Inflating with Open Strings

Memoria de Tésis Doctoral realizada por

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Abstract

Cosmic inflation allows for a natural explanation of the homegenous and isotropic universe while also allowing for the tiny fluctuations that are present in the cosmic microwave background. It can be conveniently described by a scalar field, called the inflaton, slow-rolling down its potential. The possible large distance that the inflaton has traversed during inflation makes the potential exceptionally sensitive to UV physics. In particular, integrating out heavy modes can change the shape of the potential significantly, interfering with inflation. This makes it desirable to study inflation in the context of a UV complete theory such as string theory. A striking prediction of String theory is the number of spacetime dimensions which has to be ten. Hence we have to consider string theory on spacetimes with the structure $M^4 \times Y^6$, where Y^6 is a compact six-dimensional manifold. The size and shape of the internal manifold are determined by the expectation values of scalar fields, called moduli. These moduli provide some of the heavy modes that can backreact on the inflationary potential and, hence, have to be stabilized at a high scale.

In this thesis, we consider the embedding of inflation in Type IIB string theory, with the inflaton given by the position modulus of a D7-brane, an open string. The inflaton potential is sourced by three-form fluxes in the background of the internal space. We study its multi-field dynamics, ignoring first the effects of moduli stabilization, and show that they improve the agreement between observations and predictions. We also study moduli-stabilization in both the Kähler and the complex structure sector. We show that, with a flux tuning, it is possible to have an epoch of inflation in Type IIB string theory with the inflaton an open string modulus.

In addition to embedding inflation in string theory we also consider more generally the structure of Type II string theories compactified in the presence of Dp-branes. We study a string theory realization of the Kalopor-Sorbo mechanism, which provides a field theory argument to constrain the shape of corrections to the scalar potential. For this mechanism it is essential to have couplings between the scalar fields and Minkowski four-forms. We show that in Type II string theories the entire axion scalar potential can be written in terms of these four-forms. We also study α' corrections to the effective action of Dp-branes. We provide general formulae for the leading order α' correction to the action of Dp-branes with $p = 3, 5, 7$ and provide a supergravity description of the α' corrections to the D7-brane action, allowing us to study its effects on moduli stabilization.

The original work presented in this thesis is based on

- S. Bielleman, L. E. Ibáñez, F. G. Pedro and I. Valenzuela, "multi-field Dynamics in Higgs-otic Inflation," JHEP **1601** (2016) 128, [arXiv:1505.00221 [hep-th]].
- S. Bielleman, L. E. Ibáñez and I. Valenzuela, "Minkowski 3-forms, Flux String Vacua, Axion Stability and Naturalness," JHEP **1512** (2015) 119, [arXiv:1507.06793 [hep-th]].
- S. Bielleman, L. E. Ibáñez, F. G. Pedro, I. Valenzuela and C. Wieck, "The DBI Action, Higher-derivative Supergravity, and Flattening Inflaton Potentials," JHEP **1605** (2016) 095, [arXiv:1602.00699 [hep-th]].
- S. Bielleman, L. E. Ibáñez, F. G. Pedro, I. Valenzuela and C. Wieck, "Higgs-otic Inflation and Moduli Stabilization," JHEP **1702** (2017) 073, [arXiv:1611.07084 [hep-th]].

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Contents

Chapter 1

Introducción

El objetivo último de la física es entender cómo funciona el universo. Este es un objetivo algo ambicioso, si no completamente inviable. Sin embargo, estamos avanzando en la dirección correcta. Déjennos ilustrar la evolución de nuestro entendimiento teniendo en cuenta las rocas en un campo gravitatorio. Hemos sabido por mucho tiempo que las rocas caen al suelo. Para Aristóteles, la roca cae porque su "causa final" es descansar en el suelo. Newton diría que cae porque hay una fuerza atractiva entre la roca y el suelo. Y Einstein diría que tanto la roca como el suelo siguen las geodésicas en un espaciotiempo curvo. A la vez que conseguimos una comprensión más profunda de la roca que cae, empezamos a entender que no solo cae la roca. También cae la luna, la Tierra, el sol, etc. Mientras tratábamos de entender algo que observamos, hemos aprendido mucho sobre efectos similares en escalas diferentes, además de como el universo como un todo.

Hace más de cien años, Planck utilizó el análisis dimensional para calcular lo que podríamos llamar una escala fundamental [1], una escala que sigue las constantes fundamentales. Combinó la constante gravitatoria G_{N}, la velocidad de la luz c y su propia constante $\hbar$ en un solo número con dimensiones de longitud

$$l_{\mathrm{p}} = \sqrt{\hbar G_{\mathrm{N}}/c^3} = 1.6 * 10^{-33} \text{ cm} . \tag{1.1}$$

La importancia de este cálculo difícilmente se puede exagerar. A esta escala, se espera que todas las teorías probadas experimentalmente produzcan predicciones erróneas o, simplemente, no puedan hacer predicciones. Ciertamente, para un físico moderno de alta energía que entiende la física a esta escala sería el objetivo definitivo.[1]

La teoría de cuerdas es la candidata principal para dar una descripción del universo a escala de Planck. Contiene ambas teorías de la gravedad y del campo gauge de una manera constante; por lo tanto, es una teoría finita de la gravedad cuántica. La teoría de las cuerdas postula que los objetos fundamentales del universo no son puntuales como en la teoría de campos cuánticos (TCC), sino que son objetos unidimensionales extendidos llamados cuerdas. Observamos los estados puntuales en los colisionadores sólo porque las cuerdas son extremadamente pequeñas y la energía de los colisionadores modernos es demasiado baja. La longitud típica de estas cuerdas, la longitud de la

[1] Consultar [2] para leer una interesante discusión sobre todo lo que hemos conseguido desde que Planck hiciera su cálculo.

cuerda l_s, podría estar justo fuera del alcance experimental o podría ser tan pequeña como la longitud de Planck. Estudiar la teoría de cuerdas es un ejercicio muy atractivo y gratificante. Si la teoría de cuerdas describe verdaderamente el universo, entonces nos enseña cosas nuevas al respecto. Por ejemplo, en las escalas cercanas a la escala de Planck, el universo es de diez dimensiones, podría haber otros universos o regiones de nuestro propio universo con leyes físicas diferentes (pero todavía gobernadas por la teoría de cuerdas) y, quizás lo más sorprendente, la teoría de cuerdas no solo trata de cuerdas, sino que existen otros objetos de dimensiones superiores que juegan un papel importante. Podemos estudiar la teoría que describe el universo en la escala de Planck, pero por desgracia hay una diferencia entre estudiar la teoría y entenderla en su total detalle.

1.1 Una historia breve del universo

En lugar de mirar las escalas más pequeñas, podemos centrarnos en las más grandes. El siglo pasado ha abierto nuestra visión del universo. Décadas después de que Planck hiciera su cálculo, la observación del Hubble [3] convenció a los científicos de que nuestro universo es más grande que la Vía Láctea. El descubrimiento de la radiación cósmica de fondo (CMB) de Penzias y Wilson [4] nos mostró por primera vez el resplandor del Big Bang. En las últimas décadas la cosmología ha entrado verdaderamente en la era de la precisión con el descubrimiento de la expansión acelerada [5, 6] y las observaciones detalladas de la CMB [7–9]. Esto ha llevado al modelo estándar de cosmología, el modelo de ΛCDM, que describe la dinámica del universo. Esto supone un universo lleno de una constante cosmológica (cc), radiación, bariones y materia oscura fría, que se comportan como fluidos perfectos.

En la actualidad, unos 13.700 millones de años después del Big Bang, la densidad media de energía en el universo es de aproximadamente $(1 \text{ meV})^4$. La comprensión actual de la física y la cosmología nos permite viajar en el tiempo y seguir la evolución del universo. A medida que retrocedemos en el tiempo, la densidad de energía comienza a aumentar y el tamaño del universo que podemos observar comienza a disminuir. Poco después del inicio de nuestro viaje, la densidad energética media de la materia domina sobre la densidad energética de otras fuentes. Si nos movemos más allá de la era de la formación de galaxias y las primeras estrellas, entramos en las edades oscuras. Si vamos aún más atrás en el tiempo llegamos a la CMB, como se muestra en la figura 1.1. Este es el origen del primer flujo luminoso libre y lo más lejos que podemos ver desde la Tierra. La densidad energética media es de aproximadamente 1 eV. Si vamos más atrás, entramos en el tiempo en que los electrones y los protones están en equilibrio térmico con los fotones y, avanzando, podemos ver los primeros protones que se forman a partir de un universo que llena el plasma de quarks-gluones. Un poco antes de los primeros protones, nos encontramos con la unificación electrodébil y la bariogénesis. Además de la unificación electrodébil, la física ha sido probada con experimentos y, además, hay una buena razón para creer que se produjo la bariogénesis. Si nos movemos más atrás en el tiempo tenemos que hacer conjeturas sobre la evolución anterior del universo. Si dejamos que la teoría de las cuerdas y la inflación cosmológica guíen nuestro viaje, observaríamos que, un tiempo después de la unificación electrodébil, el tamaño del universo que podemos observar comienza a aumentar. Esta era de expansión acelerada del espacio es lo que se llama inflación. Después de la inflación, observamos que se restaura la supersimetría. Cerca del Big Bang, nos dimos cuenta de que el universo es de diez dimensiones que realmente podría ver cuerdas moviéndose alrededor.

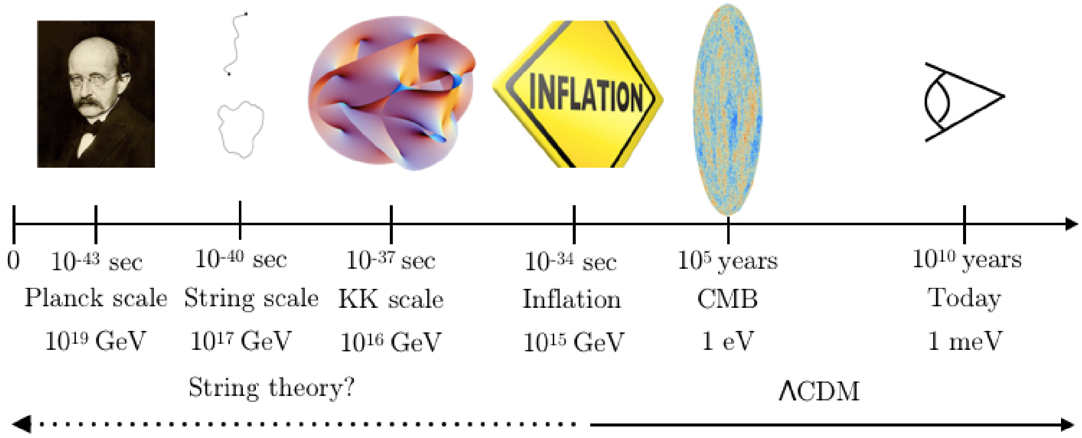

Figure 1.1: Una breve descripción de la historia de nuestro universo. El eje horizontal está marcado con el tiempo y la densidad energética. Estamos en el extremo derecho, mirando hacia la CMB y el universo primitivo.

Las observaciones del universo primitivo nos revelarían si esta historia conjeturada es verdadera o no. Podría haber pequeñas diferencias, por ejemplo, que se pudiera restaurar la supersimetria antes de la inflación, o podría haber grandes diferencias, por ejemplo, que nunca veríamos la inflación. La CMB nos da mucha información sobre el universo primitivo. Satélites como el satélite de Planck [9] miden los espectros de potencia angular de las fluctuaciones de temperatura y polarización de la CMB y no dan información directa sobre el universo primitivo. Sin embargo, estos espectros de potencia están relacionados con las perturbaciones de densidad en el universo en el momento de la inflación. Podemos aprender mucho de la dependencia de escala del espectro de potencia escalar, $n_{\mathrm{s}}-1$, y la relación de los espectros de potencia tensorial y escalar, r. La relación tensor-escalar en particular está directamente relacionada con la escala energética de la inflación. Por desgracia, todavía no se ha medido, pero hay una serie de telescopios en tierra tratando de hacerlo. Entre otros, están el BICEP2, BICEP3, el Keck Array [10] y el POLARBEAR [11]. Con suerte, habrá una nueva generación de satélites construidos específicamente para medir la relación tensor-escalar [12].

1.2 Fenomenología de las cuerdas

La teoría de cuerdas supone que los objetos fundamentales más pequeños son cuerdas unidimensionales. Estas cuerdas pueden estar abiertas o cerradas y. dependiendo de los modos vibratorios de las cuerdas, pueden manifestarse como diferentes partículas. La naturaleza en forma de cuerdas de las cuerdas sólo se puede apreciar en la escala de cuerdas o más allá, M_{s}, que es inversamente proporcional a la longitud de la cuerda, l_{s}, y normalmente cerca de la escala de Planck, M_{p}, pero podría ser mucho menor. Una de las observaciones más importantes en la historia de la teoría de cuerdas fue que siempre hay un bosón spin-2 en el sector de cuerdas cerradas sin masa. Combinado con el hecho de que las cuerdas abiertas interactivas pueden cerrarse para formar cuerdas cerradas, esto implica que la teoría de cuerdas siempre es una teoría de la gravedad. Además, los extremos de las cuerdas abiertas se comportan como partículas cargadas y, por lo tanto, las interacciones del campo gauge naturalmente también se incluyen en la teoría de cuerdas. Además de las cuerdas existen otros objetos de mayor dimensión en la teoría de cuerdas, generalmente llamados branas. Las Dp-branas son el lugar en el que terminan las cuerdas abiertas. Sin embargo, llevan su propia carga, son dinámicas y pueden convertirse en anti-Dp-branas. Las Dp-branas y las cuerdas abiertas dan una hermosa interpretación geométrica hermosa de teorías del campo gauge. Hay una teoría del campo gauge que vive en el volumen global de cada brana. Las cuerdas abiertas que terminan en el volumen global de dp-branas se comportan como partículas cargadas que viven en la dp-brana. En particular, la TCC en una sola dp-brana obedece a una simetría de campo gauge $U(1)$; en una pila de dos Dp-branas, hay una teoría de gauge $U(2)$, etc. Si dos Dp-branas se alejan una de la otra, la simetría $U(2)$ se rompe a $U(1)^2$. Esto se puede ver como un mecanismo de Higgs desde el punto de vista del volumen global de branas. La distancia entre las Dp-branas desempeña el papel de un bosón de Higgs.

La inclusión de fermiones en la teoría de cuerdas conduce naturalmente a la supersimetría (de la hoja del universo). Junto a la cancelación de anomalías, la supersimetría predice(!) que el número de dimensiones del espaciotiempo es diez. Esto está en línea con la afirmación de que la teoría de cuerdas no tiene parámetros libres. Otro ejemplo de esto es que el acoplamiento de cuerda g_s se fija por el valor de expectativa de vacío (vev) de la dilatación. La libertad aparente para elegir la escala de cuerda M_s está vinculada a la libertad de elegir unidades. La tensión aparente entre la teoría de cuerdas de diez dimensiones y nuestro universo de cuatro dimensiones puede resolverse asumiendo que seis de las dimensiones son compactas y demasiado pequeñas para observarse en experimentos. Esto se llama compactificación. Hay un gran número de formas de compactificar seis dimensiones, y existe muy poca orientación para seleccionar una compactificación específica. Para mantener el control teórico, normalmente una cantidad determinada de supersimetría tiene que sobrevivir a la compactificación y, por lo tanto, a menudo tenemos en cuenta las variedades de Calabi-Yau para el espacio interno. Si combinamos el número de variedades de Calabi-Yau con la libertad de elegir vevas internas para los campos de diez dimensiones, nos damos cuenta de que el número de distintas teorías cuatridimensionales que se pueden producir en la teoría de cuerdas es enorme. El reto de la fenomenología de las cuerdas es buscar vacuas (compactificaciones o antecedentes) que conduzcan a la física cuatridimensional compatible con las observaciones. En esta tésis nos centramos en la posibilidad de incorporar la inflación en la teoría de cuerdas. Para ello, buscamos compactificaciones que soporten una gran excursión de campo para un pequeño número de escalares paramétricos ligeros. Estos escalares son los que impulsan la inflación y se llaman inflatones. Nos centraremos en la interacción entre los inflatones y los otros escalares en la teoría; en particular, los que establecen la forma y el tamaño del espacio interno. Estos escalares se llaman módulos y es importante que no se desestabilicen durante la inflación. Esto podría conducir, por ejemplo, a la descompactificación donde una o más de las dimensiones internas se hace no compacta.

1.3 Plan de la tésis

Consideramos una clase específica de modelos de cuerdas, a saber, Tipo II compatificado sobre un orientifold con D7-branas y flujos internos. El inflatón es un módulo de cuerda abierta que se extiende entre las D7-branas. Empezamos dando una introducción a las compactificaciones y cosmología inflacionaria del orientifold tipo II en el capítulo 3. Esto nos permite introducir todos los ingredientes necesarios. Los cuatro capítulos siguientes se basarán en cuatro artículos en los que estuvo involucrado el autor de esta tésis. El capítulo 4 está dedicado a la monodromía y a las cuatro dimensiones de la teoría de cuerdas. En el capítulo 5, se discute la acción efectiva de los módulos de posición de las Dp-branas. También discutimos maneras de obtener la misma acción en teorías supersimétricas $\mathcal{N} = 1$ cuatridimensionales. En el capítulo 6 presentamos un ejemplo del tipo de compactificaciones que consideramos en esta tésis, a saber, el modelo de Higgs-otic, y discutimos su dinámica inflacionaria de dos campos. El capítulo 7 se dedica a la estabilización de los módulos de la inflación de Higgs en la estructura compleja y en el sector de Kähler. El capítulo 8 está reservado para las conclusiones. Hay tres anexo que acompañan a esta tésis. En el anexo A, damos más detalles sobre la expansión de la acción efectiva de las Dp-branas. Damos ejemplos de aplanamiento de potenciales en el anexo B. Finalmente, en el anexo C se discute un posible origen microscópico de un término μ en el superpotencial.

Chapter 2

Introduction

The ultimate goal of physics is to understand how the universe works. This is a somewhat ambitious if not completely unfeasible goal. However, we are moving in the right direction. Let us illustrate the evolution of our understanding by considering rocks in a gravitational field. We have known for a long time that rocks fall to the ground. For Aristotle a rock falls because its "final cause" is to rest on the ground. Newton would say that it falls because there is an attractive force between the rock and the ground. And Einstein would say that both the rock and the ground are following geodesics in a curved spacetime. While getting a deeper understanding of the falling rock, we start to understand that it is not just the rock that is falling. It is also the Moon, the Earth, the Sun and so forth. Through trying to understand something that we observe we have learned a lot about similar effects on different scales as well as about the universe as a whole.

Over a hundred years ago Planck used dimensional analysis to compute what we might call a fundamental scale [1], a scale that follows from fundamental constants. He combined the gravitational constant $G_{\rm N}$, the speed of light c and his own constant $\hbar$ into a single number with dimensions of length

$$l_{\rm p} = \sqrt{\hbar G_{\rm N}/c^3} = 1.6 * 10^{-33} \text{ cm} . \tag{2.1}$$

The importance of this computation can hardly be overstated. At this scale all experimentally tested theories are expected to give wrong predictions or simply fail to give predictions at all. Certainly, for a modern high-energy physicist understanding physics at this scale would be the ultimate goal.[1]

String theory is the prime candidate to give a description of the universe at the Planck scale. It contains both gravity and gauge theories in a consistent way, hence it is a finite theory of quantum gravity. String theory postulates that the fundamental objects of the universe are not point-like as in quantum field theory (QFT) but rather are extended one-dimensional objects called strings. We observe point-like states at colliders only because strings are extremely small and the energy at modern colliders is too low. The typical length of these strings, the string length $l_{\rm s}$, could be just outside of experimental reach or it could be as small as the Planck length. Studying string theory is a very engaging and rewarding exercise. If string theory truly describes the

[1] See [2] for an interesting discussion on how far we have gotten since Planck did his computation.

universe then it teaches us new things about it. For instance, at scales close to the Planck scale the universe is ten-dimensional, there could be other universes or regions of our own universe with different physical laws (but still governed by string theory) and, maybe most surprising of all, string theory is not a theory of just strings, there exist other higher-dimensional objects that play an important role. We may study a theory that potentially describes the universe at the Planck scale but unfortunately there is a difference between studying a theory and understanding it in full detail.

2.1 A short history of the universe

Instead of looking at the very smallest scales, we can focus on the very biggest ones. The last century has opened up our view of the universe. It would be decades after Planck did his computation that scientists were convinced by the observation of Hubble [3] that our universe is larger than the Milky Way. The discovery of the cosmic microwave background (CMB) by Penzias and Wilson [4] showed us for the first time the afterglow of the Big Bang. In the last decades cosmology has truly entered the precision era with the discovery of accelerated expansion [5, 6] and the detailed observations of the CMB [7–9]. This has led to the standard model of cosmology, the ΛCDM model, which describes the dynamics of the universe. It assumes a universe filled with a cosmological constant (cc), radiation, baryons and cold dark matter, that behave as perfect fluids.

Currently, about 13.7 billion years after the Big Bang, the average energy density in the universe is about $(1\ \text{meV})^4$. Current understanding of physics and cosmology allows us to journey back in time and follow the evolution of the universe. As we move back in time the energy density starts to increase and the size of the universe that we can observe starts to decrease. Shortly after the start of our journey, the average energy density of matter dominates over the energy density of other sources. If we move past the era of the formation of galaxies and the first stars we enter the dark ages. If we go even further back in time we reach the CMB, as depicted in Figure 2.1. This is the origin of the first free streaming light and the furthest we can look from Earth. The average energy density is about 1 eV. If we go further back we enter the time when electrons and protons are in thermal equilibrium with photons and, moving on, we can see the first protons being formed from a universe filling quark-gluon plasma. A bit before the first protons, we encounter electroweak unification and baryogenesis. Physics up to and including electroweak unification has been tested with experiments and, in addition, there is good reason to believe that baryogenesis occurred. If we move further back in time we have to make conjectures about the further evolution of the universe. If we let string theory and cosmological inflation guide our journey we would observe that, some time after electroweak unification, the size of the universe that we can observe starts to increase. This epoch of accelerated expansion of space is what is called inflation. After inflation we would observe that supersymmetry is restored. Close to the Big Bang we would realise that the universe is ten-dimensional and we would actually be able to see strings moving around.

Observations of the early universe would reveal to us if this conjectured history is true or not. There could be small differences, for instance, supersymmetry could be restored before inflation or there could be large differences, for instance, we would never see inflation. The CMB gives us a lot of information about the early universe. Satellites like the Planck satellite [9] measure the angular power spectra of temperature fluctuations and polarization of the CMB and do not give direct information about the early universe. However, these power spectra are related to the density perturbations

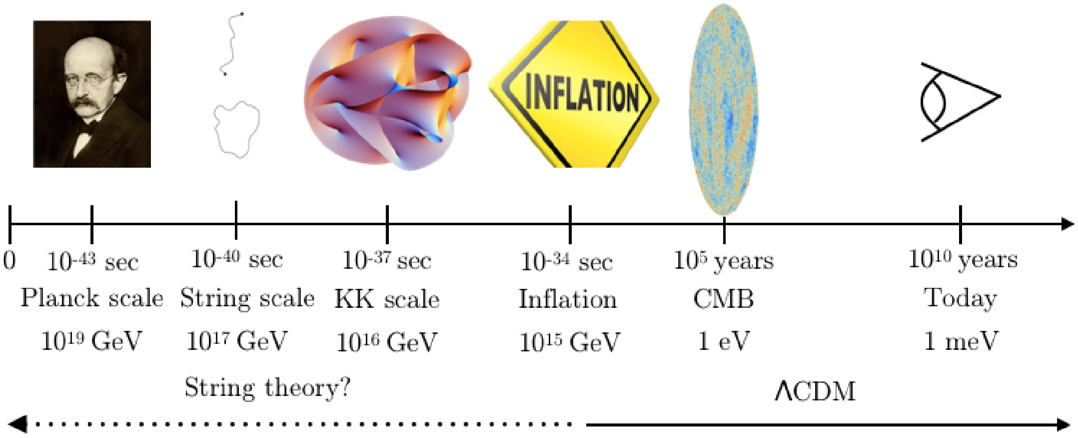

Figure 2.1: A short depiction of the history of our universe. The horizontal axis is labeled with both time and energy density. We are at the far right looking towards the CMB and the early universe.

in the universe at the time of inflation. We can learn a lot from the scale dependence of the scalar power spectrum, $n_{\rm s} - 1$, and the ratio of the tensor and scalar power spectra, r. The tensor-to-scalar ratio in particular is directly related to the energy scale of inflation. Unfortunately, it has not been measured yet but there are a number of ground-based telescopes trying to measure it. These include BICEP2, BICEP3, the Keck Array [10] and POLARBEAR [11]. Hopefully, there will be a new generation of satellites specifically build to measure the tensor-to-scalar ratio [12].

2.2 String phenomenology

String theory assumes that one-dimensional strings are the fundamental objects. These strings can be open or closed and depending on the vibrational modes of the strings they can manifest as different particles. The string-like nature of strings can only be appreciated at or beyond the string scale, $M_{\rm s}$, which is inversely proportional to the string length, $l_{\rm s}$, and typically close to the Planck scale, $M_{\rm p}$, but it could be much lower. One of the most important observations in the history of string theory was that there is always a spin-2 boson in the massless closed-string sector. Combined with the fact that interacting open strings can close to form closed strings, this implies that string theory is always a theory of gravity. In addition to this, the end-points of open strings behave like charged particles and therefore gauge interactions are also naturally included in string theory. Besides strings there exist other higher-dimensional objects in string theory, generally called branes. Dp-branes are the loci were open strings end. However, they carry their own charge, are dynamical and can annihilate on anti-Dp-branes. Dp-branes and open strings give a beautiful geometrical interpretation of gauge theories. There is a gauge theory living on the world-volume of each brane. The open strings ending on the Dp-brane world-volume behave like charged particles living on the Dp-brane. In particular, the QFT on a single Dp-brane obeys a $U(1)$ gauge symmetry, on a stack of two Dp-branes there is a $U(2)$ gauge theory, etc. If two Dp-branes move away from each other the $U(2)$ symmetry is broken to $U(1)^2$. This can be viewed as a Higgs mechanism from the brane world-volume point of view. The distance between the Dp-branes plays the role of a Higgs boson.

Including fermions in string theory naturally leads to (world-sheet) supersymmetry. In combination with anomaly cancellation, supersymmetry predicts(!) the number of spacetime dimensions to be ten. This is in line with the statement that string theory does not have any free parameters. Another example of this is that the string coupling g_s is fixed by the vacuum expectation value (vev) of the dilaton. The apparent freedom to choose the string scale M_s is linked to the freedom of choosing units. The apparent tension between ten-dimensional string theory and our four-dimensional universe can be resolved by assuming that six of the dimensions are compact and too small to observe in experiments. This is called compactification. There is a huge number of ways to compactify six dimensions and there exists very little guidance to select a specific compactification. To maintain theoretic control usually a certain amount of supersymmetry has to survive the compactification and therefore we often consider Calabi-Yau manifolds for the internal space. Combine the number of Calabi-Yau manifolds with the freedom to choose internal vevs for the ten-dimensional fields and the number of distinct four-dimensional theories that can be produced in string theory is enormous. The challenge of string phenomenology is to look for vacua (compactifications or backgrounds) that lead to four-dimensional physics compatible with observations. In this thesis we focus on the possibility of embedding inflation in string theory. To this end we look for compactifications that support a large field excursion for a small number of parametrically light scalars. These scalars are what drive inflation and they are called inflatons. We will focus on the interplay between the inflatons and the other scalars in the theory, in particular those that set the shape and size of the internal space. These scalars are called moduli and it is important that they are not destabilized during inflation. This could lead to, for example, decompactification where by one or more of the internal dimensions becomes non-compact.

2.3 Plan of the thesis

We consider a specific class of string models, namely Type II compactified on an orientifold with D7-branes and internal fluxes. The inflaton is an open string modulus stretching between the D7-branes. We start by giving an introduction to Type II orientifold compactifications and inflationary cosmology in Chapter 3. This allows us to introduce all necessary ingredients. The next four chapters will be based on four papers in which the author of this thesis was involved. Chapter 4 is dedicated to monodromy and four-dimensional four-forms in string theory. In Chapter 5 we discuss the effective action of the position moduli of Dp-branes. We also discuss ways to obtain the same action in four-dimensional $\mathcal{N} = 1$ supersymmetric theories. In Chapter 6 we introduce an example of the kind of compactifications we consider in this thesis, namely the Higgs-otic model, and we discuss its two-field inflationary dynamics. Chapter 7 is dedicated to moduli stabilization of Higgs-otic inflation in both the complex structure and Kähler sector. Chapter 8 is reserved for conclusions. There are three appendices accompanying this thesis. In Appendix A we give more details on the expansion of the effective action of Dp-branes. We give examples of flattening of potentials in Appendix B. Finally, Appendix C discusses a possible microscopic origin of a μ-term in the superpotential.

Chapter 3

Theory ingredients

There exists only one "string theory", or M-theory, which has several perturbative limits. These are the 11-dimensional M-theory supergravity and the five ten-dimensional string theories: $E_8 \times E_8$ heterotic, $SO(32)$ heterotic, Type I, Type IIA and Type IIB. There are two natural expansion parameters in string theory, the string couplings g_s and the string length l_s. The string coupling is given by the vev of a massless scalar field, the dilaton ϕ, and the string length is related to α' and M_s

$$g_\mathrm{s} = e^{\phi} \ , \quad l_\mathrm{s}^2 = \alpha' = M_\mathrm{s}^{-2} \ . \tag{3.1}$$

In an effective description of string theory, corrections with powers of g_s are quantum whereas corrections with powers of α' are due to the extended nature of the string. The different string theories have different spectra and contain different non-perturbative objects. Some are more suited for the description of certain problems than others. In particular, the heterotic theories contain gauge groups from the start. They were the most intensely studied at the start of string phenomenology. Model building in Type II string theories intensified when it was realized that it is possible to construct gauge theories on the world-volume of branes. In addition, in Type II theories moduli stabilization is better understood than in the other limits of string theory. In this thesis we are mostly concerned with orientifold compactifications of Type IIB and Type IIA theories.

We start this chapter with a discussion of Type II string theories in ten dimensions. Section 3.2 is reserved for a discussion on orientifold and flux compactifications of these theories. In Section 3.3 we present the effective action of Dp-branes and introduce open strings. Kähler moduli stabilization in Type IIB string theory is treated in Section 3.4. For a general introduction to string theory see [13, 14] and for an introduction to string phenomenology see [15]. In the second part of this chapter we discuss inflationary cosmology, starting with a motivation and the theory of single-field inflation in Section 3.5 and continuing with the perturbation theory of two-field inflation in Section 3.6. In the last section of this chapter, Section 3.7, we highlight some issues commonly found in string inflation.

3.1 Type II string theories

Type II string theories are theories of closed strings with world-sheet supersymmetry. There are two different possible boundary conditions for the world-sheet fermions, which are called the Neveu-Schwarz (NS) and Ramond (R) boundary conditions. In both theories all combinations of these boundary conditions are present. The distinction between Type IIA and Type IIB comes from two inequivalent ways to project, in a modular invariant way, the partition function of the theory. At the massless level this projection removes certain states leading to different spectra in the two theories. The NS-NS sector for both theories contains the dilaton ϕ, a 2-form B_2 and the metric. The RR sector for both theories contains p-forms C_p, with $p = 1, 3$ for Type IIA and $p = 0, 2, 4$ for Type IIB. The NS-R and R-NS sectors contain the space-time fermions. They are chiral for Type IIB and non-chiral for Type IIA. The low-energy effective action for both theories is an $\mathcal{N} = 2$ ten-dimensional supergravity.

The bosonic effective action of Type IIB is [14]

$$S_{\mathrm{IIB}} = \frac{1}{2\kappa_{10}^2}\int d^{10}x\sqrt{-g}\Bigg[e^{-2\phi}\left(R + 4\partial_M\phi\partial^M\phi - \frac{1}{2}|H_3|^2\right) - \frac{1}{2}|G_1|^2 - \frac{1}{2}|\tilde{G}_3|^2 - \frac{1}{4}|G_5|^2\Bigg] - \frac{1}{4\kappa_{10}^2}\int C_4\wedge H_3\wedge F_3\ , \qquad (3.2)$$

with

$$G_1 = F_1\ , \qquad \tilde{G}_3 = F_3 - C_0\wedge H_3\ , \qquad G_5 = F_5 - \frac{1}{2}C_2\wedge H_3 + \frac{1}{2}B_2\wedge F_3\ , \qquad (3.3)$$

$F_{p+1} = dC_p$, $H_3 = dB_2$, $2\kappa_{10}^2 = (2\pi)^7\alpha'^4$ and g is the determinant of the spacetime metric. The part of the action in the second integral is called the Chern-Simons action. To this action we should add the self-duality relation $G_5 = *_{10}G_5$, the fermionic piece and a local contribution coming from localized sources like branes and orientifold planes.

