Nonmetricity, Conformal Invariance and Inflationary Cosmology

E.A.Poberii
Friedmann Laboratory for Theoretical Physics,
University E&F, Griboedov Can. 30/32,
St.Petersburg 191023,
Russia

Abstract

We investigate the possible role of nonmetricity in cosmology within the framewirk of metric-affine space-time. It is shown that the Weyl part of nonmetricity could play the role of inflaton field. We investigate also cosmological consequences of gravitational theory based on the complete relaxing of Riemannian constraints combined with the requirement of local conformal invariance. Such a theory turns out to be independent of the choice of measuring standards. Nevertheless we demonstrate that there exists a mechanism of spontaneous gauge fixing through which the scale enters the theory. It is shown that field equations admit of a de Sitter solution with no cosmological constant. Various possible developments of the theory are discussed in brief.

1 Introduction

Modern theoretical understanding of evolution of the universe is based on cosmological applications of Einstein's general relativity (GR). In this way, in 1922, A.A.Friedmann derived equations describing an expanding universe [1]. In nowadays A.Guth [2] suggested that, before the Friedmann stage, the universe had passed through the inflationary stage during which its size was growing very rapidly. The inflationary scenario of evolution of the universe is accepted now by many cosmologists mainly because it solves many cosmological problems in elegant way. On the other hand, solving cosmological problems inflation produces new problems connected with its own existence. For instance, to obtain inflation within the framework of GR one needs to take into consideration some material field (the inflaton field) which produces and governs the process of inflation. The simplest cosmological scenarios use for this aim a minimally coupled scalar field. However, as was shown by Chernikov and Tagirov [3] such a field has no physical sense in curved space-time. Moreover, they have shown that, in curved space-time, the physical field is the conformally coupled scalar field. Just this field has valid quasiclassical limit and other good properties. Therefore with scalar inflaton we find ourselves in paradoxical situation: the physical conformally coupled scalar field could not produce inflation, and if inflation takes place it inevitably is caused by the unphysical minimally coupled scalar field. Therefore a scalar inflaton is not an appropriate cause of inflation.

The problem of conformal coupling has another side which looks as follows. The inflationary stage is described by the de Sitter geometry where the Ricci tensor $R_{\mu\nu}$ is given by $-3R_0 g_{\mu\nu}$ where R_0 =const. Hence the Einstein equations

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -8\pi G T_{\mu\nu}$$

in a de Sitter cosmology inevitably imply that $T_{\mu\nu}$ is not traceless, *i.e.*, it is not conformally invariant. Therefore the ordinary formulation of the de Sitter cosmology requires the presence of masses or at least non-conformal couplings in $T_{\mu\nu}$. This fact looks somewhat suspicious because the de Sitter cosmology should describe the early universe where the temperature is so high that matter field mass scales should not be relevant and all the matter fields at that stage can be described in a conformally invariant way. Moreover, one might think that at very high temperatures even the gravitational field should be conformally coupled. The price one has to pay for breaking the conformal invariance at the de Sitter stage is the arising of a non-vanishing, and usually huge, cosmological constant.

In this paper in order to avoid the difficulties of such kind we shall try to go beyond the framework of GR by enlarging its geometrical structure.

Really, concerning the geometrical nature of gravitation one may ask the question: why is the space-time continuum Riemannian? At the microscopical level there are no physical reasons for the existence of the Riemannian structure on the space-time continuum. The existence of locally Lorentz metric is guaranteed by special relativity while the existence of affinity is guaranteed by the weak equivalence principle. Hence the existence of the above two objects indicates that, in general, the space-time continuum is a metric-affine manifold. Ehlers *et al.* [4] analyzed the separate roles played by affinity and metric at the classical level when sufficiently large distances were considered. Let us note, however, that their analysis becomes unapplicable when one considers scales close to the Planck length where the very notions of the light cone and rigid rods lose their sense. To reach GR one must impose two *ad hoc* mathematical constraints: (i) the affinity must be metric compatible, and (ii) the affinity must be symmetric. There are no evident physical reasons to impose these conditions in microworld. A method of clarifying the physical meaning of the above constraints is to construct a more general theory by relaxing them and then examining the physical effect. Perhaps in this way one may deeper understand a mysterious connection of gravity with the space-time structure.

