Structures of K.Saito Theory of Primitive Form in Topological Theories Coupled to Topological Gravity

A. Losev*

Institute of Theoretical and Experimental Physics
B.Cheremushkinskaya 25
117259 Moscow, Russia

Abstract. Structure of topological theory coupled to topological gravity is studied on a typical example - Landau-Ginzburg theory. It is shown that all main ingredients of K.Saito theory of primitive form are implied in such gravity theory. Filtration, that he considers turns out to be filtration by degrees of Morita-Mamford classes, and can be considered as a filtration of equivariant cohomologies (equivariance with respect to rotation of local coordinate). Higher residue pairing are nothing by pairing in equivariant cohomologies induced by integration over C^d. Section is the kernel of a contact term map. Axioms on goods sections follow from the symmetry of n-point correlation functions on genus zero.

1 Introduction

Topological theories coupled to topological gravity[1, 3] seem to be interesting because:
1. They look like the simplest string-like theories, and by studying them we can get some information about what string theory is. Moreover, some of them are equivalent[1, 2, 9] to noncritical strings for $c < 1$
2. These theories naturally connect geometry of the space of complex structures[1, 6], geometry of algebraic manifolds[1, 7] and theory of integrable systems[5, 8, 17]

In this paper we try to study general formalism of topological theory coupled to gravity on example of Landau-Ginzburg model. We show that the topological matter is descibed by the ring and a linear functional on it. In the case of Landau-Ginsburg theory ring is a Jacobian ring of singularity and linear functional is

*This work was partially supported by Grant 93-02-14365 of the Russian foundation of fundamental research.

given by zero-th higher residue pairing of K.Saito [11]. This linear functional involves additional data - holomorphic top form due to fermionic anomaly. We discuss coupling theory to topological gravity and stress the problems in performing such coupling for massive theory: these problems come from the boundary of the moduli space. Then we show how in conformal theory descendents[15, 16, 10] could appear in matter theory from fields with logarithmic anomalous conformal dimension, and illustrate it on example of Landau-Ginzburg theory. We find that observables in topological thery coupled to gravity theory form filtration in the degree of gravitational descendents, and identify this filtration with the space of equivariant cohomologies with respect to rotations of local coordinates. This is the same filtration that appeared in K.Saito's theory [11]. His higher residue pairing is identified with the pairing in equivariant cohomologies induced by integration over target space. Integration over position of marked points leads to flow on the space of theories equipped with the connection in the bundle of observables (fiber of this bundle is a filtration of equivariant cohomologies). This connection arises from the contact term that we calculated up to the kernel of this map(that is a subspace in the fitration with dimension of the ring). Such a kernel is called section in K.Saito theory. Integration over positions of the marked points could be interchanged, moreover, 3-point function could be obtained in different ways. Physical requirement of consistency leads exactly to K.Saito axioms of good section, that involve higher residue pairing. K.Saito proved the integrability of connection arised from good section. To fix invariance over diffeomorphisms we choose some versal deformation of singularity. Then flows due to descendents lead to change of holomorphic top form. Finally we describe generating function for correlators in genus zero generalising algebraic solution to dispersionless KP found in this context by I.Krichever [8].

2 Topological matter

By topological matter[18, 14, 19] we mean a 2-d theory with a scalar fermionic symmetry Q, such that its square equals to zero, and with an action S_m, that on the Riemann surface with the metric g has the following form:

$$S_m(\phi, g) = S_{top}(\phi) + Q(R(\phi, g)) \tag{1}$$

Here we assume that action is invariant under diffeomorphisms that act on both matter fields ϕ and metric g; the first term in (1) is metric independent and thus we call it topological term. The second term in (1) depends on metric but is Q-trivial. Function R here and below we will call regulator.

From the form of action (1) we easily get that the energy-momentum tensor T is Q-exact.Really, if we define

$$G(x) = \frac{\delta R}{\delta g} \tag{2}$$

then

$$T(x) = \frac{\delta S_m}{\delta g(x)} = Q(G) \tag{3}$$

i.e. G is a superpatner of the energy-momentum tensor.

Let us denote by $<>^M$ correlator of local fields in the topological matter theory(here superscript M means matter , not to be confused with correlator in topological matter coupled to topological gravity, that would appear later):

$$< \Phi_1(z_1), \dots, \Phi_n(z_n) >^M = \int D\phi_m \Phi_1(z_1), \dots, \Phi_n(z_n) \exp(S_m) \qquad (4)$$

Here ϕ_m stands for fields in the matter theory, z_i denotes different points on the Riemann surface.

Since Q is a nilpotent symmetry of the theory, the exactness of T leads to independence of correlator of Q-closed local operators on metric on the Riemann surface and thus on their positions. It means that correlators of such operators are topological, i.e. depend only on the type of operators and genus of the worldsheet.

Definition. Local fields that are Q-closed are called local observables in the topological matter theory. Correlator of local observables in topological theory is called topological correlator.

It is obvious that topological correlator of a Q-exact local observable is zero, thus topological correlators are non-zero on the Q-cohomologies of the space of local operators.

These cohomologies form a ring, that we will denote J. Multiplication can be defined by putting two local observables at two different points and moving them to each other. Since energy-momentum tensor is exact, one can show that different limits of such a motion would differ on a Q-exact local field. Topological correlator could be computed by putting all observables together, i.e. multiplying them in cohomology ring J, described above. Thus, any correlator on a Riemann surface could be reduced to one-point correlator, and for n-point correlator in genus q in topological matter theory we get :

$$< \Phi_1(z_1), \dots, \Phi_n(z_n) >_q^M = < (\Phi_1 * \Phi_2 * \dots * \Phi_n) >_q^M \qquad (5)$$

where the star denotes multiplication in cohomology ring J, and $<>_q^M$ is just a linear functional on cohomologies[1].

3 Partition function in topological gravity coupled to topological matter

Following [3] we define topological gravity as a theory with the following fields: metrics g on Riemann surfaces and its superpartner Ψ, that is an element of the tangent bundle to the space of all metrics; this theory has fermionic Q_G-symmetry that acts as an external derivative:

$$Q_G(g(x)) = \Psi(x) \qquad (6)$$

[1] Using "handle gluing" operation one can show that linear functionals for all genera could be expressed just through structure constants of the ring and linear functional for genus zero

and topological part of the action is zero. Thus, topological gravity itself should be some cohomological theory on the space of all metrics on a Riemann surface. However, such a theory is ill-defined, since this space is infinitely-dimensional and non-compact. The way out is to consider equivariant cohomologies with respect to the action of some Lie group, i.e. to fix the supergroup, associated with this group[2]. We will take this group to be the product of group of conformal transformations of metrics and group of diffeomorphisms. It would reduce integration to an integral over the moduli space of conformal structures.