Similarly, the effective action of Type IIA is given by

$$S_{\mathrm{IIA}} = \frac{1}{2\kappa_{10}^2}\int d^{10}x\sqrt{-g}\Bigg[e^{-2\phi}\left(R + 4\partial_M\phi\partial^M\phi - \frac{1}{2}|H_3|^2\right) - \frac{1}{2}|G_0|^2 - \frac{1}{2}|G_2|^2 - \frac{1}{2}|G_4|^2\Bigg] - \frac{1}{4\kappa_{10}^2}\int B_2\wedge G_4\wedge G_4 - \frac{1}{4\kappa_{10}^2}\int G_0\wedge F_{10}\ , \qquad (3.4)$$

with

$$G_0 = -m\ , \qquad G_2 = F_2 - G_0B_2\ , \qquad G_4 = F_4 - C_1\wedge H_3 - \frac{1}{2}G_0B_2\wedge B_2\ . \qquad (3.5)$$

The parameter m is known as Romans mass and is regarded here as a background field strength. The Chern-Simons part of the action is given in the second integral. $\mathcal{N} = 2$ supersymmetric theories are problematic from a phenomenological viewpoint, while, on the other hand, supersymmetry prevents problematic features such as tachyons in a compactification. We consider compactifications that partly break supersymmetry in order to allow for interesting phenomenology while avoiding potential problems. However, before discussing these compactifications we discuss a different formulation of Type IIA supergravity called the democratic formalism, as this formulation is useful in Chapter 4.

The democratic formalism of Type IIA

The Type IIA action contains the RR fields C_1 and C_3. These objects couple naturally to Dp-branes with $p = 0$ and $p = 2$. However, there are more branes in the spectrum of the theory. In particular, branes with $p = 4$, $p = 6$ and $p = 8$. These couple naturally to the duals of the RR fields and the dual of the Romans mass. In ten-dimensions, the dual of a p-form gauge field is an $(8 - p)$-form gauge field. The dual fields appear in the effective actions of the previous section in the field strength through the identity $\int d^{10}x(-g)^{1/2}|F_p|^2 = \int F_p \wedge *F_p$, where $*F_p$ is the dual field strength. However, the dual fields do not carry any new degrees of freedom since they are related to the old degrees of freedom through the duality relations.

The democratic formalism treats all of these fields, including Romans mass and its dual, on equal footing, see [16–18]. Including more fields naturally increases the number of degrees of freedom and, hence, by using the democratic formalism we double the number of degrees of freedom in the RR sector of the theory. Note that we do not increase the number of fermionic degrees of freedom, so that we potentially break supersymmetry explicitly. We can take care of this issue by introducing constraint equations relating the $p-$ and $(8 - p)$-forms. The action that we write down for these fields is a pseudo-action because the constraint equations do not follow from it. The bosonic field content of the theory is given by the massive IIA NS-NS and RR sectors plus C_5, C_7 and C_9. The bosonic part of the pseudo-action is given by

$$S_{\rm Dem} = \frac{1}{2\kappa_{10}^2} \int d^{10}x \sqrt{-g} \left[e^{-2\phi} \left(R + 4\partial_M \phi \partial^M \phi - \frac{1}{2}|H_3|^2 \right) - \frac{1}{4} \sum_p |G_{p+1}|^2 \right] , \quad (3.6)$$

where p is understood to run over all RR fields including the Romans mass. The field strengths are defined as before

$$G_{p+1} = dC_p - H \wedge C_{p-2} - G_0 e^B \ . \quad (3.7)$$

The duality relations that have to be added by hand on the level of the equations of motion are

$$G_6 = - *_{10} G_4, \qquad G_8 = *_{10} G_2, \qquad G_{10} = - *_{10} G_0 \ . \quad (3.8)$$

Finally note that the Chern-Simons part of the action has been eliminated by the introduction of the additional degrees of freedom, meaning that upon expansion of the field strengths the Chern-Simons part of the standard action appears.

There is another equivalent way to write the RR part of the action which we find the most useful

$$\mathcal{L}_{\rm RR} = G_4 \wedge G_6 + G_2 \wedge G_8 + G_0 \wedge G_{10} \ . \quad (3.9)$$

We have to impose the same duality relations on the equations of motion to eliminate half of the degrees of freedom as before. This pseudo-action is equivalent to the one given in Equation (3.6) in that it gives the same equations of motion for the RR fields. Plugging the solutions to the equations of motion back in the pseudo-action to eliminate some of the fields gives the standard Type IIA action.

3.2 Orientifold compactifications of Type II theories

Compactifications of Type II string theories on Calabi-Yau three-folds lead to four-dimensional $\mathcal{N}=2$ vacua. The field content of these theories depends on the topology of the Calabi-Yau manifold, Y. In particular, the number of fields depends on the Hodge numbers, which are topological invariants of the compactification space. The Hodge number $h_{p,q}(Y)$ gives the dimension of the space of harmonic (p,q)-forms of the manifold Y. For Type IIA(B) the four-dimensional spectrum consists of a gravity multiplet, $h_{1,1}(h_{2,1})$ vector multiplets and $h_{2,1}(h_{1,1})$ +1 hypermultiplets. The deformation modes of the ten-dimensional metric give $h_{1,1}$ Kähler and $2h_{2,1}$ complex structure moduli, which are scalar fields that parametrize the shape and size of the Calabi-Yau three-fold.

We consider compactifications that break half of the supersymmetry generators. Typically, appropriate orientifold quotients of Type II Calabi-Yau compactifications reduce the four-dimensional supersymmetry to $\mathcal{N}=1$. An orientifold of a Type II compactification is defined by taking the Type II theory compactified on a Calabi-Yau and mod out the orientifold action $\Omega\mathcal{R}$, where Ω is world-sheet parity and $\mathcal{R}$ is a $\mathbb{Z}_2$ geometric symmetry of the Calabi-Yau three-fold. The fixed points of $\mathcal{R}$ define Op-planes. They are objects with opposite tension and typically opposite charge to Dp-branes that wrap a $(p-3)$-cycle of the internal space and span four-dimensional Minkowski spacetime. Op-planes source RR charges which have to be cancelled by the introduction of Dp-branes in order to satisfy tadpole cancellation conditions. In this section we first discuss Type IIA orientifolds focusing on the ingredients needed for Chapter 4. We then focus on Type IIB in the remainder of this chapter.

3.2.1 Type IIA orientifolds

In Type IIA orientifolds the orientifold action is $\Omega\mathcal{R}(-1)^{F_L}$ where F_L is the left-moving spacetime fermion number and $\mathcal{R}$ acts antiholomorphycally on the complex coordinates of the Calabi-Yau such that, $J\to -J$ and $\Omega_3\to\overline{\Omega}_3$ where J is the Kähler form and Ω_3 the Calabi-Yau three-form. The closed-string spectrum has to be truncated to states invariant under the orientifold action. This implies that the $\mathcal{N}=2$ gravity multiplet reduces to an $\mathcal{N}=1$ gravity multiplet, the $2h_{2,1}+1$ complex structure hypermultiplets reduce to $h_{2,1}+1$ chiral multiplets and the $h_{1,1}$ vector multiplets are reduced to $h^+_{1,1}$ vector- and $h^-_{1,1}$ hypermultiplets. Here $h^+_{1,1}(h^-_{1,1})$ denote the number of even(odd) $(1,1)$-forms under the $\mathcal{R}$ action.

Next we need to introduce a basis of harmonic forms on Y. We define a basis of $(1,1)$-forms ω_A, with $A=1,\cdots,h_{1,1}$, that splits into a basis for the odd and even (1,1)-forms $\omega_a\in H^{1,1}_-(Y)$ and $\omega_\alpha\in H^{1,1}_+(Y)$. Similarly we define a basis of three-forms $\{\alpha_K,\beta^K\}$ with $K=0,\cdots,h_{2,1}$. It is possible to choose α_K to be even and β^K to be odd. These forms satisfy the following orthogonality relations

$$\int_Y \omega_\alpha\wedge\tilde{\omega}^\beta=\delta^\beta_\alpha\ ,\qquad \int_Y \omega_a\wedge\tilde{\omega}^b=\delta^b_a\ ,\qquad \int_Y \alpha_K\wedge\beta^L=\delta^L_K\ . \tag{3.10}$$

Where $\tilde{\omega}^\beta$ is the $(2,2)$-form dual to ω_α. Note that their parity under the orientifold is reversed, so that $\tilde{\omega}^\alpha$ is odd. It is possible to show that both the Kähler form J and the 2-form B_2 are odd under the orientifold and hence they can be expanded as

$$J=v^a\omega_a\ ,\qquad B_2=b^a\omega_a\ . \tag{3.11}$$

Both v^a and b^a are $h^-_{1,1}$ four-dimensional scalars and are grouped in four-dimensional complex Kähler moduli

$$T^a = v^a - ib^a \ . \tag{3.12}$$

Similarly, one can expand the RR three-form

$$C_3 = A_1^\alpha \wedge \omega_\alpha + C^K \alpha_K \ , \tag{3.13}$$

where the C^K are four-dimensional scalars and the A_1^α are four-dimensional vector bosons. The scalars can be grouped with the dilaton and the surviving complex structure moduli into $h_{2,1}+1$ four-dimensional complex scalars. Compactification of Type IIA amounts to expanding the fields of the action (3.4) in the above basis and integrating over the internal dimensions. The resulting theory is a four-dimensional $\mathcal{N}=1$ supergravity. Therefore, there should exist a suitable Kähler potential and a suitable superpotential which are fixed in terms of geometric data of the compactification space.

The IIA Kähler potential

It is possible to derive the general four-dimensional Kähler potential belonging to Type IIA orientifolds. It is fully given in terms of the Kähler form J and the three-form Ω_3, which in turn depend on the Kähler and complex structure moduli, respectively. The Kähler potential, to leading order in α', is [19, 20]

$$\kappa_4^2 K_{\rm IIA} = -\log\left[\frac{4}{3}\int_Y J\wedge J\wedge J\right] - 2\log\left[2\int_Y Re(C\Omega_3)\wedge *_6 Re(C\Omega_3)\right] \ . \tag{3.14}$$

C is a normalization factor and κ_4^2 is inversely proportional to the four-dimensional Planck mass

$$\kappa_4^2 = \frac{8\pi}{M_{\rm p}^2} \ , \tag{3.15}$$

which in turn is related to the string scale and the internal volume, V_6,

$$M_{\rm p}^2 = \frac{8V_6 M_{\rm s}^8}{(2\pi)^6 g_{\rm s}^2} \ . \tag{3.16}$$

Fluxes and the IIA superpotential

Generically, the internal field strengths of the compactification have non-trivial backgrounds,

$$\int_{\gamma_I} F_p = (2\pi)^2\alpha' e^I \ , \tag{3.17}$$

where γ_I is some cycle of the compactification space and e^I is called a flux parameter. Dirac quantization implies that fluxes are quantized in units of the string scale. These fluxes induce a superpotential for the internal moduli, which is given by [19, 21]

$$W_{\text{RR-flux}} = \int_Y e^{i\mathcal{J}_c}\wedge\mathcal{F}_{\rm RR} \ , \qquad W_{\text{NS-flux}} = \int_Y \Omega_{\rm c}\wedge H_3 \ , \tag{3.18}$$

with $\mathcal{J}_c = i\sum_{a=1}^{h_{1,1}^-} T_a\omega_a$. These two contributions to the superpotential induced by fluxes from the RR and NS-NS sectors, respectively. Here $\mathcal{F}_{\rm RR}$ is a formal sum over the RR fluxes and $\Omega_{\rm c}$ is a complexification of the Calabi-Yau holomorphic three-form. Note that this superpotential depends on all the moduli for generic fluxes. We can compute the resulting scalar potential using Equations (3.14), (3.18) and the well-known formula

$$V = e^K(K^{A\overline{A}}D_A W D_{\overline{A}}\overline{W} - 3|W|^2) \ , \tag{3.19}$$

where A runs over all holomorphic coordinates, $\overline{A}$ over all the antiholomorphic ones and D_A is the Kähler covariant derivative $D_A = \partial_A + K_A$. Depending on the flux choice, the scalar potential depends on all complex structure and Kähler moduli in such a way that it is possible to find a (local) minimum for all moduli. The moduli masses have to be high enough such that they can be safely integrated out while, on the other hand, their masses have to be lower than the compactification scale such that this can be done in an effective four-dimensional theory.

Compactification on an orientifolded torus

We can make the above discussion slightly more concrete by considering Type IIA compactified on the orientifold $\mathbb{T}^6/[\Omega\mathcal{R}_A(-1)^{F_L}]$, where $\mathcal{R}_A$ reverses the direction on half of the compact dimensions. In this model there are one dilaton, three Kähler moduli and three complex structure moduli. The Kähler moduli were given before in Equation (3.12) and the dilaton and complex structure moduli are

$$S = e^{-\phi} + i\int_Y C_3\wedge\beta^0 \ , \qquad U^i = u^i - i\int_Y C_3\wedge\beta^i \ , \tag{3.20}$$

with u^i parametrizing deformations of the torus. The Kähler potential for this type of compactification, to leading order in α', is given by

$$\kappa_4^2 K_{\rm IIA} = -\log(S+\bar{S}) - \sum_{i=1}^{3}\log(U^i+\bar{U}^i) - \sum_{a=1}^{3}\log(T^a+T^a) \ . \tag{3.21}$$

We can expand the fluxes of the various fields in the appropriate bases. For the NS sector this gives

$$H_3 = \sum_{i=0}^{3} h_i\beta_i \ , \tag{3.22}$$

and for the RR sector

$$F_0 = -m \ , \qquad F_2 = \sum_{i=1}^{3} q_i\omega_i \ , \qquad F_4 = \sum_{i=1}^{3} e_i\tilde{\omega}_i \ , \qquad F_6 = e_0 dV_6 \ , \tag{3.23}$$

where dV_6 is the volume form of the three-fold. The coefficients in this expansion are integer fluxes. Plugging this expansion in Equation (3.18) and using that in this case

$$\Omega_c = iS\alpha_0 - i\sum_{i=1}^{3} U^i\alpha_i \ , \tag{3.24}$$

leads to

$$W_{\text{RR-flux}} = e_0 + i\sum_{i=1}^{3} e_i T_i - q_1 T_2 T_3 - q_2 T_1 T_3 - q_3 T_1 T_2 + imT_1 T_2 T_3 \ , \tag{3.25}$$

$$W_{\text{NS-flux}} = ih_0 S - i\sum_{i=1}^{3} h_i U_i \ . \tag{3.26}$$

Using these formulae one can derive the scalar potential for the $\mathcal{N}=1$ four-dimensional theory [19, 22]. The superpotential does not receive perturbative corrections in α', but the compactification may contain additional fluxes whose geometric origin is less well understood. Moreover, there may be non-perturbative corrections to the superpotential that we do not consider.

3.2.2 Type IIB orientifolds

The general approach to Type IIB orientifolds is analogous to the one for Type IIA orientifolds. One takes the theory compactified on a Calabi-Yau three-fold Y and mods out $\Omega\mathcal{R}$. We are mainly interested in models with O7-planes and D7-branes, implying that the geometric symmetry $\mathcal{R}$ locally acts on a compact coordinate as $z \to -z$ while leaving the other coordinates invariant. We should also add a factor $(-1)^{F_L}$ as before. The four-dimensional spectrum contains the $\mathcal{N}=1$ gravity multiplet, $h_{1,1} + h_{2,1}^{-} + 1$ chiral multiplets and $h_{2,1}^{+}$ vector multiplets. The expansion of the ten-dimensional fields can be performed as before and for the O7/D7 case reads

$$\begin{aligned} B_2 &= b^a \omega_a \ , \\ C_2 &= c^a \omega_a \ , \\ C_4 &= A_1^\kappa \wedge \alpha_\kappa + C_\alpha \tilde{\omega}^\alpha \ . \end{aligned} \tag{3.27}$$

Here $\kappa = 1, \cdots, h_{2,1}^{+}$, and a and α label the odd and even $(1,1)$-forms, respectively. As in Type IIA, the scalars of this expansion can be grouped with the geometric moduli to complete the four-dimensional supersymmetry multiplets.

Type IIB Kähler potential

The Kähler potential for Type IIB is given by [14]

$$k_4^2 K_{\text{IIB}} = -2\log(e^{-\frac{3}{2}\phi}\int J\wedge J\wedge J) - \log(S+\bar{S}) - \log(-i\int_Y \Omega_3 \wedge \Omega_3) \ , \tag{3.28}$$

where J is the Calabi-Yau Kähler form and Ω_3 is the Calabi-Yau three-form. In general, the Kähler potential is an implicit function of the moduli, however, there exists an explicit expression for the large volume Kähler potential for the overall modulus T, given by

$$k_4^2 K_{\text{IIB}} = -3\log(T+\bar{T}) - \log(S+\bar{S}) - \log(-i\int_Y \Omega_3\wedge\Omega_3) \ , \tag{3.29}$$

which has a no-scale structure for the Kähler moduli, implying that

$$K^{T\bar{T}} K_T K_{\bar{T}} = 3 \ . \tag{3.30}$$

Fluxes and the IIB superpotential

In general, in Type IIB compactifications there are fluxes as in Type IIA. These fluxes source the well-known Gukov-Vafa-Witten superpotential [23]

$$W_{\text{GVW}} = \int_Y G_3 \wedge \Omega_3 \ , \tag{3.31}$$

where $G_3 = F_3 - iSH_3$ and $S = e^{-\phi} + iC_0$. is the four-dimensional complex dilaton. This superpotential depends explicitly on S, implicitly on the complex structure moduli U^i and is independent on the Kähler moduli. In general we can split the three-form flux in an imaginary self-dual part (ISD) and an imaginary anti-self-dual part (IASD), defined by

$$*_6 G_3 = \pm i G_3 \ , \tag{3.32}$$

where a $+$ implies ISD and a $-$ implies IASD flux. We can decompose these parts further by holomorphicity,

$$G_3^{ISD} = G_{2,1} + G_{0,3} \ , \tag{3.33}$$

$$G_3^{IASD} = G_{3,0} + G_{1,2} \ . \tag{3.34}$$

We have written only the primitive components of the flux that obey $G_3 \wedge J = 0$.

A subtlety that we did not mention in the Type IIA case is that fluxes gravitate and therefore they backreact on the ten-dimensional spacetime [24]. For Type IIB compactified on a Calabi-Yau three-fold in the presence of three-form fluxes this backreaction can be treated quite explicitly. In those backgrounds the ten-dimensional metric is warped

$$ds_{10}^2 = Z^{-1/2}(y)\eta_{\mu\nu}dx^\mu dx^\nu + Z^{1/2}(y)g_{mn}dy^m dy^n \ , \tag{3.35}$$

where g_{mn} is the underlying Calabi-Yau metric. The function $Z(y)$ is called warp factor. The three-form fluxes also source the RR four-form giving a contribution to the Bianchi identity of its field strength. The general solution to the Bianchi identity for the 5-form is

$$F_5 = (1 + *_{10})d\alpha \wedge dx^0 \wedge dx^1 \wedge dx^2 \wedge dx^3 \ . \tag{3.36}$$

Here α is a function of the internal space and serves as a potential for the 5-form flux. For ISD three-form flux this implies

$$G_{3,0} = G_{1,2} = 0 \tag{3.37}$$

and the warp factor and the function α are related by

$$\alpha = Z^{-1} \ . \tag{3.38}$$

Compactification on an orientifolded torus

As in the Type IIA case we can make the above discussion more transparent by considering the theory compactified on $\mathbb{T}^6/(\Omega\mathcal{R}(-1)^{F_L})$. In this case we expand the three-form fluxes as

$$F_3 = - m\alpha_0 + e_0\beta_0 + \sum_{i=1}^{3}(e_i\alpha_i - q_i\beta_i) \ , \tag{3.39}$$

$$H_3 = \bar{h}_0\beta_0 + h_0\beta_0 - \sum_{i=1}^{3}(a_i\alpha_i + \bar{a}_i\beta_i) \ , \tag{3.40}$$

where the latin symbols denote fluxes and the greek symbols denote a basis of three-forms. This leads to the following superpotential

$$\begin{aligned} W_{\text{IIB-flux}} =& e_0 + i\sum_{i=1}^{3} e_iU_i - q_1U_2U_3 - q_2U_1U_3 - q_3U_1U_2 + imU_1U_2U_3 \\ &+ S\left[ih_0 - \sum_{i=1}^{3} a_iU_i + i\bar{a}_1U_2U_3 + i\bar{a}_2U_1U_3 + i\bar{a}_3U_1U_2 - \bar{h}_0U_1U_2U_3\right] \ , \end{aligned} \tag{3.41}$$

and the Kähler potential is given in Equation 3.21.

Supersymmetry conditions

In Type IIB the superpotential is independent of the Kähler moduli and there is a no-scale structure in the Kähler potential. As a result of this the scalar potential has no minimum for the Kähler moduli and they are not stabilized by the fluxes we have introduced here. This problem may be addressed by taking into account non-perturbative terms in the superpotential and/or α' corrections in the Kähler potential. We consider stabilization of the Kähler moduli in Section 3.4, here discuss local minima of the scalar potential for the dilaton and the complex structure moduli. Because of the no-scale structure in the Kähler potential, the scalar potential is given by

$$V = e^K K^{i\bar{j}} D_iW D_{\bar{j}}\bar{W} \ , \tag{3.42}$$

where the index runs only over the dilaton and complex structure. This scalar potential is positive definite and has Minkowski vacua for $D_iW = 0$. Minkowski vacua are obtained precisely for ISD three-form flux. In order to show this, we need the following basis of $(2,1)$ forms [25]

$$\chi_a = \frac{\partial\Omega_3}{\partial U_a} + K_{U_a}\Omega_3 \ . \tag{3.43}$$

Using this basis we can express the F-term equations as

$$D_SW = -\frac{1}{S+\bar{S}}\int_Y \bar{G}_3 \wedge \Omega_3 \ , \qquad D_{U_a}W = \int_Y G_3 \wedge \chi_a \ . \tag{3.44}$$

These equations vanish if the imaginary anti-self dual (IASD) flux is zero. This shows that Type IIB theory compactified on a Calabi-Yau orientifold with O7/D7 planes and ISD three-form flux has Minkowski vacua.

Sector	**SO(p-1)**	**(p+1)-dim field**
NS	Vector	Gauge boson A_μ
NS	Scalar	9-p real scalars φ_m
R	spinor	fermions λ_α

Table 3.1: The massless spectrum of a single Dp-brane

3.3 Dp-branes

So far we have analyzed the perturbative action of Type II string theories, which were explicitly constructed as theories of closed strings. It should be no surprise that these theories contain non-perturbative objects, since, the low-energy description of string theory is given in terms of QFT, which in general contains non-perturbative states. A particularly simple set of non-perturbative objects in Type II string theories are Dp-branes [26]. At weak string coupling, they can be described as $(p+1)$-dimensional subspaces of spacetime. The difference in definition between Type IIA and Type IIB was the particular choice of projection. This turns out to imply that p is an even number in Type IIA and an odd number in Type IIB. Each theory contains the branes that couple to the RR-forms present in the spectrum. For instance, Type IIB contains a D7-brane that couples electrically to C_8, IIA does not contain an RR-form of degree 8 and the corresponding BPS Dp-brane is also not present.

Dp-branes are surfaces on which open strings can end. The fact that the vacuum of the theory does not contain any open strings suggests that a Dp-brane configuration represents an excited state of the theory. The brane should be regarded as a dynamical object with the open-string sector describing its dynamics. This setup gives rise to gauge theories in Type II string theories. The endpoints of open strings are forced to move on the world-volume of the brane by definition. An observer living on the brane does not observe the extended nature of the strings and only see charged particles, since the endpoints are charged objects. From this it follows that the brane world-volume theory is a gauge theory. The massless spectrum of a single Dp-brane is given in Table 3.1. The spectrum contains a $U(1)$ vector supermultiplet with respect to 16 supersymmetries in $(p+1)$ dimensions, which can be viewed as the result of the dimensional reduction of the ten-dimensional $\mathcal{N}=1$ vector multiplet. An example is the D3-brane, whose world-volume theory is $\mathcal{N}=4$ super Yang-Mills (SYM) theory containing a $U(1)$ vector multiplet. Since Type II superstring theories contain 32 supersymmetries it is clear from the above discussion that branes preserve half of the sypersymmetries, hence they are BPS states with charge equal to tension. In the case of a stack of N Dp-branes the gauge group is enhanced to $U(N)$ and all particles, gauge bosons, fermions and scalars, transform in the adjoint of the gauge group.

The world-volume theory gives an effective description of the dynamics of the open strings and hence of the brane. In particular, the $(9-p)$ massless scalars φ_m are the Goldstone bosons of the translational symmetries broken by the presence of the brane. Therefore the vev of these scalars parametrizes the position of the brane in the transverse space and a non-trivial profile describes the fluctuations of the brane worldvolume on spacetime. For this reason, these scalars are often called position moduli. The bosonic action contains two pieces, known as the Dirac-Born-Infeld (DBI) action and the Chern-Simons action [27].

The Dirac-Born-Infeld action

From now on we follow the convention that M and N are ten-dimensional indices, μ and ν are spacetime indices, a and b are internal indices labelling the $(p-3)$-cycle wrapped by the brane, and m and n label the real coordinates transverse to the brane. The DBI action for a single brane is given by

$$S_{\rm DBI} = -\mu_{\rm p} \int d^{p+1}\xi e^{-\phi} \sqrt{-det(P\left[g_{\rm s}^{1/2} G - B\right] + \sigma F_{MN})} \ , \tag{3.45}$$

where the integral runs over the brane world-volume. F_{MN} is the field strength of the gauge bosons, which is assumed to vanish outside the world-volume of the brane, B_{MN} is the NS-NS 2-form, G_{MN} is the spacetime metric, ϕ is the dilaton, $\sigma = 2\pi\alpha'$. We define the pullback on the brane world-volume as follows,

$$P[G]_{ab} = G_{ab} + G_{am}\partial_b\varphi_m + \partial_a\varphi_m G_{mb} + \partial_a\varphi_m\partial_b\varphi_n G_{mn} \ . \tag{3.46}$$

The coefficient $\mu_{\rm p}$ is related to the charge and tension of the brane and is given by

$$\mu_{\rm p} = \frac{1}{(2\pi)^p(\alpha')^{(p+1)/2}} = \frac{M_{\rm s}^{p+1}}{(2\pi)^p} \ . \tag{3.47}$$

The DBI action describes the interaction of the world-volume fields with the NS-NS sector of the Type II theory. Notice that the position moduli enter the action through the pullback.

The derivation of the above action (and its non-abelian generalization) goes beyond the scope of this introduction. We do have some intuition on the form of the action. It is worth mentioning that at first order in α' we find

$$S_{\rm YM} = \frac{\sigma^2\mu_{\rm p}}{4g_{\rm s}} \int d^{p+1}\xi\sqrt{-g}F_{MN}F^{MN} \ , \tag{3.48}$$

which is the Yang-Mills action for the gauge field. This shows that the DBI action gives the Nambu-Goto action to lowest order in α' and the first correction is the standard world-volume gauge theory.

The Chern-Simons action

The Chern-Simons part of the action describes the topological couplings of the RR-fields. It is given by

$$S_{\rm CS} = \mu_{\rm p} \int P\left[\sum_{p+1} C_{p+1}\right] \wedge e^{\sigma\mathcal{F}} \ , \tag{3.49}$$

where p is in for Type IIB and even in Type IIA and we ignore curvature corrections. We can once again expand this action in α', setting B_2 to zero for clarity,

$$S_{\rm CS} = \mu_{\rm p}\left(\int C_{p+1} + \sigma\int C_{p-1}\wedge F + \cdots\right) \ . \tag{3.50}$$

This shows that, indeed, branes carry RR charges and that a Dp-brane is an electrically charged object for C_{p+1}. Furthermore, the world-volume gauge fields induce lower-dimensional D-brane charges, showing that branes can couple to all lower degree RR-forms.

We mentioned above that a stack of Dp-branes leads to a gauge group $U(N)$ with all the fields transforming in the adjoint. In that case, one has to change the derivatives in the pullback to covariant derivatives of the gauge theory and we have to keep track of the various non-trivial commutators such as $[A, \varphi]$. The fields are now $N \times N$ matrices of the adjoint. In the end the non-abelian extension of the DBI action was found to be [27, 28]

$$S_{\text{DBI}} = -\mu_{\text{p}} \int d^{p+1} e^{-\phi} \sqrt{-\det\left(P\left[E_{MN} + E_{Mi}(Q^{-1} - \delta)^{ij} E_{jN}\right] + \sigma F_{MN}\right) \det(Q_{ij})} \ , \tag{3.51}$$

where $E = g_{\text{s}}^{1/2} G - B$, $Q_{mn} = \delta_{ij} + i\sigma[\varphi_i, \varphi_k] E_{kj}$ and a symmetrized trace over the gauge indices is implicit. We discuss this action in more detail in Chapters 5 and 7 and in Appendix A.

3.4 Kähler moduli stabilization in Type IIB

The only string theory ingredient that we still have to discuss is Kähler moduli stabilization in Type IIB string theory. This is a separate issue compared to the stabilization of the other moduli because the standard Type IIB superpotential (3.31) does not depend on the Kähler moduli. We are therefore forced to introduce additional ingredients such as additional (non-geometric) fluxes, α' corrections or non-perturbative effects. In order to simplify the discussion we assume that the moduli that appear in the Gukov-Vafa-Witten superpotential are stabilized at a high scale and that $\langle W_{\text{GVW}} \rangle = W_0$ is constant.

Most mechanisms of Kähler moduli stabilization fall in two different classes, those for which the mass of the Kähler moduli are related to the scale of supersymmetry breaking like KKLT [29], Kähler uplifting [30] and the Large-Volume-Scenario [31]. And those for which this is not true such as the KL-mechanism [32]. We focus on the first class and in particular on the KKLT-mechanism because it is one of the simplest examples and it is the one relevant for the discussion in Chapter 7.

The original paper considers a single Kähler modulus T that parametrizes the volume of the internal space. The corresponding Kähler potential is then given in Equation (3.29). This is a big simplification for most compactifications but it can be viewed as a proof of principle. If we show that we can stabilize one modulus in this way then the others might be as well. In a realistic model there could be problems that we do not encounter in this simple toy example. The superpotential is

$$W = W_0 + Ae^{-\alpha T} \ , \tag{3.52}$$

where the non-perturbative term is sourced by, for example, a Euclidean D3-brane instanton or a gaugino condensate on a stack of D7-branes. A is a constant that depends on the vevs of the dilaton, complex structure moduli and possible position moduli. The value of α depends on the source of the non-perturbative term, eg. for a pure gauge gaugino condensate it is $\alpha = 2\pi/N$. It is important to note that the computation of this non-perturbative term relies on the single-instanton approximation implying that $\alpha T > 1$. The potential for T can be computed using Equation (3.19). The result has two extrema, a local maximum at $T = \infty$ and a supersymmetric AdS minimum at the point determined by $D_T W = 0$. At the AdS minimum we have the following relation between the parameters

$$W_0 = -Ae^{-\alpha T_{AdS}}(1 + \frac{2}{3}\alpha T_{AdS}) \ . \tag{3.53}$$

Throughout this work we assume that the parameters in the superpotential are real [33]. This choice implies that $\mathrm{Im}(T)$ is stabilized at the origin with the same mass as $\mathrm{Re}(T)$.

At this point we are still in an AdS vacuum, whereas for obvious reasons we want to have a Minkowski or dS vacuum. This requieres us to add a positive contribution to the scalar potential. This so-called uplift is probably the most discussed part of KKLT. It can be sourced by anti-D3-branes as in the original setup, but since then many other sources have been discussed, see [34] and references therein. We are agnostic about the precise source of the uplift and add a term

$$V_{\mathrm{up}} = e^K \Delta^2 \ . \tag{3.54}$$

Adding this term does not change the extremum at $T = \infty$, but lifts the value of the potential at the minimum which is now no longer at the point $T = T_{AdS}$. In addition, there is now a third extremum which is a local maximum situated between the two other extrema. In the vacuum one finds the following relations among the parameters [33],

$$A = -\frac{3W_0(\alpha t_0 - 1)e^{\alpha t_0}}{2\alpha t_0(\alpha t_0 + 2) - 3} \, , \qquad \Delta^2 = \frac{12\alpha^2 t_0^2 W_0^2(\alpha t_0 - 1)(\alpha t_0 + 2)}{[3 - 2\alpha t_0(\alpha t_0 + 2)]^2} \, , \tag{3.55}$$

where t_0 is the value of the Kähler modulus in the vacuum. The first equality defines t_0 in terms of the parameters in W. In this vacuum, the auxiliary field of T breaks supersymmetry, and

$$F_T \equiv e^{K/2}\sqrt{K^{T\bar{T}}} D_T W = \frac{3\sqrt{3}W_0}{4\sqrt{2}\alpha t_0^{5/2}} + \mathcal{O}\left[(\alpha t_0)^{-2}\right] \ . \tag{3.56}$$

Notice that we have expanded in inverse powers of αt_0 which is a good expansion parameter under the assumptions of KKLT. The gravitino mass is defined as $e^{K/2}W$ and is given by

$$m_{3/2} = \frac{W_0}{(2t_0)^{3/2}} + \mathcal{O}\left[(\alpha t_0)^{-1}\right] \ , \tag{3.57}$$

in the vacuum and the mass of the canonically normalized modulus is

$$m_t = 2\alpha t_0 m_{3/2} \, , \tag{3.58}$$

this proves our earlier claim that in KKLT the scale of supersymmetry breaking and the mass of the modulus are related. In KKLT the height of the maximum, or equivalently the height of the barrier separating the metastable minimum from runaway is given in Planck unites by $V_{\mathrm{barrier}} \simeq 3m_{3/2}^2$. This is of great importance later on, when the Lagrangian of the modulus is coupled to inflation. Finally, in KKLT, the scale of the parameters in W is somewhat constrained by the supergravity approximation. Consistency requires that $t_0 > 1$ which according to (3.55) implies that $W_0 \ll 1$ as long as $A \sim \mathcal{O}(1)$.