2 Nonmetricity as a Cause of Inflation

We shall work with a metric-affine space-time with no constraints imposed on the metric $g_{\mu\nu}$ or on the connection $\Gamma^{\lambda}{}_{\mu\nu}$, so that $g_{\mu\nu}$ and $\Gamma^{\lambda}{}_{\mu\nu}$ are completely independent gravitational variables. The nonmetricity tensor $W_{\lambda\mu\nu}$ is defined according to the relation

$$W_{\lambda\mu\nu} = \nabla_\lambda g_{\mu\nu}. \tag{1}$$

It is convenient to split $W_{\lambda\mu\nu}$ in the following way

$$W_{\lambda\mu\nu} = \overline{W}_{\lambda\mu\nu} + 2W_\lambda g_{\mu\nu} \tag{2}$$

where W_λ is proportional to the trace of the nonmetricity tensor and $\overline{W}_{\lambda\mu\nu}$ is its traceless part

$$W_\lambda = \frac{1}{8} W_{\lambda\rho}{}^{\rho}; \quad \overline{W}_{\lambda\rho}{}^{\rho} = 0. \tag{3}$$

Using (1) one can represent the full affine connection $\Gamma^{\mu}{}_{\nu\rho}$ in the form

$$\Gamma^{\mu}{}_{\nu\rho} = \{^{\mu}{}_{\nu\rho}\} + S^{\mu}{}_{\nu\rho} - S_{\nu}{}^{\mu}{}_{\rho} - S_{\rho}{}^{\mu}{}_{\nu} + \frac{1}{2}(W^{\mu}{}_{\nu\rho} - W_{\nu}{}^{\mu}{}_{\rho} - W_{\rho}{}^{\mu}{}_{\nu}) \qquad (4)$$

where $S^{\mu}{}_{\nu\rho}$ is the torsion tensor

$$S^{\mu}{}_{\nu\rho} \equiv \Gamma^{\mu}{}_{[\nu\rho]} = \frac{1}{2}(\Gamma^{\mu}{}_{\nu\rho} - \Gamma^{\mu}{}_{\rho\nu}). \qquad (5)$$

The curvature tensor $R^{\alpha}{}_{\beta\mu\nu}$ is defined according to

$$R^{\alpha}{}_{\beta\mu\nu} = \partial_{\mu}\Gamma^{\alpha}{}_{\beta\nu} - \partial_{\nu}\Gamma^{\alpha}{}_{\beta\mu} + \Gamma^{\alpha}{}_{\lambda\mu}\Gamma^{\lambda}{}_{\beta\nu} - \Gamma^{\alpha}{}_{\lambda\nu}\Gamma^{\lambda}{}_{\beta\mu}. \qquad (6)$$

Let us define the Ricci tensor $R_{\mu\nu}$ by the formula

$$R_{\mu\nu} = \frac{1}{2}(R^{\sigma}{}_{\mu\sigma\nu} + R_{\mu\sigma\nu}{}^{\sigma}), \quad (R_{\mu\nu} \neq R_{\nu\mu}), \qquad (7)$$

and the segmental curvature tensor $\Omega_{\mu\nu}$ by

$$\Omega_{\mu\nu} \equiv \frac{1}{4}R^{\sigma}{}_{\sigma\mu\nu} = \partial_{\nu}W_{\mu} - \partial_{\mu}W_{\nu}. \qquad (8)$$

Now, let us turn to the choice of the gravitational Lagrangian. As is well known, [5] the main problem in the metric-affine gravity with the Hilbert type Lagrangian

$$L = \alpha R; \quad \alpha = M_p^2/16\pi \qquad (9)$$

(M_p is the Planck mass) is its projective invariance, *i.e.*, this Lagrangian is invariant under the transformations

$$\Gamma^{\lambda}{}_{\mu\nu} \longrightarrow \Gamma^{\lambda}{}_{\mu\nu} + \delta^{\lambda}_{\mu}\lambda_{\nu}; \quad g_{\mu\nu} \longrightarrow g_{\mu\nu}. \qquad (10)$$

As a consequence, four degrees of freedom associated with the Weyl vector W_{μ} remain completely undetermined by the field equations obtained from this Lagrangian. It is well known [5] that for a viable gravitational theory the projective invariance must be broken. There are several ways to do this. Remind that in GR this problem is solved by imposing the metric condition $\nabla_{\lambda}g_{\mu\nu} = 0$ that implies $W_{\mu} = 0$. Here we break the projective invariance by including terms proportional to the square of the Weyl vector $W_{\mu}W^{\mu}$ into the gravitational Lagrangian. Details and justification for this approach have been given elsewhere [6]. The total gravitational Lagrangian has the form

$$L = \sqrt{-g}\left(\alpha R + \frac{k}{2}\Omega_{\mu\nu}\Omega^{\mu\nu} + U(\xi)\right) \qquad (11)$$