Still there is a problem of constructing a form to be integrated over moduli space. Originally this form was constructed from ghosts, that appear in the procedure of gauge fixing[3]. It is possible to treat ghosts, conformal factor, and their superpartners as some specific conformal topological theory coupled to "gravity" (the only thing that left from gravity is the space of moduli of complex structures). Obvious generalization is to change such a special topological theory on a general one.

We suppose that after coupling topological matter to topological gravity the total fermonic symmetry is a sum of a matter fermionic symmetry and gravitational one:

$$Q_t = Q_G + Q \tag{7}$$

thus, the action is a sum of S_{top} and Q_t-exact term,i.e.

$$S_{M+G} = S_{top} + Q_t(R) = S_m + Q_G(R) \tag{8}$$

Space of metrics forms a bundle over the moduli space of conformal structures. Fixing supergroup means integration over the section of this bundle[3].

Conformal regulator

If we take such a regulator R, that trace of G is zero, then $\exp(S_{M+G})$ as a form on the space of metrics is invariant under conformal transformations and horizontal(its contraction with the vector, tangent to the fiber is zero). Let $\sigma(m)$ be a section from the moduli space to the space of all metrics, perhaps with vertical jumps. Let σ' be a continuous hypersurface that is obtained by gluing to σ hypersurfaces W_i, that are vertical, i.e. tangent space to any point of W_i contains vertical vector. Then we define

$$Z = \int_{\sigma'} \int D\phi_m \exp(S_{M+G}) = \int_{\sigma'} < (Q_G(R))^{6q-6} >^M \tag{9}$$

Since trace of G is zero, an integral over vertical hypersurfaces is zero. Using this, one can show that for conformal regulator partition function Z does not depend on the section. In this way it could be continued on the compactified moduli space that we will consider below in applications.

[2]Consider supersymmetric system as a space of differential forms on some space; then the action of the element of the Lie algebra (associated with the Lie group) on forms is represented by taking Lie derivatives along the vector. Superpartner to this element of the Lie algebra acts on forms by internal contraction of them with this vector field

4 Anomalies

This section was done in collaboration with M.Bershadsky

Here we will make preliminary attempt to study anomalies in topological gravity. In this section we will consider non-compactified moduli space.

Naively, action (8) looks like an action in theory with fixed ordinary gauge symmetry, where we expect that nothing depends on the regulator R. Let us check what happens with expression (9) over noncompactified moduli space even for a smooth section when we a)change the section, b) change the regulator. We will find that anomaly(dependence on section and regulator) comes from the boundary of that moduli space.

Section anomaly for massive regulator

Let us define the form on the space of metrics:

$$F = \int D\phi_m \exp(S_{M+G}) \tag{10}$$

One can explicitly check that this form is closed, so its integral over the boundary of some manifold in the space of metrics is zero. If two sections from moduli to metrics coincide near the boundary of the moduli space, then they form a boundary and integral of F over these sections coincide. If two sections do not coincide, one can form a cylinder-like (vertical) hypersurface, that is a boundary: its upper and lower bases are sections ; its lateral surface L is formed by vertical segments, connecting points on two sections, that are projecting on the same point on the boundary of the moduli space.

Thus, we have the following anomaly in the massive theory

$$\int_{\sigma_1} F - \int_{\sigma_2} F = \int_L F \tag{11}$$

Anomaly in changing a regulator

Anomalous dependence of Z on R could be found as follows: one can show, that under the change of R

$$\delta F = d_G(\int D\phi_m \delta R \exp(S_{M+G})), \tag{12}$$

Let ∂M denote the boundary of the moduli space, then

$$\delta Z = \int_{\partial M} \int D\phi_m \delta R \exp(S_{M+G}) \tag{13}$$

This dependence on R is quite similar to holomorphic anomaly, found in [7].

That is why below we will consider only conformal regulators. If for some theories it is impossible to find such a regulator (it happens, for example, in topological sigma models of type A on a manifolds with non-zero first Chern class) it seems that this theory is should be considered in the asymptotically conformal limit, i.e. when trace of G is very small and we can neglect it.

5 Local observables in theories coupled to topological gravity

Like in the string theory, we will place local observables at marked points on the Riemann surface. In 2-d gravity it means that such Riemann surfaces are obtained by cutting n nonintersecting discs from the compact surface, and gluing semiinfinite flat cylinders to n boundaries. Placing local observables at marked points means that first we take cylinders to be of the finite length l, then we glue the boundary of the cylinders by a disc with local observable inserted in the middle of it, and finally we take the limit l goes to infinity. Typical example of such metrics are Penner metrics, constructed for fat graphs. Under conformal projection on a compact Riemann surface discs are projected to points (called marked points); these points never coincide.

Along with the obvious conditions on local observables, namely, local observable should be Q-closed and be primary conformal field of zero dimension, we will impose one additional condition, namely equivariance(horizontality) condition:

$$G_{L,0}\Phi = 0; G_{R,0}\Phi = 0, \tag{14}$$

here subscripts mean components of G ,acting on the left and right movers respectively. Obvious conditions are needed to preserve Q_t invariance. To make correlator in conformal theory covariant under coordinate transformations one has to multiply field of dimension Δ by the $(\det g)^{\Delta}$, so condition of being a conformal field with zero dimension is needed for Q_G invariance of the observable.

Appearance of the equivariance condition was explained in string theory[20, 21] using operator formalism. In that formalism a form on the space $Z(M)$ of complex structures of Riemann surfaces with local coordinates [3] at marked points was constructed. In the case of genus zero with n marked points, for example, the pairing of this form with the $2(n-3)$ vectors, tangent to $Z(M)$, is given by the functional integral:

$$\int D\phi_m \prod_{j=n-2}^{n} \Phi_j(P_j) \prod_{i=1}^{n-3} \int_{\gamma_i} v_i(z) G_L \int_{\gamma_i} \bar{v}_i G_R \Phi_i(P_i) \exp(S_m) \tag{15}$$

here γ_i is a contour around marked point P_i, and v_i is a holomorphic vector fields inside this contour. If this vector field equals zero at P_i, it corresponds to change of local coordinates leaving position of the marked point P_i fixed, such vectors fields are tangent to the fiber of the bundle $Z(M)$. If the vector field is nonzero at marked point, its value at P_i is identified with the tangent vector to M, and corresponds to the motion of the marked point P_i along $v_i(P_i)$ on the Riemann surface (accompanied with local coordinate).

To reduce this form to the base M one has to impose horizontality conditions, that for zero dimensional primary conformal fields corresponds to (14).

[3] By local coordinate Z_i at a marked point P_i we mean holomorphic function in a neighborhood of P_i that has a simple zero at P_i

Similarly, observable is trivial if it is of the form $Q(\Phi)$, where Φ is a horizontal field of zero conformal dimension.

