Mechanics of Spacetime with Surface Tension and Preferred Curvature

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Abstract. In previous publications, it was argued that because energy and matter are aligned in the thin increment of present time for an observer, spacetime should and must have surface tension. In this paper, we review how to apply 4-dimensional continuum mechanics with imaginary time coordinates to derive a mechanical model of spacetime with surface tension. Then, to continue model development, we discuss the concept of preferred curvature from the physical chemistry of surfaces and attempt to apply those concepts to spacetime geometry. We show how the model exhibits quantum fluctuations at the Plank scale and components resembling dark matter and dark energy at the cosmic scale.

Introduction

There have been and continue to be numerous attempts to find a unified field theory in physics. The approach taken here is to find a natural mechanism that causes spacetime itself to warp, ripple, or otherwise form small particles in geometry. The proposed mechanism is surface tension [1,2,3]. From everyday experiences, we know that surface tension results in micro-phenomena including capillary waves, corpuscles, vortices, and menisci. In a similar way, it was argued in [1,2,3] that incorporating surface tension in general relativity provides a bridge to quantum mechanics and atomic particle physics.

Surface tension of spacetime is an obscure, yet present topic in modern physics. At least nineteen references can be found in quantum and gravitational literature suggesting the importance of surface tension [4-23]. A number of works discuss surface tension effects on black hole horizons, worm holes, and stellar geometries [17,18,23]. Some novel works suggest surface tension as an analogy for the cosmological constant [13,16,18]. Others address surface tension at the boundary of an expanding universe [4,10,21]. A few suggest the importance of surface tension in quantum field theory [11,19,22]. This work is a continuation of efforts to justify the hypothesis that spacetime itself has surface tension [1,2,3].

This paper contains a summary of the surface tension of spacetime hypothesis including the postulated stress energy tensor with negative terms. Tensors describing the curvature gradient, rate of curvature, and constitutive relation for spacetime with surface tension are presented. It is shown that components of these tensors resemble wave equations in quantum field mechanics. It is also shown that on a cosmic scale, gravity has extra terms that explain dark matter and dark energy.

Stress Energy

The case for surface tension of spacetime has been argued based on statistical thermodynamics [2,3]. Surface tension is negative stress energy in all spatial directions which thermodynamically balance the presence of nearby mass. When coordinates are rotated such that the off-diagonal time terms in the stress-energy tensor are zero, time is normal to space. This orientation is called the principal reference frame in continuum mechanics, the rest frame in modern physics, and tangent space in mathematics. Governing differential equations describing the motion of curvilinear coordinates in a stress-energy field are best derived in the principal frame. The stress-energy tensor for an infinitesimal element of spacetime with surface tension in the principal frame is given by [3],

$$T_\nu^\mu = \begin{bmatrix} dP & 0 & 0 & 0 \\ 0 & -\text{♀} & 0 & 0 \\ 0 & 0 & -\text{♀} & 0 \\ 0 & 0 & 0 & -\text{♀} \end{bmatrix} \qquad (1)$$

Where dP is mass density, $-\text{♀}$ is surface energy density (a.k.a. surface tension), and $T_\nu^\mu = \Lambda^{\mu\alpha}T_{\alpha\nu}$ is the Lorentz boosted stress energy tensor from the observer's manifold to the tangent space at a point.

It has been shown using conservation of work and energy that mass and surface tension can be related thus [3],

$$\oiint_\sigma -\text{♀} d\sigma = \oint_V dPdV \qquad (2)$$

From (2) and the divergence theorem, surface tension is related to mass by [3],

$$-\nabla \cdot \text{♀} = dP \qquad (3)$$

For a polar symmetric system with central mass in laboratory coordinates, (3) can be re-written as,

$$-3\text{♀}\frac{dy^0}{R} = \frac{dP}{c^2} \qquad (4)$$

where R is the spacetime separation from the center of mass to the point where surface tension is being measured, dy^0 is the basis of time at the point.

Curvature Gradient

Lorentzian geometry and continuum mechanics are very similar fields in mathematics. Both use differential geometry of manifolds to describe curved spaces with a continuous metric tensor. The mathematics of both are inherently intrinsic as neither requires a containing space. Lorentzian geometry has a pseudo-Riemannian metric and continuum mechanics has a regular (positive definite) Riemannian metric. If one makes a coordinate substitution such that time and proper time are imaginary, then it creates a complex Hilbert space where all the tools developed for continuum mechanics can be applied to evaluate spacetime geometry.