3.5 Cosmology and inflation

In the remainder of this chapter we review some standard cosmological physics focusing on inflation [35–37]. This will be useful for our later discussions. In this section we motivate inflation using one of the problems it addresses, namely the horizon problem. We then discuss the background and perturbation theory of single-field inflation discussing the necessary steps to compute the power spectrum. In the next section we review the same computations for two-field inflation. The ultimate goal is to write the equations of motion for the scalar perturbations and the power spectra which we use to discuss a string-inspired model of inflation in Chapter 6. In the last section of this chapter we touch upon inflation in string theory in general, quickly going over a few of the principal problems.

We started this thesis with a short history of the universe, traveling back in time and moving ever closer to the Big Bang. One of the first objects that we encountered was the cosmic microwave background. We noted that this is the source of the first free streaming light in the universe. We did not mention that this light has a very uniform temperature with fluctuations only of about the order of $\mathcal{O}(10^{-5})$. To see why this is a puzzling property in the context of the hot Big Bang scenario we consider the standard Friedmann-Robertson-Walker (FRW) metric of a spatially flat universe

$$ds^2 = -dt^2 + a^2(t)dx_i dx^i \ , \tag{3.59}$$

where $a(t)$ is the scale factor of the universe. The scale factor is related to the Hubble parameter via $H = \partial_t \log a$. We can write this in conformal time given by $d\tau = a^{-1}(t)dt$ as follows

$$ds^2 = a^2(\tau)\eta_{\mu\nu}dx^\mu dx^\nu \ . \tag{3.60}$$

In conformal time the distance a photon can travel is equal to the conformal time elapsed between any two points. Thus the distance that any light can travel is given by

$$\tau = \int \frac{dt}{a(t)} = \int (aH)^{-1} d\log a \ , \tag{3.61}$$

where $(aH)^{-1}$ is the comoving Hubble radius. For a universe with an energy momentum tensor dominated by a perfect fluid, as is the case in the ΛCDM model, the comoving Hubble radius always expands and the conformal time elapsed between the Big Bang and the CMB is finite. In fact, it follows that most points in the CMB would have never been in causal contact. The homogeneity of the CMB appears to be the result of a fine-tuning of the order of 10^{-5} in the initial conditions of the universe.

The way out of this is to give the universe more conformal time. Looking at Equation (3.61) the way to do this is to introduce a period in the past where the comoving Hubble radius decreases. Because if it decreases rapidly τ gets a relatively large contribution from early times. With a sufficiently rapid decrease there is enough time for all patches in the sky to have communicated with each other, thus solving the horizon problem. It is easy to see how this leads to inflation, since it follows from the definition of the Hubble parameter that

$$\partial_t (aH)^{-1} < 0 \quad \Rightarrow \quad \partial_t^2 a > 0 \ , \tag{3.62}$$

which amounts to an accelerating universe. From the Friedmann equation it follows that the above condition is equivalent to having

$$\epsilon = -\frac{\dot{H}}{H^2} < 1 \ . \tag{3.63}$$

ϵ is one of the slow-roll parameters. We define the end of inflation as the point where $\epsilon = 1$. Another important quantity to be defined is the number of e-folds $N_e = d \log a$, which expresses the growth of the scale factor as a function of time. During inflation the number of e-folds acts as a time variable as it increases monotically. Typically we need $N_e \approx 60$ to have had successful inflation, though the exact number depends on the process of reheating [38].

Background evolution

The simplest way to realize a universe which undergoes a period with a shrinking Hubble radius is to consider Einstein gravity coupled to a real scalar field φ, called the inflaton, in an action of the form

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{2} R + \frac{1}{2} (\partial\varphi)^2 - V(\varphi) \right] \ . \tag{3.64}$$

Inflation puts restrictions on the potential $V(\varphi)$. The inflaton can be decomposed in a background and a perturbation

$$\varphi(t, x^i) = \ \varphi_0(t) + \delta\varphi(t, x^i) \ , \tag{3.65}$$

where φ_0 is the background and $\delta\varphi$ is the perturbation. From the action we can derive the equations of motion for the background evolution of the scale factor and the inflaton. Assuming an FRW universe these are given by

$$\ddot{\varphi}_0 + 3H\dot{\varphi}_0 + V_{\varphi_0} = 0 \ , \quad H^2 = \frac{1}{3} \left(\frac{1}{2} \dot{\varphi}_0^2 + V(\varphi_0) \right) \ . \tag{3.66}$$

For slow-roll inflation $\dot{\varphi}_0$ is small compared to $V(\varphi_0)$ so we see that $H^2 \approx \frac{1}{3} V(\varphi_0)$. Hence the potential acts as the Hubble parameter, implying that if it is almost constant and positive during inflation the universe is quasi-de Sitter. In order to ensure that inflation lasts a long enough time we need the potential to be constant enough and hence we need the acceleration of the inflaton to be small. This leads to the second slow-roll parameter, given by

$$\eta = -\frac{\ddot{\varphi}_0}{H\dot{\varphi}_0} = \frac{\dot{\epsilon}}{H\epsilon} \ , \tag{3.67}$$

which also has to be small (but could be negative) in order to have successful inflaton. Related to the above Hubble slow-roll parameters are the potential slow-roll parameters

$$\epsilon_v = \left(\frac{V_\varphi}{V} \right)^2 \ , \quad \eta_v = \frac{V_{\varphi\varphi}}{V} \ , \tag{3.68}$$

which for slow-roll inflation, small ϵ and small η, are related to the Hubble slow-roll parameters through $\epsilon = \epsilon_v$ and $\eta = -2\eta_v + 4\epsilon_v$.

Perturbation theory

The background evolution of the universe during inflation is governed by the equations given above. However, the small fluctuations in the CMB that ultimately lead to structure in the universe arise from perturbations of this background. This leads us to consider perturbation theory. In particular we consider perturbations of the metric, that in general can be written as

$$ds^2 = -(1+2\Phi)dt^2 + 2aB_i dx^i dt + a^2[(1-2\Psi)\delta_{ij} + E_{ij}]dx^i dx^j \ . \tag{3.69}$$

Where the metric contains scalar (Φ, Ψ), vector (B_i) and tensor (E_{ij}) perturbations. They should be supplemented by perturbations in the energy-momentum tensor. These expressions contain some gauge redundancies which make it possible to eliminate a number of degrees of freedom and consider only the gauge-invariant combinations [39]. The matter and density perturbations during slow-roll can be related to variations of the inflaton. We are mostly interested in the following gauge-invariant combination

$$\zeta = \Psi + \frac{H}{\dot{\varphi}_0}\delta\varphi \ , \tag{3.70}$$

called the comoving curvature perturbation [40]. Using some of the gauge freedom to fix $\delta\varphi = 0$, the perturbations in g_{ij} take the following form

$$\delta g_{ij} = a^2(1-2\zeta)\delta_{ij} + a^2 h_{ij} \ , \tag{3.71}$$

where h_{ij} is a traceless and transverse tensor. We first discuss the dynamics of the scalar perturbations ζ before quickly touching on those of the tensor perturbations h_{ij}. A very important result, that we do not prove, is that ζ is time-independent on superhorizon scales, implying that any quantities that are functions of ζ should be evaluated at the horizon. Modes leave the horizon when their wavenumber k satisfies $k = aH$ [41,42].

We can use either the Arnowitt-Deser-Misner (ADM) formalism [43] or the Einstein equations to write δg_{00} and δg_{0i} as a function of ζ and obtain the quadratic action for ζ. The resulting action is best written in terms of the Mukhanov-Sasaki variable [44,45]

$$v = a\frac{\dot{\varphi}}{H}\zeta \ . \tag{3.72}$$

This allows us to write the quadratic action for the perturbations as

$$S = \frac{1}{2}\int d\tau d^3x \left[(\partial_\mu v)^2 + \frac{H}{a\dot{\varphi}}\partial_\tau^2\left(\frac{a\dot{\varphi}}{H}\right)v^2\right] \ . \tag{3.73}$$

This is nothing more than the action of a harmonic oscillator with a time-dependent mass. The equation of motion for the curvature perturbation in Fourier space follows from this action and is given by

$$\partial_\tau^2 v(k) + \left(k^2 + \frac{H}{a\dot{\varphi}}\partial_\tau^2\left(\frac{a\dot{\varphi}}{H}\right)\right)v(k) = 0 \ . \tag{3.74}$$

This is known as the Mukhanov-Sasaki equation, whose general solution takes the form

$$v = \int \frac{d^3k}{(2\pi)^{3/2}}\left[a_k^- v_k e^{ik\cdot x} + a_k^+ v_k^* e^{-ik\cdot x}\right] \ . \tag{3.75}$$

Here v_k is a complex solution to the Mukhanov-Sasaki equation and the $a_k^{\pm}$ are time-independent constants. The last step we have to take before we can discuss the power spectra is the quantization of the field v. This can be done fairly straightforward by promoting the constants $a_k^{\pm}$ to creation and annihilation operators. The only subtlety comes from the selection of the vacuum, which is not unique. This is related to the fact that the background we consider is time-dependent. This issue is resolved by considering that in the far past all observable modes were deep inside the horizon for which their frequencies where nearly time-independent. We thus need to impose Bunch-Davies vacuum conditions

$$\lim_{\tau\to-\infty} v_k = \frac{1}{\sqrt{2k}} e^{-ik\tau} \ , \tag{3.76}$$

which allow us to solve the Mukhanov-Sasaki equation and give a unique solution compatible with the Bunch-Davies condition

$$v_k = \sqrt{\frac{-\pi\tau}{2}} H_\nu^{(1)}(-k\tau) \ , \quad \nu = \frac{3}{2} + \epsilon + \frac{1}{2}\eta \ , \tag{3.77}$$

where $H_\nu^{(1)}(-k\tau)$ is the Hankel function of the first kind and the index ν is to lowest order in slow-roll.

Power spectra

The power spectrum of scalar perturbations can be computed from two-point correlation functions of v_k

$$P_\zeta(k,\tau) = \frac{k^3}{4\pi^2 a^2 \epsilon} |v_k|^2 = \frac{H^2}{8\pi^2\epsilon} \ , \tag{3.78}$$

which should be evaluated at horizon crossing $k = aH$. The spectral index $n_{\rm s}$ is defined as the rate of change of the power spectrum with k as follows

$$n_{\rm s} = 1 + \frac{d\log P_\zeta}{d\log k}\Big|_{k=aH} \ , \tag{3.79}$$

which is approximately given by

$$n_{\rm s} = 1 - 2\epsilon_* - \eta_* \ , \tag{3.80}$$

where the star denotes evaluation at horizon exit. If $n_{\rm s}$ is equal to 1 then the power spectrum of scalar perturbations would be independent of k, hence it would be scale-independent.

In order to discuss the power spectrum of tensor perturbations we need to write the quadratic action of h_{ij}. This analysis is very similar to the one for scalar perturbations and we simply quote the results. We find two copies of the action (3.73) leading to a power spectrum for tensor perturbations

$$P_t(k,\tau) = \frac{4k^3}{\pi^2 a^2} |v_k|^2 = \frac{2H^2}{\pi^2} \ . \tag{3.81}$$

The last equality shows that measurement of the tensor power spectrum gives direct information of the scale of inflation. The relevant observable related to tensor perturbations is the tensor-to-scalar ratio which is defined by

$$r = \frac{P_t}{P_\zeta} = 16\epsilon_* \ . \tag{3.82}$$

This is one of the most important unmeasured quantities in cosmology. It gives a direct measure on the energy scale during inflation. We now focus our attention on the generalization of the discussion in this section to a model with two dynamical fields during inflation.

3.6 Basics of two-field inflation

In this section we study cosmological perturbations of a system of two scalar fields coupled to Einstein gravity [46–49]. The action is a simple generalization of the action studied in the previous section and takes the form

$$S = \int \sqrt{-g} d^4x \left(\frac{1}{2} R - \frac{1}{2} G_{ab} g^{\mu\nu} \partial_\mu \varphi^a \partial_\nu \varphi^b - V(\varphi) \right) , \tag{3.83}$$

where G_{ab} denotes the field-space metric and $a, b = 1, 2$. The scalar fields φ^1 and φ^2 span a two-dimensional manifold with all relevant information contained in its metric G_{ab}. The Christoffel symbols, Riemann tensor and Riemann scalar of the scalar manifold are defined in the usual way. The analysis of perturbations goes along the same line as in the previous section with a few subtleties. We assume again a background FRW spacetime, so that we can derive the background equations of motion for the scalar fields

$$\ddot{\varphi}_0^a + \Gamma^a_{bc} \dot{\varphi}_0^b \dot{\varphi}_0^c + 3H \dot{\varphi}_0^a + G^{ab} \partial_b V = 0 . \tag{3.84}$$

This expression can be written more economically using the covariant derivative $D\dot{\varphi}^a = d\dot{\varphi}_0^a + \Gamma^a_{bc} \dot{\varphi}_0^b \dot{\varphi}_0^c$. The equation of motion for the scale factor is generalized in the obvious way

$$H^2 = \frac{1}{3} \left(\frac{1}{2} \dot{\varphi}_0^2 + V \right) , \tag{3.85}$$

where $\dot{\varphi}_0^2 = G_{ab} \dot{\varphi}_0^a \dot{\varphi}_0^b$. Solutions to Equations (3.84) and (3.85) determine the background evolution of the system. One obvious complication is that the field space is two-dimensional in this case, so whereas we could think of ϵ and η as measuring the velocity and acceleration of the inflaton during inflation, we need to be a bit more careful here. The equations of motion (3.84) and (3.85) relate ϵ to the vector tangent to the trajectory in field space $\dot{\varphi}_0^2 \equiv G_{ab} \dot{\varphi}_0^a \dot{\varphi}_0^b$. This implies that η, as the derivative of ϵ, is related to the tangential acceleration of the background trajectory.

As before we have to expand the scalar and metric degrees of freedom in terms of the background quantities (φ_0^a and g_{ab}^0) and perturbations, finding the equations of motion for the gauge-invariant perturbations and solving them [50]. Before writing the equations of motion of the perturbations we have to define a local frame on the trajectory in field space given by [46, 47, 49]

$$\begin{aligned} T &= \frac{1}{\dot{\varphi}_0} (\dot{\varphi}_0^1, \dot{\varphi}_0^2) , \\ N &= \frac{1}{\sqrt{G} \dot{\varphi}_0} (-G_{22} \dot{\varphi}_0^2 - G_{12} \dot{\varphi}_0^1, G_{11} \dot{\varphi}_0^1 + G_{12} \dot{\varphi}_0^2) . \end{aligned} \tag{3.86}$$

These vectors form an orthonormal basis of the tangent space of the scalar manifold. Hence, they can be used to decompose the physically relevant quantities along the normal and tangential directions with respect to the background trajectory. In particular, the derivatives of the scalar potential can be written in this basis as $V_\varphi = T^a \partial_a V$ and $V_{\rm N} = N^a \partial_a V$ (a labels both components of the basis vectors as well as fields). The total acceleration of the fields can be derived by taking the covariant derivative of T^a, using the equations of motion and projecting with N^a. The result reads

$$\ddot{\varphi}_0 = -3H\dot{\varphi}_0 - V_\varphi \ . \tag{3.87}$$

The slow-roll parameter η needs to be generalised to capture the full dynamics in field space and we need to take the curvature of field space into account. We thus need the following two slow-roll parameters

$$\eta^a = -\frac{1}{H\dot{\varphi}_0} D\dot{\varphi}_0^a \ , \tag{3.88}$$

which measure the acceleration of the fields φ^a.

In the local (T, N) basis, the η^a are projected on the following parallel and perpendicular components

$$\eta_\parallel = -\frac{\ddot{\varphi}_0}{H\dot{\varphi}_0} \ , \quad \eta_\perp = \frac{V_{\rm N}}{H\dot{\varphi}_0} \ , \tag{3.89}$$

so that

$$\eta^a = \eta_\parallel T^a + \eta_\perp N^a \ . \tag{3.90}$$

$\eta_\parallel$ measures the tangential acceleration, responsible for the variation of the modulus of the background trajectory velocity, whereas $\eta_\perp$ measures the normal acceleration causing the background trajectory to curve.

Perturbation theory

As before, the equations of motion for the perturbations are best given in terms of the gauge-invariant Mukhanov-Sasaki variables [44, 45]

$$Q^a = \delta\varphi^a + \frac{\dot{\varphi}^a}{H}\Psi \ , \tag{3.91}$$

which can also be decomposed using the basis given in Equations (3.86) as follows

$$v^T = aT_a Q^a \ , \quad v^N = aN_a Q^a \ , \tag{3.92}$$

v_α^T are called curvature modes and v_α^N are called isocurvature modes.This is the two-field generalization of (3.72). The action of these variables can be obtained by considering the general action and expanding to second order in the perturbations. We write them in Fourier space as

$$v^{N,T}(x,\tau) = \int \frac{d^3k}{(2\pi)^{3/2}} e^{ik\cdot x} \sum_\alpha \left(v_\alpha^{N,T}(k,\tau) a_\alpha(k) + v_\alpha^{N,T*}(k,\tau) a_\alpha^\dagger(-k) \right) , \tag{3.93}$$

where, after quantization, a_α and $a_\alpha^\dagger$ are the usual creation and annihilation operators. The Greek index α labels the quantum modes of the perturbations, consistency requires that $\alpha = 1, 2$ in the two-field case. The equations of motion for the Mukhanov-Sasaki variables in this basis take the form [49]

$$\frac{d^2 v_\alpha^T}{d\tau^2} + 2aH\eta_\perp \frac{dv_\alpha^N}{d\tau} - a^2H^2\eta_\perp^2 v_\alpha^T + \frac{d(aH\eta_\perp)}{d\tau} v_\alpha^N + \Omega_{\rm TN} v_\alpha^N + (\Omega_{\rm TT} + k^2) v_\alpha^T = 0, \tag{3.94}$$

$$\frac{d^2 v_\alpha^N}{d\tau^2} - 2aH\eta_\perp \frac{dv_\alpha^T}{d\tau} - a^2H^2\eta_\perp^2 v_\alpha^N - \frac{d(aH\eta_\perp)}{d\tau} v_\alpha^T + \Omega_{\rm NT} v_\alpha^T + (\Omega_{\rm NN} + k^2) v_\alpha^N = 0. \tag{3.95}$$

We see that the two-field perturbation system consists of a set of pairwise coupled harmonic oscillators. The coupling between curvature and isocurvature modes is controlled by $\eta_\perp$ which is inversely proportional to the curvature radius of the background trajectory. It follows that the coupling is strong whenever there is a sharp turn in the background trajectory. The symmetric mass matrix Ω of Equations (3.94) and (3.95) has the following elements

$$\Omega_{\rm TT} = -a^2H^2(2 + 2\epsilon - 3\eta_\parallel + \eta_\parallel\xi_\parallel - 4\epsilon\eta_\parallel + 2\epsilon^2 - \eta_\perp^2) \ , \tag{3.96}$$

$$\Omega_{\rm NN} = -a^2H^2(2 - \epsilon) + a^2V_{\rm NN} + a^2H^2\epsilon R_\varphi \ , \tag{3.97}$$

$$\Omega_{\rm TN} = a^2H^2\eta_\perp(3 + \epsilon - 2\eta_\parallel - \xi_\perp) \ , \tag{3.98}$$

where R_φ is the Ricci scalar of the scalar manifold, and the third slow-roll parameters are defined by

$$\xi_\parallel = -\frac{\dddot{\varphi}_0}{H\ddot{\varphi}_0} \ , \quad \xi_\perp = -\frac{\dot{\eta}_\perp}{H\eta_\perp} \ . \tag{3.99}$$

The initial conditions can be determined by the same logic as in the single-field case. The system is decoupled ($\eta_\perp = 0$) when observationally relevant modes are deep inside the horizon ($\frac{k}{aH} \gg 1$) and so the initial conditions for the scalar perturbations are fixed to be Bunch-Davies

$$v_\alpha^{N,T} = \delta_\alpha^{N,T} \frac{1}{\sqrt{2k}} e^{-ik\tau} \ . \tag{3.100}$$

Note that in Equation (3.100) it is understood that $\delta_1^T = 1$, $\delta_2^N = 1$ and derivatives define the initial condition for $\frac{d}{d\tau} v_\alpha^{T,N}$. It is important to note that there are two sets of equations of motion for the Muhkanov-Sasaki variables (in total 4 equations). There are corresponding initial conditions, one for each value of α. Both sets of equations should be taken into account when computing the inflationary observables.

Power spectra

The power spectra are defined in terms of the scalar two-point functions as

$$P_\zeta(k,\tau) = \frac{k^3}{4\pi^2 a^2 \epsilon} \sum_{\alpha=1,2} v_\alpha^T(k,\tau) v_\alpha^{T*}(k,\tau) \ , \tag{3.101}$$

$$P_{\mathcal{S}}(k,\tau) = \frac{k^3}{4\pi^2 a^2 \epsilon} \sum_{\alpha=1,2} v_\alpha^N(k,\tau) v_\alpha^{N*}(k,\tau) \ , \tag{3.102}$$

where $P_\zeta(k,\tau)$ and $P_{\mathcal{S}}(k,\tau)$ denote the dimensionless power spectra for the curvature and isocurvature modes, respectively. Given that in multi-field models there can be superhorizon evolution of the perturbations, these are to be evaluated at the end of inflation. This is to be contrasted to single-field models, where the freezing of curvature perturbations on superhorizon scales means the power spectra can be evaluated at horizon exit. From Equation (3.101) one can compute the spectral index for the curvature perturbations

$$n_{\rm s} = 1 + \frac{d\,\ln(P_\zeta(k,\tau_{end}))}{d\,\ln(k)}\ , \tag{3.103}$$

as well as the amplitude at the pivot scale k_*

$$A_s = P_\zeta(k_*,\tau_{end}), \tag{3.104}$$

which in the absence of an analytical solution to Equations (3.94) and (3.95) must be computed numerically. The evolution of tensor modes is unaffected by the number of dynamical fields driving the background expansion. The amplitude of the tensor power spectrum is the same as in the single-field case, which implies the following definition of the tensor-to-scalar ratio

$$r = \frac{P_t}{P_\zeta}(k_*,\tau_{end}) = \frac{2H^2}{\pi P_\zeta(k_*,\tau_{end})}\ . \tag{3.105}$$

Besides probing the scalar and tensor power spectra, observations also put bounds on the total fraction of primordial isocurvature, defined by

$$\beta_{\rm iso} = \frac{P_{\mathcal{S}}}{P_{\mathcal{S}} + P_\zeta}\ , \tag{3.106}$$

which is only relevant in multi-field models. From the theoretical point of view the isocurvature fraction depends on the mass of the isocurvature modes ($\Omega_{\rm NN}$) and on the strength of their coupling to the adiabatic perturbations. The observational bounds on $\beta_{\rm iso}$ can vary by many orders of magnitude, depending on how primordial isocurvature is transferred to the post-inflationary Universe. In [51] it was found that the less constraining bound is of the order

$$\beta_{\rm iso} \leq 10^{-3} \tag{3.107}$$

at the end of inflation.

In addition to putting constraints on the fraction of primordial isocurvature, observations also put constraints on the non-linear non-Gaussianity parameters f_{NL}. These are constraint to be no larger than $\mathcal{O}(10)$ by observations. Producing large non-Gaussianities would, in principle, spoil the validity of an inflationary model. However, for two-field models the non-linear f_{NL} are of the order of the slow-roll parameters and hence they are suppressed, see [52], [53]. We do not consider non-Gaussianities beyond this point.

Decoupling limit and single-field observables

In order to understand how to relate the observables defined above with those of single-field inflation of the previous section let us take the decoupling limit $\eta_\perp \to 0$, such that the equations of motion reduce to

$$\frac{d^2 v_\alpha^T}{d\tau^2} + \left[k^2 + \frac{1}{\tau^2}(-2 - 6\epsilon + 3\eta_\parallel)\right] v_\alpha^T = 0\ , \tag{3.108}$$

$$\frac{d^2 v_\alpha^N}{d\tau^2} + \left[k^2 + \frac{1}{\tau^2}\left(-2 + \frac{M^2}{H^2} + \left(-3 + \frac{2M^2}{H^2}\right)\epsilon\right)\right] v_\alpha^N = 0\ , \tag{3.109}$$

where the isocurvature mass is $M^2 = V_{\rm NN} + H^2 \epsilon R$ and we have used $\tau^{-1} = aH(1-\epsilon)$ for the background evolution. Equations (3.108) and (3.109) admit solutions that are a superpositions of Hankel functions of the first and second kind, which upon imposing Bunch-Davies boundary conditions reduce to

$$v_1^T = \frac{\sqrt{-\tau\pi}}{2} \, H^{(1)}_{\nu_T}(-k\tau) \equiv v^T, \qquad \nu_T = \frac{3}{2} + 2\epsilon - \eta_{\parallel} \ , \tag{3.110}$$

$$v_2^N = \frac{\sqrt{-\tau\pi}}{2} \, H^{(1)}_{\nu_N}(-k\tau) \equiv v^N, \qquad \nu_N = \frac{3}{2}\sqrt{1 - \left(\frac{2M}{3H}\right)^2 (1+2\epsilon) + \frac{4}{3}\epsilon} \ , \tag{3.111}$$

up to overall unimportant phases, with $v_2^T = v_1^N = 0$. The tangential mode is just the same as for the single-field case. On superhorizon scales, in the decoupling limit, one can show that the curvature perturbations are frozen as in the pure single-field case, since $Q^T = v^T/a \propto a^0$ while the isocurvature perturbations decay as $Q^N = v^N/a \propto a^{\nu_N - 3/2} \sim a^{-\frac{2M^2}{9H^2}}$, to zeroth order in slow roll, leaving only the single-field limit.

In order to make contact with observations one can compute the dimensionless power spectrum of curvature perturbations in this limit, finding

$$P_\zeta = \frac{k^3}{4\pi^2 a^2 \epsilon} |V_{\rm T}|^2 \to \left(\frac{k}{aH}\right)^{3-2\nu_T} \frac{H^2}{8\pi\epsilon} \ , \tag{3.112}$$

on superhorizon scales, which implies the following definitions for the spectral index and the amplitude

$$n_{\rm s} - 1 = 3 - 2\,\nu_T = -4\,\epsilon + 2\,\eta_{\parallel} = -2\epsilon - \eta \qquad \text{and} \qquad A_s = \frac{H^2}{8\pi^2\epsilon} \ . \tag{3.113}$$

Given that in the $\eta_\perp = 0$ limit the curvature perturbations are frozen once they leave the horizon, these observables can be evaluated at horizon exit, when $k = aH$.

By performing a similar computation for the isocurvature modes we can show that the amplitude of the isocurvature power spectrum at horizon crossing is the same as for the curvature modes. As noted above, due to the fact that $\nu_N < 3/2$, the isocurvature perturbations decay on superhorizon scales with a rate controlled by the ratio M^2/H^2. This implies that the isocurvature fraction at the end of inflation scales as

$$\beta_{\rm iso} \sim \frac{|v^N|^2}{|v^T|^2} = a^{-\frac{4M^2}{9H^2}} \tag{3.114}$$

and is therefore suppressed if at some stage during observable inflation $M \geq H$. The tensor-to-scalar ratio is given in the decoupling limit by its single-field expression

$$r = \frac{A_t}{A_s} = 16\,\epsilon_* \ . \tag{3.115}$$

3.7 String inflation

Now that we have discussed the theoretical ingredients needed for this thesis we end with a short discussion on string inflation, see eg. [34, 54–57] for more detailed discussions. When we introduced the tensor-to-scalar ratio r we mentioned that a measurement would give direct data about the energy scale of inflation. Besides the energy scale, r also contains information about the field excursion of the inflaton through the so-called Lyth bound [58]

$$\Delta\phi \geq \frac{1}{4}\sqrt{\frac{r}{0.01}}\ , \tag{3.116}$$

which can be derived under rather general assumptions. This bound in particular implies that if $r \geq 0.01$ then the field excursion of the inflaton was super-Planckian during inflation. Models with super-Planckian displacement are called large-field models and are the only class we consider in this thesis. Rather than a problem, the Lyth bound gives a motivation of embedding inflation into string theory. We discussed inflation disconnected from other physics, but this is in general not the case. Other physics has to be integrated out, which, generically, gives extra terms in the effective Lagrangian. These terms can depend on (large) powers of the inflaton field and, because of the trans-Planckian field values during inflation, they could dominate the Lagrangian. In principle, a full understanding of the connection between inflation and other physics is needed. In particular, knowledge of the embedding of the model in string theory is needed to show that the inflationary Lagrangian does not get modified significantly. To ensure that the inflaton potential survives the integrating out of heavy physics, it is usually constructed in such a way that it has properties, such as symmetries, that protect it. We consider such a property in Chapter 4, where we discuss monodromic potentials in string compactifications.

We focus on string inflation models embedded in Type II string theory. They can be split in two classes depending on the origin of the inflaton: the inflaton can be a closed-string field or an open-string field. Models where the inflaton is a closed-string field can be decomposed further into models where the inflaton is a geometric modulus and into models where the inflaton is a zero mode of one of the ten-dimensional NS or RR forms. Examples of the first class of closed-string inflation are Kähler moduli inflation [59,60] and Fibre inflation [61]. On the other hand, examples of models where the inflaton is one of the axions of the closed-string moduli are Race-track inflation [62], N-flation [63] and axion-monodromy [64,65]. Examples of models where the inflaton is an open-string modulus are brane/anti-brane inflation [66,67], Wilson line inflation [68], inflation using D3/D7-branes [69] and models with only Dp-branes [70–72].

Let us comment on a fairly generic problem in string inflation, and, more generally, supergravity, called the η-problem. The problem takes a lot of different forms but it generically leads to a potential with the slow-roll parameter η unfit for inflation. We illustrate this with the simple example of a complex inflaton in $\mathcal{N} = 1$ supergravity. We can expand the the e^K factor of the F-term potential around a reference point, which we take to be the origin, to find

$$V(\phi) = V(0)(1 + \phi\bar{\phi} + \cdots)\ , \tag{3.117}$$

where the dots indicate terms coming from expansions of the Kähler and superpotential not in the exponential of the F-term potential. Unless the terms in the dots cancel the quadratic term in ϕ, the inflaton has a large mass and $\eta \approx 1$. Without some fine-tuning of the parameters in the model this cancellation does not happen and inflation is ruined. All successful models of string inflation either admit a bit of fine-tuning or assume that the inflaton does not appear in the Kähler potential such that the above expansion does not apply. This can be achieved by a complex field that appears as $\phi + \bar{\phi}$ in the Kähler potential, the complex part of ϕ is then the inflaton candidate, and is the situation for the class of models discussed in Chapters 6 and 7.

Besides the η-problem there exist a number of other challenges in string inflation. One of the most basic properties that all successful models should have is the following hierarchy of masses

$$H < M_{\mathrm{KK}} < M_{\mathrm{s}} < M_{\mathrm{p}} \ ,$$

which gives sufficient theoretical control. This hierarchy implies that we can compactify a ten-dimensional supergravity theory, without having to worry about string states, and that we do not have to worry about Kaluza-Klein (KK) states during inflation. Even if this mass hierarchy can be achieved it is, in general, difficult to obtain a model with one field parametrically lighter than other fields in the spectrum. If this is not then case then it is necessary to consider models with more than one inflaton such as the one discussed in Chapter 6.

A related issue is the generic presence of moduli in the spectrum of a string compactification. If the inflaton mass is of the same scale as the masses of (some of) the moduli then interactions between these fields during inflation could destabilize the model. It is therefore necessary to stabilize the moduli at a high mass scale and to integrate them out consistently. It is not sufficient to set the moduli to their vevs even if they have a high mass since, during inflation, their expectation values depend on the inflaton. This backreaction generically leads to run-away behaviour for the inflaton at large-field values. It has to be checked explicitly that a period of inflation is still possible after the moduli have been integrated out. We discuss moduli stabilization in length in Chapter 7.

Chapter 4

Monodromy and four-forms

In the present chapter we discuss the role played by Minkowski four-forms, understood as the field strengths of corresponding three-forms, in four-dimensional models. We discuss how these four-forms provide a potential for scalar fields that exhibit monodromy in the so-called Kalopor-Sorbo mechanism. We address how monodromy helps alleviate some of the problems of embedding large-field inflation in a UV-complete theory. To connect with string theory we study in a systematic way the appearance of Minkowski four-forms in four-dimensional Type II, $\mathcal{N} = 1$ vacua. We find that in these vacua RR and NS closed-string fluxes are in one-to-one correspondence with Minkowski four-forms. The full dependence of the flux scalar potential on RR and NS axions always goes through combinations of Minkowski three-forms. As a result the scalar potentials of string flux vacua have a branched structure. As in the Kalopor-Sorbo field theory model, gauge invariance of the four-forms combined with the duality symmetries of the compactification constrain the corrections to the axion potential to come suppressed by powers of the cut-off scale of the effective theory.