Gravitational descendents

Sometimes it happens[16] that theory contains a local field A, that has conformal dimension zero, and such that:

$$(G_{L,0} - G_{R,0})A = 1 \tag{16}$$

Consider A inserted at i-th marked point as a form on the principal $U(1)$ bundle $E_i(M)$ of phases of local coordinate at i-th point over space of complex structures. Condition of zero conformal dimension means that A is invariant along the fiber. Condition (16) means that restriction of A to the fiber produces an invariant volume form on it. Thus, A is a connection form on this bundle, and its Q derivative is a curvature of this bundle. Bundle $E_i(M)$ is canonically associated with the bundle $T_i(M)$ of tangent planes at marked point, and holomorphic line bundle $L_i(M)$ of holomorphic one forms at marked point. For example, the map from bundle of local coordinates to $L_i(M)$ is given by

$$Z_i \to dZ_i(P_i)$$

So, $Q(A)$ is a first Chern class of L_i or the Euler class of T_i.

Such strange fields as connections appear rather naturally in topological theories by the following mechanism. In conformal theories the fermionic symmetry Q could be decomposed as a sum of two symmetries, Q_L and Q_R, that act on the left and the right movers respectively:

$$\{Q_L, G_L\} = T_L; \{Q_R, G_R\} = T_R \tag{17}$$

This decompositions corresponds to decomposition of the external derivative on the moduli space on the holomorphic and antiholomorphic one. Suppose we have a local field H that has logarithmic conformal dimension, namely:

$$T_{L,0}(H) = T_{R,0}(H) = 1 \tag{18}$$

Such a field could be interpreted as a logarithm of the hermitean metric on the bundle L_i. Then natural connection could be constructed as

$$A = (Q_L - Q_R)(H) \tag{19}$$

So first Chern class of the line bundle L_i (by tradition denoted as $\sigma_1(1)$) takes the form

$$\sigma_1(1) = Q_L Q_R(H) \tag{20}$$

Fields with logarithmic conformal dimension could be introduced into the theory from the very beginning, like conformal factor of the metric[3] (or by coupling a scalar field to the curvature of the background metric). Such fields have normal(or classical) logarithmic dimensions.

It is interesting that there is a phenomena of getting anomalous logarithmic conformal dimension due to normal ordering in Landau-Ginsburg theory, that we will consider below.

6 Structure of the Landau-Ginzburg model

Landau-Ginzburg model[14, 4, 13] is a type B twisted sigma model on C^d (non-compact flat space) with holomorphic superpotential $W(X)$ (that prevents fields from flying away). To describe its action we need also a Kahler potential that we will take to be flat . Moreover, we start with potential, that gives nonsingular metric on the full C^d, i.e. with

$$K(X, \bar{X}) = \sum_{A=1}^{d} X^A \bar{X^A} \tag{21}$$

Action in this theory could be found from the twisted version of the superfield formalizm, and is a sum of kinetic D-term, arizing from the Kahler potential, F-term, described by superpotential $W(X)$, and $\bar{F}$ term , described by superpotential $\bar{W}(\bar{X})$. Note, that we do not insist on $\bar{W}$ being complex conjugate to W. Let $X, \psi_z, \psi_{\bar{z}}, F_{z\bar{z}}$ be components of the chiral B type twisted superfield $\hat{X}$, and $\bar{X}, \rho, \rho_*, \bar{F}$ are components of twisted antichiral superfield $\hat{\bar{X}}$; here all fields of antichiral supermultiplet are worldsheet scalars, while two ψ fields form a worldsheet one form, and auxillary field F is a worldsheet two-form. Then the Lagrangian of the F-term is a topological one:

$$L_{top} = \frac{\partial^2 W}{\partial X^A \partial X^B} \psi^A \wedge \psi^B + \frac{\partial W}{\partial X^A} F^A \tag{22}$$

Lagrangian of D-term is conformal and equals to

$$L_{conf} = X^A \Box \bar{X^A} + \psi^A \bar{\partial} \rho^A + \psi^A \partial \rho_*^A + F_{z\bar{z}}^A \bar{F^A} \tag{23}$$

and the $\bar{F}$ term depends on the conformal factor of the metric:

$$L_{massive} = \sqrt{\det g} \left(\frac{\partial^2 \bar{W}}{\partial \bar{X^A} \partial \bar{X^B}} \rho^A \wedge \rho^B + \frac{\partial \bar{W}}{\partial \bar{X^A}} \bar{F^A} \right) \tag{24}$$

From the superfield formalizm we get that the theory for a given complex structure of the worldsheet has two scalar fermionic symmetries that we will call Q_L and Q_R. The offshell symmetry Q_L acts as follows:

$$\begin{gathered} Q_L(X) = 0,\ Q_L(\psi_z) = \partial_z X,\ Q_L(\psi_{\bar{z}}) = 0,\ Q_L(F) = \partial_z \psi_{\bar{z}} \\ Q_L(\bar{X}) = \rho,\ Q_L(\rho) = 0,\ Q_L(\rho_*) = -\bar{F},\ Q_L(\bar{F}) = 0 \end{gathered} \tag{25}$$

while Q_R acts as

$$\begin{gathered} Q_R(X) = 0,\ Q_R(\psi_z) = 0,\ Q_R(\psi_{\bar{z}}) = \partial_{\bar{z}} X,\ Q_R(F) = \partial_{\bar{z}} \psi_z \\ Q_R(\bar{X}) = \rho_*,\ Q_R(\rho_*) = 0,\ Q_R(\rho) = \bar{F},\ Q_R(\bar{F}) = 0 \end{gathered} \tag{26}$$

From these formula we see that the sum $Q = Q_L + Q_R$ acts as a scalar independently on complex structure of the worldsheet and can be taken as a Q-symmetry of the topological matter theory.

One can check that the massive part of Lagrangian (that depends on $\bar{W}$) is exact, conformal is exact up to the total derivative, and topological term is closed up to the total derivative. Thus, the second and third terms could be considered as regulators, and the first as a density of the topological action.

Above we stressed that there are problems in defining massive topological theories coupled to topological gravity, that is why below we will take $\bar{W}$ equal to zero, or ,equivalently , consider sections of the almost zero area.