where k is a dimensionless constant, $\xi = W_{\mu}W^{\mu}$ and $U(\xi)$ is a function (a "potential") which is not projectively invariant. Such form of the Lagrangian could be arisen, as well, in the result of a spontaneous breakdown of the scale invariance in the initially conformally invariant theory in a metric-affine space-time (see section 3). In any way we can regard the function $U(\xi)$ as an effective potential analogous to that arising for a scalar field in modern cosmology. Equations of motion following from this Lagrangian, after some transformations, can be represented in the form

$$\widetilde{R}_{\mu\nu} - \frac{1}{2}g_{\mu\nu}\widetilde{R} = -\frac{1}{2\alpha}T_{\mu\nu}; \qquad (12)$$

$$\widetilde{\nabla}_{\sigma}\Omega^{\sigma}{}_{\nu} + 2U'(\xi)W_{\nu} = 0; \qquad (13)$$

$$\Gamma^{\mu}{}_{\nu\lambda} = \{^{\mu}{}_{\nu\lambda}\} - \delta^{\mu}_{\nu}W_{\lambda}; \qquad (14)$$

$$S_{\mu\nu\lambda} = g_{\mu[\nu}W_{\lambda]}; \qquad (15)$$

$$\overline{W}_{\lambda\mu\nu} = 0. \qquad (16)$$

where the tilde above a letter denotes the ordinary Riemannian part of corresponding geometrical object, and the prime denotes the derivative with respect to ξ. The stress-energy tensor $T_{\mu\nu}$ is equal to

$$T_{\mu\nu} = k\left[\Omega_{\mu}{}^{\lambda}\Omega_{\nu\lambda} - \frac{1}{4}g_{\mu\nu}\Omega_{\alpha\beta}\Omega^{\alpha\beta}\right] - g_{\mu\nu}U + 2U'W_{\mu}W_{\nu}. \qquad (17)$$

As it is seen from the above equations a remarkable feature of the theory with the Lagrangian (11) is that the Weyl vector here plays the role of a source of the Riemannian part of curvature. On the other hand, just the Riemannian part of connection governs the dynamics of the Weyl vector field W_{μ} as it is seen from Eq. (13). Due to this equation there is no place for the projective invariance. One may consider this theory to be equivalent to usual GR with some external massive vector field. But an essential difference is that here this field is internal and is a part of the full nonmetrical connection, as it is seen from Eq. (14). Moreover, this field via the algebraic relation (15) also determines the torsion properties of the metric-affine space-time.

Let us consider what cosmological consequences follow from the theory described by the Lagrangian (11). The nonmetrical properties essential for cosmology are contained in the stress-energy tensor (17) of the Weyl vector field. In general the stress tensor of this field need not be isotropic, so we may assume that space-time will not be Friedmann-Robertson-Walker type. Instead, we may take an anisotropic metric. Let us take a Bianchi type-I metric

$$ds^2 = -dt^2 + a^2(t)(dx^2 + dy^2) + b^2(t)dz^2. \tag{18}$$

Let W_z be the nonzero spatial component of the Weyl vector field. We shall be interested in homogeneous solutions, so that $W_\mu = W_\mu(t)$. This leads to the fact that in the metric (18) Eq. (13) for W_μ implies that $W_t = 0$ for such solutions, and we have for the only nonzero component W_z the following equation:

$$\ddot{W}_z + \left[2\frac{\dot{a}}{a} - \frac{\dot{b}}{b}\right]\dot{W}_z + \frac{2}{k}U'W_z = 0. \tag{19}$$

The gravitational equations (12) for this metric are:

$$2\frac{\dot{a}\dot{b}}{ab} + \frac{\dot{a}^2}{a^2} = \frac{1}{2\alpha}\varrho, \tag{20}$$

$$\frac{\ddot{a}}{a} + \frac{\dot{a}\dot{b}}{ab} + \frac{\ddot{b}}{b} = -\frac{1}{2\alpha}p_x = -\frac{1}{2\alpha}p_y, \tag{21}$$

$$2\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} = -\frac{1}{2\alpha}p_z \tag{22}$$

with the conservation law

$$\dot{\varrho} + \left[2\frac{\dot{a}}{a} + \frac{\dot{b}}{b}\right]\varrho + 2\frac{\dot{a}}{a}p_x + \frac{\dot{b}}{b}p_z = 0, \tag{23}$$