From continuum mechanics of thin shells, we know that the curvature gradient of spacetime (rotation of curvilinear coordinates in a stress-energy field) between two observations separated by $d\tau$ is given by [1,2,3],

$$\dot{u}^{\mu}_{\nu} = -\frac{1}{\varrho}\begin{bmatrix} \varrho\nabla^2 y^0 + \frac{dP}{dy^0} & 0 & 0 & 0 \\ 0 & \frac{\partial\varrho}{\partial y^1} & 0 & 0 \\ 0 & 0 & \frac{\partial\varrho}{\partial y^2} & 0 \\ 0 & 0 & 0 & \frac{\partial\varrho}{\partial y^3} \end{bmatrix} = \begin{bmatrix} \frac{\partial u^0}{\partial\tau} & 0 & 0 & 0 \\ 0 & \frac{\partial u^1}{\partial\tau} & 0 & 0 \\ 0 & 0 & \frac{\partial u^2}{\partial\tau} & 0 \\ 0 & 0 & 0 & \frac{\partial u^3}{\partial\tau} \end{bmatrix} \qquad (5)$$

where y^{ν} are curvilinear coordinates at a point, u^{μ} is the motion of coordinates over the period of observation given by $\frac{\partial y^{\mu}}{\partial\tau}$.

Rate of Curvature

In this work, we use the Green-Lagrange definition of rate of curvature, which is similar to applying a small perturbation to the metric as given by,

$$\left(1 + 2D^{\mu}_{\nu}d\tau\right)ds^{\nu 2} = \widetilde{ds}^{\mu 2} \qquad (6)$$

where the terms in parenthesis represent the change in the metric over the period of observation, $\dot{g}$. For small strains, $ds^{\nu} \approx \widetilde{ds}^{\mu}$, (6) is approximated by linearized engineering strain, ergo change in basis over initial basis.

$$D^{\mu}_{\nu}d\tau = \frac{\widetilde{ds}^{\mu} - ds^{\nu}}{ds^{\nu}} \qquad (7)$$

In continuum mechanics of thin shells, the rate of curvature tensor is given by,

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$$D_\nu^\mu = \begin{bmatrix} \frac{1}{2}\sum_{k=1}^{3}\left(\frac{\partial u^0}{\partial y^k}\right)^2 & \frac{1}{2}\sum_{k=1}^{3}\left(\frac{\partial u^0}{\partial y^k}\right)\left(\frac{\partial u^k}{\partial y^1}\right) & \frac{1}{2}\sum_{k=1}^{3}\left(\frac{\partial u^0}{\partial y^k}\right)\left(\frac{\partial u^k}{\partial y^2}\right) & \frac{1}{2}\sum_{k=1}^{3}\left(\frac{\partial u^0}{\partial y^k}\right)\left(\frac{\partial u^k}{\partial y^3}\right) \\ \frac{1}{2}\sum_{k=1}^{3}\left(\frac{\partial u^0}{\partial y^k}\right)\left(\frac{\partial u^k}{\partial y^1}\right) & \frac{\partial u^1}{\partial y^1} + \left(\frac{\partial u^0}{\partial y^1}\right)^2 d\tau & \frac{1}{2}\left(\frac{\partial u^1}{\partial y^2} + \frac{\partial u^2}{\partial y^1}\right) + \frac{1}{2}\left|\left(\frac{\partial u^0}{\partial y^1}\right)\left(\frac{\partial u^0}{\partial y^2}\right)\right| d\tau & \frac{1}{2}\left(\frac{\partial u^1}{\partial y^3} + \frac{\partial u^3}{\partial y^1}\right) + \frac{1}{2}\left|\left(\frac{\partial u^0}{\partial y^1}\right)\left(\frac{\partial u^0}{\partial y^3}\right)\right| d\tau \\ \frac{1}{2}\sum_{k=1}^{3}\left(\frac{\partial u^0}{\partial y^k}\right)\left(\frac{\partial u^k}{\partial y^2}\right) & \frac{1}{2}\left(\frac{\partial u^2}{\partial y^1} + \frac{\partial u^1}{\partial y^2}\right) + \frac{1}{2}\left|\left(\frac{\partial u^0}{\partial y^2}\right)\left(\frac{\partial u^0}{\partial y^1}\right)\right| d\tau & \frac{\partial u^2}{\partial y^2} + \left(\frac{\partial u^0}{\partial y^2}\right)^2 d\tau & \frac{1}{2}\left(\frac{\partial u^2}{\partial y^3} + \frac{\partial u^3}{\partial y^2}\right) + \frac{1}{2}\left|\left(\frac{\partial u^0}{\partial y^2}\right)\left(\frac{\partial u^0}{\partial y^3}\right)\right| d\tau \\ \frac{1}{2}\sum_{k=1}^{3}\left(\frac{\partial u^0}{\partial y^k}\right)\left(\frac{\partial u^k}{\partial y^3}\right) & \frac{1}{2}\left(\frac{\partial u^3}{\partial y^1} + \frac{\partial u^1}{\partial y^3}\right) + \frac{1}{2}\left|\left(\frac{\partial u^0}{\partial y^3}\right)\left(\frac{\partial u^0}{\partial y^1}\right)\right| d\tau & \frac{1}{2}\left(\frac{\partial u^3}{\partial y^2} + \frac{\partial u^2}{\partial y^3}\right) + \frac{1}{2}\left|\left(\frac{\partial u^0}{\partial y^3}\right)\left(\frac{\partial u^0}{\partial y^2}\right)\right| d\tau & \frac{\partial u^3}{\partial y^3} + \left(\frac{\partial u^0}{\partial y^3}\right)^2 d\tau \end{bmatrix} \quad (8)$$