Then theory turns out to be conformal. Really, after exclusion of auxillary fields, we find that

$$F = 0;\ \bar{F^A} = \frac{\partial W(X)}{\partial X^A} \tag{27}$$

and after substitution of (27) we get a theory with the Lagrangian:

$$X^A \Box \bar{X^A} + \psi_z^A \bar{\partial} \rho^A + \psi_{\bar{z}}^A \partial \rho_*^A + \frac{\partial^2 W}{\partial X^A \partial X^B} \psi^A \wedge \psi^B \tag{28}$$

This theory is really conformal even at the quantum level, since the only non-quadratic term could not be contracted with itself by propagator, and thus, it only changes the classical equations of motion in the theory. For fields from chiral supermultiplet they look exactly like in the free theory

$$\Box X = 0;\ \partial_z \psi_{\bar{z}} = 0;\ \partial_{\bar{z}} \psi_z = 0, \tag{29}$$

while for fields from antichiral supermultiplet they are a little bit different:

$$\begin{aligned} \Box \bar{X^C} &= \frac{\partial^3 W}{\partial X^A \partial X^B \partial X^C} \psi^A \wedge \psi^B \\ \partial_z \rho_*^B &= \frac{\partial^2 W}{\partial X^A \partial X^B} \psi_z^A \\ \partial_{\bar{z}} \rho^B &= \frac{\partial^2 W}{\partial X^A \partial X^B} \psi_{\bar{z}}^A \end{aligned} \tag{30}$$

Using these equations of motion one can prove that theory contains two conserved fermionic currents:

$$J_L = \rho^A \partial_z X^A + \frac{\partial W}{\partial X^A} \psi_{\bar{z}}^A \tag{31}$$

$$J_R = \rho_*^A \partial_{\bar{z}} X^A + \frac{\partial W}{\partial X^A} \psi_z^A \tag{32}$$

Note, that each of these currents contains both holomorphic and antiholomorphic components along the worldsheet, and thus is conserved as a current of a global symmetry, i.e. $dJ = 0$. These currents generate symmetries Q_L and Q_R respectively. Being conformal, this theory also contains holomorphic and antiholomorphic energy-momentum tensors T_L and T_R,and their superpartners G_L and G_R, that have the standard form of the free theory, for example for the left-movers:

$$T_L = \partial_z X^A \partial_z \bar{X^A} + \psi_z^A \partial_z \rho^A \tag{33}$$

$$G_L = \psi_z^A \partial_z \bar{X}^A \tag{34}$$

These fields are holomorphic:

$$\partial_{\bar{z}} T_L = \partial_{\bar{z}} G_L = 0 \tag{35}$$

Similar expressions are valid for the right-movers; moreover, action of symmetries $Q_{L,R}$ on $G_{L,R}$ are standart:

$$Q_L(G_L) = T_L,\ Q_R(G_L) = 0 \tag{36}$$

Summing up all properties of the theory, we may say that topological LG theory with *nonhomogenious* superpotential is nonunitary conformal, has traceless superpartner of the energy-momentum tensor, and two fermionic symmetries(scalars for a given complex structure). These fermionic symmetries and superpartners of energy-momentum tensor commute like BRST-symmetries and b fields in ordinary string theories, so it is possible to construct a measure on the moduli space exactly like in string theory. At the same time, fermionic symmetries are global, not holomorphic, that is why there are no holomorphic conserved currents, apriory no notion of anomaly in the current and this theory in general could not be obtained by twisting of $N = 2$ superconformal theory(sometimes such theories could be obtained from twisted superconformal $N = 2$ theory by perturbing the action with the perturbation, that violates twisted $N = 2$ superconformal algebra).

7 Calculation of the ring and the linear functonal in LG topological matter theory

The ring of observables

Let us first qualitevly estimate the structure of the ring and the linear functional on it. From offshell supersymetry transformations it is clear that all polynomials in fields X are Q-closed fields. Since $\bar{F}$ is Q-exact in the offshell algebra, and in the process of eliminating the auxillary fields we found (27), that $\bar{F}$ equals to the gradient of W, we can expect, that ring of observables is the factor ring over the gradient ideal. Really, one can show that there are no other Q-cohomologies, thus

$$J = \frac{C[X]}{\{\frac{\partial W}{\partial X^A}\}} \tag{37}$$

Homotopy to deal with zero modes

We will calculate k-point correlator in conformal ($\bar{W} = 0$) version of the theory. Before we start calculation, we should notice that after substituting polynomials of X into the fuctional integral, the integral is ill defined: namely an integral over bosonic zero modes implies that the integral is infinite, while an integral over the zero modes of ρ fermions turns integral to be zero; so we have typical "zero times infinity" problem. To solve this problem we will use instead one of polynomials an element in the same Q-cohomology class, namely, for some function $\bar{V}(\bar{X})$, we construct homotopy:

$$P(X) \to \exp(\tau Q(\frac{\partial \bar{V}}{\partial \bar{X}^A}\rho^A))P(X) \tag{38}$$

We are mostly interesed in the case when τ tends to zero. At the point $\tau = 0$ we have the initial problem. Consider τ very small, so we could ignore interaction of nonzero modes, that is proportional to τ, and pick up only contribution from zero modes, that should appear(as we expect) in the zero-th order in τ. The only restriction on function $\bar{V}(\bar{X})$ is that the integral over bosonic zero modes should converge. For example we can take for $\bar{V}$ complex conjugate to $W(X)$.

Anomaly in the measure and holomorphic top form

Before we worked with the LG theory on the level of the action. To compute functional integral we need the measure. In ordinary nontwisted sigma models measure is defined by the metric $G_{A\bar{B}}$ on the target space and the metric on the worldsheet. After twisting situation is different. Really, it is impossible to construct measure on the fermions with the help of metric on the target space since fermions, that are holomorphic tangent vectors to the target space (ψ fermions) are one-forms on the worldsheet, while fermions that are antiholomorphic tangent vectors(ρ fermions), are scalars on the worldsheet. That is why it seems impossible to construct a non-degenerate scalar product (something like an integral of $\psi^A\rho^{\bar{B}}G_{A\bar{B}}$) not only because holomorphic square root of worldsheet metric is needed, but also because on a surface different from the torus there are different number of ψ and ρ fields and the difference is $2(g-1)$, where g is the genus of the surface.

Moreover, one would expect that in topological theories nothing depends on the worldsheet metric, since energy-momentum tensor is Q-exact. Typically this is a result of existance of equal number of bosons and fermions with the same target-space properties, so that formal measure $DbosonDfermion$ written in some coordinates is well defined as a prescription to integrate over the space of all bosons and consider fermions as differentials of corresponding bosons(in other words, to define such a measure one can choose some coordinates on the space of bosons, this induces coordinates on the space of fermions, and under changes of such coordinates measure is invariant, since Jacobians cancel each other).

In the case of type B twist, this takes place in the space of antichiral fields: one can really define a formal measure on the space of $\bar{X}, \rho, \rho_*, \bar{F}$ since all of them are worldsheet scalars. But it is not the case for the chiral sector: X, ψ, F - the difference between numbers of bosons and fermions (in any reasonable regularization) is $2(1-g)$. That is why to define a measure in the functional integral we need an additional data[14, 12, 19] *holomorphic top form* Ω. We expect, the answer on genus g to depend on this form like $\Omega^{2(1-g)}$. If we still want to define a measure on bosons in terms of metric $G_{A\bar{B}}$, we have to say that measure is defined on the ψ -fermions themself by Ω, on ρ fermions themself by $\bar{\Omega}$, and measures on bosons and fermions correspond to each other if

$$\Omega\bar{\Omega} = \det(G_{A\bar{B}}) \tag{39}$$

Moreover, holomorphic top form needs to be holomorphic to preserve supersymmetry of the measure (we will see how this condition appears in calculations of the correlators). Note, that on the compact target spaces compatibility between holomorphic top form and metric(together with nondegeneracy of metric) means that the target space is a Calaby-Yau manifold.

