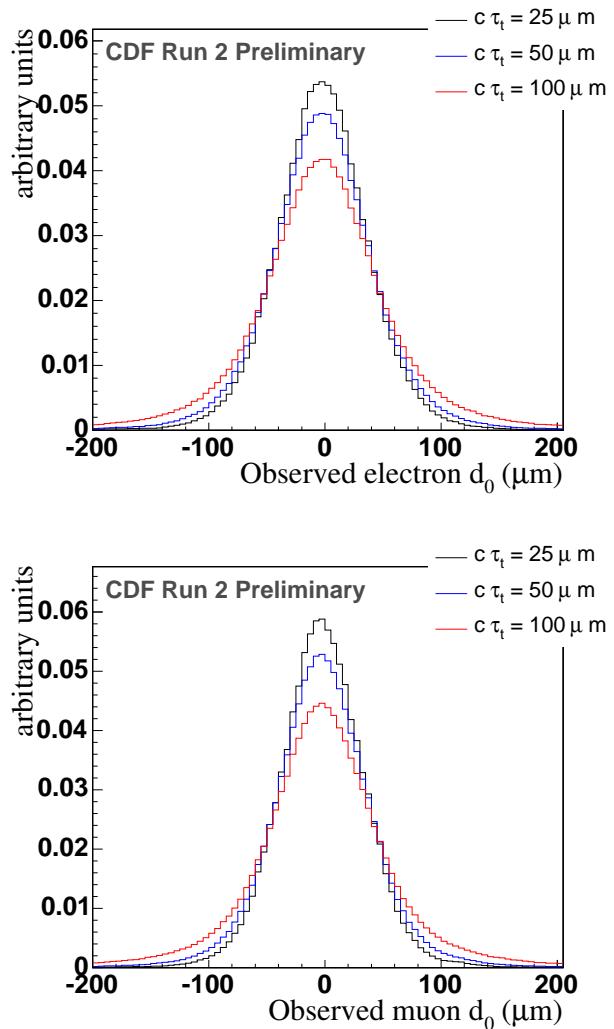


Fig. 53. Templates for $t\bar{t}$ signal simulated for different top lifetime electron and muon tracks impact parameter distribution.

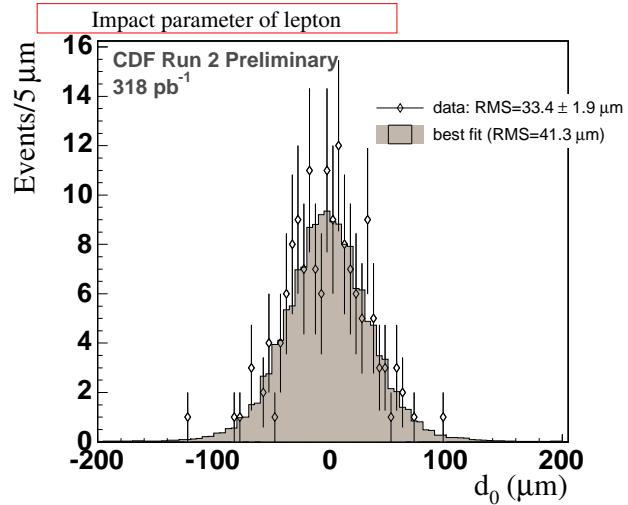


Fig. 54. A comparison of impact parameter distribution of leptons for $t\bar{t}$ candidates and best fit by templates