where the energy density ϱ and pressures p_x, p_z for the Weyl field are

$$\varrho = \frac{k}{2}\frac{\dot{W}_z^2}{b^2} + U, \tag{24}$$

$$p_x = \frac{k}{2}\frac{\dot{W}_z^2}{b^2} - U, \tag{25}$$

$$p_z = -\varrho + 2\xi U'. \tag{26}$$

Let us note that different inflation scenarios within the framework of GR for a material vector field have been considered by Ford [7]. In our approach only a chaotic-type scenario is acceptable because in this case the nonmetricity field tends to zero at the end of inflation. Thus consider the case when initially W_z was nonzero and slowly varying field (so that one can neglect the time derivative terms in (24)-(26)) but at late times it evolves toward zero. To obey these requirements the potential $U(\xi)$ must have a minimum at $\xi = 0(\xi = W_z^2/b^2)$. Moreover it must be sufficiently flat for large ξ in order that the duration of inflation would be sufficiently long. With these assumptions supposing that

$$U \gg 2\xi U' \tag{27}$$

we obtain $p_x = p_y \approx p_z \approx -\varrho$, and the universe rapidly comes to the quasi-de Sitter stage

$$a(t) = b(t) = e^{Ht} \tag{28}$$

with the slowly varying Hubble parameter

$$H = (U/6\alpha)^{1/2}. \tag{29}$$

Let us consider an explicit form of a suitable potential $U(\xi)$:

$$U(\xi) = U_0 \ln\left(1 + \frac{m^2\xi}{2U_0}\right) \tag{30}$$

where one may consider U_0 as a lower boundary of inflation below which the inequality (27) is broken and the potential becomes rapidly falling. The potential (30) mathematically has no upper boundary. Nevertheless one can consider its physical upper boundary

$$U_{max} \approx M_p^4, \tag{31}$$

so that inflation takes place in the region

$$U_0 \lesssim U \lesssim U_{max}. \tag{32}$$

Due to the high degree of flatness of the potential (30) the process of inflation may be sufficiently long and the value of expansion during inflation may be extremely large. However in order to obtain adequate inflation one needs considerable fine-tuning.

The field W_z during inflation evolves according to the following equation

$$\ddot{W}_z + H\dot{W}_z + \frac{2}{k}U'W_z = 0. \tag{33}$$

For large ξ ($\xi \gg U_0 m^{-2}$) $U' \approx 0$, so the field is a slowly falling (at most linearly) function. On the other hand, for small ξ ($\xi \ll U_0 m^{-2}$) one has $U \approx 1/2m^2\xi$ and the field W_z obeys the equation

$$\ddot{W}_z + H\dot{W}_z + \frac{m^2}{k}W_z = 0. \tag{34}$$

In the case

$$\lambda^2 = 4\frac{m^2}{k} - H^2 > 0 \tag{35}$$

this equation has a solution in the form of damping oscillations which can be written as

$$W_z = A\exp(-\frac{1}{2}Ht)\sin\frac{1}{2}\lambda(t-B) \tag{36}$$

where A and B are two arbitrary constants. Thus one may consider the region $0 \lesssim U \lesssim U_0$ as corresponding to the reheating stage. In order to obtain some quantitative information about this stage one needs to take into consideration the interaction of matter with torsion and nonmetricity. However in the presence of material sources of torsion and nonmetricity the resulting equations will be essentially different from (12)-(16). Let us note that due to the fact that W_z tends to zero, the stress tensor of the Weyl field vanishes, so the problem of anisotropy at late times does not arise in this scenario.

In the example considered above we have assumed that inflation goes with the same rates in all directions ($a(t) = b(t) = \exp Ht$). But there is no need for this assumption in cosmology. All that is needed is that the universe expands by the factor greater than 10^{28} in all directions. However this process may go anisotropically, *i.e.*, at different rates along different axes but under condition that at later stages the anisotropy disappears. The process of anisotropic inflation for the material vector field was considered in detail by Ford [7]. It may be relevant also in the case of inflation driven by vector nonmetricity.

As we have seen from the above consideration, nonmetricity in its Weylian form could have played the role of geometrical inflaton field in metric-affine cosmology. By the end of the process nonmetricity disappears, so that space-time becomes Riemannian (or Riemann-Cartan type in the presence of material torsion sources). As it is easy to see, here the Weyl nonmetricity produces the cosmological term responsible for inflation because the conformal invariance is broken from the very beginning (by the Einstein-Hilbert term in the Lagrangian). In the next section we consider a more general situation when the conformal invariance is preserved but the de-Sitter stage nevertheless exists.

3 Scale-Covariant Metric-Affine Cosmology

In this section we represent main cosmological results of scale-covariant metric-affine gravity. For details we refer the reader to [8]. Let us first introduce the main mathematical notions to be used in further consideration.