Calculations

We will compute in genus zero $< P_1, \ldots, P_n >^M$ on C^d with the metric

$$G_{A\bar{B}} = \delta_{A\bar{B}}$$

and holomorphic top form

$$\Omega = \prod_{A=1}^{d} dX^A$$

One can easily see that determinants on nonzero modes cancel each other and we are left with the finite dimensional integral over zero modes:

$$< P_1, \ldots, P_n >^M = \int d\rho d\rho_* dX d\bar{X} P_1(X) \ldots P_n(X)$$
$$\exp(\tau \rho^{\bar{A}} \rho_*^{\bar{B}} \frac{\partial \bar{V}}{\partial \bar{X}^A \partial \bar{X}^B} - \tau \frac{\partial \bar{V}}{\partial \bar{X}^A} \frac{\partial W}{\partial X^A}) \tag{40}$$

If $d = 1$, then an integral could be calculated as follows. Consider a manifold M obtained from the full C by cutting out small discs around the points, where derivative of $W(X)$ equals to zero, and substitute integral over C on integral over M. On M the integral takes the following form

$$< P_1, \ldots, P_n >^M = \int_M \bar{\partial}(P_1(X) \ldots P_n(X) \frac{\exp(-\tau \partial W \bar{\partial} \bar{V})}{\partial W}) \tag{41}$$

Thus, integral over M reduces to boundaries of M, and we get

$$< P_1, \ldots, P_n >^M = \int_\Gamma \frac{P_1(X) \ldots P_n(X)(dX)^2}{dW} \tag{42}$$

Here Γ is a contour that goes around zeroes of dW. Note, that dX^2 in the numerator stands for holomorphic top form to the second power, as we expected before.

Note, that result we obtained is independent of τ. If we choose nonholomorphic top form, it would not be the case because Q symmetry would be breaken by measure.

In the general case calculations become a little bit more tricky. Nevertheless they could be done using the following observation of Vafa [13]. One can show that not only functional integral but also finite dimensional integral is independent of τ and $\bar{V}$. Then we can take $\bar{V}$ equal to complex conjugated superpotential, put τ to infinity. We observe that the integral gets localized on the critical points of W, and we get the following result:

$$< P_1, \ldots, P_n >^M = \int_\Gamma \frac{P_1(X) \ldots P_n(X)(\Omega)^2}{\prod_{A=1}^{d} \partial_{X^A} W dX^A} \tag{43}$$

Here Γ is defined by equations

$$|\partial_{X^A} W| = c$$

for some small positive number c.

Moreover, here we restored dependence on the holomorphic top form, so correlator here is put in a coordinate invariant form.

Thus, we justified that ring of observable is a factor over the gradient ideal of superpotential and linear functional is determined by a holomorphic top form.

8 Descendents and equivariant cohomology

Descendents constructed from matter fields

In order to find descendents, consider the following Q-exact field

$$\Phi = P_A(X)\frac{\partial W(X)}{\partial X^A} \tag{44}$$

where P_A are some polynomials. One can check that this field is a local observable in a theory coupled to topological gravity(apriori it could be zero obzervable).Note the following fact:

$$\Phi = Q_L Q_R(P_A(X)\bar{X}^{\bar{A}}) \tag{45}$$

while field in the round brackets has nontrivial logarithmic conformal dimension due to contraction of fields X and $\bar{X}$. Namely:

$$T_{L,0}(P(X)_A\bar{X}^{\bar{A}}) = T_{R,0}(P_A(X)\bar{X}^{\bar{A}}) = \frac{\partial P_A(X)}{\partial X^A} \tag{46}$$

For example, for $P_A(X) = X^B\delta_{AB}$ observable Φ corresponds to the first Chern class of the line bundle L_i.

More physically, since correlator between X and $\bar{X}$ is singular, operation of placing these two fields at the same marked point is ill-defined. It is natural to regularize it by point splitting. Making a point splitting means to choose a nonzero vector from a tangent plane, and smooth point splitting everywhere on the moduli space means picking up a nonzero vector field from the bundle $T_i(M)$(or smooth section of $E_i(M)$). The obstacle to do it is just the Euler class of this bundle.

Higher descendents

Now let us consider the general polynomial observable $P(X)$. In order to show that such an observable could represent powers of the first Chern classes we use the following trick. Consider the tensor product of the initial LG system with the trivial one (described in terms of N fields Y_i, the number N of fields Y can be as big as we wish) with the total superpotential

$$W_{coupled}(X,Y) = W(X) + \sum_{C=1}^{N} Y_C^2/2 \tag{47}$$

The matter ring in this theory is the same as in initial one, and if an observable does not contain fields Y, we can simply omit them. At the same time logarithmic divergency in the field

$$P_A(X)\bar{X}^{\bar{A}} - \bar{Y}_1 Y_1 \frac{\partial P_A(X)}{\partial X^A}$$

equals to zero, thus in the space of observables in topological theory coupled to topological gravity:

$$P_A(X)\frac{\partial W(X)}{\partial X^A} \sim (Y_1)^2 \frac{\partial P_A(X)}{\partial X^A} \tag{48}$$

where $\sim$ means that the left and right hand sides of above expression differ by the zero observable.

In other terms [16],

$$P_A(X)\frac{\partial W(X)}{\partial X^A} - (Y_1)^2 \frac{\partial P_A(X)}{\partial X^A} = Q(P_A(X)\rho_A - Y_1\rho_1 \frac{\partial P_A(X)}{\partial X^A}) \tag{49}$$

where ρ_1 is a fermion in the antichiral supermultiplet of $\bar{Y}_1$, and [4]

$$G_{0,-}(P_A(X)\rho_A - Y_1\rho_1 \frac{\partial P_A(X)}{\partial X^A}) = 0 \tag{50}$$

It may happen that polynomial in X in the right hand side of the equation (48) also belongs to gradient ideal of $W(X)$, then this procedure could be applied once again, but now we will compensate the logarithmic dimension with the help of Y_2 fields, e.t.c. Thus, in principle it could happen, that some polynomial $P(X)$ equals in the space of observables to:

$$P(X) \sim \prod_{C=1}^{k} (Y_C)^2 \tilde{P}(X) \tag{51}$$

Then, we represent $(Y_C)^2$ as $Q_L(Q_R(Y_C\bar{Y}_C)$ and split Y_C and $\bar{Y}_C$ along some section v_C of cotangent bundle.

Taking first functional integral over all Y we get the measure on the moduli space that has support on the common zero of all these sections. In the rest of the integral observable $\tilde{P}$ is inserted at the marked point we consider. In cohomologies of the moduli space the support of the measure before integration over X is the cycle dual to the k-th self intersection of the first Chern class. In such a situation we write $P = \sigma_k(\tilde{P})$.