Temporal terms above were misrepresented in earlier works [1,2,3] and have been corrected here.

The rate of curvature tensor compares closely to the Einstein tensor in general relativity. We know from mathematics of elasticity that the rate of curvature tensor for thin shells is the push forward of the Lie derivative of the metric plus the Gaussian curvature. We also know from general relativity that the Lie derivative of the metric is the 3-dimensional extrinsic curvature and that Gaussian curvature is the Ricci scalar times the metric. Thus, the rate of curvature of spacetime IS the Einstein tensor in tangent (3+1) space.

$$D_\nu^\mu = \frac{1}{2}\mathcal{L}_\tau g_\nu^\mu + \frac{1}{2}|\kappa_\nu \kappa^\mu| \qquad (9)$$

$$D_\nu^\mu = K_{3D} + \frac{1}{2}|\kappa_\nu \kappa^\mu| \qquad (10)$$

$$D_\nu^\mu = R_\nu^\mu + \frac{1}{2}R g_\nu^\mu \qquad (11)$$

Constitutive Relation

A covariant constitutive relationship between stress-energy and rate of deformation enforces locality, creates a map over all configurations (histories), and establishes conservation laws for energy, linear momentum, angular momentum, and other symmetries. In continuum mechanics, the general constitutive relationship is given by the tensor version of Hooke's law,

$$T_\nu^\mu = C_{\beta\nu}^{\mu\alpha} D_\beta^\alpha \qquad (12)$$

where $C_{\beta\nu}^{\alpha\mu}$ is a 4x4x4x4 matrix known as stiffness or elasticity tensor. To ensure that spacetime remain smooth and continuously differential, the elasticity tensor must be orthonormal, invertible, and non-degenerate. An anisotropic constitutive relation was proposed in [2] in an attempt to unify gravitational and quantum fields. However, [3] explained that an anisotropic elastic tensor is unnecessary for unification if the relationship between mass and stored surface energy is taken into consideration at minimum curvature.

For the current work, we shall define the constitutive relation between stress-energy and rate of deformation of spacetime as a uniform constant in accord with Einstein,

$$T_\nu^\mu = \frac{c^4}{8\pi G} D_\nu^\mu \qquad (13)$$

Preferred Curvature

In the mechanics of thin shells, particularly biological systems, there is a concept known as preferred curvature wherein the lowest negative energy state (greatest surface tension) of a thin shell has a minimum preferred curvature. It is suggested here, that the minimum radius of curvature of spacetime is the Plank length, λ_p.

