The ρ- and λ-mode excitations in the diquark-antidiquark ($[cc][\bar{c}\bar{c}]$) tetraquark System

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Introduction

The exploration of exotic hadrons such as tetraquarks has become a significant area of interest in recent years. The LHCb experiment unveiled the discovery of the first tetraquark state comprising all charm quarks $cc\bar{c}\bar{c}$ in 2020 [1]. This finding was indeed confirmed by other experiments including ATLAS [2] and CMS collaborations [3]. This experimental breakthrough has opened new avenues and presented both opportunities and challenges for theorists and experimentalists alike. The study of such states is crucial for enhancing our understanding of the strong interaction within Quantum Chromodynamics (QCD), particularly in the heavy quark sector. In this study, we focus on the P-wave excitations in the diquark-antidiquark structure examining both the ρ-mode related to the motion between diquark and antidiquark and the λ-mode within the diquark-antidiquark pair using the relativistic quark model.

Theoretical Approach

To investigate the P-wave masses of $cc\bar{c}\bar{c}$ tetraquark state as a bound-state system of dicharm and anti-dicharm, we employed a relativistic quark model utilizing a static non-Coulombic power-law potential of the form[4]

$$V_{d\bar{d}}(r) = Ar^{\nu} + V_0 \quad (1)$$

Here, A represents the strength of the potential and V_0 is a constant negative potential depth. To obtain the binding energy of the diquarks (equivalent to the anti-diquark), we solve the Dirac equation. The wave functions

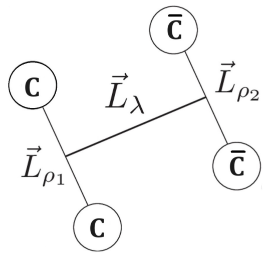

FIG. 1: Excitation configuration of the compact diquark-antidiquark tetraquark system.

satisfy the Dirac equation (with $\hbar = c = 1$) expressed as,

$$[E^{Dir} - \vec{\alpha}\cdot\vec{P} - m_d - V(r)]\psi^{Dir}(\vec{r}) = 0 \quad (2)$$

Hence, the corresponding energy eigenvalue is given by,

$$\epsilon^{Dir} = (E^{Dir} - M_d - V_0)[(2A)^{\frac{-2}{\nu}}(E^{Dir} + M_d)]^{\frac{\nu}{\nu+2}}.$$

Where E^{Dir} represents the diquark (equivalent to antidiquark) binding energy of the diquark-antidiquark bound state system. So, the Dirac bound-state masses of the Tetraquark $T_{d\bar{d}}$ system is given by,

$$M^{Dir}_{d\bar{d}} = E^{Dir}_d + E^{Dir}_{\bar{d}} + \left\langle V^{d\bar{d}}_{j_1 j_2}\right\rangle + \left\langle V^{d\bar{d}}_{LS}\right\rangle + \left\langle V^{d\bar{d}}_{T}\right\rangle \quad (3)$$

Here, $\left\langle V^{d\bar{d}}_{j_1 j_2}\right\rangle$, $\left\langle V^{d\bar{d}}_{LS}\right\rangle$ and $\left\langle V^{d\bar{d}}_{T}\right\rangle$ represents the spin-spin, spin-orbit and spin-tensor interaction potential respectively, introduced specifically to resolve the degeneracy within the P-wave masses of the system [5].

As shown in Fig. 1, the orbital angular momentum in a compact tetraquark system can be decomposed as:

$$\vec{L} = \vec{L}_{\rho_1} + \vec{L}_{\rho_2} + \vec{L}_\lambda \quad (4)$$

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where $\vec{L}_{\rho_1}(\vec{L}_{\rho_2})$ is the intrinsic orbital angular momentum in the diquark $[cc]$ (antidiquark $[\bar{c}\bar{c}]$) field, while $\vec{L}_\lambda$ is the relative orbital angular momentum between diquark and antidiquark fields.

The P-wave tetraquark can be in either the $\rho-$mode excitation ($\vec{L}_{\rho_1}+\vec{L}_{\rho_2}=\vec{L}_\rho=1, \vec{L}_\lambda=0$) or $\lambda-$mode excitation ($\vec{L}_\lambda=1, \vec{L}_\rho=0$). The P-wave excited $cc\bar{c}\bar{c}$ tetraquarks are the excitations with $\vec{L}_\lambda+\vec{L}_\rho=1$. There exist several different excitation structures for the P-wave tetraquarks : $(\vec{L}_\lambda, \vec{L}_\rho\{\vec{L}_{\rho_1}, \vec{L}_{\rho_2}\}) = (1,0\{0,0\})$, $(0,1\{1,0\})$ and $(0,1\{0,1\})$. We study all these P-wave tetraquarks with the same structures and quantum numbers.

Results, Summary and Conclusion

In the framework of the relativistic quark model, we investigate the masses of different excitation modes in the P-wave tetraquark states ($[cc][\bar{c}\bar{c}]$) using a static non-Coulombic power-law potential. The potential model parameters fitted for this study are presented in Table I.

TABLE I: potential model parameters

	power ν	confining strength $A(GeV^{\nu+1})$	Potential depth $V_0(GeV)$
$cc\bar{c}\bar{c}$	0.1	5.831	-6.012

The predicted P state masses of fully charm tetraquarks are tabulated in Table II. To check the consistency and reliability of our model, we compare our results with experimental thresold masses (from the recent PDG) as well as other available theoretical predictions.

Through an analysis of the overall mass spectra with available experimental and other theoretical findings, we can conclude that the potential employed here, a static non-Coulombic plus power-law potential is effective in determining the P-wave mass of fully charm tetraquark states. Our results show that internal excitations play a crucial role in determining energy levels. For P-wave states, the mass hierarchy is $M_{(0,1\{1,0\})}$ ($M_{(0,1\{0,1\})}$) $<$ $M_{(1,0\{0,0\})}$. The results show that tetraquark masses with relative excitation modes, particularly the λ-mode excitations are closer to the threshold masses.

TABLE II: P-wave masses of tetraquark $cc\bar{c}\bar{c}$

states $N^{2S+1}L_J$	J^{PC}	Excitation modes $(\vec{L}_\eta, \vec{L}_\rho\{\vec{L}_{\rho_1}, \vec{L}_{\rho_2}\})$	$M_{cc\bar{c}\bar{c}}$	M_{thr} [6]	[7]
1^1P_1	1^{--}	$(1,0\{0,0\})$ $(0,1\{1,0\})$ $(0,1\{0,1\})$	6531 6372	6553	6521
1^3P_0	0^{-+}	$(1,0\{0,0\})$ $(0,1\{1,0\})$ $(0,1\{0,1\})$	6496 6336	6398	6476
1^3P_1	1^{-+}	$(1,0\{0,0\})$ $(0,1\{1,0\})$ $(0,1\{0,1\})$	6517 6358	6494	6507
1^3P_2	2^{-+}	$(1,0\{0,0\})$ $(0,1\{1,0\})$ $(0,1\{0,1\})$	6559 6401	6539	6545
1^5P_1	1^{--}	$(1,0\{0,0\})$ $(0,1\{1,0\})$ $(0,1\{0,1\})$	6489 6329	6508	6463
1^5P_2	2^{--}	$(1,0\{0,0\})$ $(0,1\{1,0\})$ $(0,1\{0,1\})$	6531 6372	6607	6514
1^5P_3	3^{--}	$(1,0\{0,0\})$ $(0,1\{1,0\})$ $(0,1\{0,1\})$	6595 6437	6653	6570

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