The "state" representation and equivariant cohomology

The effect observed above has the following nice presentation. Let us represent local fields as differential forms of the type $(0, m)$, that take value in

[4]It is sufficient to consider $G_{0,-} = G_{0,L} - G_{0,R}$,since sum of G correspond to dilatations of local coordinate, i.e. to noncompact direction in the structure group of the bundle. It is wellknown that noncompact directions are always contractible, i.e. structure group of the bundle could be always reduced to its compact subgroup

antisymmetric powers of holomorphic vector bundle, namely, $\rho_+ = \rho + \rho_*$ corresponds to $d\bar{X}$, $\nu_A = G_{A\bar{B}}(\rho^{\bar{B}} - \rho_*^{\bar{B}})$ corresponds to ∂_A. Multiplication in the sence of field theory corresponds to wedge multiplication of forms.

In such representation operators Q and $G_{0,-}$ take the following form:

$$Q = \bar{\partial} + \partial_A W dX^A \tag{52}$$

$$G_{0,-} = dX^A \partial_A \tag{53}$$

Here dX^A acts by contraction with the polyvector of the bundle. It is easy to see,that $G_{0,-}$ is not a differentiation with respect to natural multiplication, and that is why observables in topological theory coupled to topological gravity do not form a ring under this product.

It is convenient to use another representation of the same space of local operators. Namely, introducing new fermionic variables θ^A,dual to ν_A, and making fermionic Laplace-like transformation with the measure determined by Ω we obtain (p,q) forms. For example, functions of X correspond to holomorphic top forms,specifically:

$$P_i(X) \to \Omega_i = P_i(X)\Omega \tag{54}$$

Formulas (52,53) look the same but now dX means wedge product with the differential.

This transformation is not so unnatural as it looks like. It somehow represents states, corresponding to local fields, inserted in the middle of the small disk. Really, one can compute the fermionic number of the disc, that is a semi-sphere and find that it equals d - the dimension of the manifold(it is in agreement with the anomaly, that equals $2d$ that we observed on the sphere). It is nice that on "states" actions of Q and $G_{0,-}$ are independent of Ω and depend only on W. Moreover, on the "states" the operation of rotation of local coordinate is represented by a vector field, and $G_{0,-}$ has a geometrical meaning of contraction with such a vector. That is why "states" corresponding to observables are equivariant cohomologies, namely, they are cohomologies of the operator $D(z)$:

$$D(z) = D_1 - zD_2 = Q - zG_{0,-} = \bar{\partial} + \partial W - z\partial \tag{55}$$

K.Saito higher residue pairing as pairing in equivariant cohomologies

In terms of equivariant cohomology it is possible to interpret K.Saito higher residue pairing, as a pairing on equivariant cohomologies (in the "state" picture) induced by integration of the ordinary wedge product of the forms over C^d. If we take representation of cohomologies as holomorphic top form we could not get the answer since product of holomorphic top forms looks like zero but they are not in an integrable class. To cure the situation we can do homotopy like in (38), but now with $\{D(z), \bar{\partial}_A \bar{W} \nu^{\bar{A}}\}$. We will obtain seria in z, the i-th term in that seria was called K.Saito as $K^{(i)}$.The zero-th term would be ordinary residue, and the first term would be the first higher residue pairing $K^{(1)}$, that has the following form:

$$K^{(1)}(\Omega_1, \Omega_2) = \int_\Gamma \frac{(P_1 \partial_{X^B} P_2 - P_2 \partial_{X^B} P_1)\Omega^2}{\partial_{X^B} W \prod_{A=1}^d \partial_{X^A} W dX^A} \tag{56}$$

Here P_i is connected with Ω_i like in (54).

Filtration on the space of equivariant cohomologies

Cohomology of $D(z)$ naturally form a $D[z]$ module. With this module we can assosiate filtration on the space of observables.

If in cohomologies of $D(z)$ $\Phi_1 = z^k\Phi_2$, then as we observed above Φ_1 is k-th gravitational descendent of Φ_2, this is denoted as $\Phi_1 = \sigma_k(\Phi_2)$. Let us define filtration in the space H of equivariant cohomologies due to degree of descendents: class of equivariant cohomologies belongs to the H_k subspace of the space H_0 it contains k-th descendent. Thus we have

$$H = H_0 \supset H_1 \supset H_2 \supset \dots \tag{57}$$

In Landau-Ginsburg theory it is obvious, that

$$H_i/H_{i+1} = J \tag{58}$$

But apriori there is no natural splitting of this filtration.

It can happen that in some theory descendents could not be constructed from the matter fields. For example, this happens in the sigma model of type B on Calabi-Yau manifold. In that case

$$D(z) = \bar{\partial} + z\partial \tag{59}$$

and from $\partial\bar{\partial}$-Lemma it follows that there are no descendents constructed from matter fields in this model. In such case to reproduce gravitational descendents we have to couple theory to additional theory with trivial ring of observables, for example, to Landau-Ginsburg theory with quadratic superpotential.

9 Connection on the space of theories

Consider n-point correlator on a Riemann surface with n marked points. It turns out that this correlator could be reduced to a $n-1$ point correlator in a nearby theory.

Really, moduli space of Riemann surfaces of genus g with n marked points (for $g = 0$,$n > 3$; for $g = 1$,$n > 1$) almost everywhere (except boundaries, where two marked points instead of collision decouple on a sphere) is fibered over the moduli space with $n-1$ marked points. The fiber is a Riemann surface with $n-1$ marked points. The idea is to integrate over the fiber, and this would give shift to a nearby theory. Including properly the boundary contribution we get a connection in the space of observables fibered over the space of theories.

Far from the boundary coordinate of n-th marked point could be considered as a moduli, this moduli corresponds to $G_{L,-1}G_{R,-1}P(X)(z_n)$ contribution to the functional integral (see (15)), the -1 means expansion in $z - z_n$), that gives

$$\partial_A \partial_B P(X)(z_n)\psi_z(z_n)^A \psi_{\bar{z}}(z_n)^B d^2 z_n$$

contribution, and thus integral over fiber symply corresponds to the shift in the action:

$$W \to W + tP$$

in the first order in t.

Calculation of boundary contribution(where n-th point decouples on a sphere together with i-th point) is more tricky, because we have to calculate the integral [3]:

$$|C(P_n, P_i) >= \int_{\tau_0}^{\infty} d\tau \int d\phi \exp(-\tau T_{0,+}) \exp(\phi T_{0,-}) G_{0,+} G_{0,-}$$
$$P_n(X(1))|P_i(X(0)) > \qquad (60)$$

Here $|P_i(X(0)) >$ is a state that represents a disc with polynomial $P_i(X)$ inserted at the middle, and polynomial field P_n is inserted at point 1. We will take τ_0 large enough, so that the disc covered by this integration has radius $\exp(-\tau_0)$ and is relatively small.