We study first Type IIA but also present analogous results for Type IIB, for both the open- and closed-string sector. Similar conclusions hold in this setup, but the final result is less engaging and some of the interesting structure of the Type IIA case is not apparent. In both cases we find that the full closed-string axion potential can be written through couplings with Minkowski four-forms. In case of the open-string sector we find that the world-volume potential of the position modulus of a D7-brane can be written in terms of four-forms and therefore exhibits the same structure.

The structure of this chapter is as follows. In Section 4.1 we recall important facts about Minkowski four-forms. In Section 4.2 we introduce the Kalopor-Sorbo mechanism by discussing the simplest field-theory example. In Section 4.3 we study the structure of Minkowski four-forms in Type IIA orientifold compactifications with RR and NS fluxes. We perform the dimensional reduction starting from the ten-dimensional democratic Type IIA action and focus on the couplings of the Minkowski four-forms. We show that they behave as auxiliary fields of moduli and that they are invariant under a class of discrete symmetries involving both RR and NS axion shifts as well as internal flux transformations. In the toroidal case we consider the action of $R \leftrightarrow 1/R$ dualities. Finally, we discuss the effect of geometric fluxes in the compactification and show that the same conclusions hold. In Section 4.4 we discuss four-forms in Type IIB orientifolds and in Section 4.5 we consider some of the results of Sections 4.3 and 4.4 in connection to four-dimensional supergravity. In Section 4.6 we show how four-forms may arise from the open-string sector, by dimensionally reducing the duals of the world-volume F_2 gauge field strengths. Here we focus on the example of Type IIB with D7-branes.

4.1 Minkowski four-forms

Consistency of Poincaré invariance in field theory implies that the possible Lorentz structure of massless fields is quite limited. The standard model, for example, contains particles fermions with spin 1/2, a scalar and vector bosons that are one-forms. However, extensions of the standard model may contain particles with different spins, still allowed by Poincaré invariance. In this section we will show that there is interesting physics related to three-form gauge fields.

Generically, the action of a three-form includes the terms

$$S = -\frac{1}{2}\int \sqrt{-g}|F_4|^2 + S_{\text{bound}} + S_{\text{mem}} + S_{\text{int}} \ , \tag{4.1}$$

where $F_4 = dC_3$ is the four-form field strength of the three-form. S_{bound} includes boundary terms that do not modify the equations of motion and will not play a part in our discussion. S_{mem} describes the coupling of the three-form to possible membranes in the theory. These membranes are, for instance, Dp-branes or NS-branes in Type II compactifications. Finally, S_{int} allows for possible couplings between the three-form and scalar fields. This is the part of the action that we will be mostly concerned with throughout this chapter. If there are no membranes nearby and ignoring possible interaction terms for the moment, the equation of motion of C_3, $d(\sqrt{-g}F_4) = 0$, force F_4 to be constant, at least locally,

$$F^{\mu\nu\rho\sigma} = f\epsilon^{\mu\nu\rho\sigma} \ . \tag{4.2}$$

As a result, there is no local dynamics and putting C_3 on-shell adds a contribution to the cosmological constant [73]. As pointed out in [74], in string theory f, the value of the four-form is quantized in units of the membrane charge, $f = qn$ with n integer. This is slightly different from the usual Dirac quantization condition that requires that the integral of the four-form over some appropriate cycle be quantized.

It is possible for f to change when membranes are present [73]. We have to consider S_{mem}, which describes the coupling of C_3 to a membrane

$$S_{\text{mem}} \ = \ q\int_{D_3} C_3 \ , \tag{4.3}$$

where D_3 is the membrane world-volume. Membrane nucleation is a mechanism to change the value of f and hence the cosmological constant. In this way the picture arises that different patches of the universe have different values of the cosmological constant. In this setup we are surrounded by domain walls seperating us from regions with other (potentially dangerous) values of the cosmological constant. A difficulty with the proposal as it is formulated in [74] is that within string theory we will generally not have $S_{\text{int}} = 0$ and we cannot separate the issue of the value of the cosmological constant from that of moduli stabilization. One expects the four-forms to couple to the moduli, making the situation far more complicated. However, we will not address the application of three-forms to the issue of the cosmological constant in this thesis.

4.2 The Kaloper-Sorbo mechanism

We now turn to axion monodromy and the role four-forms play in this discussion. Kaloper and Sorbo [75,76] showed that four-forms in field theory provide a natural way to gauge the shift symmetry of an axion and induce a quadratic potential stable under corrections to the action that arise, for instance, from integrating out heavy modes. An example of a situation where this is particularly important is in models of large-field inflation, where the inflaton has trans-Planckian field excursions. The Kaloper-Sorbo mechanism is an example of a situation where there is a bilinear coupling between a scalar field and a four-form in the action. Let us consider the simplest example of this mechanism. The relevant part of the action is

$$S_{KS} = \int_M -\frac{1}{2}|\partial\phi|^2 - \frac{1}{2}|F_4|^2 + \mu\phi F_4 \ , \tag{4.4}$$

with μ a parameter with dimensions of mass. The identity $dC_3 = F_4$ can be imposed by adding a Lagrange multiplier term to the action. The equation of motion for the four-form is easy to find

$$d(*_4F_4 - \mu\phi) = 0 \ . \tag{4.5}$$

Its solution reads

$$*_4F_4 = f + \mu\phi \ , \tag{4.6}$$

where f is an integration constant. Since this is a constraint equation we can directly plug it back in the action. The axion gains a potential

$$V(\phi) = \frac{1}{2}(f + \mu\phi)^2 \ , \tag{4.7}$$

which is essentially the square of the four-form field strength.[1] The minimum of the potential lies at $\phi_{min} = -f/\mu$. As before f is interpreted as the four-form vev. The axion still has a shift symmetry,

$$\phi \to \phi + 2\pi n \ , \qquad f \to f - 2\pi\mu n \ , \tag{4.8}$$

where a shift of ϕ is compensated by a shift of the four-form. Once f is fixed the shift symmetry is broken spontaneously. This shows that $V(\phi)$ is not a scalar potential but rather a family of scalar potentials or different branches parametrized by the value of f. Fixing f amounts to choosing a single branch of this family. After gauge fixing we are left with a single quadratic potential for ϕ.

What makes this elaborate construction of a simple quadratic potential interesting is that the symmetries protect the potential from cut-off suppressed corrections. Before integrating out the four-form, gauge invariance of C_3 and the shift symmetry of ϕ force the corrections to appear in powers $(F_4^2/M^4)^n$, with M the ultraviolet cut-off of the theory, rather than arbitrary powers of ϕ. Couplings of ϕ to other fields should only come in terms of derivatives of ϕ. Thus, in this simple model, corrections to the potential should appear as powers of V_0/M^4. This is crucial for the stability of large-field inflation.

[1] The correct sign of the potential is obtained after taking boundary terms into account, see [77].

The Kaloper-Sorbo Lagrangian is a field theory avatar of the monodromy inflation models of [64, 64, 65, 70, 78–95] that are based on string theory. In those models large-field inflation is attained by coupling an axion-like field to an external source of energy, like, for example, a brane tension. Upon a period the field gets a shift in energy, so that the field does not come to the same point but rather performs a large trans-Planckian excursion. In [78] it has been explicitly shown how a structure analogous to that of the Kaloper-Sorbo Lagrangian appears in specific string constructions. In the following sections we will see to what extent couplings between four-forms and scalar fields appear in flux compactifications of Type II string theories.

4.3 Four-forms in Type IIA orientifolds

We now turn to describe how four-dimensional four-forms appear in Type IIA orientifold compactifications. The compactification of ten-dimensional massive Type IIA string theory on a Calabi-Yau three-fold in the presence of background fluxes has been thoroughly studied in, for example, [19, 22, 96–98]. Here we perform the compactification while keeping track of all Minkowski four-forms that appear upon dimensionally reducing the ten-dimensional RR and NS fields. This leads to a new formulation of the full scalar potential in terms of Minkowski four-forms and the intriguing result that the full dependence of the flux scalar potential on RR and NS axions comes only through couplings to the four-forms. We make use of the symmetries of Type II string theory to show that corrections to this extended result can once again only appear in powers of the scalar potential itself. Ultimately, this means that we find a generalization of the simple field-theory example discussed in the previous section.

4.3.1 Four-forms, RR and NS fluxes in IIA orientifolds

We consider Type IIA string theory in the democratic formalism discussed in Section 3.1. The massless bosonic matter content in the NS sector contains as usual the metric, dilaton and two-form B_2. The p-form fields C_p coming from the RR sector have $p = 1, 3, 5, 7, 9$. The corresponding gauge invariant field strengths are defined as [16, 22]

$$G_p = dC_{p-1} - H_3 \wedge C_{p-3} + \mathcal{F} e^B \ , \tag{4.9}$$

where $\mathcal{F}$ is a formal sum over all the RR fluxes. Remember that the relevant parts of the action are given by

$$S = S_{\rm RR} + S_{\rm NS} + S_{\rm loc} \ , \tag{4.10}$$
$$\tag{4.11}$$

with

$$S_{\rm RR} = \frac{1}{4k_{10}^2} \int_{R^{1,3}\times Y} G_4 \wedge G_6 + G_2 \wedge G_8 + G_0 \wedge G_{10} \ , \tag{4.12}$$

$$S_{\rm NS} = -\frac{1}{4k_{10}^2} \int_{R^{1,3}\times Y} e^{-2\phi} H_3 \wedge *_{10} H_3 \tag{4.13}$$

$S_{\rm loc}$ refers to the contribution from localized sources like D6-branes and O6-planes and $G_0 = -m$. We compactify this theory an orientifolded Calabi-Yau three-fold Y such that there are O6 planes and the four-dimensional effective theory has $\mathcal{N} = 1$ supersymmetry. The basis of harmonic forms is given in Equations (3.10). We keep track of all the four-form field strengths in four dimensions by expanding $F_p = dC_{p-1}$ in terms of internal fluxes and Minkowski four-forms

$$F_0 = -m \ , \quad F_2 = \sum_i q_i \omega_i \ , \quad F_4 = F_4^0 + \sum_i e_i \tilde{\omega}_i$$
$$F_6 = \sum_i F_4^i \omega_i + e_0 dvol_6 \ , \quad F_8 = \sum_a F_4^a \tilde{\omega}_a \ , \quad F_{10} = F_4^m dvol_6 \ . \tag{4.14}$$

Here a and i run from 1 to $h_-^{1,1}$. We use them to distinguish the different four-forms. The parameters e_0, e_i, q_i, m refer to the internal RR fluxes on Y which will induce a scalar potential for the moduli of the compactification. In addition we get $2h_-^{(1,1)} + 2$ Minkowski four-forms labelled by F_4^0, F_4^i, F_4^a and F_4^m. There are also four-forms coming from the dual of the NS two-form B_2

$$H_3 = \sum_{K=0}^{h_{2,1}^-} h_K \beta_K \ , \qquad H_7 = \sum_K H_4^K \wedge \alpha_K \ , \tag{4.15}$$

from which we obtain $h_{2,1}^+ + 1$ additional Minkowski four-forms H_4^K. The axions arise from the expansion of B_2 and C_3

$$B_2 = \sum_a b^a \omega_a \ , \quad C_3 = \sum_K c_3^K \alpha_K \tag{4.16}$$

where b_i and c_3^I are four-dimensional scalars. They correspond to the axionic part of the complex supergravity fields T^a, S and $U^{\tilde{K}}$, as discussed in Section 3.2.

Scalar potential

We start by analyzing $S_{\rm RR}$, which we have to supplement with the duality relations given in Equation (3.8). Plugging Equations (4.14)-(4.16) into the above RR action and integrating over the internal dimensions we obtain the following effective scalar potential, from the RR sector, in four dimensions

$$V_{\rm RR} = -\frac{1}{2}\left[F_4^0\left(e_0 + b^i e_i + \frac{1}{2}k_{ijk}b^i b^j q_k - \frac{m}{6}k_{ijk}b^i b^j b^k\right) + \right.$$
$$\left. + F_4^i\left(e_i + k_{ijk}b^j q_k - \frac{1}{2}m k_{ijk}b^j b^k\right) + F_4^a(q_a - m b_a) - k m F_4^m\right] \ , \tag{4.17}$$

where k is the volume of the internal space and k_{ijk} are the triple intersection numbers, which are related by

$$k = \frac{1}{6}k_{ijk}v^i v^j v^k \ . \tag{4.18}$$

Here the v^i are the real part of the Kähler moduli. Adding $V_{\rm NS}$,, we see, from the full scalar potential, that the four-forms couple to the functions

$$\begin{aligned}
\rho_0 &= e_0 + b^i e_i + k_{ijk}\frac{1}{2}q_i b^j b^k - \frac{m}{6}k_{ijk}b^i b^j b^k - h_0 c_3^0 - h_i c_3^i \ , \\
\rho_i &= e_j + k_{jkl}b^k q^l - \frac{m}{2}k_{jkl}b^k b^l \ , \\
\rho_a &= q_b - m b_b \ , \\
\rho_m &= -m \ ,
\end{aligned} \tag{4.19}$$

which we define for later convenience.

Interestingly, we can write this scalar potential in a convenient form using the duality relations given in Equation (3.8). They relate the internal fluxes, axions and four-forms. For instance, expanding G_4 and G_6 into Minkowski and internal parts and imposing the duality relations we find an expression for $*_4F_4^0$ in terms of the fluxes and axions. Similarly, for the other Minkowski four-forms

$$\begin{aligned} *_4F_4^0 &= \frac{1}{k}(e_0 + e_i b^i + \frac{1}{2}k_{ijk}q^i b^j b^k - \frac{m}{3!}k_{ijk}b^i b^j b^k - h_0 c_3^0 - h_i c_3^i) \ , \\ *_4F_4^i &= \frac{g^{ij}}{4k}(e_j + k_{ijk}b^j q^k - \frac{m}{2}k_{ijk}b^j b^k) \ , \\ *_4F_4^a &= 4kg^{ab}(q_b - mb_b) \ , \\ *_4F_4^m &= -m \ , \end{aligned} \tag{4.20}$$

where $g_{ij} = \frac{1}{4k}\int \omega_i \wedge *\omega_j$ is the metric of the Kähler moduli space. This allows us to write the scalar potential in the form

$$V_{\rm RR} = -\frac{1}{2}\Big[-kF_4^0 \wedge *F_4^0 + 2F_4^0\rho_0 - 4kg_{ij} * F_4^i \wedge F_4^j + 2F_4^i\rho_i - \\ -\frac{1}{4k}g_{ab}F_4^a \wedge *F_4^b + 2F_4^a\rho_a + kF_4^m \wedge *F_4^m\Big] \ . \tag{4.21}$$

Here Equations (4.20) arise as equations of motion for the three-forms.

We can perform the same analysis for the NS part of the action. The kinetic term for the NS field leads to the following contribution

$$V_{\rm NS} = \frac{1}{2}e^{-2\phi}c_{KL}H_4^K H_4^L \ , \tag{4.22}$$

where $c_{KL} = \int \beta_K \wedge *\beta_L$ is the metric of the complex structure moduli space. Duality between H_3 and H_7 leads to a relation between the Minkowski four-form and the NS internal flux,

$$*_4H_4^I = h^I \ . \tag{4.23}$$

Finally, the contribution from localized sources can be written as [22]

$$V_{\rm loc} = \sum_a \int_\Sigma T_a\sqrt{-g}\ e^{-\phi} \ , \tag{4.24}$$

where T_a is the tension of the source and Σ its world-volume. Assuming that tadpole cancellation is satisfied, this contribution can be related to the fluxes and the real part of the moduli such that [22]

$$V_{\rm loc} = \frac{1}{2}e^K v_i v_j v_k k_{ijk}(mh_0 s - mh_i u^i) \ , \tag{4.25}$$

where s, u_i, v_i the real parts of the S, U_i, T_i moduli, respectively. Combining all pieces and using Equation (4.20) we find the following scalar potential

$$V = \frac{k}{2}|F_4^0|^2 + 2k\sum_{ij} g_{ij}F_4^i F_4^j + \frac{1}{8k}\sum_{ab} g_{ab}F_4^a F_4^b + k|F_4^m|^2 + \frac{1}{2s^2}\sum_{IJ} c_{IJ}H_4^I H_4^J + V_{\rm loc} \ , \tag{4.26}$$

or equivalently

$$V = \frac{1}{2k}(e_0 + e_i b^i + \frac{1}{2} q_i k_{ijk} b^j b^k - \frac{1}{6} m k_{ijk} b^i b^j b^k)^2 + \\ + \frac{g^{i\bar{j}}}{8k}(e_i + q^k k_{ikl} b^l - \frac{1}{2} m k_{ikl} b^k b^l)(e_j + q^m k_{jmn} b^n - \frac{1}{2} m k_{jmn} b^m b^n) + \\ + 2k g_{ij}(q^i - m b^i)(q^j - m b^j) + k m^2 + \frac{1}{2s^2} \sum_{IJ} c_{IJ} h^I h^J + V_{\rm loc} \quad (4.27)$$

which is the scalar potential obtained previously in the literature [96]. It can also be recovered from an $\mathcal{N} = 1$ four-dimensional effective Kähler potential and superpotential, see [19]. We would like to stress that the full axionic part of the scalar potential can be written in terms of the above couplings to Minkowski four-forms and it is always positive definite.

It is worth mentioning a subtlety regarding the process of integrating out the four-form. Considering (4.21), the equation of motion for the four-form implies

$$d(*_4 F_4 - \rho) = 0 \rightarrow *_4 F_4 - \rho = c \ , \quad (4.28)$$

where c is a constant and ρ one of the functions depending on the axionic moduli defined in (4.19). This would imply a shift on the four-form background leading to new terms in the scalar potential that cannot be recovered from the supergravity description. These shifts agree with the results of [99–101], for which a four-form acting as an auxiliary field implies a shift on the scalar potential with respect to the standard supergravity formula. While valid from a pure effective four-dimensional approach, our four-forms descend from RR and NS fields which are related, at the classical level, by Hodge duality. In fact, we have seen that the Hodge dualities relate the four-form backgrounds and the internal fluxes forcing this extra shift to vanish.

4.3.2 Symmetries

The reason for writing the scalar potential in terms of four-forms is that it allows us to see the same structure as in the simple Kalopor-Sorbo example, Equation (4.4). It is also important that the resulting scalar potential has the right symmetries to constrain corrections to the effective action. The effective scalar potential (4.26) indeed has the right shift and duality symmetries. In particular, the scalar potential is invariant under simultaneous shifts of the fluxes and the axions, which correspond to shifts on the axionic components of the Kähler and complex structure moduli. A shift on the Kähler axion given by

$$b_i \rightarrow b_i + n_i \ , \quad (4.29)$$

combined with

$$m \rightarrow m = \rho_m \ , \quad (4.30)$$
$$q_a \rightarrow q_a + n_a m = \rho_a(b_i = -n_i) \ , \quad (4.31)$$
$$e_i \rightarrow e_i - k_{ijk} q^j n^k - \frac{m}{2} k_{ijk} n^j n^k = \rho_i(b_i = -n_i) \ , \quad (4.32)$$
$$e_0 \rightarrow e_0 - e_i n_i + \frac{1}{2} k_{ijk} q^i n^j n^k + \frac{m}{6} k_{ijk} n^i n^j n^k = \rho_0(b_i = -n_i) \ , \quad (4.33)$$

leaves the scalar potential invariant and relates equivalent vacua. These transformations were first introduced in the toroidal orientifold of [97]. They are expected to be part of the duality symmetries of any Calabi-Yau orientifold. In the mirror Type IIB picture this corresponds to a shift on the complex structure of the torus. Notice that the above transformations leave invariant each four-form independently, as expected from higher-dimensional gauge invariance. Therefore, the derivation of this group of transformations is more intuitive in this formulation in terms of four-forms than in the supergravity description. They also correspond to a generalization of the Kaloper-Sorbo shift symmetry underlying the axion monodromy inflationary models.

Analogously, the scalar potential is also invariant under shifts on the complex structure moduli of the form

$$c_3^I \to c_3^I + n^I \ , \tag{4.34}$$

combined with a shift in the fluxes

$$e_0 \to e_0 + h_I n_I \ , \tag{4.35}$$

corresponding to the mirror of Type IIB $\mathrm{SL}(2,\mathbb{Z})$ shifts. Also in this case, the four-forms remain invariant independently. Note that this is not enough to guarantee that corrections to the effective action appear in powers of the scalar potential. Rather these shifts imply that corrections appear in powers of the four-forms. This is clearly not sufficient since, even though the scalar potential itself may have a lower value than the cut-off, separate terms may not. We require symmetries that relate the different four-forms. In a toroidal compactification we can make use of the well-known T-dualities.

T-dualities

Let us consider Type IIA compactified on a orientifolded torus, and focus on the three Kähler moduli. Consider the effect of performing two or more T-duality transformations over the system. Given a basis of two-forms ω_i such that the Kähler form can be written as usual

$$J = \sum_{i=1}^{3} v^i \omega_i \ , \tag{4.36}$$

we can perform two T-duality transformations along the two real directions of the Poincaré-dual two-cycle of some ω_i. In particular, if a T-duality transformation is performed along $i = 3$ we obtain again a Type IIA theory in which

$$v^3 \to \frac{1}{v^3} \tag{4.37}$$

and the other two fields v^i with $i \neq 3$ remain invariant. In this case v^3 corresponds to the area of the two-torus along which we perform the two T-duality transformations. Let us assume for simplicity an isotropic compactification such that the triple intersection number is $k_{ijk} = 1$ if all the indices are different, and zero otherwise. The volume of the manifold transforms as

$$k = \frac{1}{6} k_{ijk} v^i v^j v^k = v^1 v^2 v^3 \to \frac{v^1 v^2}{v^3} \tag{4.38}$$

The metric is given in general by

$$g_{ij} = -\frac{1}{4}\left(\frac{k_{ij}}{k} - \frac{1}{4}\frac{k_i k_j}{k^2}\right) , \quad g^{ij} = -4k\left(k^{ij} - \frac{v^i v^j}{2k}\right) , \tag{4.39}$$

and transforms under the two T-duality transformations as

$$\frac{g^{33}}{8k} \leftrightarrow \frac{1}{4k} , \quad \frac{g^{11}}{8k} \leftrightarrow kg_{22} , \quad \frac{g^{22}}{8k} \leftrightarrow kg_{11} , \quad 2kg_{33} \leftrightarrow \frac{k}{2} . \tag{4.40}$$

The RR part of the scalar potential is invariant under this T-duality if the functions defined in (4.19) are also interchanged

$$\rho_0 \leftrightarrow \rho_i \quad \text{if } i = 3 \tag{4.41}$$

$$\rho_i \leftrightarrow \rho_a \quad \text{if } i \neq a \neq 3 \tag{4.42}$$

$$\rho_a \leftrightarrow \rho_m \quad \text{if } a = 3 \tag{4.43}$$

Therefore, T-duality exchanges Minkowski four-forms with each other. Recall that each four-form comes from dimensionally reducing the field strength of the different higher-dimensional RR fields. One can confirm that the result matches with the known transformation rules for the RR fields under T-duality,

$$C_3 \leftrightarrow C_5 \quad \text{if } C_5 \text{ propagates along the T-dual direction} \tag{4.44}$$

$$C_5 \leftrightarrow C_7 \quad \text{if } C_7 \text{ (but not } C_5\text{) propagates along the T-dual direction} \tag{4.45}$$

$$C_7 \leftrightarrow C_9 \quad \text{if } C_9 \text{ (but not } C_7\text{) propagates along the T-dual direction} . \tag{4.46}$$

Finally, if the internal manifold is $\mathbb{T}^6$ we can perform a T-dual transformation along all the internal dimensions, obtaining

$$k \leftrightarrow \frac{1}{k} , \quad \frac{g^{ij}}{8k} \leftrightarrow kg_{ij} , \tag{4.47}$$

and the potential is invariant if

$$\rho_0 \leftrightarrow \rho_m , \qquad \rho_i \leftrightarrow \rho_a , \tag{4.48}$$

consistent with the transformation rules for the RR fields. Note that T-dualities relate the different four-forms in such a way that only the full scalar potential V_{RR}, involving all four-forms, is invariant under all dualities and shift symmetries. In particular, this implies that corrections to the scalar potential should come in powers of the scalar potential, not in powers of the separate four-forms. This is particularly useful when discussing large-field inflation, since in that case the energy density is well below the Planck scale whereas the field excursion is super Planckian.

4.3.3 Four-forms and geometric fluxes in toroidal Type IIA orientifolds

It is known that beyond standard RR and NS other, less studied NS fluxes may be present. These appear as geometric fluxes in toroidal models in the context of Scherk-Schwarz reductions. Geometric fluxes can be seen as non-trivial components of the curvature two-form. See [22, 98] and references therein for a more thorough discussion of geometric fluxes. In this section we will explore whether the addition of these fluxes changes the above discussion.

First, we are interested to see how the presence of geometric fluxes change the four-forms described in Equations (4.20). For simplicity we assume geometric fluxes on a factorized six-torus $\otimes_{i=1}^{3}\mathbb{T}_i^2$, with O6-planes wrapping three-cycles. In addition we assume there is a $\mathbb{Z}_2\times\mathbb{Z}_2$ orbifold twist so that only diagonal moduli survive projection. In this case we are left with three Kähler moduli and four complex structure moduli (including the complex dilaton). In this setting there are twelve geometric fluxes ω_{NK}^{M} that are convienently written in a three-vector a_i and a 3×3-matrix b_{ij}, see [15, 98] for notation.

Geometric fluxes can be used to convert a p-form into a $(p+1)$-form

$$(dX)_{N_1\ldots N_{p+1}} = \omega^{K}_{[N_1 N_2}X_{N_3\ldots N_{p+1}]K} \ , \tag{4.49}$$

denoted by $\omega\cdot X$. In particular,

$$\omega\cdot B = b^i a_i\beta_0 - b^i b_{ij}\beta^j \quad \text{and} \quad \omega\cdot C_3 = -\tilde{\omega}^i a_i c_0 + \tilde{\omega}^i b_{ij} c^j \ . \tag{4.50}$$

Using these relations it is possible to define a generalized operator $\tilde{d} = d+\omega$ that acts on forms instead of the standard differential. In this way geometric fluxes change the field strengths of B_2, C_3 and C_5 as follows [22]:

$$G_4 \quad \to \quad F_4 + \omega\cdot C_3 - H\wedge C_1 - \omega\cdot B_2\wedge C_1 + \mathcal{F}e^B \ , \tag{4.51}$$

$$G_6 \quad \to \quad F_6 - H\wedge C_3 - \omega\cdot B_2\wedge C_3 + \mathcal{F}e^B \ , \tag{4.52}$$

$$H_3 \quad \to \quad H_3 + \omega\cdot B_2 \ . \tag{4.53}$$

Substituting these field strengths in the Type IIA action and integrating over the internal dimensions as before we find an extra coupling in the NS sector,

$$-\int_Y e^{-2\phi}\omega\cdot B\wedge H_7 \quad = \quad \frac{e^{-2\phi_4}}{k}\left(b^i a_i H_4^0 - b^i b_{ij} H_4^j\right) \ , \tag{4.54}$$

and two in the RR sector,

$$-\int_Y G_4\wedge G_6 \quad = \quad F_4^0\left(b^i b_{ij} c^j - b^i a_i c^0\right) - F_4^i(b_{ij}c^j - a_i c^0) \ . \tag{4.55}$$

In this way the four-forms are modified as follows,

$$\begin{aligned}
*_4F_4^0 &= \frac{1}{k}[e_0 + b^i e_i - \frac{1}{6}m k_{ijk}b^i b^j b^k + \frac{1}{2}k_{ijk}q_i b^j b^k - h_0 c_3^0 - h_i c_3^i + b^i b_{ij} c_3^j - b^i a_i c_3^0] \ ,\\
*_4F_4^i &= \frac{g^{ij}}{4k}[e_j + k_{jkl}b^k q^l - \frac{m}{2}k_{jkl}b^k b^l + b_{jk}c_3^k - a_j c_3^0] \ ,\\
*_4H_4^0 &= h^0 + b^i a_i \ ,\\
*_4H_4^i &= h^i - b_j b^{ji} \ .
\end{aligned}$$

One can show that the scalar potential obtained from these four-forms and Equation (4.26) can also be obtained from the superpotential given in [22, 98].

Symmetries

Naturally we need to wonder if a shift in the axions can still be compensated by a shift in the flux. Since the geometric fluxes mix the NS and RR axions we need to shift them simultaneously. Shifting

$$b^i \quad \to \quad b^i + n_b^i \ , \tag{4.56}$$

$$c^J \quad \to \quad c^J + n_c^J \ , \tag{4.57}$$

in combination with

$$h^0 \;\to\; h^0 - a_i n_b^i \;, \tag{4.58}$$

$$h^i \;\to\; h^i + n_b^j b_{ji} \;, \tag{4.59}$$

$$e_j \;\to\; e_j + a_j n_c^0 - b_{jk} n_c^k \;, \tag{4.60}$$

$$e_0 \;\to\; e_0 + h_i n_c^i + h_0 n_c^0 + n_b^i b_{ij} n_c^j - n_b^i a_i n_c^0 \;, \tag{4.61}$$

and in combination with the shifts of the previous section, leaves all four-forms invariant separately. Since the four-forms themselves are still related through T-dualities this shows that introducing a more general flux background does not change the overall structure of the scalar potential found in Equation (4.26), nor does it change its symmetries in a significant way. All in all, the general structure for four-forms we described above remains in the presence of geometric fluxes.

4.4 Four-forms in Type IIB orientifolds

In this section we concentrate on four-forms descending from the closed-string sector of Type IIB orientifolds with RR and NS fluxes. We will see that similar results as in the Type IIA case hold. In Type IIB the dilaton and fluxes naturally combine into complex objects, leading to complex four-forms in the effective theory. Compared to Type IIA, the structure in Type IIB [20, 102] is slightly simpler because only the NS three-form H_3 and RR three-form F_3 play a role in the context of Calabi-Yau orientifolds. It is convenient to define the complex three-form

$$G_3 \;=\; F_3 \;-\; iSH_3 \;, \tag{4.62}$$

as in Chapter 3. The relevant piece of the action for our discussion is the kinetic terms of the RR and NS two-forms, which in complex notation may be written as

$$S_{IIB} = -\frac{1}{2k_{10}^2}\int_{\mathbb{R}^{1,3}\times Y} \frac{1}{3!}\frac{1}{S+S^*} G_3 \wedge *\bar{G}_3 \tag{4.63}$$

where $*\bar{G}_3 = \bar{G}_7$. As we did in Type IIA, we can now expand G_7 in terms of internal harmonics with coefficients given by Minkowski four-forms and fluxes. We consider only primitive IASD G_3 fluxes. These fluxes induce supersymmetry-breaking F-terms as discussed in Chapter 3. The contribution from primitive ISD G_3 fluxes does not depend on the moduli, but rather appears combined with the contribution from localized sources in the tadpole cancellation conditions which we do not consider. The relevant expansion of G_7 is given by

$$\overline{G}_7 \;=\; \overline{G}_4^0 \wedge\; \overline{\Omega} \;+\; \overline{G}_4^a \wedge \chi_a \;, \quad a = 1, .., h_{2,1} \;, \tag{4.64}$$

where Ω is the Calabi-Yau three-form, and χ_a is the basis of three-forms given in Equation (3.43). Here G_4^0 and G_4^a are complex Minkowski four-forms which may be written in terms of NS and RR pieces F_4, H_4 as

$$G_4^0 = F_4^0 - iSH_4^0 \quad \text{and} \quad G_4^a = F_4^a - iSH_4^a \;. \tag{4.65}$$

We can substitute the expansion of G_7 in the Type IIB action. The term $G_3 \wedge \bar{G}_7$ induces couplings of the four-forms to the axions and fluxes. This term splits according to the expansion given in Equation (4.64). The piece proportional to G_4^a gives the coupling

$$\frac{1}{S+S^*}\sum_a \overline{G}_4^a \int_Y G_3 \wedge \left(\frac{\partial\Omega}{\partial U_a} \ + \ K_{U_a}\Omega\right) = \frac{1}{S+S^*}\sum_a \overline{G}_4^a \ D_a \int_Y G_3 \wedge \Omega \tag{4.66}$$

$$= \frac{1}{S+S^*}\sum_a \overline{G}_4^a \ D_a W_{GVW} \ , \tag{4.67}$$

where W_{GVW} is the Gukov-Vafa-Witten superpotential. We have used Equation (3.43) and D_a are the Kähler covariant derivatives with respect to the complex structure fields U_a. The remaining piece of $G_3 \wedge \bar{G}_7$ is proportional to the four-form G_4^0,

$$\frac{1}{S+S^*}\overline{G}_4^0 \int_Y G_3 \wedge \overline{\Omega} = -\overline{G}_4^0 \ (\overline{D_S W}_{GVW}) \ . \tag{4.68}$$

We observe that the four-forms couple to the covariant derivatives of the Gukov-Vafa-Witten superpotential, implying that upon integrating them out we are left with the positive definite part of the scalar potential. Hence we can already conclude that we find the same structure as in Type IIA.