Generalized conformal transformations in a metric-affine space have the form [8]:

$$g'_{\mu\nu} = e^{2\lambda(x)}g_{\mu\nu}; \tag{37}$$
$$W'_\mu = W_\mu + \partial_\mu\lambda(x); \tag{38}$$
$$\overline{W}'^{\lambda}{}_{\mu\nu} = \overline{W}^{\lambda}{}_{\mu\nu}; \tag{39}$$
$$S'^{\lambda}{}_{\mu\nu} = S^{\lambda}{}_{\mu\nu}. \tag{40}$$

A localized geometrical quantity A transformed under (37) according to the law $A' = e^{n\lambda(x)}A$ is called to be of power n. If A further behaves as a tensor relative to the ordinary coordinate transformations, it is called a *co-tensor of power n*. If $n = 0$ it is called an *in-tensor*. By definition $g_{\mu\nu}$ is a co-tensor of power 2, while $g^{\mu\nu}$ is a co-tensor of power -2. It is easy to check that, if A_μ is a co-vector of power n, the quantity

$$\overset{*}{\nabla}_\lambda A_\mu \overset{def}{\equiv} \partial_\lambda A_\mu - \Gamma^\sigma{}_{\mu\lambda}A_\sigma - nW_\lambda A_\mu, \tag{41}$$

called the *co-covariant derivative* of A_μ, is also a co-tensor of the same power n. Now we are ready to construct scale-covariant gravitational theory in terms of co-covariant objects.

The requirement of scale invariance in general means that we want to have a possibility to choose an arbitrary standard of length, or *gauge*, at each space-time point. This leads to the idea of gauge transformation of metric (37) where $\lambda(x)$ is an arbitrary function of the coordinates. Therefore we want our theory to satisfy the following two conditions:

- It is locally Lorentz invariant;
- It is locally conformally invariant.

The second requirement forbids the presence of the Hilbert-Einstein term $\sim R$ in the action integral. This fact was known still to Weyl [9]. For this he was forced to take an action involving the square of the curvature scalar $\sim R^2$. This led to unsatisfactory higher order field equations.

Later, Dirac [10] returned to Weyl's theory, but modified it. In order to preserve the Hilbert-Einstein term he introduced a co-scalar β of power -1, so that the action with the term $\sim \beta^2 R$ was conformally invariant. In view of the possibility of gauge transformations, the function β is arbitrary. By a suitable transformation $\beta \rightarrow \beta' = \exp(-\lambda)\beta$ one can make $\beta = 1$. This gives the so-called "Einstein gauge", since the formalism of GR corresponds to $\beta = 1$.

In our approach we shall follow Dirac's ideas. Thus in our manifold there exist at least three fundamental notions: distance, parralelism, and gauge. The notion of gauge is introduced in the meaning of the conformal transformation such as Weyl's original point of view, so there exists the notion of gauge only when there exists the notion of distance. Consequently the notion of parallelism is independent of not only the notion of distance but also of gauge. The freedom of gauge will be described by the co-scalar β of power -1. Therefore the Lagrangian density, in general, will depend on the gravitational variables $g_{\mu\nu}, \Gamma^{\lambda}{}_{\mu\nu}$ and on the scalar field β. Let us emphasize here an essential difference from standard scalar-tensor theories: the field β in our approach *does not* represent an additional gravitational variable while in scalar-tensor theories [11] it *does*. In our approach β expresses only the freedom in choice of units.

For simplicity here we do not consider the material Lagrangian referring the reader to [8] where a comlete theory is considered. Following Dirac [10],

let us take the gravitational Lagrangian density $\mathcal{L}_g$ in the form

$$\mathcal{L}_g = \sqrt{-g}\left(\beta^2 R - \frac{1}{4}\Omega_{\mu\nu}\Omega^{\mu\nu} + kg^{\mu\nu}\overset{*}{\nabla}_\mu\beta\overset{*}{\nabla}_\nu\beta + \lambda\beta^4\right) \tag{42}$$

where $\beta(x)$ is a co-scalar of power -1, k and λ are dimensionless constants (numbers). Let us emphasize that each term of $\mathcal{L}_g$ separately is conformally invariant contrary to the case of a conformal scalar field in the Riemannian space where the conformal invariance for the scalar field is fulfilled only for the total Lagrangian with special choice of a constant by the R term.