Commuting G operators with P_n for finite τ still could be considered as the same perturbation of the action on the small disc that we considered before, so the infinity in the upper limit of the integral is really important, since the effect we are trying to find could be considered as a delta-function contribution from infinity.

First note, that field P_n could be placed at zero, really , derivative of this field with respect to the worldsheet coordinate has conformal dimention 1 and is suppresed like $\exp(\tau_0)$. Thus, $C(P_n, P_i)$ depends only on product of polynomials.

The problem of computation of the resulting expression comes from the fact, that G acting on the state give zero, but integral over τ up to infinity of the state with conformal dimension zero is divergent.

The only known way to calculate such an integral is to regularize it. This can be easily done when the product of polynomials belongs to the gradient ideal, i.e. for some field Ψ:

$$P_i P_n = Q(\Psi) \qquad (61)$$

Then, we take Q from Ψ outside. Commutation of Q with $G_{0,-}$ is irrelevant, since it gives $T_{0,-}$, and integral over compact circle gives zero(this is true in all regularizations), so the only contribution comes from commutation of Q with $G_{0,+}$. (Q acting outside seems to be irrelevant, because it acts on the G_0^- exact state). So after taking integral over ϕ we get:

$$|C(P_n, P_i) >= \int_{\tau_0}^{\infty} d\tau \exp(-\tau T_{0,+}) T_{0,+} G_{0,-}|\Psi > \qquad (62)$$

This integral could be easily regularized(see also [10]) by substituting zero energy state $G_{0,-}|\Psi>$ by state with non-zero energy E. After computation of the integral we will put E to zero. We are left with the integral:

$$|C(P_n, P_i)>=\int_{\tau_0}^{\infty} d\tau \exp(-\tau E)E(G_{0,-}|\Psi>) = \exp(-\tau_0 E)(G_{0,-}|\Psi>) \quad (63)$$

Note, that main contribution to such an integral came from the region $\tau \sim 1/E$, that is infinity regularized by E. Now, putting E to zero we obtain the final answer:

$$|C(P_n, P_i)>= G_{0,-}|\Psi> \qquad (64)$$

In other terms, if

$$|P_n P_i>=|\sigma_1(P_{n,i})>,$$

then

$$|C(P_n P_i)>=|P_{n,i}>.$$

We see, that contact term is completely defined as a linear operator C from the space of observables H to itself , such that it maps H^i on H^{i-1} for $i > 0$.

Definition of a section

The kernel V of the contact term map C in the space of observables H we call a section.

From derivation of the contact term we see that up to section it can be described in terms of filtration of decsendents, that in "state" representation (54) does not need the fixation of the holomorphic top form Ω. Below we will see from compatibility conditions that in "state" representation section is also independent of this form. That is why below we will mostly use this representation. Operator representation (i.e. representation in terms of polynomials), is needed when we want to assosiate to observable a shift in W.

10 Recursion relations and properties of the section

Above we constructed connection in the bundle of observables over the space of theories. It allows us to write the following recursion relations for correlators (if $g = 0, n > 3$, if $g = 1, n > 1$):

$$< \Omega_1, \ldots, \Omega_n >_{W,\Omega}= \frac{d}{dt} < \Omega_1, \ldots, \Omega_{n-1} >_{W+tP_n,\Omega} |_{t=0} +$$
$$+\sum_{i=1}^{n-1} < \Omega_1, \ldots, C(P_n, \Omega_i), \ldots, \Omega_{n-1} >_{W,\Omega} \qquad (65)$$

Here P_n is connected with Ω_n as in (54).

Connection given by contact term is really connection on equivariant cohomologies

To show this statement we have to prove that $D(z)$ trivial fields are mapped to themself by connection, induced by contact term. Note, that since all such fields are Q exact, we do not need to involve notion of section. Such a proof could be given at a formal level of equivariant cohomologies, we will show how it works in LG case. Really, contact term

$$C(P, z\partial\Omega_i - \partial W\Omega_i) = \partial P\Omega_i = \delta_P(z\partial\Omega_i - \partial W\Omega_i) \tag{66}$$

thus

$$(z\partial\Omega_i - \partial W\Omega_i) + \delta_P(z\partial\Omega_i - \partial W\Omega_i) = z\partial\Omega_i - \partial(W + \delta_P W)\Omega_i \tag{67}$$

So we can rewrite recursion relation as follows:

$$< \Omega_1, \ldots, \Omega_n >_{W,\Omega} = \frac{d}{dt} < \Omega_1(t), \ldots, \Omega_{n-1}(t) >_{W+tP_n,\Omega} |_{t=0} \tag{68}$$

where $\Omega_i(t)$ is a result of trasport of a class of equivariant cohomologies Ω along the line $W + tP$ with connection, given by contact term.

Now we will analize the properties of this connection, that come from physical consistency conditions.

Symmetry of the four-point function and first K.Saito higher residue pairing

The four-point function on the sphere could be reduced to the three-point function(that we know how to calculate, since it is given by an answer in the topological matter theory) either by integration of the position of the first maked point or by integration over the second.

After some calculation one can show that the difference equals to

$$K^{(1)}(V(P_1P_3), V(P_2P_4)) - K^{(1)}(V(P_1P_4), V(P_2P_3)) \tag{69}$$

that is why from the symmetry of the four-point function we get:

$$K^{(1)}(V_1, V_2) = 0 \tag{70}$$

for any two elements of a section. Note, that this is the first axiom for a good section in the K.Saito theory of the primitive form.

Motion of punctures in different order and second K.Saito axiom of good section

The origin of additional requirements on section comes from the fact that we can integrate first over the position of the first point, then over the position of the second, but we can also change the order and start from integration over position of the second point. To see this effect we have to apply recursion relation twice, and we get:

$$\begin{aligned} < \Omega_1, \ldots, \Omega_n >_W = \frac{d}{dt_1}\frac{d}{dt_2} < \ldots, \Omega_i + t_1 C_W(P_1\Omega_i) + t_2 C_W(P_2\Omega_i) + \\ t_1 t_2 C_W(C_W(P_1P_2)) + [t_1t_2C_W(P_2C_W(P_1\Omega_i)) + t_2C_{(W+t_1P_1)}(P_2\Omega_i)] \\ >_{(W+t_1P_1+t_2P_2+t_1t_2C_W(P_1P_2))} |_{t=0} \end{aligned} \tag{71}$$

The only asymmetric in 1 and 2 term stands in the square brackets, consistency demands that antysymmetric part of this term should equal to zero.

