ON THE SIGNIFICANCE OF THE DILATATIONS FOR HIGH ENERGY PHYSICS

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(Presented by H. A. KASTRUP)

The physical significance of the dilatations

$$x^{\mu'} = \varrho x^{\mu}, \quad \mu = 0, 1, 2, 3; \quad \varrho \text{ real and} > 0, \quad (1)$$

is still controversial and to a large extent unknown. Because this transformation group transforms a given value of a physical quantity continuously into other values of the same quantity, the low energy nuclear physics with its discontinuous energy spectra for instance seems to be beyond the scope of the dilatations (1).

Nevertheless it may be that the dilatations become essential when the continuous kinetic energies of atomic particles are so large that the rest masses and all other discontinuous energy states of the particles are negligible. If this hypothesis turns out to be right, then one would have an elegant and powerful method of describing, for instance, the asymptotic behaviour of scattering quantities at very high energies [1].

This hypothesis of the asymptotic dilatation invariance at very high energies has, among others, the following consequences.

1. It can easily be seen that the conserved quantity D, which belongs to the continuous transformations (1), for free particles has the form

$$D = Et - \vec{r}\vec{p}, \quad E = (p^2 + m^2)^{1/2}, \quad (2)$$

where $\vec{r}$ is the position of the particle at the time t, $\vec{p}$ its momentum and E its energy.

In the wave picture D is the phase of the wave at the centre of the wave packet. Therefore we call D the «Central Phase» of the particle. Since $\vec{r} = \vec{p}/E \cdot t + \vec{b}$, one sees immediately that the quantity D is indeed a constant of motion at very high energies!

In reaction processes, which are characterized by dilatation invariant interactions, the sum of the Central Phases of the particles, which are involved in the scattering reaction, should therefore be the same before and after the scattering. Combining this asymptotic conservation law with the Eisenbud — Wigner formulation of the particle retardation in scattering processes by means of phase shifts [2], one gets for the phase shifts $\delta_l(p)$ in relativistic potential scattering:

$$\lim_{p\to\infty} \delta_l(p) = \text{const} \neq 0. \quad (3)$$

This is in agreement with exact solutions of the Klein — Gordon and Dirac equations with Coulomb- and other potentials [3].

2. We illustrate the consequences of the dilatations (1) for the relativistic scattering theory in the case of two scalar particles with equal masses, which are scattered elastically and have the four momenta p_1, p_2 before and the four momenta p_1', p_2' after the scattering. The method used in this case may be applied also to inelastic processes, partial waves etc.

The differential elastic cross section in the c. m. system is

$$d\sigma_{el} = \frac{1}{s} |F(s, t)|^2 d\Omega; \quad s = (p_1 + p_2)^2,$$
$$t = (p_1 - p_1')^2 = -2q^2(1 - \cos\vartheta),$$
$$q = |\mathbf{p}_1|.$$

Further, let M be the largest rest mass, which has to be considered in the scattering process — M is the rest mass of the scattered particles or the rest mass of an intermediate state.

A four momentum p is transformed under the dilatations (1) at very high energies into $\varrho^{-1}p$ and the arguments of the relativistic scattering amplitude $F(s, t)$ therefore go over into $\varrho^{-2}s$ and $\varrho^{-2}t$. The essential question now is, in which way $F(s, t)$ itself is transformed. We discuss the following two cases:

a) $s \gg M^2$, $-t \lesssim M^2$ (forward scattering). Our first assumption in this case is that an asymptotic expansion of $F(s, t)$ in s exists with the leading term $F_0(s, t)$. If the dilatations (1) are of importance for the scattering amplitude at very high energies, then this must express itself in the structure of $F_0(s, t)$ with regard to its s-dependence. Nothing can be said about the t-dependence of $F(s, t)$ since $-t$ is of the order of magnitude of the rest masses, for which there is generally no dilatation invariance.

Now we make the following, somewhat formal hypothesis, the validity of which has to be checked by experiment: As the physical quantities usually form an irreducible representation of the symmetry group connected with them, we suppose that the structure of $F_0(s, t)$ is closely related to irreducible representations of the dilatations: the homogeneous functions. Therefore we make the assumption that $F_0(s, t)$ is an homogeneous function in s:

$$F_0(s, t) = \beta(t)\, s^{\alpha(t)}. \qquad (4)$$

Obviously α and β may be functions of t but nothing can be said in the framework of the dilatations about their special t-dependence. α even may be independent of t.