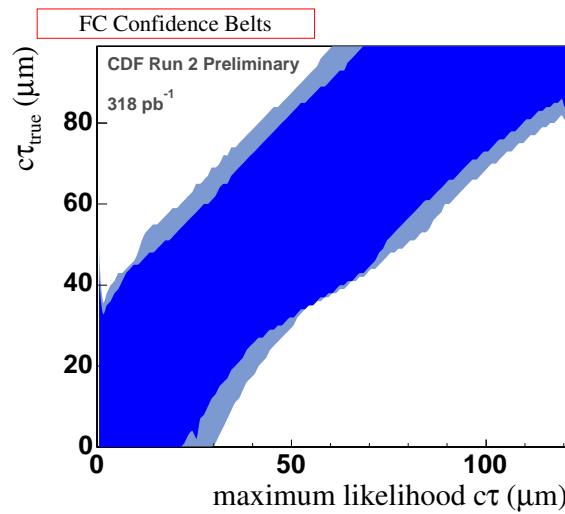


Fig. 55. Feldman-Cousins confidence level bands between measured and true top $c\tau$ measurement

Top quark width measurement

Question of top quark width is addressed by another CDF analysis [74] based on statistics of integrated luminosity $\approx 1 \text{ fb}^{-1}$.

Idea behind this analysis is to use top mass reconstruction and templates for different top width and to fit it to data. Analysis was done on statistics 1 fb^{-1} sample, lept. + jets channel. Main difference in event selection in a comparison with section 3.1.2 is that 4 jets are required but in case when 2 jets are b-tagged there is applied relaxed criterion on 4-th jet selection. In this case fourth jet is required to have $E_T > 8 \text{ GeV}$ only (each of other at least $E_T > 15 \text{ GeV}$). Selection passed 171 events with one jet b-tagged (expected signal / background ratio 3.7) and 82 events with at least 2 b-tagged jets ($S/B = 10.6$).

Top mass reconstruction follows well tested procedure [46] used in the past many times for reconstruction of top quark mass by a comparison of reconstructed observable top mass with templates created for preset values of top mass parameter. In top quark width study top quark mass is considered known, fixed to $M_{top} = 175 \text{ GeV}/c^2$ and templates are produced for range of different top width Γ_t^{input} . Top quark width parameter is appropriately set in generator PYTHIA [16] and generated events pass through detector simulation and event reconstruction chain. Finally, above selection procedure is applied and by using observable top mass reconstruction procedure [46] top mass observable is reconstructed. Distribution of top mass observables (m_t^{reco}) for a given Γ_t^{input} forms a template. Templates are normalized and parametrized and probability density function $P_s(m_t^{reco}; \Gamma_t^{input})$ is created.

Examples of parametrization for Γ_t^{input} 1.5, 30, and 50 GeV is shown in Fig. 56. An a priori

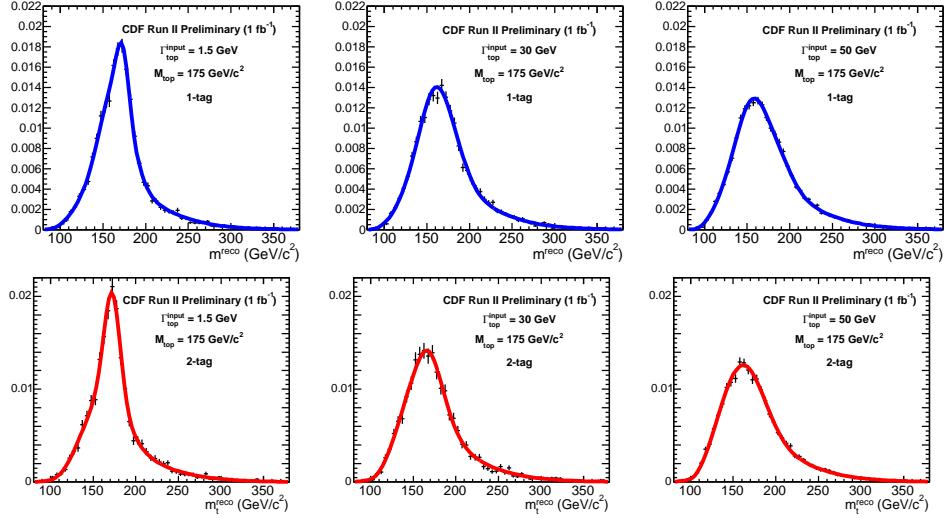


Fig. 56. Parametrization fits to the reconstructed top mass distributions for Γ_t^{input} 1.5, 30, and 50 GeV. Top: 1- b-tag sample, Bottom: 2-b-tag sample.

estimate for background composition is used to obtain m_t^{reco} shapes for background.