Suppose that $\Omega_i = V_i$, i.e. is an element of a good section. Suppose that $P_1 = 1$, then term in square brackets equals to zero. Changing 1 and 2 we get:

$$C_{(W+tP_2)}(V_i + tC(P_2V_i)) = 0 \tag{72}$$

in the first order in t. This is the second K.Saito axiom on the good section, that could be reformulated as follows:

Good section is invariant under connection determined by itself

Note, for general P_1 and P_2 the symmetry of the twise applied recursion relation means that connection, determined by good section, is flat. It was really shown in [11].

Invariance under diffeomorfisms and change of Ω

In our previous considerations we implied that after integration over position of a marked point the observable P simply shifts superpotential. Since observables could be of any degree it makes picture not so nice, namely we add terms to the superpotential, that change its behaviour at infinity, and we have to treat them perturbatively. It is possible to get rid of almost all of these terms because observables from H_1 that have form RdW in "state" representation look like $R^A \partial_A W$ as polynomials (index is raised with the help of the form Ω); they shift superpotential on a term that could be killed by diffeomorphism:

$$X^A \to X^A + R^A(X)$$

This would change holomorphic top form: $\Omega \to \Omega - dR$. One can check [22] that the total effect of contact term and diffeomorphism leave observables in the "state" representation invariant. It justifies our assumption that section in "state" representation is independent of Ω.

All this means, that we can restrict ourself to some particular versal deformation of W, we will call it $W(u)$, and represent any observable as a sum of two terms: change of W along versal deformation and diffeomorphism. Explicetely, in the state representation for any Ω_i we have:

$$\Omega_i = \delta_i(W(u)) + R_i dW \tag{73}$$

and recursion relation take the following form:

$$<\Omega_1, \dots, \Omega_n>_{W,\Omega} = \frac{d}{dt} <\Omega_1(t), \dots, \Omega_{n-1}(t)>_{W+t\delta_n W, \Omega - tdR_i} |_{t=0} \tag{74}$$

with $\Omega_i(t)$ understood as a parallel transport along the base of versal deformation in direction of $\delta_n W$.

Generating function for correlators

To write down the formula, containing all correlators, let us introduce the notion of formal exponent:

$$<; \exp \Phi> = <> + <\Phi> + 1/2 <\Phi, \Phi> + 1/6 <\Phi, \Phi, \Phi> + \dots \tag{75}$$

We are going to consider correlators in presence of formal exponent. If recursion relations are self consistent, they imply, that:

$$< \Omega_1, \dots, \Omega_n; \exp(t_i\Omega_i) >_{W,\Omega} = < \Omega_1(t), \dots, \Omega_n(t) >_{W(u(t)),\Omega(t)} \qquad (76)$$

naimely, recursion equations could be considered as vector fields on the space of theories equipped with the connection in the bundle of observables, and formal exponent just integrates them. Parameters t become coordinates on the space of theories.

Relation (76) is self-similar,taking derivative with respect to t we get:

$$\Omega_i(t) = \frac{\partial W}{\partial t_i}\Omega(t) - z\frac{\partial \Omega(t)}{\partial t_i} \qquad (77)$$

and

$$\frac{\partial \Omega_j(t)}{\partial t_i} = C_W(t)(\frac{\partial W}{\partial t_i}, \Omega_j(t)) \qquad (78)$$

The second equation implies that contact terms form a flat connection in the bundle of observables over base of versal deformation. Such a connection as we understood is nothing else that K.Saito connection. Suppose we know its flat sections $\Omega_i(u)$. Then it turns out that it is possible to solve first equation, namely:

$$(t_i + t_i^0)\Omega_i(u) = z\Omega(t) \qquad (79)$$

where numbers t_i^0 are choosen to satisfy initial condition: $t_i^0\Omega_i = z\Omega$. Algebraic solution of this equation is generalization of the algebraic solution of dispersionless KP [8].

After solving this equation we know generating function for correlators in genus 0:

$$< \Omega_1, \Omega_2, \Omega_3; \exp(t_i\Omega_i) >_{W,\Omega} = \int_\Gamma \frac{\Omega_1(t)\frac{\partial W}{\partial t_2}\Omega_2(t)}{\prod_{A=1}^d \partial_{X^A} W dX^A} \qquad (80)$$

The author is deeply indebted to R.Dijkgraaf,A.Gerasimov, V.Fock, I.Krichever, S.Kharchev, A.Marshakov, A.Mironov, A.Morozov, N.Nekrasov, I.Polyubin, A.Rosly, K.Saito, A.Varchenko for fruitful discussions.

References

[1] E.Witten, *Nucl.Phys*B340 (1990) 281

[2] R.Dijkgraaf and E.Witten, *Nucl.Phys* **B342** 486

[3] E.Verlinde and H.Verlinde,*Nucl.Phys.* **B348** (1991) 457

[4] R.Dijkgraaf, E.Verlinde, H.Verlinde, Proc. of the Trieste Spring School 1990, M.Green et al. eds., (World Scientific, 1991)

[5] S.Kharchev et.al.,*Nucl.Phys.*, **380B** (1992) 181

[6] M.Kontsevich,*Comm. Math. Phys.***147** (1992) 1

[7] M.Bershadsky et al , *Holomorphic Anomalies in Topological Field Theories*preprint HUTP-93/A008

[8] I.M.Krichever,*Comm. Math. Phys.***152**(1993) 539

[9] K.Li,*Nucl.Phys.* **354B**,(1991) 711

[10] R. Dijkgraaf, talk on RIMS conference "Infinite dimensional analisis",July,1993,

[11] K.Saito, Publ.RIMS, Kyoto Univ. 19 (1983) 1231

[12] S.Checotti and C.Vafa,*Nucl.Phys***367B**(1991) 359

[13] C. Vafa, *Mod.Phys.Lett.***A6(1990)** 337

[14] E.Witten,*Mirror Manifolds And Topological Field Theory*,in S.-T.Yau,ed., *Essays On Mirror Manifolds* (International Press, 1992)

[15] A.Losev,*Descendants Constructed From Matter Fields And K. Saito Higher Residue Pairing In Topological L-G Theories Coupled To Topological Gravity*, preprint TPI-MINN-40T (May,1992) ; A.Losev, *TMP(Teoreticheskaya i matematicheskaya Fizika,in English)***vol.95,n.2**(1993) 307

[16] T.Eguchi et al,*Topological Strings, Flat Coordinates And Gravitational Descendants*, University of Tokyo preprint UT 630, Febr.1993

[17] B.Dubrovin, *Nucl. Phys.* **379B** (1992) 627

[18] E.Witten, Comm.Math.Phys.117(1988)353,

[19] C.Vafa, in *Essays on Mirror Manifolds*,ed. by S.T.Yau,International Press, 1992

[20] L.Alvarez-Gaume et al.,*Nucl.Phys.***B303**(1988) 455

[21] J.Distler and P.Nelson, Comm.Math.Phys.138(1991) 273

[22] A.Losev and I.Polyubin *On connection between Landau Ginzburg gravity and integrable theories*, hepth 9305079