We can complete the description of the four-dimensional action by considering the ten-dimensional kinetic term of the seven-form. This yields the four-dimensional quadratic piece

$$\frac{\kappa}{S+S^*}(|G_4^0|^2 - G_4^a \overline{G}_4^b G_{a\bar{b}}) \ , \tag{4.69}$$

where

$$\kappa = \int_Y \Omega \wedge \overline{\Omega} = i e^{-K_{c.s.}(U_a)} \tag{4.70}$$

with $K_{c.s.}(U_a)$ the Kähler potential of the complex structure moduli and $G_{a\bar{b}}$ is the metric of the complex structure fields given by

$$G_{a\bar{b}} = -\frac{\int_X \chi_a \wedge \overline{\chi}_b}{\int_X \Omega \wedge \overline{\Omega}} \ , \tag{4.71}$$

Collecting all pieces, the ten-dimensional action (4.63) reduces to the following four-dimensional effective Lagrangian in terms of the Minkowski four-forms,

$$\mathcal{L}_{\text{IIB}} = \frac{1}{S+S^*}\left(\kappa(|G_4^0|^2 - G_4^a \overline{G}_4^b G_{a\bar{b}}) - \overline{G}_4^0 (S+S^*)\overline{D_S W}_{GVW} + \sum_a \overline{G}_4^a D^a W_{GVW}\right) \ . \tag{4.72}$$

In analogy to the Type IIA case, the full axion scalar potential, excluding the contribution from localized sources, can be written in terms of the Minkowski four-forms coupling to axions. The potential for the axions can be found by integrating out the four-forms, which is done by solving the equations of motion. We find the following solutions

$$*_4 G_4^{\bar{b}} = - i e^{K_{c.s.}} G^{a\bar{b}} \left(D_a W_{GVW} + (f_4 - iSh_4)_a\right) \ , \tag{4.73}$$

$$*_4 G_4^0 = - i e^{K_{c.s.}} \left((S+S^*)\overline{D_S W}_{GVW} + (f_4 \ - \ iSh_4)_0\right) \ ,$$

where $f_4^{a,0}, h_4^{a,0}$ are RR and NS integration constants. We observe that the complex four-forms $G_4^{a,0}$ are proportional to the F-terms and thus are associated to the auxiliary fields of the complex structure and dilaton. However, they also include a shift associated to the Minkowski four-form backgrounds. In Type IIA we argued that the classical Hodge dualities forced the shifts to vanish, identifying the constant background terms of the Minkowski four-forms with the internal fluxes of the magnetic duals. The analogy here would be to set f_4 and h_4 equal to zero with the argument that the internal fluxes parametrizing the G_3 background are enough to account for all degrees of freedom.

By inserting Equations (4.73) in the Lagrangian (4.72) we find the following scalar potential

$$V = e^{K_S + K_{c.s.}} \left(|(S+S^*)\overline{D_S W} + g_0|^2 + K^{a\bar{b}} |D_a W - g_a|^2 \right) \tag{4.74}$$

where we have used the tree-level result $K_S = -\log(S+S^*)$ and grouped the shifts into $g_{0,a} \equiv (f_4 - iSh_4)_{0,a}$. If the shifts vanish, we recover the standard formula for the $\mathcal{N}=1$ supergravity scalar potential. Note that, due to the no-scale structure, after using the equations of motion for the four-forms, we obtain a positive definite scalar potential.

Symmetries

There is a set of symmetries present in this setup, similar to the Type IIA case. The transformations studied in Section 4.3.2, relating different vacua of Type IIA compactified in a Calabi-Yau three-fold, are present in Type IIB compactified in the mirror Calabi-Yau. The transformations studied for Type IIA can be directly translated to the case at hand. For example, the discrete shift of the Kähler axion of Type IIA given by Equation (4.29) corresponds to a shift on the complex structure of the mirror Type IIB. The shift on the Kähler axion left the effective theory invariant if it was combined with a shift of the internal fluxes. The same is true in the Type IIB case. While in Type IIA the description in terms of four-forms offered an intuitive picture about these transformations, since they leave each four-form invariant, the situation in Type IIB is less transparent. Here we only have the four-forms descending from G_7 so we cannot decompose the scalar potential into smaller invariant pieces. In the end, in Type IIB, we find the generalization of the shift symmetry of axion monodromy models and the Kaloper-Sorbo Lagrangian. In other words, in Type IIA and Type IIB theories there exist symmetries that are the generalization of the Kalopor-Sorbo mechanism. We remark, once again, that the full appearance of the axionic moduli in terms of couplings to the Minkowski four-forms is expected to severely constrain the form of α' and perturbative corrections of the Lagrangian.

Non-geometric fluxes

Before closing this section, let us make a few remarks about non-geometric fluxes [103, 104] in toroidal Type IIB orientifolds. The existence of non-geometric fluxes is inferred from consistently applying T-dualities to flux compactifications of Type II theories. It is known that they induce additional terms in the superpotential. Type IIB orientifolds allow only for so-called Q-type non-geometric fluxes, in the notation of [104]. These fluxes have index structure Q_M^{NP} with antisymmetric upper indices and they are odd under the $O(3)$ orientifold involution of the compactification. The effect of the Q-fluxes on $W_{\rm GVW}$ is captured by the replacement

$$G_3 = (F_3 - iSH_3) \longrightarrow G_3 + \mathcal{Q}\mathcal{J} \ , \tag{4.75}$$

where the four-form $\mathcal{J}$ is given by

$$\mathcal{J} = i \sum_{i=1}^{3} T_i \tilde{\omega}_i \ , \tag{4.76}$$

with T_i the three Kähler moduli and

$$(\mathcal{Q}\mathcal{J})_{MNP} = \frac{1}{2} Q^{AB}_{[M} (\mathcal{J})_{NP]AB} \ . \tag{4.77}$$

Going back to the four-forms in Type IIB, Equation (4.67) is then modified as follows,

$$\frac{1}{S+S^*} \sum_a \overline{G}^a_4 \ D^a \int_Y (G_3 + \mathcal{Q}_{\mathcal{J}}) \wedge \Omega \ . \tag{4.78}$$

This is nothing but the Kähler derivative of the $W_{\rm GVW}$ after the inclusion of non-geometric fluxes. It seems that also in the presence of this class of non-geometric fluxes the structure of the Minkowski four-forms acting as auxiliary fields in the effective action persists.

4.5 Four-forms in supergravity

In this section we take a little detour from the general discussion of this chapter to focus on interesting connections between the results obtained above and supersymmetric theories with four-form auxiliary fields. Massless four-forms may be embedded into $\mathcal{N}=1$ supersymmetric multiplets, in which they naturally appear as auxiliary fields of non-minimal versions of the $\mathcal{N}=1$ chiral multiplet [77, 99–101, 105–112]. Essentially, in this formalism, one or both of the real auxiliary fields in a supermultiplet is replaced by corresponding four-forms. Similarly, one can formulate non-minimal $\mathcal{N}=1$ supergravity multiplets with one or two real scalar auxiliary fields replaced by four-forms.

The connection to the discussion in the previous section of this chapter and the possibility of having four-form auxiliary fields in supermultiplets becomes obvious once we consider the supersymmetric action in more detail. In [100] the globally supersymmetric action of a non-minimal chiral multiplet S including one four-form auxiliary field is discussed. The corresponding superfield may be defined as

$$S = -\frac{1}{4} \overline{D}^2 V \ , \tag{4.79}$$

where V is a real multiplet with the same content as a standard vector multiplet, but with the vector field replaced by $\epsilon_{\mu\nu\rho\sigma} C^{\nu\rho\sigma}$. S can be expanded in terms of the Grassmann variables as follows,

$$S = M + i\theta\sigma^\mu\bar{\theta}\partial_\mu M + \frac{1}{4}\theta\theta\overline{\theta\theta}\Box M + \sqrt{2}\theta\lambda + \frac{i}{\sqrt{2}}\theta\theta\bar{\theta}\overline{\sigma}^\mu\partial_\mu\lambda + \theta\theta(D+iF) \ , \tag{4.80}$$

with $F = \epsilon_{\mu\nu\rho\sigma} F^{\mu\nu\rho\sigma}$ and D an auxiliary real scalar. On-shell, this multiplet contains one complex scalar M and one Weyl fermion λ. Such a non-minimal chiral superfield

S can have couplings such as,

$$S_{\rm W} = \int d^2\theta d^2\overline{\theta} S^a \overline{S}^a + \int d^2\theta W(S) + \int d^2\overline{\theta} W^*(\overline{S}) \tag{4.81}$$

$$= |\partial M|^2 + D^a D^a + F^a F^a + W_a(D^a + iF^a) + W_a^*(D^a - iF^a) \ , \tag{4.82}$$

where $W_a = \partial_{S_a} W$. Using the equations of motion for the gauge field C_3 associated with the four-form, one finds $F^a = \text{Im}(W_a) + f_a$, with f_a a constant. As a result, the scalar potential has the form [99–101]

$$V_{\rm S} = |W_a + i f_a|^2 \ . \tag{4.83}$$

This agrees with the result obtained for standard chiral multiplets with the replacement $W_a \to W_a + i f_a$. This resembles the structure we found below Equation (4.74) for the Type IIB scalar potential. However, the above supermultiplet is not enough to describe the general structure we found. In particular, the shift obtained from the supermultiplet is real whereas the shift we found in the Type IIB compactification is complex, suggesting replacing both auxiliary fields with four-forms. In addition, our result is based on a supergravity potential.

Type IIA RR superpotential

In Type IIA we have found in total $2h^{11}_-$ four-forms, denoted above as F_4^i and F_4^a, which act as auxiliary fields for the h^{11}_- Kähler moduli of the compactification. Similar to Type IIB, this means that the supersymmetry multiplets associated to the Kähler moduli should contain two four-forms, each acting as an auxiliary field. On the other hand, there are h^3_+ four-forms H_4^I associated to the h^3_+ complex structure fields. In this case the associated supersymmetry multiplets only include one four-form auxiliary field, like the multiplets discussed in [100]. In addition, there are two four-forms F_4^0 and F_4^m which could be associated to the $\mathcal{N} = 1$ supergravity complex scalar auxiliary field. The relation imposed by the equations of motion between the four-forms and the moduli of the compactification is interesting. Considering Equations (4.20), the Minkowski four-forms satisfy

$$\text{Re}[W] = kF_4^0 + v_a F_4^a \ , \qquad \text{Im}[W] = \frac{1}{2} k_i F_4^i + kF_4^m \ , \tag{4.84}$$

where W is the $\mathcal{N} = 1$ Type IIA RR superpotential given by

$$W = e_0 + i e_a T^a - \frac{1}{2} k_{abc} q^a T^b T^c + \frac{1}{6} i m k_{abc} T^a T^b T^c \ . \tag{4.85}$$

It would be interesting to understand if this structure is a consequence of the possible identification of four-form fields as auxiliary fields of the moduli/gravity multiplets. More generally, it would seem that non-minimal $\mathcal{N} = 1$ supergravity formulations, with auxiliary field scalars replaced by Minkowsk four-forms, as in [99, 100, 105–109], could be the appropriate formulation to describe the multi-branched nature of string flux vacua.

4.6 Minkowski four-forms and open-string moduli

So far in this chapter we have focussed on the closed-string sector of Type II theories. We have seen that the full RR and NS axion dependence of the flux scalar potential can be written in terms of these four-forms. With the introduction of branes we also introduce open strings in the spectrum. It is a natural question to ask if the monodromic structure of the closed-string sector arises in the open-string sector. In this section we address this issue for the D7-brane moduli sector of a Type IIB orientifold compactification. We show that the scalar potential of the position modulus of the brane can be written in terms of a Kaloper-Sorbo coupling of the scalar to a Minkowski four-form field arising from the magnetic open-string field strength. This way we can use the standard Kaloper-Sorbo symmetry properties to argue, from an effective perspective, that higher-order corrections are under control and do not spoil the effective theory.

In the open-string sector of Type II string theory, Minkowski four-forms may arise from the dual magnetic potentials of the world-volume gauge fields of Dp-branes. The spectrum of the brane world-volume theory was discussed in Section 3.3. In particular, for a D7-brane the magnetic dual of the one-form gauge potential is a five-form , denoted by A_5, whose field strength can be expanded in terms of internal harmonics as

$$F_6 = iF_4 \wedge \bar{\omega}_2 - i\bar{F}_4 \wedge \omega_2 \ . \tag{4.86}$$

ω_2 is a (2,0)-form associated to the complex position modulus Φ of the D7-brane. Φ can be expanded in terms of internal harmonics as $\Phi = \phi\omega_2$ where ϕ is the four-dimensional complex position modulus. Notice that, unlike the four-forms coming from the closed-string sector, F_4 is a complex Minkowski four-form.

Next let us consider fluxes which induce a non-trivial potential for ϕ. We are going to focus on a single brane compactified on a six-dimensional torus but the analysis could be generalised to multiple branes on more general spaces. In this section we only consider ISD G_3 bulk fluxes. These fluxes induce a B-field on the brane world-volume given by [70, 113–117]

$$B_2 = \frac{g_s\sigma}{2i}(G^*\Phi - S\bar{\Phi})\omega_2 + cc. \ \ , \tag{4.87}$$

where we have denoted the non-supersymmetric ISD (0,3)-flux as $G \equiv G_{\bar{1}\bar{2}\bar{3}}$ and the supersymmetric (2,1)-flux as $S \equiv \epsilon_{3jk}G_{3\bar{j}\bar{k}}$ (see appendix A for details on the notation).

The relevant part of the DBI action, to leading order in α', is given by [70, 113, 117]

$$\mathcal{S}_{DBI} = \mu_7\sigma \int_{\mathbb{R}^{1,3}\times S_4} \frac{1}{2}(B_2 + \sigma F_2) \wedge *_8(B_2 + \sigma F_2) \ . \tag{4.88}$$

Plugging the decomposition (4.86) into the above Lagrangian and performing the dimensional reduction we obtain

$$\int_{S_4} F_6 \wedge *_8 F_6 \ = 2|F_4|^2 \int_{S_4} \omega_2 \wedge *_4\bar{\omega}_2 \ , \tag{4.89}$$

$$\int_{S_4} B_2 \wedge F_6 \ = \frac{1}{2}g_s\sigma\left(F_4(G^*\phi - S\phi^*) + \bar{F}_4(G\phi^* - S^*\phi)\right)\int_{S_4} \omega_2 \wedge \bar{\omega}_2 \ , \tag{4.90}$$

leading to the following four-dimensional effective Lagrangian

$$\mathcal{L}_4 = \mu_7\sigma\rho \ \left(|F_4|^2 - \frac{1}{2}g_s\sigma\left(F_4(G^*\phi - S\phi^*) + \bar{F}_4(G\phi^* - S^*\phi)\right)\right) + \ldots \tag{4.91}$$

Here $\rho = \int_{S_4}\omega_2 \wedge *_4\bar{\omega}_2$ and we have used the identity $*_4\omega_2 = -\omega_2$. The scalar potential can be obtained after integrating out the four-form. We find

$$V_4 = \mu_7\sigma\rho \ \left|f - \frac{1}{2}g_s\sigma(G^*\phi - S\phi^*)\right|^2 \ , \tag{4.92}$$

where f is an integration constant which can be identified with the magnetic flux F_2. The above expression reflects the branched structure of the scalar potential as the D7-brane moves along a cycle in the internal space. The potential is invariant under shifts on the position modulus if they are combined with the corresponding shift on F_2 flux. This shift symmetry underlies the typical multi-branch structure of a Kaloper-Sorbo Lagrangian. The idea again is that the underlying shift symmetry and the gauge invariance of the four-form protects the potential from dangerous higher order corrections. Once a specific branch is chosen, which means that the flux background is fixed, the position modulus with its monotonic potential may play the role of an inflaton field with trans-Planckian excursion. Let us note that we recover only half of the complete scalar potential because we have omitted the Chern-Simons part of the action, which, because of supersymmetry will give the same contribution as in Equation (4.92).

One can think of exploring a similar structure within Type IIA orientifolds containing D6-branes. Here, the magnetic gauge field is a four-form A_4, which has to be expanded in a basis of 1-forms on the $D6$-brane three-cycles in order to yield a Minkowski three-form field. See [118] for a discussion on this subject.

Chapter 5

The Dirac-Born-Infeld action and supergravity

In this chapter we discuss the effective action of Dp-branes in Type IIB toroidal orientifolds, with $p = 3, 5, 7$. We focus on the α' corrections to the scalar action coming from the DBI action. These scalars parametrize the position of the Dp-brane in the internal space. We demonstrate that, under certain conditions, the first-order correction to the action takes a general form, independent of the dimensionality of the brane. The α' corrections always carry powers of derivatives of the position modulus and we find that, in general, the scalar potential does not receive any corrections. Schematically we find that the Lagrangian of the complex Dp-brane position moduli ϕ_i has the on-shell structure

$$\mathcal{L} = -\left[1 + aV(\phi_i)\right] \partial_\mu \phi_i \partial^\mu \bar{\phi}_i - V(\phi_i) + \mathcal{O}(\partial_\mu^4) \, , \tag{5.1}$$

where V is the leading-order scalar potential and a is a positive constant proportional to the inverse fourth power of the string scale, $M_{\rm s}$. The DBI action captures all higher-dimensional corrections involving arbitrary powers of single derivatives and the scalars themselves, but not terms with more than one derivative acting on a single scalar. Hence the structure we find is exact in powers of the scalar field and, in particular, in powers of the potential. Let us note that more general fluxes may break this simple structure. The violation of this structure will be important in the discussion of Chapter 7.

We discuss how to capture the first-order α' corrections in globally $\mathcal{N} = 1$ supersymmetric models via corrections to the Kähler potential. We find higher-order operators that correct the Kähler potential in such a way that we find an action that matches the DBI action. If we consider a single complex modulus, then the operator which turns out to be most relevant in this discussion has the form

$$\int d\theta^2 d\bar{\theta}^2 |\Phi|^2 \partial_\mu \Phi \partial^\mu \bar{\Phi} \ , \tag{5.2}$$

where $\Phi = \phi + i\theta\sigma^\mu\bar{\theta}\partial_\mu\phi + \theta^2 F + \frac{1}{2}\theta^2\bar{\theta}^2\Box\phi$ denotes a chiral multiplet with its fermionic component set to zero. An important property of this operator is that no terms proportional to $|V|^2$ arise and hence that the scalar potential does not receive any corrections.

Describing the DBI action in terms of supersymmetric higher-derivative operators allows for an embedding into an $\mathcal{N}=1$ supergravity formulation. This is the main motivation for this excercise. While the procedure of coupling Kähler potentials to gravity is straightforward, see [119], the computations can become quite tedious. Including α' corrections at the supergravity level allows us to study their effect on moduli stabilization. In this chapter we focus our discussion of the theory on the different terms in the Lagrangian that we obtain in this way.

The structure of this chapter is as follows. In the next section we study the structure of the effective action for Dp-brane moduli in Type IIB toroidal compactifications. We analyze in detail the cases of D3-, D5-, and D7-branes and display the bosonic action up to fourth order in derivatives. In Section 5.2 we discuss higher-derivative operators in globally supersymmetric theories and describe how the result obtained in the previous section can be written in terms of these operators. In Section 5.3 we discuss the supergravity embedding of a specific operator, discussing which terms are important for applications to inflation. In the related appendix B, we discuss the effect of the non-canonical kinetic term in models of Dp-brane inflation like the one of [70]. For any potential that we consider the effect of the α' corrections is always to flatten the potential and lower the tensor-to-scalar ratio. This is expected since, for positive a, the correction acts as a friction term dissipating energy from the system. We follow the conventions of [119].

5.1 α' corrections to the Dp-brane moduli action

In this section we discuss the effective action of position moduli for Type IIB Dp-branes up to second order in α'. These corrections are important when studying the dynamics of a Dp-brane system. In particular, the cosmological evolution of a model of inflation could receive corrections to its predictions and its stability along the inflaton trajectory could be affected. The corrections to the four-dimensional effective theory for the bosonic open-string fields of Dp-branes can be derived from the DBI and CS actions describing the world-volume deformations of the brane. In the case of compactifications for which the internal profile of the scalar fields is constant and the compactification to four dimensions is trivial, we can compute the effective action without much difficulty. We will consider this kind of compactifications in order to keep the computations manageable.

We demonstrate under which circumstances the schematic structure Equation (5.1) arises for the open-string fields of a system of Dp-branes in Type IIB orientifold compactifications. The general form of the DBI action for Dp-branes was given earlier in Equation (3.51). We repeat it here for convenience [27, 28]

$$S=-\mu_{\mathrm{p}}\int d^{p+1}\xi e^{-\phi}\sqrt{-\det(P[E_{MN}+E_{Mi}(Q^{-1}-\delta)^{ij}E_{jN}]+\sigma F_{MN})\det(Q_{mn})}\,, \tag{5.3}$$

where, as usual,

$$\sigma=2\pi\alpha'\,,\qquad E_{MN}=g_{\mathrm{s}}^{1/2}G_{MN}-B_{MN}\,,\qquad Q_{mn}=\delta_{mn}+i\sigma[\varphi_m,\varphi_p]E_{pn}\;, \tag{5.4}$$

and the φ_m are the real position moduli. They can be grouped in complex moduli ϕ_i where the index i runs over the complex dimensions normal to the brane.

Assuming a toroidal compactification and the vanishing of tensors with mixed internal/external indices, we can expand the determinant and the square root of the DBI action to get an effective four-dimensional action. This computation is done in appendix A. We omit all terms involving gauge bosons and Wilson lines. For our purposes the interesting part of the expansion of the DBI action is

$$\mathcal{L} = -\frac{\mu_{\rm p} V_{p-3}}{Z} f(\phi) \left(1 + Z\sigma^2 \sum_i \partial_\mu \phi_i \partial^\mu \bar{\phi}_i - \frac{1}{2} Z^2 \sigma^4 \left[\sum_{i \neq j} (\partial^\mu \phi_i \partial_\mu \bar{\phi}_j)(\partial^\nu \phi_j \partial_\nu \bar{\phi}_i) \right.\right.$$
$$\left.\left. + \sum_{i,j} (\partial_\mu \phi_i \partial^\mu \phi_j)(\partial_\nu \bar{\phi}_i \partial^\nu \bar{\phi}_j) \right] + \dots \right) , \quad (5.5)$$

with

$$f(\phi) = \sqrt{\det(g_{\rm s}^{1/2} g_{ab} + \sigma F_{ab} - B_{ab}) \det(g_{mn} + i\sigma[\varphi_m, \varphi_p](g_{\rm s}^{1/2} g_{pn} - B_{pn}))} \ . \quad (5.6)$$

Here $\mu_{\rm p}$ and V_{p-3} denote the tension of the brane and the volume wrapped by the brane, respectively, and Z is the warp factor. Note that no term of the form $(\partial_\mu \phi \partial^\mu \bar{\phi})^2$ is present in the effective action after the square root expansion when we consider a single complex scalar, for example, for a D7-brane.

After we redefine the scalar fields to absorb the global factors in Equation (5.5), subtract the orientifold tension cf. [70, 78], and identify $V(\phi) = a^{-1}(f(\phi) - 1)$ we find that in all cases the bosonic action has the structure of Equation (5.1). The constant a includes the remaining global factors and is proportional to $(\mu_{\rm p} V_{p-3})^{-1}$. Let us stress that the Lagrangian (5.5) includes all α' corrections arising from higher-order terms containing only powers ϕ^n and no derivatives. It is an expansion in $\partial\phi$ up to second order and it does not include second- or higher-order derivatives of ϕ. As a result, the effective action can only be trusted as long as the speed and acceleration of the brane remain small compared to a. This is precisely the case for slow-roll inflation.

The four-dimensional scalar potential comes from the function $f(\phi)$, given in Equation (5.6), which is a function on the internal space. In general, it is an infinite series in powers of α'. However, as shown in appendix A, for certain fluxes the determinants yield a perfect square, simplifying the computation. The fluxes that preserve a certain amount of supersymmetry at the string scale are precisely the ones that give this simple structure. For these fluxes taking the square root of the determinant is trivial and the scalar potential is given by the leading-order scalar potential V_0. In other words, all higher-order terms in α' vanish and the potential is simply $V = V_0$. However, the stringy nature of the action does leave a trace in the effective theory because the kinetic terms for the scalar fields are non-canonical. The prefactor of the kinetic term is indeed given by $(1 + aV_0)$, with a showing the stringy nature of the correction. Let us stress that the structure given in Equation (5.1) is quite general and valid beyond the supersymmetric configurations. The advantage of these configurations is that one can replace V by the well-known leading-order result V_0. In general, the scalar potential receives corrections, but, these corrections also appear in the kinetic term, implying that the structure given in Equation (5.1) is preserved. In the case of D5- and D7-branes, this structure relies on the assumption that we can factor the determinant in an internal and external piece. This is characteristic of toroidal compactifications. We have not considered how this generalizes to other compactifications.

In addition to the DBI part of the action coming, there is a contribution from the CS action. In settings where the fluxes preserve supersymmetry in the vacuum, the scalar potential from the CS action is equal to the potential from the DBI action, leading to a factor of two in front of the scalar potential, but not in the correction to the kinetic term which comes exclusively from the DBI action. The structure of the scalar potential depends, through the specific form of $f(\phi)$, on the Dp-brane under consideration and on the flux background. Before moving on to the next section we summarize the dependence on the brane dimensionality by considering D7-, D3-, and D5-branes separately.

D7-branes

This case is of the most interest for this thesis, and in a way it is also the simplest because there is only one complex scalar field ϕ. As always, this scalar transforms in the adjoint representation of the gauge group of the system of D7-branes. One can obtain more phenomenologically relevant quantum numbers, for example the Standard Model gauge group and bifundamentals, if the branes are located at orbifold singularities, cf. [70]. In the presence of three-form closed-string fluxes G_3, the position of the branes can be stabilized due to the flux-induced B-field on the brane which yields a non-vanishing F-term scalar potential for ϕ. This potential comes from the first determinant in Equation (5.6) which reads

$$\det\left(g_{ab} + Z^{-1/2} g_{\rm s}^{-1/2} \mathcal{F}_{ab}\right) = \det(g_{ab}) \left[1 + Z^{-1} g_{\rm s}^{-1} \mathcal{F}^2 + Z^{-2} g_{\rm s}^{-2} \frac{1}{4} \left(\mathcal{F} \wedge \mathcal{F}\right)^2\right] , \quad (5.7)$$

where $\mathcal{F}_{ab} = \sigma F_{ab} - B_{ab}$. Whenever $\mathcal{F}$ is a selfdual or anti-selfdual two-form, $\mathcal{F} = \pm *_4 \mathcal{F}$, we have

$$(\mathcal{F} \wedge \mathcal{F})^2 = (\mathcal{F} \wedge *_4 \mathcal{F})^2 = \left(\mathcal{F}^2 dV_{S_4}\right)^2 = (\mathcal{F}^2)^4 , \quad (5.8)$$

where dV_{S_4} is the volume form of the cycle wrapped by the brane, and hence

$$f(\phi)^2 = g_{\rm s}^2 Z^2 \left(1 + \frac{1}{2} Z^{-1} g_{\rm s}^{-1} \mathcal{F}_{ab} \mathcal{F}^{ab}\right)^2 , \quad (5.9)$$

a perfect square. This is the case for a configuration with only imaginary self-dual closed-string fluxes including $(0,3)$-form and $(2,1)$-form fluxes denoted by G and S, respectively, see [113–117, 120, 121] and appendix A. For these fluxes the B-field is a $(2,0)+(0,2)$-form. Far from being isolated or useless cases, the G and S fluxes are the fluxes which solve the ten-dimensional supergravity equations of motion in a Calabi-Yau compactification at the classical level [24], as reviewed in Chapter 3. The F-term scalar potential in terms of these fluxes, after absorbing constant factors, reads [70]

$$V(\phi) = \frac{g_{\rm s}}{2Z^2} |G^* \phi - S \bar{\phi}|^2 . \quad (5.10)$$

In addition to the position modulus, the world-volume theory contains two complex Wilson lines such that all of the scalar components of the $\mathcal{N} = 4$ theory are present. The second determinant in Equation (5.6) leads to a D-term given by

$$\det(Q_{ij}) = 1 + g_{\rm s} \sigma^2 Z [\phi, \bar{\phi}]^2 . \quad (5.11)$$

For simplicity we consider D-flat configurations and neglect this contribution to the scalar potential. The generalization to non-vanishing D-terms is trivial and does not change any of our conclusions. The leading order effective Lagrangian is of the form given in Equation (5.1) with V given by Equation (5.10) and $a = (V_4 \mu_7 g_{\rm s})^{-1}$.

D3-branes

In the case of spacetime filling D3-branes only the second determinant in Equation (5.6) is present since all world-volume indices are spacetime indices. Notice that in this case the factorization between internal and external determinants always exists, regardless of the specific compactification. At leading order the square of Equation (5.6) is given by

$$\begin{aligned}\det(\delta_{mn} + i\sigma g_s^{1/2} Z^{1/2}[\varphi_m, \varphi_n]) &= 1 - 2\sigma^2 g_s Z \sum_{i<j} [\phi_i, \phi_j]^2 - \sigma^2 g_s Z \sum_{i,j} [\phi_i, \bar{\phi}_j]^2 + \dots \\ &= 1 + \sum_i |F_i|^2 + \sum_i D_i^2 + \dots ,\end{aligned} \tag{5.12}$$

where the dots include higher-order terms in α'. It is remarkable that in the absence of D-terms the above determinant can again be written as a perfect square,

$$f(\phi)^2 = \det(\delta_{mn} + i\sigma g_s^{1/2} Z^{1/2}[\varphi_m, \varphi_n]) = \left(1 - \sigma^2 g_s Z \sum_{i<j} [\phi_i, \phi_j]^2\right)^2 , \tag{5.13}$$

implying (5.1) with $a = \mu_3^{-1} Z$ and $V = \sum_{i<j} g_s [\phi_i, \phi_j]^2$. This structure is partially broken if we introduce warping and fluxes. The situation is slightly more subtle since, as described in [113–117, 120, 121], the local equations of motion force the internal warping and five-form background to be non-vanishing. One can then locally expand the warp factor around the position of the brane as [114, 115]

$$Z^{-1/2} = Z_0^{-1/2} + \frac{1}{2}\sigma^2 K_{mn} \varphi^m \varphi^n + \dots , \tag{5.14}$$

where K_{mn} is the second derivative of the warp factor with respect to the real scalar fields. This induces an additional contribution to the scalar potential which does not appear multiplying the kinetic term. Therefore, in the presence of non-constant warping the correction to the kinetic term is given by only a part of the scalar potential.

D5-branes

The result for D5-branes is a combination of the two cases considered above. Both determinants in Equation (5.6) contribute to the F-term scalar potential. Once again, the computation is simple in a purely supersymmetric configuration with no D-terms or fluxes. In that case,

$$f(\phi)^2 = \det(\delta_{mn} + i\sigma g_s^{1/2} Z^{1/2}[\varphi_m, \varphi_n]) = \left(1 - 4\sigma^2 g_s Z [\phi_1, \phi_2]^2\right)^2 , \tag{5.15}$$

where ϕ_1 and ϕ_2 are the two complex fields parameterizing the position of the D5-brane in the transverse space, which we have assumed to be a T^4 for simplicity. We thus once more obtain a Lagrangian of the form given in Equation (5.1) with $a = \mu_5^{-1} V_2^{-1} g_s^{-1/2} Z^{1/2}$ and $V = (\mu_5 V_2 \sigma^2)^{-1} Z^{-1/2} g_s^{1/2} [\phi_1, \phi_2]^2$.

5.2 Supersymmetric higher-derivative operators

In this section we discuss how to write a supersymmetric version of the four-dimensional effective action of a D7-brane compactified on a toroidal orientifold. The lesson of the previous section is that the DBI action yields a very particular effective action for the D7-brane position moduli, given in Equation (5.1). This action may be viewed as a first-order Lagrangian $\mathcal{L}_0$ with a correction $\mathcal{L}_1$ that is suppressed by a cut-off scale Λ

$$\mathcal{L} = \mathcal{L}_0 + \frac{1}{\Lambda^4}\mathcal{L}_1 \ . \tag{5.16}$$

We aim to find an $\mathcal{N} = 1$ supersymmetric theory that produces this action, which is equivalent to finding an appropriate Kähler potential and superpotential. In general, the four-dimensional effective Kähler potential may depend on the superfields and their derivatives, i.e.,

$$\int d\theta^2 d\bar{\theta}^2 K(\Phi_i, \bar{\Phi}_i; D_\alpha \Phi_i, \bar{D}_{\dot{\alpha}}\bar{\Phi}_i; \partial_\mu \Phi_i, \partial_\mu \bar{\Phi}_i, \dots) + \left(\int d\theta^2 W(\Phi_i) + \text{h.c.}\right) , \tag{5.17}$$

where D_α denotes the usual supersymmetric covariant derivative and Φ denotes a chiral multiplet with its fermionic component set to zero. The standard Kähler and superpotential produce $\mathcal{L}_0$, but to produce $\mathcal{L}_1$ we need to consider higher-order corrections to the theory. Higher-order corrections to the superpotential are generally model-dependent and involve, for example, higher powers of the superfields. We are looking for a correction to the kinetic term of the scalar field and therefore it makes sense to find a correction to the Kähler potential that has four chiral superfields. The number of covariant derivatives is fixed by the maximum number of ordinary derivatives in the Lagrangian. The anticommutator of two covariant derivatives is proportional to an ordinary derivative, and the D-term of the canonical Kähler potential already contains two ordinary derivatives. Thus the relevant correction describing the DBI action should contain four covariant derivatives. In this section we consider the structure of globally supersymmetric operators while in Section 5.3 we discuss the coupling to supergravity. Higher-dimensional operators involving chiral superfields have been studied in the past in supersymmetry and supergravity [122–135].