The reconstructed top mass distributions from data are compared to signal and background templates using unbinned likelihood fit. Definition of likelihood is given below.

$$\mathcal{L} = \mathcal{L}_{shape} \times \mathcal{L}_{bg} \quad (56)$$

$$\mathcal{L} = \frac{e^{-(n_s+n_b)}(n_s+n_b)^N}{N!} \prod_{i=1}^N \frac{n_s P_{sig}(m_i; \Gamma_{top}) + n_b P_b(m_i)}{n_s + n_b} \quad (57)$$

$$-ln(\mathcal{L}_{bg}) = \frac{(n_b - n_b^{exp})^2}{2\sigma_{n_b}^2} \quad (58)$$

Where n_s, n_b are expected number of signal and background events, N is the observed number of events with observed top mass per event m_i . The likelihood \mathcal{L}_{shape} is the joint probability density for sample of N reconstructed events with observed top mass m_i and fraction of background $n_b/(n_s + n_b)$. Gaussian term \mathcal{L}_{bg} constrains the number of background events to expected number of background events n_b^{exp} within uncertainty σ_{n_b} . Γ_{top} is determined by maximum likelihood fit to the observed top mass distribution. Method was tested on samples with fixed top width and fitted width distribution is shown in Fig. 57. To set a limit on top width

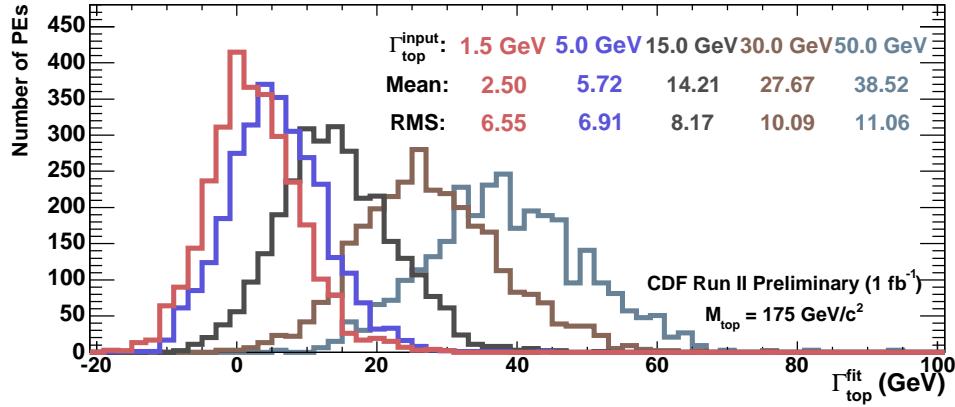


Fig. 57. Fitted width distribution for 3000 pseudo-experiments using MC samples with various Γ_t^{input}

Feldman-Cousins prescription [75] is employed. The FC prescription guarantees a physically meaningful result even in case when fitted values are close to unphysical or in unphysical region (e.g. $\Gamma < 0$). As an ordering principle for selecting acceptance region and creating acceptance bands likelihood ratio is used. Likelihood ratio is defined:

$$R(x) = \frac{P(x|\Gamma_0)}{P(x|\Gamma_{max})}$$

Where $R(x)$ is a likelihood ratio at x , Γ_0 is the given input top width and Γ_{max} is the input top width that yields the maximum likelihood among all the possible width for a measured width x . The systematic uncertainties are incorporated to the confidence bands by convolution probability

density function of $\Delta\Gamma$ due to systematics with fitted width function. The new width function is used to create the confidence bands with systematics.