If we define a discrete mass, m, as the mass density integrated over the volume of three sphere with a minimum radius equal to the Plank length, then the right side of (2) becomes,

$$\oint_V dPdV = 3m \qquad (14)$$

If we integrate surface energy density (a.k.a. surface tension) over the surface of a three sphere of Plank radius, then the left side of (2) becomes,

$$\oiint_\sigma -\text{♀} d\sigma = -12\text{♀}\pi\lambda_p^2 \qquad (15)$$

By equivalence of work and energy (2), we see,

$$-\text{♀} = \frac{m}{4\pi\lambda_p^2} \qquad (16)$$

Quantum Mechanics Analogs

General relativity is a geometric theory. Quantum mechanics is interpreted as a probabilistic theory. Unification of general relativity and quantum mechanics requires a geometric analog for quantum mechanics. The differential geometry of continuum mechanics exists in the tangent space at all points on a manifold. If a manifold describes all possible positions, the tangent space represents all possible velocities. In an earlier section, it was explained that making a coordinate substitution such that time and proper time are imaginary, creates a positive definite complex Hilbert space. The tangent space at all points in spacetime is an orthogonal linear vector space with valid inner product consisting of state vectors $|dx_{\mu}>$ and corresponding dual vectors $< \frac{\partial}{\partial x_\mu}|$ that establish the expectation value of stress energy and curvature fields. Imposing a constitutive relation between stress energy and curvatures ensures that the space is continuously differential, separable, and complete.

Many continuum mechanics equations can be written in quantum mechanics notation. For example, (3) can be re-written as,

$$< \frac{\partial}{\partial \tau}|\text{♀}|d\tau > = dP$$

Mass density at a point is the expectation value of bras and kets formed by the spacetime separation between observations and the surface tension operator. In order to unify general relativity and quantum mechanics, it is important to recognize the mathematical similarities between differential geometry and Hilbert spaces.

In the remainder of this section, we examine other quantum mechanics analogs that arise in the geometry of spacetime with surface tension. The foregoing continuum mechanics tensors have some immediately recognizable wave equation components. The time-time component in the curvature gradient, (5), in laboratory coordinates is,

$$\frac{\partial^2 y^0}{\partial {y^1}^2}+\frac{\partial^2 y^0}{\partial {y^2}^2}+\frac{\partial^2 y^0}{\partial {y^3}^2}+\frac{1}{c^2}\frac{dP}{dy^0 \text{♀}}=\frac{1}{c^2}\frac{\partial^2 y^0}{\partial \tau^2} \qquad (17)$$

where time and proper time have been changed to natural units by multiplying by ic. For a vacuum, (17) reduces to the wave equation,

$$\frac{\partial^2 y^0}{\partial {y^1}^2}+\frac{\partial^2 y^0}{\partial {y^2}^2}+\frac{\partial^2 y^0}{\partial {y^3}^2}=\frac{1}{c^2}\frac{\partial^2 y^0}{\partial \tau^2} \qquad (18)$$

This equation describes capillary waves, which in general relativity, represent ripples in the coordinate of time that follow the overall curvature of the metric. The waves are massless and move at the speed of light regardless of frequency and wavelength. This expression is analogous to the Klein-Gordon equation in quantum mechanics.

For the case where temporal acceleration is zero, (17) reduces to,

$$\frac{\partial^2 y^0}{\partial {y^1}^2}+\frac{\partial^2 y^0}{\partial {y^2}^2}+\frac{\partial^2 y^0}{\partial {y^3}^2}=-\frac{1}{c^2}\frac{dP}{dy^0 \text{♀}} \qquad (19)$$

which is the classic Young-Laplace equation that describes corpuscular formation. The first three terms are the principal curvatures in the time direction. In a polar symmetric system with radius of curvature R, (19) becomes,

$$-3\text{♀}\left(\frac{dy^0}{R}\right)=\frac{dP}{c^2} \qquad (20)$$

which is the same expression intuitively derived and shown in (4).

Another quantum fluctuation is recognized in the spatial diagonal components of the deformation gradient (8). The dispersion equation is obtained by replacing the stress-energy with the corresponding rate of deformation in accord with the constitutive relation (13),

$$\frac{c^4}{8\pi G}\left(\frac{\partial^2 u^j}{\partial {y^j}^2}\right)=-\text{♀}\;\frac{\partial u^j}{\partial \tau} \qquad (21)$$

where the third order term has been omitted for simplification. For small particles with radius equal to the Plank length, λ_p, stored surface energy can be represented by a discrete mass, m, via (16). Changing to laboratory units, and noting that ${\lambda_p}^2=\frac{\hbar G}{c^3}$ results in,

$$-\frac{\hbar}{2m}\left(\frac{\partial^2 u^j}{\partial {y^j}^2}\right)=i\;\frac{\partial u^j}{\partial \tau} \qquad (22)$$

which is analogous to the one-dimensional Schrödinger wave equation with four-velocity replacing probability density. Surface energy and the corresponding spatial strains in spacetime move in accord with a Schrödinger-like dispersion equation.