Derivatives of F

In global $\mathcal{N} = 1$ supersymmetric theories it is well-known that $V = |F|^2$, this observation is helpful since it allows us to rule out a number of operators. Any operator that contains a term proportional to $|F|^4$ is ruled out since it is expected to give a correction to the effective Lagrangian as V^2 which we do not find from the DBI action.

We will observe that all operators that have no $|F|^4$ have terms proportional to derivatives of F. Such terms seem to imply that the auxiliary field propagates. This would be unacceptable since we know from the DBI action that no such extra bosonic fields should be present, and it breaks SUSY explicitly. However, as emphasized in [126], derivative terms of auxiliary field are artefacts of the effective field theory description. Theories with higher-derivative corrections like (5.27) must be UV completed above the cut-off scale Λ. The momenta of auxiliary fields with kinetic terms from higher-derivative operators are larger than Λ and are hence irrelevant in the EFT. This argument is supported by the fact that UV-complete theories, such as string theory, should be free of ghosts and propagating auxiliary fields. To see that we do not have too many degrees of freedom more explicitly, note that the lowest-dimensional action contains the standard bosonic pieces

$$\mathcal{L} \supset -|F|^2 - \left(F\frac{\partial W}{\partial \phi} + \text{h.c.}\right) , \tag{5.18}$$

to which we have to add a kinetic term coming from the correction

$$\mathcal{L} \supset \frac{1}{\Lambda^2}|\partial F|^2 \,. \tag{5.19}$$

To obtain the canonically normalized kinetic term for F we need to redefine $\tilde{F} = F/\Lambda$. We thus get

$$\mathcal{L} \supset -m_{\tilde{F}}^2|\tilde{F}|^2 - m_{\tilde{F}}\left(\tilde{F}\frac{\partial W}{\partial \phi} + \text{h.c.}\right) \,, \tag{5.20}$$

with $m_{\tilde{F}} = \Lambda$. Thus, actually the scalar field $\tilde{F}$ has a mass of the same order as the cut-off scale and should decouple below the scale Λ. One has to be careful though, since integrating out $\tilde{F}$ is not equivalent to setting $m_{\tilde{F}} \to \infty$, due to the presence of the dimensionful coupling of $\tilde{F}$ to ϕ in the above expression. It does imply that in an effective action we can neglect all terms proportional to $\partial_\mu \tilde{F}$.

Dimension 8 operators

A list of supersymmetric operators with the desired amount of fields and derivatives was proposed in [125]. The relevant operators can be written in terms of component bosonic fields as follows, cf. (8)-(13) in [125],

$$\begin{aligned}
\mathcal{O}_1 = |\Phi|^2 D^2\Phi \bar{D}^2\bar{\Phi} = \quad & 16|\phi|^2\Box\phi\Box\bar{\phi} + 20|F|^2\bar{\phi}\Box\phi + 20|F|^2\phi\Box\bar{\phi} + 16|F|^4 \\
& - 8|\phi|^2\partial_\mu F\partial^\mu\bar{F} + 8\bar{\phi}F\partial_\mu\phi\partial^\mu\bar{F} - 8\bar{\phi}\bar{F}\partial_\mu\phi\partial^\mu F \\
& - 8|F|^2\partial_\mu\phi\partial^\mu\bar{\phi} + 4|\phi|^2F\Box\bar{F} + 4|\phi|^2\bar{F}\Box F \\
& + 8\phi\bar{F}\partial_\mu\bar{\phi}\partial^\mu F - 8\phi F\partial_\mu\bar{\phi}\partial^\mu\bar{F} \,, \qquad (5.21)\\
\mathcal{O}_2 = \bar{\Phi}\bar{D}^2\bar{\Phi}(D\Phi)^2 = \quad & 16\partial_\mu\phi\partial^\mu\phi\bar{\phi}\Box\bar{\phi} - 16|F|^2\bar{\phi}\Box\phi + 16|F|^2\partial_\mu\bar{\phi}\partial^\mu\bar{\phi} \\
& - 16|F|^4 + 16\bar{\phi}\bar{F}\partial_\mu\phi\partial^\mu F - 16\bar{\phi}F\partial_\mu\phi\partial^\mu\bar{F} \,, \qquad (5.22)\\
\mathcal{O}_3 = |\Phi|^2 D\bar{D}\bar{\Phi}\bar{D}D\Phi = \quad & - 8|\phi|^2\partial_\mu\phi\partial^\mu\Box\bar{\phi} - 8|\phi|^2\partial_\mu F\partial^\mu\bar{F} - 8|F|^2\partial_\mu\phi\partial^\mu\bar{\phi} \\
& 8(\partial_\mu\phi\partial^\mu\bar{\phi})^2 + 8\phi\partial_\mu\bar{\phi}(\partial_\nu\bar{\phi}\partial^\mu\partial^\nu\phi - 8\partial_\nu\phi\partial^\mu\partial^\nu\bar{\phi}) \\
& - 8\bar{\phi}F\partial_\mu\phi\partial^\mu\bar{F} - 8\phi\bar{F}\partial_\mu\bar{\phi}\partial^\mu F \,, \qquad (5.23)\\
\mathcal{O}_4 = \Phi^2 D\bar{D}\bar{\Phi}D\bar{D}\bar{\Phi} = \quad & - 4|\partial_\mu\phi\partial^\mu\phi|^2 - 4\phi\Box\phi\partial_\mu\bar{\phi}\partial^\mu\bar{\phi} - 4\phi^2\partial_\mu\partial_\nu\bar{\phi}\partial^\mu\partial^\nu\bar{\phi} \\
& - 16\phi\partial_\mu\phi\partial_\nu\bar{\phi}\partial^\mu\partial^\nu\bar{\phi} - 4\phi^2\partial_\mu\Box\bar{\phi}\partial^\mu\bar{\phi} \\
& - 32\phi F\partial_\mu\bar{\phi}\partial^\mu\bar{F} \,, \qquad (5.24)
\end{aligned}$$

where $\Box = \partial_\mu\partial^\mu$. These are dimension-eight operators which, when they appear in a four-dimensional action, are divided by the mass scale Λ^4. In addition to these operators, there are the complex conjugates $\overline{\mathcal{O}}_2$ and $\overline{\mathcal{O}}_4$. Notice that we did not include (14) and (15) of [125] because, after partial integration, they are proportional to $\mathcal{O}_4$ and $\overline{\mathcal{O}}_4$, respectively. Operators $\mathcal{O}_3$, $\mathcal{O}_4$, and $\overline{\mathcal{O}}_4$ contain the $|F|^4$-free operators. In particular, any $|F|^4$-free linear combination of $\mathcal{O}_1$, $\mathcal{O}_2$, and its complex conjugate can be expressed in terms of these operators. While comparing supersymmetric operators to the DBI action one has to keep in mind that the latter does not capture higher-derivative contributions involving multiple derivatives of the scalar fields, for example terms containing $\Box\phi$ and $\partial_\mu\partial_\nu\phi$. Ignoring these, we obtain

$$\frac{\mathcal{O}_3}{\Lambda^4} = \frac{8}{\Lambda^4}\left[(\partial_\mu\phi\partial^\mu\bar{\phi})^2 - |\phi|^2\partial_\mu F\partial^\mu\bar{F} - |F|^2\partial_\mu\phi\partial^\mu\bar{\phi} - \bar{\phi}F\partial_\mu\phi\partial^\mu\bar{F} - \phi\bar{F}\partial_\mu\bar{\phi}\partial^\mu F\right] , \tag{5.25}$$

$$\frac{\mathcal{O}_4}{\Lambda^4} = -\frac{4}{\Lambda^4}\left[|\partial_\mu\phi\partial^\mu\phi|^2 - 8\phi F\partial_\mu\bar{\phi}\partial^\mu\bar{F}\right] . \tag{5.26}$$

Partial integration of the quartic kinetic terms introduces an ambiguity here, since terms with second derivatives can be written as first derivatives and vice versa. This ambiguity is manifest in a free coefficient of the four-derivative terms in the two expressions above. This makes the quartic kinetic terms not meaningful in the comparison with the DBI action. Thus, the strongest constraint on possible operators is indeed the absence of $|F|^4$. Taking this freedom into account, all operators without $|F|^4$ can be written as

$$c_1\mathcal{O}_3 + c_2\left(\mathcal{O}_4 + \overline{\mathcal{O}}_4\right) . \tag{5.27}$$

Therefore, this includes all operators that, after partial integration, yield the correction of the form (5.1) up to terms containing derivatives of F. This leads to the conclusion that, ignoring the quartic kinetic terms, the operators $\mathcal{O}_4$ and $\overline{\mathcal{O}}_4$ above may be ignored and the operator $\mathcal{O}_3$ is left with the only desired piece

$$\mathcal{O}_3 = -\frac{8}{\Lambda^2}|\tilde{F}|^2\partial_\mu\phi\partial^\mu\bar{\phi} + \mathcal{O}\left((\partial_\mu\phi)^4\right) . \tag{5.28}$$

A possible point of confusion is that one might argue that $\tilde{F}$ decouples completely in the effective action and the relevant part of the operator $\mathcal{O}_3$ does not survive. However, it is easy to convince oneself that this is not the case due to the second term in (5.20). Indeed, as shown in Figure 5.1, one can draw a tree-level Feynman diagram with a vertex stemming from (5.28) and two $\tilde{F}$ propagators. The latter end in vertices provided by the second piece in (5.20). In the effective action limit with $(\partial_\mu\tilde{F}) \ll m^2_{\tilde{F}}$ the propagator of $\tilde{F}$ is approximately $-1/m^2_{\tilde{F}}$ so that, in the end, we are left with

$$\mathcal{O}_3 = -\frac{8}{\Lambda^4}\left|\frac{\partial W}{\partial\phi}\right|^2\partial_\mu\phi\partial^\mu\bar{\phi} + \mathcal{O}\left((\partial_\mu\phi)^4\right) . \tag{5.29}$$

In conclusion, we find that the following supersymmetric action

$$\mathcal{L} = \int d\theta^2 d\bar{\theta}^2|\Phi|^2\left(1 + \frac{c_1}{\Lambda^4}D\bar{D}\bar{\Phi}\bar{D}D\Phi\right) + \left(\int d\theta^2 W(\Phi) + \text{h.c.}\right) , \tag{5.30}$$

gives the correct effective action

$$\mathcal{L} = -\left(1 + \frac{8c_1}{\Lambda^4}\left|\frac{\partial W}{\partial\phi}\right|^2\right)\partial_\mu\phi\partial^\mu\bar{\phi} - \left|\frac{\partial W}{\partial\phi}\right|^2 + \mathcal{O}\left(\Box\phi, \partial_\mu\partial_\nu\phi, (\partial_\mu\phi)^4\right) . \tag{5.31}$$

Using the identity $D_\alpha\bar{D}_{\dot{\alpha}}\bar{\Phi} = \{D_\alpha, \bar{D}_{\dot{\alpha}}\}\bar{\Phi} = -2i\sigma^\mu_{\alpha\dot{\alpha}}\partial_\mu\bar{\Phi}$ we can write the supersymmetric action in the more transparent fashion

$$\mathcal{L} = \int d\theta^2 d\bar{\theta}^2|\Phi|^2\left(1 + \frac{8c_1}{\Lambda^4}\partial^\mu\Phi\partial_\mu\bar{\Phi}\right) + \left(\int d\theta^2 W(\Phi) + \text{h.c.}\right) , \tag{5.32}$$

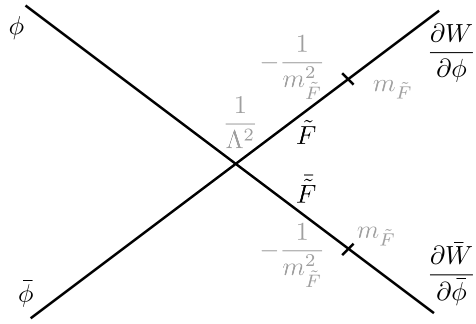

Figure 5.1: Feynman diagram that leads to the presence of (5.29) in the effective action. The interaction is similar to the interaction of Fermi's weak interaction theory. In both cases, the fact that the coupling constant is inversely proportional to the cut-off scale signals the breakdown of the effective theory.

which is the main result of this section. This is the action given in Equation (5.1) if identify $a = c_1/\Lambda^4$ which leads to

$$\frac{c_1}{\Lambda^4} = \frac{1}{V_4 \mu_7 g_\mathrm{s}} = \frac{16\pi^3 \alpha_G}{g_\mathrm{s} M_\mathrm{s}^4}\,, \tag{5.33}$$

where α_G is the gauge coupling of the theory living on the D7-brane and we have used

$$\alpha_G = \frac{8\pi^4 g_\mathrm{s}}{V_4 M_\mathrm{s}^4}\,. \tag{5.34}$$

This makes the stringy nature of the higher-derivative correction more manifest. The operators found in this section are interesting since they give us information on how to embed the non-canonical kinetic terms of the DBI action into a supersymmetric action. Our result can be easily generalized to the case of multiple scalar fields which may appear in Dp-brane configurations in different compactifications. In the next section we discuss the coupling of the action to supergravity.

5.3 Supergravity higher-derivative operators

In this section we discuss the generalization of the previous section to local supersymmetry. This can be done along the lines of [126] and [119], coupling the Kähler potential and superpotential to the $\mathcal{N} = 1$ gravity multiplet. It is known that the interaction between the dynamical closed-string modes and the open-string inflationary sector is not captured by the DBI and CS actions. As a result the DBI action does not describe gravity, i.e. it is only relevant for $M_\mathrm{p} \to \infty$. The non-minimal coupling in the effective action produces a flattening of the potential in canonical frame. We have already managed to give a supersymmetric description of this flattening. With the result of this section, we can now capture this effect in supergravity. A motivation to find an $\mathcal{N} = 1$ supergravity description of the effective theory is to be able to model the consequences of the α' corrections to the study of closed-string moduli stabilization. In Chapter 7 we will study the effects of moduli stabilization in an inflation model with a D7-brane.

For concreteness, we focus on a single chiral superfield corresponding to the position modulus of a D7-brane in a toroidal setting, as in the model discussed in Chapter 6. The Kähler potential given in Equation (3.21) is modified with the addition of the open-string modulus. For an isotropic compactification with two of the complex structure moduli stabilized, the Kähler potential, at leading order in α', reads [70, 136–139]

$$K = -\log\left[(S+\bar{S})(U+\bar{U}) - \frac{1}{2}(\Phi+\bar{\Phi})^2\right] - 3\log[T+\bar{T}]\,, \tag{5.35}$$

Assuming that the potential is minimized when $D_S W = D_U W = 0$, the dominant source of supersymmetry breaking is the auxiliary field of T, which leads to a soft mass for the D7-brane matter field Φ. The superpotential in this case is given by

$$W = W_0 + \mu\Phi^2\ . \tag{5.36}$$

Both contributions W_0 and μ in the superpotential are required to match the DBI result, (5.10). The precise matching between the fluxes and the supergravity parameters at leading order in α' was worked out in [70].

Notice the shift-symmetric structure of the Kähler potential for the position modulus contained in Φ, which leads to an approximate continuous shift symmetry in the scalar potential broken by fluxes. This flat direction in the Kähler potential is not only present in toroidal compactifications, but also in generic Calabi-Yau compactifications in the large complex structure limit, and it is expected to be preserved by all perturbative corrections to K, so in particular by our higher-derivative correction. This leads us to write the correction of the last section as

$$\Delta K = \frac{1}{(S+\bar{S})(U+\bar{U})}\frac{8c_1}{\Lambda^4}\left[(\Phi+\bar{\Phi})^2\partial_\mu\Phi\partial^\mu\bar{\Phi}\right]\ . \tag{5.37}$$

After integrating out the auxiliary field and ignoring the quartic kinetic terms the result is equivalent to (5.31). The scaling with the axio-dilaton and the complex structure moduli is required by modular invariance of the Kähler potential.

Coupling to supergravity

The coupling of a Kähler potential and superpotential to gravity is well understood. Specifically, the curved-space supergravity Lagrangian involving a Kähler potential and a superpotential reads

$$\mathcal{L} = \int \mathrm{d}^2\Theta\ \mathcal{E}\left[\frac{3}{8}(\bar{\mathcal{D}} - 8R)e^{K(\Phi,\bar{\Phi})} + W(\Phi)\right] + \text{h.c.}\,. \tag{5.38}$$

Here $\mathcal{E}$ is the chiral density, $(\bar{\mathcal{D}}-8R)$ is the chiral projector that ensures that the integral over superspace gives a supersymmetric Lagrangian and R is the supergravity superfield containing the supergravity multiplet $(\mathcal{R}, b_\mu, M)$. This way of coupling (arbitrary) Kähler and superpotentials to gravity was extensively applied in a systematic study of higher-derivative operators in supergravity in [140].

Note that, unlike the globally supersymmetric case with only a single mass scale Λ, once we couple to gravity we are bound to deal with two cut-off scales: Λ and M_p. In the following, it is crucial to keep track of both of them as they determine the relevant terms in $\mathcal{L}$. We find it useful to reinstate the factors of M_p in the results of [140] to discuss the relevance of each term in the effective theory. The mass dimension of M

and b_μ is three and that of the curvature, $\mathcal{D}_\mu$, and F is two. In accordance with the notation of [140], we write the correction to the Kähler potential as

$$\Delta K = \frac{\mathcal{T}}{\Lambda^4}\partial_\mu\Phi\partial^\mu\bar{\Phi} \ , \tag{5.39}$$

where

$$\mathcal{T} = \frac{|\Phi+\bar{\Phi}|^2}{6(S+\bar{S})(U+\bar{U})} \ . \tag{5.40}$$

The component expansion of the Lagrangian in the Jordan frame[1] can now be written as,

$$\delta\hat{\mathcal{L}}/\sqrt{|g|} = -\frac{1}{2}\Omega\mathcal{R} - \delta V + \mathcal{L}^{(4\text{-der})} + \mathcal{L}^{(2\text{-der})} \ , \tag{5.41}$$

where

$$\Omega = \frac{4\mathcal{T}}{\Lambda^4}|\partial\Phi|^2 \, , \tag{5.42}$$

$$\delta V = -\frac{4\mathcal{T}}{3\Lambda^4 M_{\text{p}}^4}|F|^2|M|^2 \, , \tag{5.43}$$

$$\begin{aligned}\Lambda^4\mathcal{L}^{(2\text{-der})} = {} & \mathcal{T}_{\bar{\Phi}}\left[\frac{1}{M_{\text{p}}^2}MF(\partial_\mu\bar{\Phi})^2 + \frac{1}{M_{\text{p}}^2}\bar{M}\bar{F}|\partial_\mu\Phi|^2 - 6\bar{F}\partial_\mu F\partial^\mu\bar{\Phi} - \frac{1}{M_{\text{p}}^2}4i|F|^2 b^\mu\partial_\mu\bar{\Phi}\right] \\ & - 3\mathcal{T}_{\Phi\bar{\Phi}}|F|^2|\partial_\mu\Phi|^2 - \mathcal{T}\left(\frac{1}{3M_{\text{p}}^4}|\partial_\mu\Phi|^2|M|^2 + \frac{4}{3M_{\text{p}}^4}|F|^2 b_a b^a + 3|\partial_\mu F|^2\right) \\ & + \mathcal{T}\left(\frac{1}{M_{\text{p}}^2}FM\Box\bar{\Phi} + \frac{1}{M_{\text{p}}^2}M\partial_\mu F\partial^\mu\bar{\Phi} - \frac{1}{M_{\text{p}}^2}F\partial_\mu M\partial^\mu\bar{\Phi}\right) \\ & + \frac{4}{3}\mathcal{T}ib^\mu\left(\frac{1}{M_{\text{p}}^4}FM\partial_\mu\bar{\Phi} + \frac{3}{M_{\text{p}}^2}\bar{F}\partial_\mu F\right) + \text{h.c.}\, ,\end{aligned} \tag{5.44}$$

and

$$\begin{aligned}\Lambda^4\mathcal{L}^{(4\text{-der})} = {} & -3\mathcal{T}_{\bar{\Phi}}\left[|\partial_\mu\Phi|^2\left(\Box\bar{\Phi} + \frac{2}{3M_{\text{p}}^2}ib^\mu\partial_\mu\bar{\Phi}\right) + \frac{2}{M_{\text{p}}^2}\partial^\mu\bar{\Phi}\,\partial^\nu\Phi\,\mathcal{D}_\mu\mathcal{D}_\nu\bar{\Phi}\right] \\ & - 3\mathcal{T}_{\Phi\bar{\Phi}}|\partial_\mu\Phi|^2(\partial_\mu\bar{\Phi})^2 + 3\mathcal{T}\partial^\mu\Phi\partial^\nu\bar{\Phi}\left[\mathcal{R}_{\mu\nu} + \frac{2}{9M_{\text{p}}^4}b_\mu b_\nu + \frac{2}{3M_{\text{p}}^3}i\mathcal{D}_\nu b_\mu\right] \\ & - 3\mathcal{T}\partial_\mu\Phi\,\mathcal{D}^\mu\left(\frac{1}{M_{\text{p}}}\Box\bar{\Phi} + \frac{2}{3M_{\text{p}}^3}ib^\nu\partial_\nu\bar{\Phi}\right) + \text{h.c.}\, .\end{aligned} \tag{5.45}$$

Two facts stand out in this contribution to the action. First, there is a correction to the scalar potential proportional to $|M|^2|F|^2$. This is noteworthy because one of the guiding principles in the determination of the Kähler potential in the previous section was the absence of corrections to V. There is no contradiction with our previous result, though, since $\delta V \to 0$ in the limit where $M_{\text{p}} \to \infty$, where the result matches the flat space result from the DBI action. Second, there are non-minimal couplings between the scalar Φ and the Ricci tensor and scalar. While the coupling to the Ricci scalar can be dealt with via a simple Weyl rescaling, that is not the case for the coupling to $\mathcal{R}_{\mu\nu}$.

[1]The standard derivation of the two-derivative supergravity Lagrangian leads to a gravitational coupling of the form $e^{K/3}\mathcal{R}$, cf. [119]. This is the frame we choose for the moment.

Dominant terms and final effective Lagrangian

Let us count dimensions to determine which of these terms dominate in the action. In the component expansion in (5.42)–(5.45) we find operators up to dimension twelve, with the following suppressions: $1/\Lambda^4$, $1/(\Lambda^4 M_\mathrm{p}^2)$, and $1/(\Lambda^4 M_\mathrm{p}^4)$. Since in the standard $\mathcal{N}=1$ supergravity action we have terms of the order $1/M_\mathrm{p}^n$, with $n=0,2,4$, we focus on the terms up to mass dimension eight. This truncation is justified since, at higher-order, additional terms in $\mathcal{L}$ can be sourced by higher-order corrections to K which we do not consider. Moreover, as in the flat-space case, let us consider $\partial_\mu F=0$, since the dynamics of F are an artefact of the effective field theory description. Then only three terms survive in the higher-derivative correction of the Lagrangian,

$$\Lambda^4 \delta\hat{\mathcal{L}}/\sqrt{|g|} = -\frac{1}{3}\mathcal{R}(\Phi+\bar\Phi)^2|\partial_\mu\Phi|^2 - |F|^2|\partial_\mu\Phi|^2 + \frac{1}{2}(\Phi+\bar\Phi)^2\partial^\mu\Phi\partial^\nu\bar\Phi\mathcal{R}_{\mu\nu}\,. \quad (5.46)$$

Notice that we have absorbed overall coefficients of $\mathcal{O}(1)$ and a possible constant a into the definition of Λ. As a result, the inclusion of the correction ΔK yields the same result as in global supersymmetry, plus nontrivial curvature couplings. And even these two additional terms can be made irrelevant by considering a model where the real part of Φ is stabilized at zero, since both are proportional to $\mathcal{T}$ and thus to $\mathrm{Re}(\Phi)$. This is exactly the case in the Higgs-otic model discussed in Chapter 6. We will see that, after Kähler moduli stabilization is taken into account, only the lightest field $\mathrm{Im}(\Phi)$ is excited during inflation, while $\mathrm{Re}(\Phi)$ remains stabilized at the origin so that the coupling to $\mathcal{R}_{\mu\nu}$ would not be relevant.

We can now recast the action into the form most similar to the one in the rigid limit via a conformal transformation to Einstein frame

$$\mathcal{L}/\sqrt{|g|} = \mathcal{L}_0 - \frac{1}{\Lambda^4}e^K|F|^2|\partial_\mu\Phi|^2 \;, \quad (5.47)$$

where

$$\mathcal{L}_{(0)}/\sqrt{|g|} = -\frac{1}{2}M_\mathrm{p}^2\mathcal{R} - K_{\Phi\bar\Phi}\,\partial_\mu\Phi\partial^\mu\bar\Phi - V_0 \;, \quad (5.48)$$

is the usual supergravity Lagrangian, with V_0 being the F-term potential. Note again that (5.47) is much simpler than our starting point, and is essentially the global result of (5.31). If, as mentioned above, we choose a superpotential that leads to a quadratic scalar potential as in the DBI and CS actions, we can find the on-shell kinetic terms using (5.35),

$$\mathcal{L}_\mathrm{kin} = -\left(K_{\Phi\bar\Phi} + \frac{1}{\Lambda^4}e^K|F|^2\right)|\partial_\mu\Phi|^2 \quad (5.49)$$

$$= -\left(\frac{1}{2} + 3a\frac{s\mu^2\varphi^2}{8t_0^3}\right)(\partial_\mu\varphi)^2, \quad (5.50)$$

where $\varphi=\sqrt{2/s}\;\mathrm{Im}(\Phi)$, $s=\langle(S+\bar S)(U+\bar U)\rangle$, and $t_0=\langle T\rangle$. A crucial assumption in the above derivation is the absence of IASD fluxes, which, if present, induce additional terms in the DBI and CS actions. As we will see later, the inclusion and stabilization of moduli fields, in particular Kähler moduli, calls for the inclusion of non-perturbative effects which act as IASD flux in the bulk. Consequently, the supergravity embedding of this more complex system requires a deviation from the ideas presented in this section, making the identification of the interesting operators a significantly more difficult task.

To summarize, we have shown in this chapter that the DBI and CS actions with ISD flux can be effectively described by the Kähler potential in (5.35). Coupling to gravity does not, in the end, make the Lagrangian more complicated as long as the cut-off scale is much larger than the dynamical scale of inflation, and $\mathrm{Re}(\Phi) = 0$ during inflation. However, we have not taken the effect of Kähler moduli stabilization into account. We will see later that this has a significant impact on our results.

Chapter 6

Higgs-otic inflation and two-field dynamics

In the last two chapters of this thesis we focus the discussion towards a single model of string inflation. In this chapter we introduce Higgs-otic inflation and we discuss its inflationary predictions. The Higgs-otic model seeks to unify inflation with the Higgs mechanism in an economic way. In [84] it was proposed that the neutral Higgs system of the MSSM with supersymmetry broken at a large scale of order $\sim 10^{13}$ GeV could drive cosmic inflation. Unlike the Higgs inflation proposal [141], this model considers a minimal coupling of the Higgs fields to gravity. This proposal is quite economical since it addresses several issues simultaneously. It provides stability for the Higgs scalar potential at the right scale, is consistent with the observed value of the Higgs mass and a neutral Higgs component acts as a complex inflaton field. The inflaton has a trans-Planckian field range and leads to a flattened version of chaotic inflation with a quadratic potential.

Neglecting the effects of moduli stabilization, the Higgs-otic model is a two-field inflaton system. In [70] a study of the cosmological observables focused only on the curvature perturbations and ignoring possible two-field effects like the generation of isocurvature perturbations. This is an important issue since two-field effects can, in principle, substantially modify the cosmological observables. Furthermore, the Planck satellite has provided strong bounds on isocurvature perturbations [9]. In this chapter, we perform a systematic analysis of the observables in the Higgs-otic two-field inflation system. We find that, as expected, curvature and isocurvature perturbations form a coupled system and there is super-horizon evolution of the curvature perturbations. This leads, in general, to a relative increase of curvature perturbations and consequently to a reduction of the tensor-to-scalar ratio, r, compared to the computation in [70]. The allowed range of $n_{\rm s}$ is decreased and is centered around the region allowed by Planck data with a tensor-to-scalar ratio in the range $r = 0.08 - 0.12$. Moreover, the isocurvature component is always suppressed at the end of inflation, consistent with Planck upper bounds.

The structure of this chapter is as follows. We introduce the Higgs-otic model in the next section, in which the relevant definitions and the inflaton potential are described. Section 6.2 presents the predictions of Higgs-otic inflation for three representative points in the parameter space of the induced soft terms. The latter are determined by a real positive parameter $0 \leq A \leq 1$, with $A = (m_H^2 - m_h^2)/(m_H^2 + m_h^2)$, H, h being the neutral Higgs scalars driving inflation [70]. The first case ($A = 0.83$) corresponds to the canonical Higgs-otic model in which the lightest scalar field at the minimum of the potential (at scale $M_{\rm SS}$) can be identified with the SM Higgs field. The second case ($A = 0.7$) analyses how those results are changed if there is new physics slightly modifying the Higgs-otic setting. For completeness, we present a third case with $A = 0.2$ in which the inflaton cannot be identified with the MSSM Higgs fields but could be relevant in extensions of the MSSM.

6.1 Higgs-otic inflation

In this section we discuss a model of inflation that is embedded in string theory, called Higgs-otic inflation in the original work [70]. Higgs-otic inflation refers to theories in which the inflaton is a complex scalar giving rise to gauge symmetry breaking through the Higgs mechanism in the vacuum, while, for large field values, it drives slow-roll inflation. The most obvious and natural candidate for this scalar is the Standard Model Higgs field itself, as described in [70]. Nevertheless the same idea may be applied to other beyond the Standard Model fields. This makes this class of models viable even if the inflaton is not the Higgs boson, as may be required by observational or theoretical constraints. In this thesis, Higgs-otic inflation serves as the prime example of an inflationary theory embedded in string theory.

One of the original benchmark models of [70] is a Type IIB compactification with O3/O7-planes and RR and NS three-form fluxes. This model features a compact orientifold with a local geometry of the form $(X \times \mathbb{T}^2)/Z_4$, where X is some complex two-fold which is wrapped by a stack of D7-branes at the singularity. The orientifold yields a four-dimensional $\mathcal{N} = 1$ supersymmetric gauge theory, as discussed in Section 3.3. The zero modes of the eight-dimensional Φ and A_μ give rise to the MSSM field content.

Some of the D7-branes may leave the singularity and travel through the bulk around the two-torus $\mathbb{T}^2$ while still satisfying tadpole cancellation conditions. When this happens the field Φ develops a vev and the $U(N)$ gauge symmetry is broken. The background closed-string fluxes give rise to a monodromy potential for the position moduli of the D7-branes, see Section 4.6, which has a D-flat direction. The three-form fluxes are in general the primitive ISD and IASD components of G_3. For ISD fluxes we distinguish between the $(0,3)$-form flux $G = G_{\bar{1}\bar{2}\bar{3}}$ and the $(2,1)$-form flux $S = \epsilon_{\bar{3}\bar{j}\bar{k}} G_{\bar{3}jk}$. G breaks supersymmetry in the vacuum and S gives rise to μ-terms in the $\mathcal{N} = 1$ superpotential, as discussed in Section 3.2.

Observations fix the mass of the inflaton to be of the order of 10^{13} GeV, and hence they fix the mass of the inflaton candidate Φ. In this realization of Higgs-otic inflation the supersymmetry-breaking scale $M_{\rm SS}$ is also of the order of 10^{13} GeV, consistent with the scenario of intermediate supersymmetry breaking discussed in [142–148] and consistent with a Higgs mass of 126 GeV [80]. The structure of mass scales in the original Higgs-otic setting is summarized in Figure (6.1).