For analysis of data likelihood eq. (56) is used separately for single b-tag sample and 2 b-tag sample. Fit for single b-tag sample was $\Gamma_{top}^{fit} = -1.9$ GeV and for 2-b-tag sample $\Gamma_{top}^{fit} = -7.4$ GeV. A comparison of data and fit is shown in Fig. 58. Product of likelihoods for single b-tag sample and 2-b-tag sample is used for the fit of top width based on both samples. Result is $\Gamma_{top}^{fit} = -4.86$ GeV.

All fit results are in unphysical region and mean that top width is small, close to 0 within resolution. In Fig. 59 95% Confidence Level bands produced by Feldman-Cousins prescription taking into account systematic errors are presented. Fitted value Γ_{top}^{fit} for combined sample is marked there. It follows that $\Gamma_{top}^{fit} = -4.86$ GeV can be interpreted with 95 % CL as upper limit $\Gamma_{top} < 12.7$ GeV or top lifetime $\tau_{top} > 5.2 \cdot 10^{-26}$ s.

We can combine two independent measurement - top lifetime and top width measurement to set 95% CL for upper and lower limit on top quark lifetime.

It follows that in 95% CL experimentaly determined top mean lifetime is in range

$$5.2 \cdot 10^{-26} < \tau_{top} < 1.75 \cdot 10^{-13} \text{ s.}$$

This result is consistent with lifetime expected for top mass $175 \text{ GeV}/c^2$ from Standard Model $\approx 4 \cdot 10^{-25} \text{ s!}$