Still another quantum fluctuation follows from the fluidity of spacetime, the constitutive relationship (13), and the shear strain or off diagonal spatial terms in (8), given by,

$$\frac{\partial u^i}{\partial y^j}+\frac{\partial u^j}{\partial y^i}+\left|\left(\frac{\partial u^0}{\partial y^i}\right)\left(\frac{\partial u^0}{\partial y^j}\right)\right|d\tau=0 \qquad (23)$$

In continuum mechanics, this is the Navier-Stokes equation for a vortex. There are three vortex solutions with half integer indices: unstable short-lived rotational vortices, stable irrotational vortices with field lines ending at a corpuscle, and a stable pair of oppositely rotating irrotational vortices entangled in time. A vortex itself does not carry mass, but does cause strain and deformation of spacetime thereby altering geodesics.

As a final comment on analogies with quantum field theory, consider the constitutive relationship itself. According to (13) and (1), spatial components of stress-energy are related to rate of deformation by,

$$-\text{♀}\, \delta^i_j = \frac{c^4}{8\pi G}\, D^i_j \qquad (24)$$

We can replace stored surface energy using the relationship (16) for discrete particles of Plank radius to obtain in laboratory units,

$$\frac{m\, \delta^i_j}{D^i_j} = -i\, \frac{\hbar}{2} \qquad (25)$$

The units of m are mass-energy. The units of D^i_j are the inverse of proper time. If we square both sides and apply the Cauchy–Schwarz inequality, then essentially,

$$m d\tau \geq \frac{\hbar}{2} \qquad (26)$$

The constitutive relationship for discrete particles of curved spacetime with surface tension at the Plank radius is analogous to the Heisenberg uncertainty principle.

Gravitational Geometry

Simply by adding and subtracting ♀ from the time-time term, the stress-energy tensor with surface tension (1) can be rewritten as,

$$T^\mu_\nu = \begin{bmatrix} dP + \text{♀} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} - \text{♀} g^\mu_\nu \qquad (28)$$

In this form, surface tension appears as an additional mass term, $+$♀, and a negative cosmological constant, $-$♀g^μ_ν. In cosmology applications, the additional mass term could be interpreted as “dark matter” and the negative cosmological constant as “dark energy”. Ordinary luminous matter, dP, and “dark matter”, $+$♀, are interrelated by (3) and (4). A correspondence between luminous matter and dark matter is to be expected. Eloquent arguments for such a correspondence are given in [24].

In a prior work [3], it was shown that replacing surface tension in (28) with its equivalent mass relation (4), results in an equation for orbital velocity given by,

$$v = \sqrt{GM\left(\frac{1}{R} + \frac{1}{4dy^0c^2}\right)} \qquad (29)$$

where the first term represents Newton’s contribution and the second term is the effect of surface tension. The time scalar, dy^0, can be set equal to unity for non-relativistic speeds.

Orbital velocities from (29) were compared with galaxy rotation curves for fifteen galaxies in [3]. Good correlation was found if the mass-to-light ratios of sample galaxies is set to values much lower than previously accepted. If the best-fit mass-to-light ratios are plotted relative to the square root of total galaxy luminosity, then an inverse correlation is found with an R^2 value of 0.96. When this relationship is re-inserted into the orbital velocity equation and the limit is taken at large values of R, the Tully-Fisher relation is derived.

Conclusions

It has been shown in this work that including surface tension in the treatment of spacetime results in quantum fluctuations that are analogous to quantum field mechanics at the minimum preferred curvature of Plank scale. It also has been shown that including surface tension in orbital dynamics at the cosmic scale can serve as an explanation for dark matter and dark energy.