The asymptotic form (4) has been discussed in the past few years to a large extent in connection with the complex angular momenta [4]. The group theoretical relation between these two aspects lies in the property that the spatial rotations and the dilatations (1) commute and therefore may be diagonalized simultaneously.

b) $s_1 - t \gg M^2$. In this case the same arguments as above lead to the conclusion that the leading term $F_{00}(s, t)$ in the asymptotic expansion of $F(s, t)$ in s and t now is homogeneous in s and t:

$$F_{00}(\varrho^{-2}s, \varrho^{-2}t) = \varrho^k F_{00}(s, t).$$

Because of our hypothesis that in the asymptotic region all rest masses and any other elementary length as, for example, coupling constants with a dimension of a length are unimportant s and t are therefore the only carriers of a dimension of a length and for that reason k must be equal to zero, since $F(s, t)$ is dimensionless.

We have therefore

$$F_{00}(s, t) = F_{00}(s/t). \qquad (5)$$

From this it follows for fixed scattering angle $\vartheta \neq 0, \pi$ that

$$d\sigma_{el} \sim A(\cos\vartheta) \cdot \frac{1}{s} d\Omega \quad \text{for } s \to \infty.$$

These examples show, how one may test experimentally, whether or not the dilatations (1) become essential at very high energies in the cases of strong, electromagnetic and weak interactions.

In the framework of the Lagrange formalism only theories with dimensionless coupling constants seem to be compatible with dilatation invariance at very high energies as, for example, Quantum Electrodynamics and the theory of weak interactions with the Intermediate Boson.

Systematic experimental deviations from the above and similar consequences of the dilatations (1) would show that there are fixed length, which are essential even at very high energies and which therefore destroy the dilatation invariance.

A rough estimation by means of the conserved quantity D shows that the dilations should become essential at c. m. energies about 50 times as large as the rest energies of the particles involved. The possibility of an experimental test of the above and similar consequences is therefore at present most promising for electromagnetic interactions particularly for Colliding-Beam experiments. In the case of strong and weak interactions the accelerator energies, which are available at present, seem to be too low. The known, but rather inexact results of cosmic ray physics have not yet been analyzed with regard to the dilatation invariance.

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DISCUSSION

J. Vigier

The quanity ϱ was first introduced by Weyl. This amounts to introduction of torsion in space-time. Now the problem is whether this does not modify the irreducible representations of the Lorentz group. So your assumption about asymptotic behaviour is only an assumption before you demonstrate this. Now can you enlarge on that?

H. A. Kastrup

In contrary to Weyl, who considered ϱ as a space-time function, our ϱ is a constant and therefore, I suppose these troubles do not occur in this way. But there are some problems in connection with this question, which have to be investigated further.

G. Marx

What can be said about the asymptotic behaviour of the total cross-section?

H. A. Kastrup

According to the optical theorem one has

$$\sigma_{tot}(s) = \frac{4\pi}{q\sqrt{s}} ImF(\varrho,\ t=0)$$

Therefore, to make predictions, one has to know the value of α (t = 0). But, as I pointed out, in the framework of dilatation symmetry nothing can be said about this value.

D. D. Ivanenko

The question of the asymptoticdilatation invariance is apparently not quite clear if we do not analyze the extent of the nonanalytic dependence on some parameter. It suffices to recall the case of Tusek's group $(\gamma_5)_H$ or the gauge group in electrodynamics. The groups either exists or not. the invariance cannot vanish suddenly.

H. A. Kastrup

The more mathematical form of the problem, which arises in connection with dilatation invariance is the following: let a physical quantity A is given with an eigenvalue spectrum, which consists of a discontinuous and a continuous part. If one then defines a group, which transforms the eigenvalues of the continuous part in to other values of this part, one has exactly the situation which occurs in the case of the dilatations. The question is now: Is it possible by means of that transformation group, to derive additional properties for those quantities which are functions $f(A)$ of the quantity. This should be the case at least in an approximate sense. As far as I know this problem has not been investigated in a rigorous mathematical way. From the physical point of view one should expect that one can derive additional useful relations, in the same or similar way as I have demonstrated in my paper. In that case, I think, an exact definition is given, how to define dilatation invariance in an asymptotic sense. On other cases further investigations are necessary.