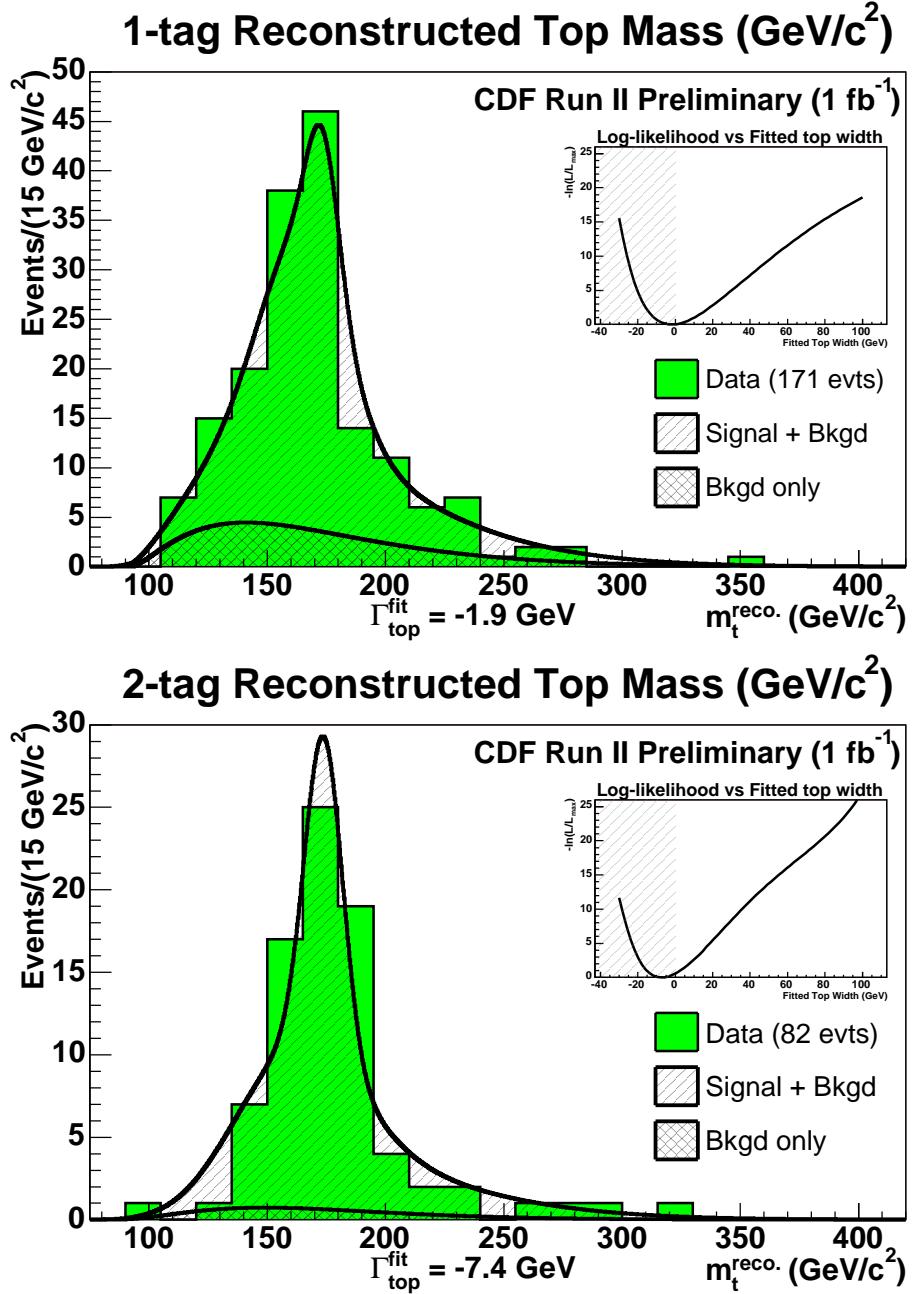


Fig. 58. Likelihood fit of top width to reconstructed top mass distribution of data. Upper plot for 1-b-tagged sample, lower plot for 2-b-tagged sample.