In the absence of material sources equations of motion for the conformally covariant gravity take a simple form:

$$\overline{W}_{\lambda\mu\nu} = 0; \tag{43}$$

$$\beta^2\left(R_{(\mu\nu)} - \frac{1}{2}g_{\mu\nu}R\right) = \frac{1}{2}\left(\Omega_\mu{}^\sigma\Omega_{\nu\sigma} - \frac{1}{4}g_{\mu\nu}\Omega_{\alpha\beta}\Omega^{\alpha\beta}\right) +$$

$$+k\left(\frac{1}{2}g_{\mu\nu}g^{\sigma\rho}\overset{*}{\nabla}_\sigma\beta\overset{*}{\nabla}_\rho\beta - \overset{*}{\nabla}_\mu\beta\overset{*}{\nabla}_\nu\beta\right) + \frac{1}{2}g_{\mu\nu}\lambda\beta^4; \tag{44}$$

$$\widetilde{\nabla}_\sigma\Omega^\sigma{}_\nu = 2k\beta\overset{*}{\nabla}_\nu\beta; \tag{45}$$

$$\beta S_{\nu\mu\lambda} = g_{\nu[\lambda}\overset{*}{\nabla}_{\mu]}\beta; \tag{46}$$

$$\beta R - k\left[g^{\rho\sigma}\overset{*}{\nabla}_\rho\overset{*}{\nabla}_\sigma\beta - 2S^\sigma\overset{*}{\nabla}_\sigma\beta\right] + 2\lambda\beta^3 = 0. \tag{47}$$

Eq.(43) restricts our space-time to be of Weyl-Cartan type. Note that Eq.(47) is not an independent equation and is a consequence of the other equations an of Bianchi's identities and hence the scalar field β is not determined by the field equations. This gauge freedom is the result of confomal invariance. Therefore a complete set of equations in this case reduces to four equations (43), (44), (45) and (46).

Using Eq.(46) one can represent the term $\beta^2 R$ in the form

$$\beta^2 R = \beta^2\widetilde{R} - 6\widetilde{\nabla}_\sigma(\beta\partial^\sigma\beta) + 6g^{\rho\sigma}\overset{*}{\nabla}_\rho\overset{*}{\nabla}_\sigma\beta + 6\beta^2 W^\sigma W_\sigma - 8\beta^2 W^\sigma S_\sigma, \tag{48}$$

where $\widetilde{R}$ is the Riemannian part of R.

Substituting this into the action integral, after removing the total divergence, we obtain for the gravitational action without matter

$$S = \int \sqrt{-g} \left(\beta^2 \widetilde{R} + (k+6) g^{\rho\sigma} \overset{*}{\nabla}_\rho \overset{*}{\nabla}_\sigma \beta + 6\beta^2 W^\sigma W_\sigma - \right.$$

$$\left. -8\beta^2 W^\sigma S_\sigma + \lambda\beta^4 - \frac{1}{4}\Omega^{\mu\nu}\Omega_{\mu\nu} \right) d^4x. \tag{49}$$

In the Einstein gauge where $\beta = \beta_0$ =const we obtain

$$S = \int \sqrt{-g} \left(\beta_0^2 \widetilde{R} + k\beta_0^2 W^\sigma W_\sigma + \lambda\beta_0^4 - \frac{1}{4}\Omega^{\mu\nu}\Omega_{\mu\nu} \right) d^4x. \tag{50}$$

From this one can see that in this gauge our theory corresponds to Einstein's gravity interacting with a massive vector field. The essential difference is that this vector field is a part of the full connection of the Weyl-Cartan space-time

$$\Gamma^\lambda{}_{\mu\nu} = \{^{\lambda}_{\mu\ \nu}\} - \delta^\lambda_\mu W_\nu. \tag{51}$$

We shall call hypothetic particles corresponding to the massive Weyl field as "weylons".

By fixing the gauge we have introduced into the theory a standard of length which can be expressed via the Planck length l_{pl} as

$$l_{pl} = \frac{1}{4\sqrt{\pi}\beta_0}.$$

Correspondingly the mass of the weylon expressed via the Planck mass M_{pl} is

$$m_{weyl} = \frac{M_{pl}}{4}\sqrt{\frac{k}{\pi}}.$$

It is important to remark that by fixing the gauge we do not break the conformal invariance explicitly. Really, one can choose another constant value for β, *e.g.* $\beta = \beta_0' \neq \beta_0$ and obtain the same result (50) with β_0 replaced by β_0'. All that was happened is that standards of measurements were changed that, by the very sense of gauge theory, did not influence physical laws.

Until the gauge is not fixed it is meaningless to speak about massive particles in the usual sense. However the above fixation of gauge was done by hand. One may ask then: is there anything in the world that could fix the gauge in a conformally invariant theory? Let us investigate this issue using the analogy with a spontaneous symmetry breaking. This possibility can be realized if the potential for the scalar field β admits minima different from $\beta = 0$. At first glance it is not possible since, in general, as it is seen from (49), the potential for β contains only the term $\lambda\beta^4$. Consider, however, a de Sitter cosmology which is described by space-time of a constant Riemannian curvature. In the de Sitter space-time we have

$$\widetilde{R}_{\mu\nu} = -3R_0 g_{\mu\nu}; \tag{52}$$

$$\widetilde{R} = -12R_0 \tag{53}$$

where R_0 =const.