The dynamics of the fields living on the D7-branes are described by the DBI and the CS actions, as discussed in Section 3.3. For G and S fluxes this leads to the following

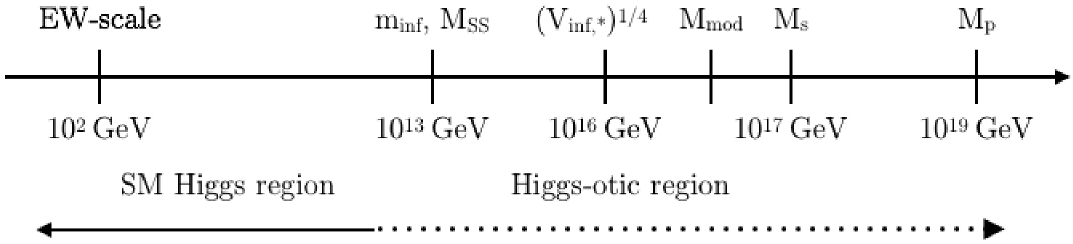

Figure 6.1: Energy scales in the original Higgs-otic inflation scenario [70]. Below 10^{13} GeV the light degrees of freedom in the Higgs sector are given by the $SU(2)$ doublet H_L. Above this scale $SU(2)$ is broken and they lie within the neutral components of h and H.

effective action for the complex four-dimensional position modulus ϕ

$$\mathcal{L}_{\text{kin}} = -\partial_\mu\phi\partial^\mu\bar{\phi}\left\{1 + \frac{1}{4Z^2V_4\mu_7}\left[(|G|^2 + |S|^2)|\phi|^2 - \bar{G}\bar{S}\phi^2 + \text{c.c.}\right]\right\} \tag{6.1a}$$

$$V = \frac{g_s}{2Z^2}|G^*\phi - S\bar{\phi}|^2\,, \tag{6.1b}$$

as described in appendix A. For configurations with only ISD fluxes the contribution in front of the kinetic term is exactly proportional to the F-term scalar potential and is given by

$$f(\phi) = 1 + \frac{1}{2V_4\mu_7 g_s}V_F\ , \quad \text{with} \quad V_F(\phi) = \frac{g_s}{2Z}|G^*\phi - S\bar{\phi}|^2\ . \tag{6.2}$$

We assume $Z = 1$ in the rest of this chapter. This action can be obtained from an $\mathcal{N} = 1$ description which we discussed in Chapter 5.

The above action concerns the $U(N)$ adjoint in the world-volume of D7-branes. However, it still applies after an orbifold projection that converts the adjoint scalars into a set of bifundamentals, some of which can be identified with the Higgs field. In the setup of [84], there is a stack of six D7-branes giving an initial $U(6)$ gauge group, after the orbifold projection a $U(3) \times U(2) \times U(1)$ gauge group is projected out. One linear combination of the $U(1)$'s can be identified with hypercharge, whereas the other two are anomalous and become massive. In this setup, the adjoint Φ contains doublets surviving projection, i.e.

$$\Phi = \begin{pmatrix} \mathbf{0}_3 & & \\ & \mathbf{0}_2 & H_u \\ & H_d & 0 \end{pmatrix}\ , \tag{6.3}$$

where H_u and H_d can be identified with the MSSM Higgs fields. Plugging this decomposition in Equation (6.1b) and taking the trace over gauge indices we obtain the scalar potential

$$V_F = \frac{Z^{-2}g_s}{2}\left[(|G| - |S|)^2|h|^2 + (|G| + |S|)^2|H|^2\right]\ , \tag{6.4}$$

where we define

$$h\ = \frac{e^{i\gamma/2}H_u - e^{-i\gamma/2}H_d^*}{\sqrt{2}} \quad , \quad H\ = \frac{e^{i\gamma/2}H_u + e^{-i\gamma/2}H_d^*}{\sqrt{2}}\ , \tag{6.5}$$

with $\gamma = \pi - \text{Arg}(GS)$. The model is essentially two copies of chaotic inflation with a quadratic potential and non-canonical kinetic terms.

To discuss different limits of this model it is useful to define the real variable A which controls the relative size of supersymmetric (S) versus non-supersymmetric (G) fluxes, i.e.,

$$A = \frac{2|SG|}{|G|^2 + |S|^2} . \tag{6.6}$$

Note that $0 \leq A \leq 1$. This parameter may also be written in terms of the masses of the above defined neutral scalars,

$$\frac{m_H}{m_h} = \sqrt{\frac{1+A}{1-A}} . \tag{6.7}$$

There are two interesting limits in which the two-field scalar potential, (6.4), becomes effectively a single-field potential. The first is when $|G| = |S|$, which corresponds to $A = 1$. In this limit the field h becomes massless. For a Higgs doublet to remain as a SM Higgs much below the inflaton mass scale, we would need to be close to that parameter point. In this limit, the Higgs field H is the one producing inflation. The second interesting limit is when supersymmetry is preserved in the vacuum, corresponding to $G = 0$ and, hence, $A = 0$.

If we insist on identifying one of the doublets with the SM Higgs it must remain massless below the supersymmetry-breaking scale $\simeq 10^{13}$ GeV. This requires fine-tuning of the fluxes G and S. However, one has to take into account the running of masses in between the UV scale (the compactification/string scale $M_{\rm s}$) and the supersymmetry-breaking scale $M_{\rm SS} \simeq 10^{13}$ GeV. Taking into account this effect, it was found in [70] that $A \simeq 0.83$ for one of the doublets to the Standard Model Higgs. This value of A follows from the assumption that the light scalar surviving down to the EW scale is to be identified with the SM Higgs, and that no extra degrees of freedom enter the particle spectrum beyond those of the MSSM. It is, however, conceivable that extra light particles are present in an extension of the MSSM, modifying the running of the Higgs' masses between the compactification/string scale and the supersymmetry-breaking scale $M_{\rm SS}$. If this is the case then other values of A can be compatible with the identification of the Higgs sector and the inflaton sector. Keeping this possibility in mind we analyze Higgs-otic inflation in three representative points in parameter space, $A = 0.83$, $A = 0.7$ and $A = 0.2$ in this chapter.

Another important ingredient of Higgs-otic inflation is the D-term potential with contributions from both the $U(1)$ charges and the $SU(2)$ charges of $H_{\rm u}$ and $H_{\rm d}$. It was shown in [70] that out of the initial four real neutral scalars one becomes massive due to the D-term potential, while another one, corresponding to a Goldstone boson, is eaten up by Z^0, thereby completing a massive $\mathcal{N} = 1$ vector multiplet. Therefore, only two real scalars remain massless before introducing fluxes, corresponding to $|h|$ and $|H|$ in the basis of (6.5). For inflation we thus have to consider the two real degrees of freedom $h \equiv |h|$ and $H \equiv |H|$, or equivalently defining the D-term flat direction as

$$\sigma = |H_u| = |H_d| \quad , \qquad H_u = e^{i\theta} H_d^* , \tag{6.8}$$

we can rewrite the potential as

$$V(\sigma, \theta) = M_{\rm SS}^2 \left(1 - A\cos\tilde{\theta}\right) \sigma^2 , \tag{6.9}$$

where we define

$$M_{\rm SS}^2 \equiv V_4 \mu_7 g_{\rm s} |\hat{G}|^2 , \quad \text{with} \quad |\hat{G}|^2 \equiv \frac{1}{Z 2 V_4 \mu_7}(|G|^2 + |S|^2) , \tag{6.10}$$

where $\hat{G}$ gives the magnitude of the flux present and $\tilde{\theta} = \theta - Arg(GS)$.

In angular variables the relevant piece of the action may be written as

$$\mathcal{L}_{4d} = f(\sigma,\theta)\left(2(\partial_\mu\sigma)^2 + \frac{\sigma^2}{2}(\partial_\mu\theta)^2\right) - M_{\rm SS}^2(1 - A\cos\tilde{\theta})\sigma^2 \ , \qquad (6.11)$$

with $f(\sigma,\theta) = 1 + \frac{1}{2}(V_4\mu_7 g_{\rm s})^{-1}V(\sigma,\theta)$. The above Lagrangian with the potential given in Equation (6.9) describes the inflationary dynamics studied in this chapter. One should not forget that the kinetic terms are not canonical and, in general, for multiple fields there does not exist a transformation that makes the metric flat everywhere on the moduli space. Therefore, to perform a complete analysis we use the general two-field Lagrangian with the field-space metric

$$G_{ab} = \begin{pmatrix} 4f(\sigma,\theta) & 0 \\ 0 & \sigma^2 f(\sigma,\theta) \end{pmatrix} \ , \qquad (6.12)$$

and use the generalized expressions of the cosmological observables for non-canonical kinetic terms and multiple fields introduced in Chapter 3.

6.2 Multi-field dynamics of Higgs-otic inflation

We have reviewed the relevant formalism for the computation of inflationary observables in two-field models in Chapter 3 and we have introduced the Higgs-otic model in the previous section. This model yields two-field inflation before moduli stabilization is taken into account. We keep the total amount of flux fixed at $\hat{G} = 1$ but, as mentioned in the previous section, analyze the model for three different values of A. For a string scale of the order of 10^{17} GeV this leads to a supersymmetry-breaking scale of $M_{\rm SS} \simeq 10^{13}$ GeV. However, we will also show the results for a large range of $\hat{G}$ values at the end of this chapter.

6.2.1 Higgs-otic regime: $A = 0.83$, $\hat{G} = 1$

In this section, we present the results for the inflationary observables corresponding to the canonical flux choice $A = 0.83$ and $\hat{G} = 1$. For this point in parameter space, at the supersymmetry-breaking scale $M_{\rm SS} \simeq 10^{13}$ GeV, there exist a heavy Higgs that can be integrated out and a light (approximately massless) Higgs, to be identified with the Standard Model Higgs field. As mentioned in Section 6.1, at the ultraviolet (string) scale the two fields have comparable masses (in fact for $A = 0.83$ one has $m_H/m_h = 3.28$), implying that inflationary dynamics driven by such a Higgs sector necessarily is multi-field in nature and that the observables are better estimated via the methods reviewed in Section 3.6.

We present in Figure 6.2 several trajectories in the (σ, θ) plane as well as the evolution of the $\eta_\perp$ parameter, defined in Equation (3.89), for those same trajectories. Recalling that $\eta_\perp$ is proportional to the inverse curvature radius of the background trajectory [49], we observe that the marked turns in the trajectories generate clear peaks in $\eta_\perp$. These peaks are sharper and higher for trajectories where the turning takes place close to the end of inflation. These trajectories have large initial values for θ, denoted by θ_0. The multi-field effects, arising through the $\eta_\perp$ controlled coupling between curvature and isocurvature modes, are more severe the earlier the turn takes place. So, even though the coupling is stronger for trajectory C than for trajectory

Trajectory	A	B	C	D	E	F	G	H
$P_\zeta/P_0\vert_{k_{60}}$	1.01	1.01	1.02	1.03	1.07	1.22	1.76	1.716
$\log_{10}\beta_{\rm iso}(k_{60})$	-3	-3	-5	-8	-13	-16	-18	-20
$P_\zeta/P_0\vert_{k_{50}}$	1.01	1.01	1.02	1.04	1.09	1.30	2.00	1.33
$\log_{10}\beta_{\rm iso}(k_{50})$	-3	-3	-5	-8	-13	-16	-18	-19

Table 6.1: Ratio between the amplitude of the curvature perturbations at the end of inflation and the single-field estimate of Equation (3.113) for the trajectories of Figure 6.2. $A = 0.83$ and $\hat{G} = 1$.

H, its effects are more pronounced in the latter since the isocurvature and curvature modes are coupled at earlier times, when there is more isocurvature power.

The effects of the multi-field dynamics on the amplitude of the scalar curvature perturbations are illustrated in Figure 6.3 where we observe that the power transfer from isocurvature to curvature is maximized the earlier the turn takes place. The impact of multi-field effects on the scalar amplitude for trajectories A-E is minimal, with the single-field estimate of Equation (3.113) providing a good approximation to the full result. For trajectories F-H the superhorizon evolution of the curvature perturbations driven by isocurvature power transfer implies that Equation (3.113) underestimates the amplitude by as much as 80%. These results are summarized in Table 6.1, where we also present the estimates for the primordial isocurvature fraction, $\beta_{\rm iso}$, at the end of inflation given in terms of the wave numbers at 50 and 60 e-folds, k_{50} and k_{60}. We observe that $\beta_{\rm iso}$ varies by many orders of magnitude, being larger for late turning trajectories (large θ_0) where power transfer between isocurvature and curvature perturbations is less efficient and the attenuation of isocurvature power is mostly driven by its decay on superhorizon scales.

In the same way as the multi-field effects can lead to an underestimation of the scalar amplitude, they can also impact other inflationary observables, in particular, the tightly constrained spectral index and the tensor-to-scalar ratio. The single- and multi-field estimates for these quantities are plotted as functions of the initial condition θ_0 in Figure 6.4. Starting with the tensor-to-scalar ratio, we observe that the effect of the multi-field dynamics is to flatten the peak and therefore to bring the results more in line with the PLANCK 2016 constraint of $r < 0.071$. This effect is partially due to the tensor modes being unaffected by these effects and in part due to the fact that the single-field estimate for the amplitude of the scalar perturbations is a bad approximation for trajectories that turn early. Therefore by underestimating the amplitude of the scalar fluctuations, the single-field formula overestimates the tensor-to-scalar ratio by

$$r \quad = \quad 16\,\epsilon_* \frac{P_0}{P_\zeta}(k_*, \tau_{end}) \ . \tag{6.13}$$

For the spectral index, we observe again that the multi-field estimate is considerably sharper than what one would expect by applying single-field techniques. We observe that the peak at low θ_0 is absent and that the trough is shallower. We recall that the single-field estimate for $n_{\rm s}$, Equation (3.113), is obtained by taking the decoupling limit $\eta_\perp \to 0$. This condition is clearly violated by the early-turning trajectories, for which $\eta_\perp$ peaks as the scalar modes leave the horizon. We therefore conclude that the peak in

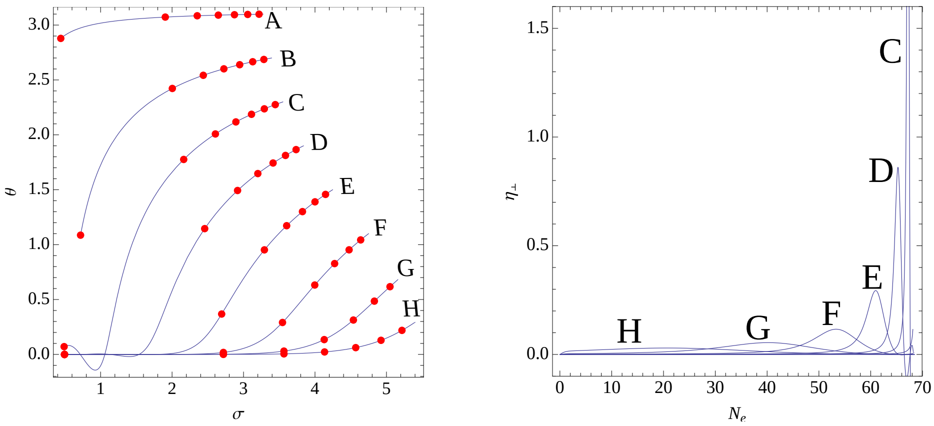

Figure 6.2: Left: The last 68 e-folds for some representative inflationary trajectories. Red dots mark 10 e-folds intervals on each trajectory. Right: Evolution of $\eta_\perp$ for the different trajectories. $A = 0.83$ and $\hat{G} = 1$.

the low θ_0 region is spurious.[1] To understand the change in n_s for larger θ_0 trajectories it is useful to rewrite Equation (3.94) as

$$\frac{d^2 v_\alpha^T}{d\tau^2} + \underbrace{(\Omega_{TT} - a^2H^2\eta_\perp^2 + k^2)v_\alpha^T}_{\text{elastic force}} = \underbrace{-2aH\eta_\perp \frac{dv_\alpha^N}{d\tau} - \frac{d(aH\eta_\perp)}{d\tau}v_\alpha^N - \Omega_{TN}v_\alpha^N}_{\text{external force}} \ .$$

We see that the equation of motion for the curvature perturbation is equivalent to a frictionless harmonic oscillator with a "time" dependent proper frequency subject to an external force whose magnitude is set by the isocurvature perturbation. The effects of a turn in the background trajectory, which gives rise to the external force, is more pronounced on k-modes for which the ratio between the external force and the elastic force is larger. This ratio is well approximated by the simpler relation between

$$R(k) \simeq \frac{v_\alpha^N(k)}{v_\alpha^T(k)} \ . \tag{6.14}$$

Since the amplitude of different k-modes around the pivot scale is affected differently by a turn in the background trajectory, there is superhorizon evolution of the spectral index for the curvature perturbations. From the solutions in the decoupling limit ($\eta_\perp = 0$) one finds that on superhorizon scales and before the turn

$$v_\alpha^T \propto k^{-\nu_T} = k^{n_s^0/2-2} \qquad \text{and} \qquad v_\alpha^N \propto k^{-\nu_N} \ , \tag{6.15}$$

where n_s^0 denotes the curvature spectral index before the turn in the trajectory and is assumed to be $n_s^0 < 1$. As for the isocurvature perturbations, one may expand

$$\nu_N \sim \frac{3}{2} - \frac{1}{3}(1+2\epsilon)\left(\frac{M}{H}\right)^2 + \epsilon < \frac{3}{2} \ , \tag{6.16}$$

[1] Comparing the results for the single-field estimates of Figure 6.4 and those of [70] we see that they differ in the low θ_0 range, where [70] has no peak. This difference can be traced back to how one generalizes $\eta = \frac{V''}{V}$ for multi-field cases. If one takes η to be the smallest eigenvalue of $\frac{G^{ij}V_{ij}}{V}$ then indeed there is no peak. However we use here a different, and more accurate, prescription which can be indeed derived from the decoupling limit, as we argued in Section 3.6.

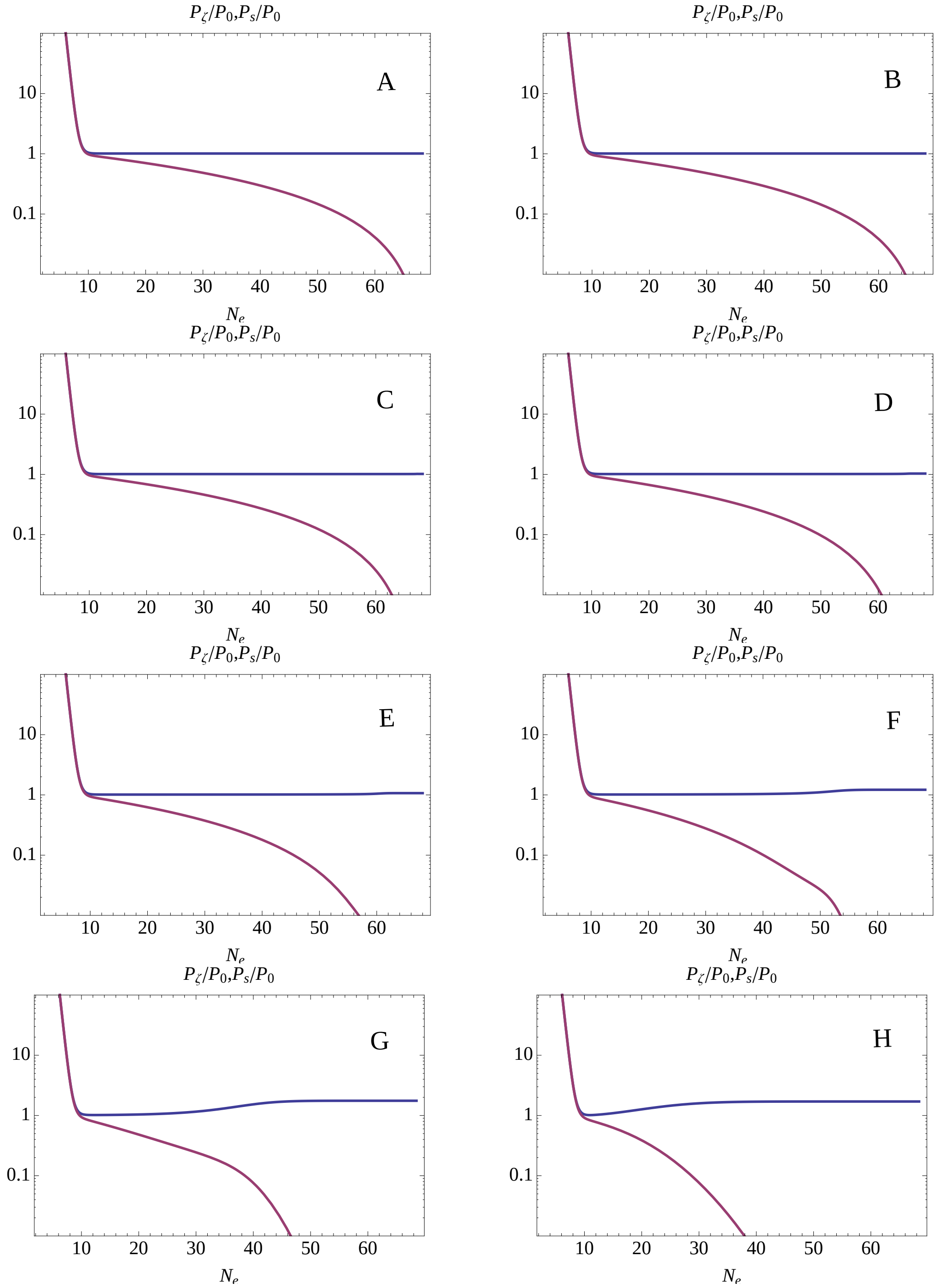

Figure 6.3: Evolution of the curvature (blue) and isocurvature (red) two point functions for k_{60} for the trajectories of Figure 6.2. Curvature and isocurvature power are normalized to the single-field estimate $P_0 = \frac{H^2}{8\pi^2\epsilon}\big|_*$.

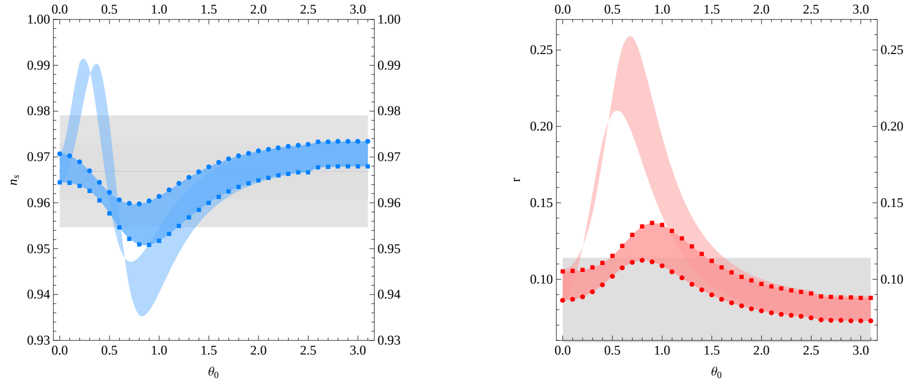

Figure 6.4: Spectral index (left) and tensor-to-scalar ratio (right) for $\hat{G} = 1$ and $A = 0.83$ superimposed with the PLANCK 2015 constraints $n_s = 0.96688 \pm 0.0061$ and $r < 0.114$ (grey band). The slightly transparent curve corresponds to the single-field estimate while the one surrounded by the data points corresponds to the multi-field results. Circles: $N_e = 60$, squares: $N_e = 50$.

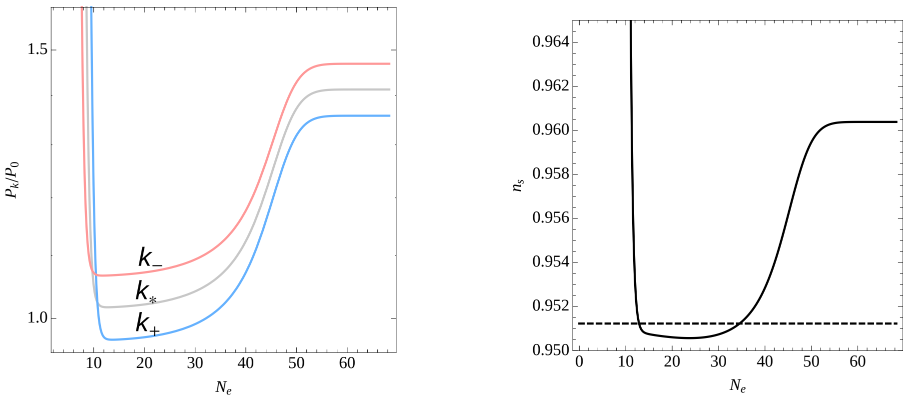

Figure 6.5: Time evolution of the curvature perturbations and of the spectral index on scales $k_* = k_{60}$ for the case $A = 0.83$, $\hat{G} = 1$ and $\theta_0 = 0.9$.

which leads to

$$R(k) \propto k^{1/2 - n_s^0/2} = k^{\alpha}, \qquad \alpha > 0 \; . \tag{6.17}$$

To understand how this changes the spectral index, consider a pair of k-modes around the pivot scale k_*: $k_- < k_* < k_+$. It follows that since

$$\frac{R(k_-)}{R(k_+)} \propto \left(\frac{k_-}{k_+}\right)^{\alpha} < 1 \; ,$$

the k_+ mode power is more enhanced than the k_- mode power, resulting in a spectral index closer to unity

$$n_s^{end} > n_s^0 \; . \tag{6.18}$$

This behaviour can be clearly observed in Figure 6.5, where the k_- power is less enhanced than the k_+, resulting in a more even distribution of power and in an increase in the spectral index. Note that, before the turn ($N_e < 50$), the single-field estimate is

Trajectory	A	B	C	D	E	F	G	H
$P_\zeta/P_0\|_{k_{60}}$	1.01	1.01	1.02	1.04	1.09	1.20	1.40	1.22
$\log_{10}\beta_{\rm iso}(k_{60})$	-4	-4	-5	-9	-10	-9	-12	-13
$P_\zeta/P_0\|_{k_{50}}$	1.01	1.01	1.02	1.05	1.11	1.26	1.43	1.16
$\log_{10}\beta_{\rm iso}(k_{50})$	-3	-3	-5	-9	-9	-11	-12	-13

Table 6.2: Order of magnitude of the isocurvature fraction at the end of inflation for the trajectories of Figure 6.6. $A = 0.7$ and $\hat{G} = 1$.

actually a good approximation to the full result and that it only fails due to the sharp turn in the background trajectory that causes conversion of isocurvature into curvature power.

6.2.2 Modified Higg-otic regime: $A = 0.7$, $\hat{G} = 1$

The motivation to consider $A = 0.83$ was based on the assumption that there is no new physics between the EW scale and inflation. However, as discussed in Section 6.1, this is not necessarily the case. In this section, we analyze the case $A = 0.7$, which corresponds to a mass ratio $m_H/m_h = 2.38$ at the string scale. In Figure 6.6 we present sample background trajectories and the corresponding evolution of the $\eta_\perp$ parameter. Comparing with the results of the previous section we see that the trajectories are straighter and that the $\eta_\perp$ peaks are less pronounced and located at later times. This implies that the differences between the exact results and the single-field estimates for the observables should be less pronounced than for the $A = 0.83$ point. That is indeed the case, as can be seen from comparing Figures 6.4 and 6.7. Though less pronounced, the disparity between single-field estimates and the full results is still important as the effect of the multi-field dynamics is to bring the observables more in line with the current constraints on $n_{\rm s}$ and r: the variation in the spectral index is damped and the tensor-to-scalar ratio is significantly reduced. In fact $n_{\rm s}$ is comfortably inside the 2σ band and the r is reduced to the point that it complies with the observational upper bound for all initial θ_0. This highlights the importance of properly estimating the observables at a time of ever increasing measurement precision. In Table 6.2 we present the isocurvature fraction at the end of inflation as well as the comparison between the single-field estimate for the curvature amplitude and the numerical result for the various trajectories.

6.2.3 Almost single-field regime: $A = 0.2$, $\hat{G} = 1$

The Higgs-otic model also features regimes in which the connection between inflation and MSSM Higgs physics is absent or hard to realize. In these cases, where the fluxes are such that A is substantially different from the canonical value of 0.83, inflation can still be driven by the D7-brane position modulus if the inflaton is associated to other degrees of freedom. Indeed, for small A the values of the Higgs masses m_h, m_H are too close to each other for the running from the string scale down to the supersymmetry-breaking scale to be sufficiently strong to yield an (approximately) massless SM doublet at $M_{\rm SS}$, but one could still identify the complex inflaton with other degrees of freedom in some extension of the MSSM. Note, in particular, that if supersymmetric particles are found at LHC, the canonical Higgs-otic scenario with $A = 0.83$ would be ruled out,

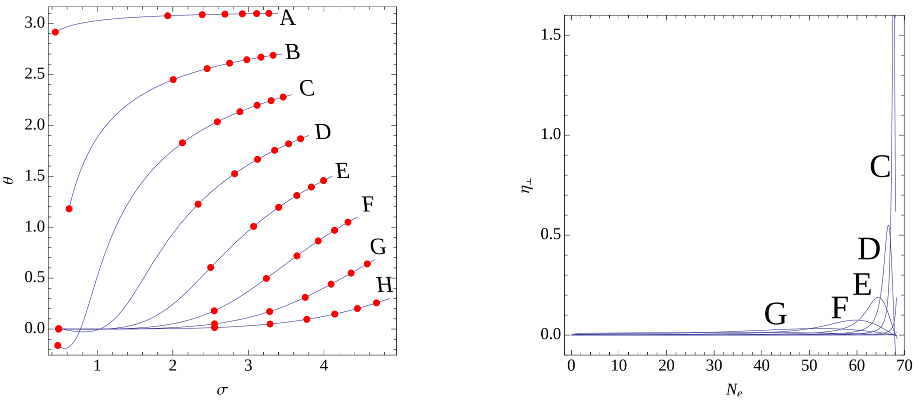

Figure 6.6: Left: The last 68 e-folds for some representative inflationary trajectories. Red dots mark 10 e-folds intervals on each trajectory. Right: Evolution of $\eta_\perp$ for the different trajectories. $A = 0.7$ and $\hat{G} = 1$.

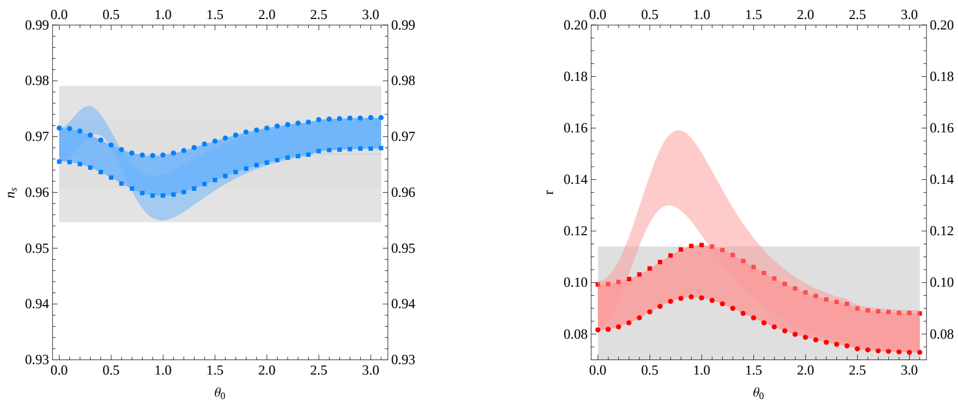

Figure 6.7: Spectral index (left) and tensor-to-scalar ratio (right) for $\hat{G} = 1$ and $A = 0.7$ superimposed with the PLANCK 2015 constraints $n_s = 0.96688 \pm 0.0061$ and $r < 0.114$ (grey band). The slightly transparent curve corresponds to the single-field estimate while the one surrounded by the data points corresponds to the multi-field results. Circles: $N_e = 60$, squares: $N_e = 50$.

since it assumes a large supersymmetry-breaking scale $M_{\rm SS} \simeq 10^{13}$ GeV. In this case, the inflaton could be identified with extra scalars which could have supersymmetric masses at the inflation scale $\simeq 10^{13}$ GeV.

While the search for specific MSSM extensions with this structure is quite interesting, the inflationary dynamics may be studied in a model-independent manner assuming that such new non-Higgs degrees of freedom correspond again to the position moduli of D7-branes. We study here for comparison the case with $A = 0.2$ (corresponding to $m_H/m_h = 1.22$). In Figure 6.8 we present the background evolution for sample trajectories in such a regime. In this case we see that the trajectories are essentially straight along the σ direction with only slight turning in the last 10 e-folds of expansion. This is in accordance with the fact that θ becomes massless in the limit of vanishing A. The straight trajectories imply that $\eta_\perp$, being inversely proportional to the curvature radius, vanishes everywhere except at the very end of inflation (where the mild turning takes place) as can be seen in Figure 6.8.

We expect the single-field estimates of Equations (3.113) and (3.115) for the infla-

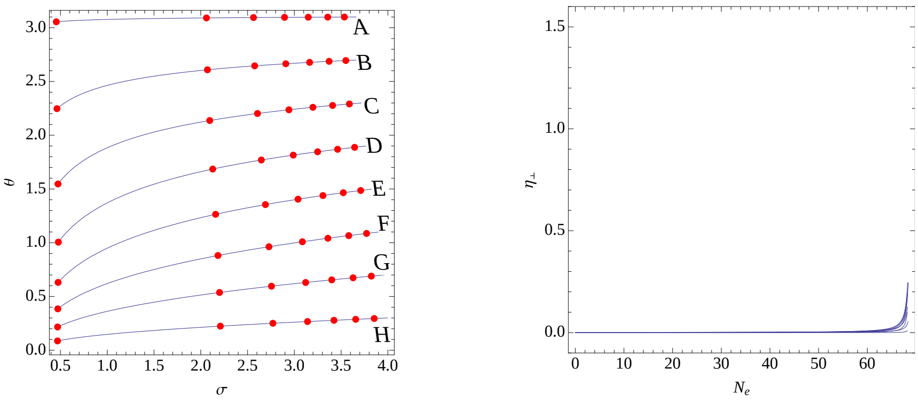

Figure 6.8: Left: The last 68 e-folds for some representative inflationary trajectories. Red dots mark 10 e-folds intervals on each trajectory. Right: Evolution of $\eta_\perp$ for the different trajectories. $A = 0.2$ and $\hat{G} = 1$.