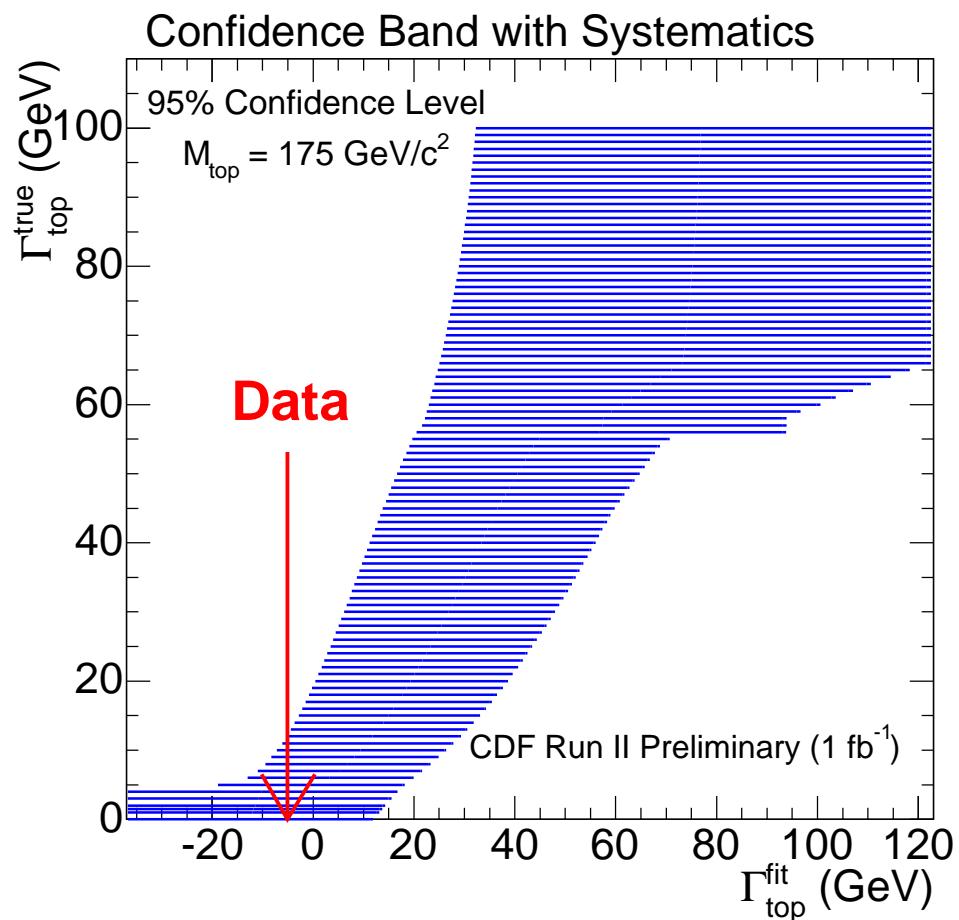


Fig. 59. Feldman-Cousins 95% Confidence Level bands. Mapping of fitted top width values to the range of corresponding true top width.

4 Concluding remarks

Over the past years was invested tremendous effort to the measurement of top quark physical parameters. Ingenious methods have been developed to push experimental limits as far as possible. Standard Model was challenged and up to now successfully passed all challenges. Still both experiments CDF and D0 did not say a final word. It is realistic expectation that at the end of the run II accumulated integrated luminosity will be around $6\text{-}8 \text{ fb}^{-1}$. What kind of improvements can be still expected and what kind of questions will need to be answered by successors of TEVATRON - LHC and ILC?

Top quark pair candidate sample is also region where new signal outside of the standard Model domain can be present. Therefore it is necessary carefully and critically analyze this sample. With larger statistics it will be possible to make detailed statistically significant study based on specific subsamples. e.g. separate analysis for electron sample, muon sample.

Let's try to predict what will happen with topics discussed in this paper when integrated luminosity will be $6\text{-}8 \text{ fb}^{-1}$.

$t\bar{t}$ cross section - uncertainty will be dominated by systematic errors.

Top quark mass - uncertainty will be dominated by systematic errors and it will be necessary to implement new ideas to push them down. For a long time most significant systematic error - jet energy scale is by *in situ* calibration reasonably under control. It can be pushed down inversely proportionally to square root of the statistics. Also combination with $Z \rightarrow b\bar{b}$ can be explored. That means that other systematic errors will dominate and there does not seem to exist similar solution like with jet energy scale. Statistical error of combined measurement is already below systematic error. New ideas are needed. One possibility is to concentrate on methods which methodically do not provide best resolution but have advantage of smaller systematic errors.

Single top - stage of discovery is expected to be achieved (5σ). However detailed study of this channel will be probably left to LHC. Interesting topics can be measurement of top spin polarization and measurement of the top width. Channel is very sensitive to a new physics

Top charge - definite answer achieved different ways in different channels. In two hypothesis scenario - rejection of the wrong hypothesis with at least 4σ significance.

W helicity - a two dimensional model independent measurement of helicity fractions separately for electrons and muons in lept. + jets and dilepton channel, uncertainties factor two smaller than present

Spin - Spin correlations - measurement of the correlation coefficient, consistency with Standard Model prediction challenged

Top quark lifetime and width - improvements in the top mass methodology can push upper limit on top width to a more challenging values. Also combination with single top cross section can help. Direct lifetime measurement is saturated by resolution of detector element (silicon vertex detector). One can hardly expect improvement from this side.

What was mentioned is straightforward extrapolation but there is also space for surprises.

What will be left for experiments which take over from TEVATRON? Agenda of these experiments is focused on Higgs. But huge statistic of top production (pair and single) gives an opportunity for detailed studies which are impossible at TEVATRON. e.g. Spin-Spin correlations measurements with few percent precision [76], measurement of cross section of $t\bar{t}\gamma$ channel (this way is direct measurement of the absolute value of the top charge possible).

Precision measurements of top quark mass based on measurement of threshold $t\bar{t}$ production at

International Linear Collider (ILC) will push uncertainty almost order of magnitude lower, ratio on $R = \frac{\sigma(e^+e^- \rightarrow \text{hadrons})}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)}$ after $t\bar{t}$ threshold will give definite support for true value of top charge. LHC should start this year (2008) therefore interesting results can be expected soon.

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