In this case the term $\beta^2\widetilde{R}$ can be included into the potential $V(\beta)$, so that it has the form

$$V(\beta) = -12R_0\beta^2 + \lambda\beta^4. \tag{54}$$

For the minimum β_0 of this potential we have

$$\beta_0^2\lambda = 6R_0, \tag{55}$$

so that our model describes either de Sitter or anti-de Sitter geometry depending on the sign of λ. In both cases the potential can be represented in the form

$$V(\beta) = \lambda\beta^4 - 2\lambda\beta_0^2\beta^2 \tag{56}$$

with the quartic and quadratic terms having opposite relative signs independent of the sign of λ. Therefore, in this case, the gauge is rigidly fixed by the geometry via Eq.(55) and just the geometry is responsible for arising of a scale.

Let us write down equations of motion (44)-(47) for this case. Because these equations are conformally covariant they are valid for a particular choice of a gauge. Substituting (52), (53) and (55) into them, after simple calculations, we obtain respectively

$$T^{weyl}_{\mu\nu} = 0; \tag{57}$$

$$\widetilde{\nabla}_\sigma \Omega^\sigma{}_\nu = 2k\beta_0^2 W_\nu; \tag{58}$$

$$S_{\nu\mu\lambda} = g_{\nu[\lambda}W_{\mu]}; \tag{59}$$

$$\widetilde{\nabla}_\sigma W^\sigma - W_\sigma W^\sigma = 0, \tag{60}$$

where $T^{weyl}_{\mu\nu}$ is the energy momentum tensor of the massive Weyl field:

$$T^{weyl}_{\mu\nu} = \frac{1}{2}\left(\Omega_\mu{}^\sigma\Omega_{\nu\sigma} - \frac{1}{4}g_{\mu\nu}\Omega_{\alpha\beta}\Omega^{\alpha\beta}\right) + k\beta_0^2\left(\frac{1}{2}g_{\mu\nu}W_\sigma W^\sigma - W_\mu W_\nu\right). \tag{61}$$

Combining (58) with (60) we find the relation

$$W_\mu W^\mu = 0, \tag{62}$$

so that (57) is identically satisfied.

One can take $W_\mu = 0$ as a solution to (62). In this case the space-time continuum becomes Riemannian because torsion also vanishes as it is seen from (59). Therefore the solution with the de Sitter form of the Riemannian part of curvature is the *vacuum solution* of conformally invariant gravity in a metric-affine space-time just as the Schwarzschield solution is the vacuum solution for Einstein gravity within the framework of Riemannian geometry. It is useful to compare this result with the situation in the previous section where the conformal invariance was broken explicitly from the very beginning. There, as a result, the energy momentum tensor of weylons was responsible for arising of cosmological constant producing the de Sitter stage, whereas the Weyl vector field itself played the role of the inflaton field. The situation presented here is quite different.

Let us note that if one takes into consideration matter without hypermomentum one can check that the field equations also admit a de Sitter solution with no cosmological constant [8].

It is important to remark that direct consequence of conformal invariance is that all the masses arising in such a theory are proportional to β_0. Therefore the ratio of masses is independent of the choice of gauge β_0. The same is valid for lengths and time intervals. This means that such a theory describes physical phenomena independently of the choice of measuring standards. This property is very natural and important for physical theory because any real measuring process represents nothing more than a comparison of the measured quantity with some quantity accepted as a standard, *i.e.*, only the ratio of quantities has direct physical sense. An explanation of these ratios, however, lies beyond the ability of the conformal invariant theory.

4 Concluding Remarks

We have shown that the approach based on the complete relaxing of Riemannian constraints combined with the requirement of local scale invariance represents a consistent framework for gravitational theory. The consequence of conformal invariance is that the theory describes the world in which only ratios of physical quantities have direct physical sense independently of the choice of conformal gauge. The other consequence is that the theory intrinsically possesses a de Sitter solution with no cosmological constant. We also have shown that there exists a natural mechanism of spontaneous gauge fixing, in the result of which the universe acquires an absolute standard of units. However this mechanism works under very special conditions. The problem of searching a plausible breaking mechanism for conformal symmetry in general is not so simple. In this connection it is of interest an attempt recently undertaken in this direction by Wood and Papini [12]. They introduced "atoms" as small classical regions in surrounding space-time with nonmetricity where the Weyl vector is zero, and therefore the conformal invariance inside such "atoms" is broken. In their approach just small regions of space-time ("atoms") produce an absolute standard of units and, moreover, they also determine the global geometry of space-time. In our approach an opposite point of view was proposed, according to which just the global geometry of space-time provides us with an absolute standard of units. Whether this scope is correct further investigations will show.