References

[1] Perko, H.A. (2016) "Introducing Surface Tension to Spacetime", Journal of Physics: Conference Series, Vol. 845, 01203
[2] Perko, H.A. (2018) "Gravitation in the Surface Tension Model of Spacetime", Journal of Physics: Conference Series, Vol. 1239, 012010
[3] Perko, H.A. (2021) "Dark Matter and Dark Energy: Cosmology of Spacetime with Surface Tension", Journal of Physics: Conference Series, Vol. 1956, 012004
[4] Davidson, A. (2018) "Hydrogen-like Spectrum of Spontaneously Created Universes with deSitter Ground State", Physics Letters, B780, 29, arXiv:1708.00987
[5] Culetu, H. (2017) "Thin Shell Embedded in de Sitter Space", arXiv:1705.10381
[6] Freidel, L. and Yokokura, Y. (2015) "Non-Equilibrium Thermodynamics of Gravitational Screens", Classical and Quantum Gravity, arXiv:1405:4881
[7] Freidel, L. (2013) "Gravitational Energy, Local Holography and Non-Equilibrium Thermodynamics", arXiv:1312.1538
[8] Barut, A.O. and Pavsic, M. (1994) "Radiation Reaction and the Electromagnetic Energy-Momentum of Moving Relativistic Charged Membranes", Physics Letter B, 45-50, arXiv:1301.4837
[9] Kumar, H., Alam, S. and Ahmad, S. (2013) "Surface Tension with Normal Curvature in Curved Space-Time", General Relativistic Gravity, 45, 125-142, arXiv:1211.0128
[10] Wang, C.H., Wu, Y.H. and Hsu, S.D. (2012) "Quantum Fluctuations in Planar Domain-Wall Space-Times: A Possible Origin of Primordial Preferred Direction", Physics Letters B, 713, 6-9, arXiv:1107.1762
[11] Codello, A. and Zanusso, O. (2011) "Fluid Membranes and 2d Quantum Gravity", Physics Review D, 83, 125021, arXiv:1103.1089
[12] Culetu, H. (2010) "Boundary Stress Tensors for Spherically Symmetric Conformal Rindler Observers", Journal of Korean Physics Society, 57, (1), 17-22, arXiv:1001.4740
[13] Katti, R., Samuel, J. and Sinha, S. (2009) "The Universe in a Soap Film", Classical and Quantum Gravity, 26, 135018, arXiv:0904.1057

[14] Avelino, P.P., Martins, C.J.A.P., and Sousa, L. (2008) “Dynamics of Biased Domain Walls and the Devaluation Mechanism”, Physics Review D, 78:043521, arXiv:0805.4013
[15] Cardoso, V. and Gualtieri, L. (2006) “Equilibrium Configurations of Fluids and their Stability in Higher Dimensions”, Classical and Quantum Gravity, 23:7151-7198, arXiv:061004
[16] Samuel, J. and Sinha, S. (2006) “Surface Tension and the Cosmological Constant”, Physics Review Letters, 97:161302, arXiv:0603804
[17] Lobo, F.S.N. (2003) “Thin Shells Around Traversable Wormholes”, Proceedings of APCTP Winter School and Workshop: Quantum Gravity, Black Holes, and Wormholes, Pohang School of Environmental Engineering, South Korea, 11-14, arXiv:0401083
[18] Lobo, F.S.N. and Crawford, P. (2004) “Linearized Stability Analysis of Thin-Shell Wormholes with a Cosmological Constant”, Classical and Quantum Gravity, 21, 391-404, arXiv:0311002
[19] Graham, N., et al. (2003) “Casmir Energies in Light of Quantum Field Theory”, Physics Letters B, 572, 196-201, arXiv:0207205
[20] Schmidt, H.J. (1984) “Surface Layers in General Relativity and Their Relation to Surface Tensions”, General Relativistic Gravity, 16:1053-1061, arXiv:0105106
[21] Guendelman, E.I. and Portnoy, J. (1999) “The Universe Out of an Elementary Particle?”, Classical and Quantum Gravity, 16:3315-3320, arXiv:9901066
[22] Hajicek, P. and Bicak, J. (1997) “Gauge Invariant Hamiltonian Formalism for Spherically Symmetric Gravitating Shells”, Physics Review D, 56:4706-4719, arXiv:9706022
[23] Callaway, D.J.E. (1996) “Surface Tension, Hydrophobicity, and Black Holes: The Entropic Connection”, Physics Review E:53:3738-3744, arXiv:9601111
[24] Mannheim, P.D. (2005) “Alternatives to Dark Matter and Dark Energy”, *The Astrophysical Journal*, 0505266v2