Consider now the question about the physical meaning that could be given to the gauge field β. In our approach β expresses only our freedom in choice of measuring standards. However, β determines all the masses in the universe in a unique way. This circumstance prompts the thought that, generally speaking, β might play the role of the universal mass function through which Mach's principle could enter into the theory. Such an interpretation of β in the framework of Riemannian geometry was actively developed by Narlikar [13]. As a result he obtained quite different cosmology in which, in order to explain observational data, there is no need in the inflationary stage and, moreover, there are no space-time singularities. For details we refer the reader to his original works [13, 14]. It should be noted that such an interpretation can be applied in the case of metric-affine space-time as well. Moreover it can be done in a consistent and elegant

8 Заказ 636

way. To see this consider the geometrical structure of our theory. It is evident that the field β has no geometrical origin and is brought into the theory from outside. The only need in this field is to ensure the conformal invariance of the theory. However one may try to connect it with the geometry of space-time. Really, in order to ensure conformal invariance one may use the term $\overline{W}^{\mu}\overline{W}_{\mu}R$ instead of $\beta^2 R$. In this way no non-geometrical quantities enter into the theory, and the mass function mentioned above will be connected with the proper nonmetricity of space-time. First step in this direction was done by Obata *et al.* [15]. Taking the nonmetricity tensor to be linear in the metric tensor they obtained a theory which under a special choice of the mass function resembles either the Brans-Dicke or the Hoyle-Narlikar theory. However these results were obtained under condition that the Weyl vector has the gradient form, so that the tensor $\Omega_{\mu\nu}$ vanishes identically in their approach. In more general case when $\Omega_{\mu\nu}$ is nonzero equations of motion will be more complicated. Nevertheless such an approach, as we think, might represent a perspective direction in the development of metric-affine gravity.

At last let us concern in brief the fate of nonmetricity in the evolving universe. There is no direct experimental evidence in favor of its presence in space-time surrounding us today. Therefore it should disappear during evolution of the universe, or at least its influence could manifest itself in some unexplained up to now physical phenomena. Here we have shown that nonmetricity could have played the role of the inflaton field and had disappeared when inflation came to the end. Another possibility was proposed in the recent paper of Israelit and Rosen [16] in which "weylons" were interpreted as the dark matter of the universe. In any way the gravitational theory in a metric-affine space-time offers reach possibilities to researchers, and further investigations are needed in order to elucidate the role of nonmetricity, if any, in surrounding physical world.

Acknowledgments

The author is grateful to the participants of A.A.Friedmann Laboratory gravitational seminar for useful discussions.

References

[1] A.A.Friedmann. *Zeitschr.fur Phys.* **10,** 377 (1922).

[2] A.H.Guth, *Phys. Rev.* D **23** 347 (1981).

[3] N.A. Chernikov and E.A.Tagirov, *Ann. Inst. H. Poincaré,* **9A,** 109 - 41 (1968).

[4] J.Ehlers, F.A.E.Pirani and A.Schild, In: *General Relativity,* ed. O'Raifertaigh, Oxford University Press, London (1972).

[5] F.W.Hehl, E.A.Lord and L.L.Smalley, *Gen. Rel. Grav.* **13** 1037 (1981).

[6] E.A.Poberii, *Friedmann Laboratory for Theoretical Physics, Preprint* FL-030692 (1992) (Submitted to "Classical and Quantum Gravity").

[7] L.H.Ford, *Phys. Rev.* D **40** 967 (1989).

[8] E.A.Poberii, *Friedmann Laboratory for Theoretical Physics, Preprint* FL-010593 (1993) (Submitted to "General Relativity and Gravitation").

[9] H.Weyl, *Raum, Zeit, Materie, vierte erweiterte Auflage,* Berlin: Springer (1921).

[10] P.A.M.Dirac, *Proc. R. Soc. Lond.,* **A333,** 403 (1973).

[11] C.Brans and R.H.Dicke, *Phys. Rev.,* **124,** 925 (1961).

[12] W.R.Wood and G.Papini, *Phys. Rev.* **D45,** 3617 (1992).

[13] J.W.Narlikar, *Phys. Rev.* **D32,** 1928 (1985).

[14] J.V.Narlikar, *Lectures on General Relativity and Cosmology,* London: Macmillan Press LTD (1979).

[15] T.Obata, H.Oshima and J.Chiba, *Gen. Rel. Grav.* **13,** 3133 (1981).

[16] M.Israelit and N.Rosen, *Found. Phys.* **22,** 555 (1992).