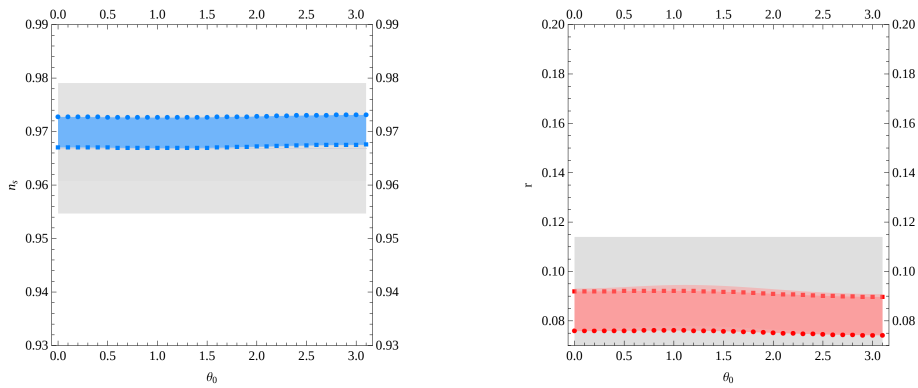

Figure 6.9: Spectral index (left) and tensor-to-scalar ratio (right) for $\hat{G} = 1$ and $A = 0.2$ superimposed with the PLANCK 2015 constraints $n_s = 0.96688 \pm 0.0061$ and $r < 0.114$ (grey band). The slightly transparent curve corresponds to the single-field estimate while the one surrounded by the data points corresponds to the multi-field results. Circles: $N_e = 60$, squares: $N_e = 50$.

tionary observables to provide a good approximation to the full result. In fact, if one employs the multi-field formalism in the computation of the observables and compares it with the single-field estimates, one finds that there is agreement at the level of a few percent. This is displayed in Figure 6.9 where the naive single-field bands track the exact results to the point of being almost indistinguishable.

Since isocurvature is practically decoupled from curvature throughout the observable inflationary range, the isocurvature fraction at the end of inflation as estimated by $\beta_{\rm iso}$ is larger that in the previous cases, as can be seen in Table 6.3. This is due to the fact that, unable to transfer power to the curvature mode, all the isocurvature modes can do in their superhorizon evolution is to decay slowly. Note that even though this almost single-field regime gives rise to the largest isocurvature fraction, it is still below the highest upper bound on $\beta_{\rm iso}$ derived from the latest PLANCK 2016 data [51].

Trajectory	A	B	C	D	E	F	G	H
$P_\zeta/P_0\|_{k_{60}}$	1.01	1.01	1.01	1.02	1.02	1.02	1.02	1.01
$\log_{10}\beta_{\rm iso}(k_{60})$	-4	-4	-5	-4	-5	-6	-6	-6
$P_\zeta/P_0\|_{k_{50}}$	1.01	1.01	1.02	1.02	1.03	1.03	1.02	1.02
$\log_{10}\beta_{\rm iso}(k_{50})$	-4	-4	-4	-4	-5	-5	-6	-6

Table 6.3: Ratio between the amplitude of the curvature perturbations at the end of inflation and the single-field estimate of Equation (3.113) and amplitude of isocurvature perturbations for the trajectories of Figure 6.8. $A = 0.2$ and $\hat{G} = 1$.

6.2.4 Varying $\hat{G}$

Finally let us discuss the effects of varying $\hat{G}$ over the results. As we commented in Section 6.1, $\hat{G}$ parametrises the total amount of flux quanta in Planck units, which determines in turn the ratio between the supersymmetry-breaking scale and the string scale. In particular,

$$\hat{G} = (V_4\mu_7)^{-1/2}G_3 \simeq \frac{n}{M_{\rm p}} \sim 5\frac{M_{\rm SS}}{M_{\rm s}^2} \tag{6.19}$$

with n being the flux quanta. We take the string scale of order $M_{\rm s} \sim 10^{16} - 10^{17}$ GeV, as suggested by gauge unification. An isotropic compactification with $n \sim \mathcal{O}(1)$ then implies $M_{\rm SS} \sim 10^{13}$ GeV. But it should be noticed that the parameter n in general receives contributions from a large number of 3-cycles so that large cancellations can take place that lead to $n \ll 1$, lowering the scale of supersymmetry breaking. However, $n \ll 1$ is problematic if one wants to satisfy the experimental bounds on density scalar perturbations. A lower supersymmetry-breaking scale may lead to a too low amplitude of the scalar power spectra in this inflationary model. The best fit indeed corresponds to $M_{\rm SS} \sim 10^{13}$, which is the typical scale for flux-induced supersymmetry breaking obtained by assuming the flux quanta to be of order one. On the other hand, $n \gg 1$ (and thereby a higher supersymmetry-breaking scale) makes the potential energy trans-Planckian at the beginning of inflation, which is inconsistent with the effective field theory approach. Throughout this chapter we have shown the results for the representative value $\hat{G} = 1$ in Planck units. Here we also show the results for other possible values of $\hat{G}$ for completeness. In Figure 6.10 we plot our results in the $n_{\rm s} - r$ plane superimposed over the experimental Planck exclusion limits. The data correspond to $A = 0.83$ and arbitrary $\hat{G}$. Notice that $\hat{G}$ not only enters in the absolute value of the potential, but also in the field redefinition to get canonical kinetic terms. The bigger $\hat{G}$ is, the stronger is the flattening of the potential and the results are closer to those of linear inflation. This is the reason why our results actually interpolate between quadratic (small $\hat{G}$, negligible flattening) and linear (big $\hat{G}$, strong flattening). However, as we already said, those closer to linear inflation are in better agreement with both density scalar perturbations and tensor-to-scalar ratio constraints from Planck. In Figure 6.10 the color pattern from red to blue refers to the density of points, being the red regions the most populated. This could have been anticipated from Figure 6.4, where most of the initial conditions gave rise to values of r and $n_{\rm s}$ closer to the single-field prediction. The spreading of the results to smaller values of the spectral index is due to the freedom on the choice of the initial conditions, but as we said the

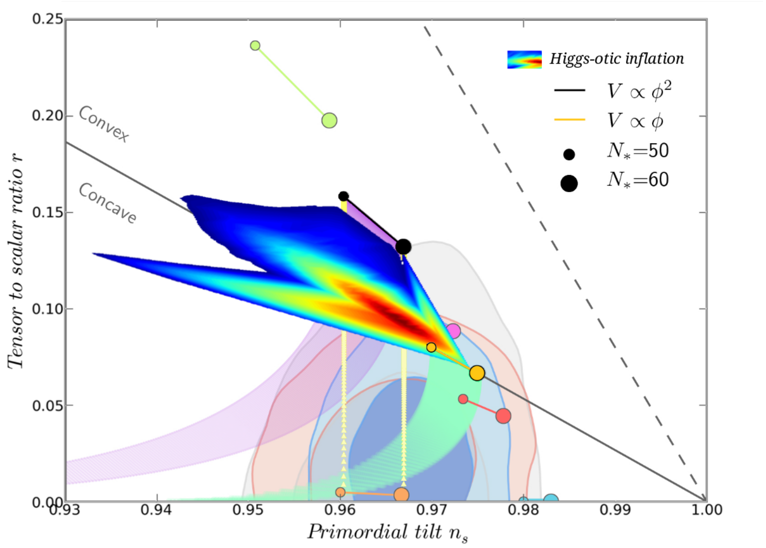

Figure 6.10: Tensor to scalar ratio and spectral index for Higgs-otic inflation with $A = 0.83$ and arbitrary $\hat{G}$. The data is superimposed over the recent Planck exclusion limits [51]. The color pattern (from red to blue) corresponds to (higher or lower) density of initial condition points . There is a lower cutoff on the density required to be plotted (or equivalently in the level of fine-tuning allowed) missing around 10 % of the points.

blue region corresponds to very fine-tuned values of θ_0 and the majority of the points is localised at $n_s \simeq 0.965$ and $r \simeq 0.08 - 0.12$.

Chapter 7

Moduli stabilization in Higgs-otic inflation

In this chapter we extend the study of large-field D7-brane inflation models, with particular emphasis on Higgs-otic inflation. We have studied this model from a local perspective based on the DBI and CS actions. We have seen that ISD three-form fluxes in the background yield a monodromy and a mass term for the position modulus, which give a quadratic inflaton potential. However, this potential is flattened at large-field values due to non-trivial kinetic terms arising from the DBI action, leading to a linear potential in the large-field regime. A realistic global model requires more ingredients, in particular, it requires a global compactification of the D7-brane system with stabilized moduli. The aim of the present chapter is to discuss the effects of moduli stabilization on the inflationary dynamics of Higgs-otic inflation.

Moduli stabilization in Type IIB is best understood in terms of its low-energy $\mathcal{N}=1$ supergravity theory, cf. [24,29]. Thus, we need an $\mathcal{N}=1$ description of the microscopic setup leading to Higgs-otic inflation, including sources which stabilize all moduli. We discussed corrections to the supergravity theory in Chapter 5 aimed a capturing the effect of the non-trivial kinetic term of the D7-brane from the DBI action. We showed that, with only ISD three-form fluxes, the leading-order α' correction to the action can be captured by higher-derivative corrections to the Kähler potential. This correction was obtained before taking moduli stabilization into account. We will see that stabilizing the Kähler moduli requires us to turn on additional (IASD) fluxes, making the identification of the correct higher-derivative operator more difficult. IASD three-form fluxes are necessary to describe non-perturbative contributions in the superpotential, which in turn are needed to stabilize Kähler moduli. However, even after the inclusion of these fluxes the correct kinetic term can be found by expressing the DBI action in terms of the correct supergravity variables. This can be done by a matching between the scalar potential obtained from the expanded DBI action, including ISD and IASD fluxes, and the supergravity Lagrangian after stabilizing the Kähler modulus.

In order to study moduli stabilization and its backreaction on the inflaton, we consider a simplified KKLT-like setup in which the dilaton and complex structure moduli are stabilized supersymmetrically by fluxes and already integrated out. A single overall Kähler modulus is then stabilized by non-perturbative effects and an appropriate de Sitter uplift is added [29]. It turns out that there are important backreaction effects which substantially modify the structure of the inflaton scalar potential derived in Chapter 6. Similar to our analysis here, the backreaction of stabilized moduli was studied in [149] for supersymmetric stabilization, and in [33,150] for the KKLT mechanism and other models in which the Kähler moduli break supersymmetry. Eventually, the modulus backreaction leads to an additional flattening of the effective potential and a limit to the field excursion of the inflaton. At the same time, the background fluxes must be chosen such that the mass of the inflaton is much smaller than that of the modulus. If this can be achieved, 60 or more e-folds of slow-roll inflation are possible. The tensor-to-scalar ratio for Higgs-otic inflation then lies in the range $r \simeq 0.04 - 0.08$.

In addition to the backreaction induced by the Kähler moduli, there is a backreaction coming from stabilizing the complex structure moduli and dilaton. Such a backreaction is known to modify the kinetic term of the inflaton, modefying the canonical field range [151]. It was shown in [152] that this problem is particularly severe in models where the inflaton is a closed-string axion. We will show that this backreaction is less severe when the inflaton is an open-string scalar. The backreaction does occur, but due to the extra freedom introduced by the open-string sector, the reduction of the canonical field range can by controlled.

The structure of this chapter is as follows. In Section 7.1 we discuss the supergravity embedding of the Higgs-otic model discussed in Section 6.1. In Section 7.2 we combine the open-string sector of the Higgs-otic model with a single Kähler modulus in a KKLT-like setting. We study the associated backreaction on the inflaton potential following the analysis in [33], and show that, for an appropriate choice of flux parameters, consistent slow-roll inflation is achieved with a stable Kähler modulus. Moreover, we translate the non-trivial kinetic terms to the supergravity language and give numerical examples with the predicted CMB observables of the canonical inflaton variable. In Section 7.3 we investigate the additional backreaction induced by the complex structure moduli and the dilaton. Using flux stabilization as in [24], we show that the inflaton field range is almost unaffected when an appropriate mass hierarchy between the inflaton and moduli is achieved via a flux choice. We discuss the flux choices needed for Kähler and complex structure moduli stabilization and how they are related. Appendix C accompanies this chapter, in it we illustrate that moduli-stabilizing fluxes may also yield μ-terms for open-string moduli in a simple toroidal orientifold setting.

7.1 Supergravity embedding of Higgs-otic inflation

Before moving to moduli stabilization of Higgs-otic inflation we have to discuss its supergravity embedding. The Kähler potential and superpotential were already given in Equations (5.35) and (5.36), when the model is compactified on a isotropic torus. We repeat them here for completeness

$$K = -\log\left[(S+\bar{S})(U+\bar{U}) - \frac{1}{2}(\Phi+\bar{\Phi})^2\right] - 3\log\left[T+\bar{T}\right] , \tag{7.1a}$$

$$W = \mu\Phi^2 + W_0 + Ae^{-\alpha T} , \tag{7.1b}$$

where we have added a non-perturbative piece to the superpotential to allow for Kähler moduli stabilization, see Section 3.4. S and U denote the axio-dilaton and a complex structure modulus, respectively. Furthermore, we assume that A is a constant that does not depend on the inflaton. In a realistic compactification it is possible for A to depend on the inflaton. However, the computation of the functional dependence goes beyond the scope of this work. The effect on the inflationary dynamics was recently discussed in [153].

We postpone further discussion of α' corrections to the effective action until Section 7.2.3. This Kähler potential and superpotential arise, for example, when the cycle wrapped by the brane is a torus, two of the three complex structure moduli are assumed to be stabilized by fluxes, and the three Kähler moduli of the tori are identified. Furthermore, we assume S and U to be stabilized supersymmetrically at a high scale. Therefore, we can write the product of the vacuum expectation values of the complex structure modulus and the dilaton as s and we work with the effective Kähler potential

$$K = -\log\left[s - \frac{1}{2}(\Phi + \bar{\Phi})^2\right] - 3\log\left[T + \bar{T}\right] . \tag{7.2}$$

We come back to the stabilization of the complex structure modulus in Section 7.3. Before we turn to the more complicated non-Abelian setup, let us consider the simple case of a single brane. In this case the position modulus does not split into the bifundamental Higgses and there is a single complex scalar field, ϕ, parameterizing the motion of the brane away from the orientifold singularity. Using the Kähler potential and superpotential given above we can use the standard supergravity formula to compute the scalar potential

$$V(t,\varphi) = \frac{1}{8st^3}\left[\frac{4}{3}\alpha t A e^{-2\alpha t}\left(3A + \alpha t A + 3W_0 e^{\alpha t}\right) + 2\left(-\alpha t A e^{-\alpha t} + s\mu\right) s\mu\varphi^2\right] , \tag{7.3}$$

where $\varphi = \mathrm{IM}(\phi)$ is the canonically normalized inflaton field and t denotes the real part of T. We assume that both μ and W_0 are real valued. It was shown in e.g. [33] that for complex parameters it is always possible to use field redefinitions and Kähler transformations to go to a frame where the results are qualitatively the same as in the case considered here.

Next, we consider the case when the position modulus splits in a set of bifundamentals. The off-diagonal fluctuations of the transverse field ϕ correspond then to the MSSM Higgs doublets H_u and H_d and can yield inflation, as in Equation (6.3). The Kähler potential and superpotential now depend in the following way on the fields [70]

$$K = -\log\left[s - \frac{1}{2}(H_\mathrm{u} + \bar{H}_\mathrm{d})(\bar{H}_\mathrm{u} + H_\mathrm{d})\right] - 3\log\left[T + \bar{T}\right] , \tag{7.4a}$$

$$W = \mu H_\mathrm{u} H_\mathrm{d} + W_0 + A e^{-\alpha T} . \tag{7.4b}$$

We do not want to discuss the backreaction of T just yet but rather focus on the connection between this formalism and the one obtained from the DBI action, so we neglect the T-dependent piece in W and we assume that $D_T W \neq 0$ which leads to a perfect no-scale cancellation of the term proportional to $-3|W|^2$. This yields the following positive definite scalar potential,

$$V = \frac{1}{8st_0^3}\left[(W_0^2 + 2s\mu W_0 + 2s^2\mu^2)(|H_\mathrm{u}|^2 + |H_\mathrm{d}|^2)\right.$$
$$\left. + W_0(W_0 + 2s\mu)(H_\mathrm{u} H_\mathrm{d} + \bar{H}_\mathrm{u}\bar{H}_\mathrm{d}) + \ldots\right] , \tag{7.5}$$

where T is supposed to be stabilized at t_0 with a large mass. The ellipsis denotes higher-order terms which are positive definite and unimportant. We neglect the backreaction of the supersymmetric stabilization of S and U. We can use the diagonal basis of Equation (6.5) to write the scalar potential as

$$V = \frac{1}{4st_0^3}\left[s^2\mu^2|h|^2 + (W_0 + s\mu)^2|H|^2 + \dots\right] , \tag{7.6}$$

again neglecting unimportant higher-order interaction terms. Comparing this to Equation (6.4) motivates us to identify the fluxes and the supergravity parameters in the following way

$$G = \frac{ZW_0}{\sqrt{4g_s st_0^3}}, \quad S = -\frac{Z(W_0 + 2s\mu)}{\sqrt{4g_s st_0^3}}, \tag{7.7}$$

In terms of the D-flat directions σ and θ the scalar potential can be written as

$$\begin{aligned} V &= \frac{1}{4st_0^3}\left[s^2\mu^2 + (W_0 + s\mu)^2 + W_0(W_0 + 2s\mu)\cos\theta\right]\sigma^2 \\ &= \frac{s^2\mu^2 + (W_0 + s\mu)^2}{4st_0^3}\left(1 + A\cos\theta\right)\sigma^2 , \end{aligned} \tag{7.8}$$

where A was defined in Equation (6.6) in terms of the fluxes but is given here equivalently as $A = W_0(W_0 + 2s\mu)/(s^2\mu^2 + (W_0 + s\mu)^2)$.

7.2 Kähler moduli stabilization and backreaction

In this section we extend the supergravity setup of Higgs-otic inflation by an explicit treatment of the backreaction of the Kähler modulus on inflationary physics. We first focus on the backreaction of a single volume mode T, stabilized via the setup of KKLT for a single brane as in the previous section. This shows many of the important features present in the slightly more involved Higgs-otic model. Next, we compute the backreaction of the stabilized Kähler modulus on the inflationary scalar potential of Higgs-otic inflation, following the analysis in [33]. We combine these results with those of Section 5.1 to account for the non-trivial kinetic term obtained from the DBI action.

Since our goal is to stabilize all relevant closed-string moduli in Higgs-otic inflation, we must consider extensions of the original setup of Section 6.1, in which only ISD fluxes were included. From the perspective of the supergravity theory we have to consider non-perturbative terms to the superpotential. In Type IIB, in the absence of non-geometric fluxes, these non-perturbative terms are required to stabilize Kähler moduli as discussed in Chapter 3. From the perspective of the world-volume DBI and CS actions, we have to take modifications into account to account for the non-perturbative superpotential. The superpotential term is sourced by a gaugino condensate which backreacts through the ten-dimensional supergravity equations on the local closed-string background, inducing IASD flux on the bulk [154]. Upon adding this flux in the computation of the effective theory arising from the DBI and CS actions, we find that both the kinetic term and the scalar potential are indeed modified.[1] The result is

$$\mathcal{L}_{\text{kin}} = -\partial_\mu\phi\partial^\mu\bar\phi\left\{1 + \frac{1}{4ZV_4\mu_7}\left[(|G|^2 + |S|^2 + |D|^2)|\phi|^2 - \bar G(\bar S - D)\phi^2 + \text{c.c.}\right]\right\} \tag{7.9a}$$

$$V = \frac{g_s}{4Z}\left(2|G^*\phi - S\bar\phi|^2 + \bar G D\phi^2 + G\bar D\bar\phi^2\right) . \tag{7.9b}$$

[1] It is possible to consider even more general flux, see [155] for the effects in a similar model.

As in Section 6.1 the potential is quadratic in ϕ and there is a non-trivial piece in the kinetic term which leads to a flattening of the effective inflaton potential. This piece is indeed proportional to the scalar potential contribution from the DBI action. When the IASD flux is equal to zero, which is when D vanishes, the scalar potential from the DBI action is equal to the CS contribution, so the correction to the kinetic term can be written as proportional to the full scalar potential itself. However, in the presence of the IASD flux, D, the contributions to the action from the DBI action and CS action are different. Since the correction to the kinetic term is only sensitive to the DBI action, the structure we studied in Chapter 5 is broken. This forces us to obtain the $\mathcal{N} = 1$ D7-brane action not from a corrected Kähler potential but rather from a matching between the above scalar potential and the scalar potential obtained from the corresponding low-energy $\mathcal{N} = 1$ theory after Kähler moduli stabilization. We discuss this in Section 7.2.3.

7.2.1 Backreaction and effective potential

Using the Kähler potential and superpotential given in Equations (7.1) and (7.2) we computed the scalar potential

$$V(t,\varphi) = \frac{1}{8st^3}\left[\Delta^2 + \frac{4}{3}\alpha t A e^{-2\alpha t}\left(3A + \alpha t A + 3W_0 e^{\alpha t}\right) + 2\left(-\alpha t A e^{-\alpha t} + s\mu\right) s\mu\varphi^2\right], \tag{7.10}$$

where we have added an uplift, Δ. Assuming real superpotential parameters, both the axion of T and $\mathrm{Re}(\phi)$ are stabilized at the origin with a large mass, so we can safely neglect them in the following, see [33] for details. In fact, we have already set $\mathrm{Re}(\phi) = 0$ in the above scalar potential. During inflation the Kähler modulus, T, and the position modulus, ϕ, are coupled in the Lagrangian. More specifically, for real superpotential parameters only the volume, $\mathrm{Re}(T)$, and the inflaton, $\mathrm{Im}(\phi)$, interact. The interaction terms between t and φ imply that, even if t is much heavier than φ, during inflation the minimum of the modulus potential is inflaton dependent. We assume that t traces its minimum adiabatically. This assumption is justified as long as a large mass hierarchy is present. Integrating out t at its φ-dependent value then leads to additional terms in the effective potential for φ. This is what we refer to as a backreaction of the modulus field.

We use Equations (3.55) to eliminate A and Δ and expand the potential in terms of $t = t_0 + \delta t(\varphi)$, where t_0 denotes the modulus vev after inflation. We are thus treating inflation as a perturbation of moduli stabilization which is allowed as long as there is a reasonable mass hierarchy between the inflaton and the volume. Unfortunately, we cannot minimize V and solve for $\delta t(\varphi)$ analytically to all orders. Instead we expand the potential to second order in the volume perturbation $\delta t(\varphi)$ and minimize afterwards. This allows us to solve for $\delta t(\varphi)$ and we find

$$\frac{\delta t(\varphi)}{t_0} = \frac{s\mu\varphi^2}{2\alpha t_0 W_0} + \mathcal{O}(H^2/m_t^2)\,, \tag{7.11}$$

where m_t was defined in Equation (3.58). As stressed in [33] the mass of the volume must be bigger than H throughout the inflationary period to guarantee stability of the Kähler modulus. Therefore, it is instructive, and indeed allowed, to expand all relevant quantities in powers of μ/W_0, as we have done above. For consistency of the expansion around t_0 we must demand that (7.11) is small compared to one. This leads to an important constraint on the parameters of the superpotential,

$$\frac{W_0}{\mu} > \frac{s\varphi^2}{2\alpha t_0}\,. \tag{7.12}$$

Solving the equations of motion explicitly shows that the modulus is lifted over the KKLT barrier at the point $\delta t(\varphi) = t_0$ and the theory decompactifies. Hence, the theory is only well-behaved as long as $\delta t(\varphi) \ll t_0$.

Now, while in the effective regime determined by the condition given in Equation (7.12), we can integrate out the modulus by inserting (7.11) into $V(t,\varphi)$. This yields the effective potential for the inflaton which reads, to leading order in αt_0 and H/m_t,

$$V_{\text{eff}}(\varphi) = \frac{1}{4t_0^3}\left(s\mu^2\varphi^2 + \frac{3}{2}\mu W_0\varphi^2 - \frac{3}{8}s\mu^2\varphi^4\right) + \ldots\,. \tag{7.13}$$

The first term is the supersymmetric mass term for φ which was present before moduli stabilization, and which is the term driving inflation in the model of Section 6.1. The third comes from the negative definite part of the scalar potential, $-3|W|^2$, which is surprising since this term is normally cancelled by the no-scale structure of the Kähler modulus in the Kähler potential. The backreaction in combination with the interaction term between t and φ interfere with the no-scale cancellation during inflation in the effective theory. Hence, it is dangerous to neglect moduli stabilization in string theory models of large-field inflation. In fact, the theory would be lost if this was the end of the story, since for field values $\varphi > 1$ the theory would destabilize. What potentially saves the theory is the second term in (7.13). In the regime required by (7.12), i.e. $W_0 \gg \mu$, it is bigger than the first term and, for φ below a certain value, bigger than the third, so it can drive inflation. Notice that the allowed field range is effectively determined by the ratio W_0/μ. Since W_0 is required to be bigger than μ we can neglect the supersymmetric mass term and write the relevant potential as follows,

$$V_{\text{eff}}(\varphi) = \frac{3}{8t_0^3}\mu W_0\varphi^2\left(1 - \frac{s\mu}{4W_0}\varphi^2\right) + \ldots\,. \tag{7.14}$$

In a sense, this is a quadratic potential with a correction term scaling as $H/m_{3/2}$ or H/m_t, as naively expected. The effective potential V_{eff} has a maximum at $\varphi_{\text{c}}^2 = 2W_0/s\mu$ and, because we must require that $\varphi_\star < \varphi_{\text{c}}$ for inflation to be successful, this leads to a parameter constraint

$$\frac{W_0}{\mu} > \frac{s\varphi_\star^2}{2}\,, \tag{7.15}$$

which is slightly more restrictive than the one in (7.12). This constraint forces a flux tuning to which we will come back at the end of Section 7.3.

Finally, it is not obvious that the second and third terms in (7.13) only arise after minimizing with respect to T. They should vanish in case the non-perturbative term in (7.1b) is absent. That this indeed happens can be seen very clearly in case we use the equations of motion for T to eliminate the parameter W_0 instead of A in (3.55). We then obtain

$$V_{\text{eff}}(\varphi) = \frac{1}{4t_0^3}\left(s\mu^2\varphi^2 - \alpha t_0 A e^{-\alpha t_0}\mu\varphi^2 + \ldots\right) + \ldots\,. \tag{7.16}$$

Clearly, the new dominant mass term vanishes if $A = 0$, in which case $V(t)$ has no minimum.

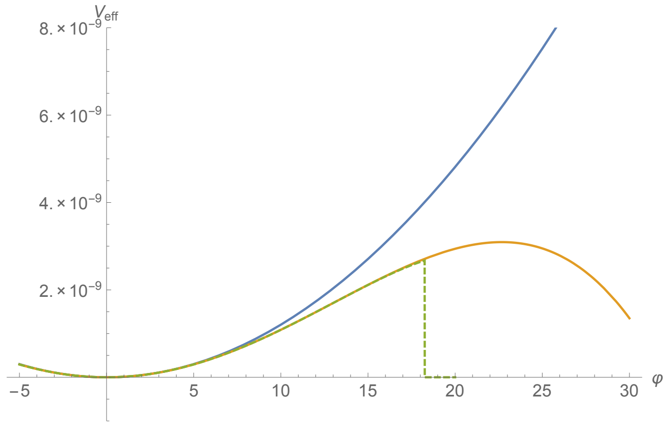

Figure 7.1: Effective inflaton potential obtained analytically via the second-order expansion in $\delta t(\varphi)$ (orange line) and numerically to all orders (green dashed line), in comparison with the naive quadratic potential (blue line). The flattening effect of integrating out T is evident. The orange curve is obtained from the result (7.14) with all higher-order terms in $(\alpha t_0)^{-1}$ taken into account.

A parameter example

Let us consider a specific set of flux parameters to illustrate our findings so far. We need to choose our parameters in such a way that they satisfy both (7.15) and yield the correct normalization of the scalar perturbations on the would-be inflationary trajectory. In Figure 7.1 we have displayed the effective inflaton potential for the following parameter choice,

$$W_0 = 0.005\,, \quad \mu = W_0/400\,, \quad s = 1\,, \quad t_0 = 10\,, \quad \alpha = 2\pi/5\,, \tag{7.17}$$

in comparison with a purely quadratic potential as obtained in Chapter 6 (blue line). The leading-order effective potential (orange line) has a local maximum at the critical value, $\varphi_{\rm c} \approx 23$. It is no surprise that the position of the maximum is very close to the point where T is destabilized and $\delta t(\varphi) \approx t_0$. At this point the effective theory we obtained from integrating out T breaks down. Beyond $\varphi_{\rm c}$ the modulus can no longer be integrated out and we obtain a theory which decompactifies. This is clear after considering the green dashed line in Figure 7.1 which is the effective inflaton potential after integrating out T numerically to all orders. Notice the good agreement with the analytic result obtained from the second-order expansion of the potential. The point where the curve drops is the point where the minimum in the modulus direction disappears and the theory decompactifies. However, to the left of the maximum value, 60 e-folds of inflation may take place. We return to the details of the inflationary phase in Section 7.2.3. Last but not least, let us consider the full scalar potential in the t-φ plane. This is shown in Figure 7.2 for the same parameter values as above. It clearly shows the flat valley along the minimum of T in which slow-roll inflation can take place. However, it also highlights the amount of fine-tuning of initial conditions that is necessary to allow for inflation without destabilization of T. Of course, the necessary amount of fine-tuning can be reduced by increasing the tuning between W_0 and μ, which pushes $\varphi_{\rm c}$ to larger values.

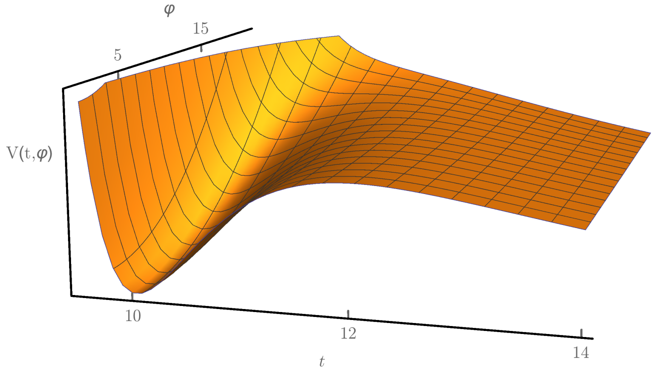

Figure 7.2: Scalar potential in the $t - \varphi$ plane. Evidently, the initial conditions must be very fine-tuned to allow for 60 e-folds of slow-roll inflation without destabilizing t.

7.2.2 Kähler moduli stabilization in Higgs-otic inflation

We can now treat moduli stabilization along the same lines as above in Higgs-otic inflation defined by the Kähler and superpotential given in (6.4). We consider the full theory with H_u, H_d, and T being dynamical. In the vacuum, after inflation, T is stabilized by the KKLT mechanism at t_0. During inflation, as in the single-field example above, T couples to both Higgs fields and we must expand $T = t_0 + \delta t(H_\mathrm{u}, H_\mathrm{d})$ and minimize with respect to δt to integrate out the modulus consistently. This, again, leads to new terms for both Higgs fields at the level of the scalar potential. After a bit of work we find for the effective potential in terms of H_u and H_d

$$\begin{aligned} V = \frac{1}{8st_0^3}\Big[&(W_0^2 + 2s\mu W_0 + 2s^2\mu^2)(|H_\mathrm{u}|^2 + |H_\mathrm{d}|^2) + W_0(W_0 - s\mu)(H_\mathrm{u}H_\mathrm{d} + \bar{H}_\mathrm{u}\bar{H}_\mathrm{d}) \\ &- \mu(W_0 + s\mu)(|H_\mathrm{u}|^4 + |H_\mathrm{d}|^4) - \frac{3}{2}s\mu W_0(H_\mathrm{u}^2H_\mathrm{d}^2 + \bar{H}_\mathrm{u}^2\bar{H}_\mathrm{d}^2) \\ &- \frac{1}{2}s\mu(5W_0 + 2s\mu)(|H_\mathrm{u}|^2H_\mathrm{u}H_\mathrm{d} + |H_\mathrm{u}|^2\bar{H}_\mathrm{u}\bar{H}_\mathrm{d} + |H_\mathrm{d}|^2H_\mathrm{u}H_\mathrm{d} + |H_\mathrm{d}|^2\bar{H}_\mathrm{u}\bar{H}_\mathrm{d}) \\ &- 5s\mu(W_0 + s\mu)|H_\mathrm{u}H_\mathrm{d}|^2\Big] + \dots , \end{aligned} \tag{7.18}$$

once more to leading order in αt_0 and H/m_t. This is the two field analog of (7.13). Notice that most of the quartic terms are now negative and thus are potentially relevant. We can compare the first line to the result before moduli stabilization given in (7.5) to see what happened. Due to the re-appearance of a part of $-3|W|^2$ in the effective theory, there is an additional mixed mass term $-3s\mu W_0(H_\mathrm{u}H_\mathrm{d}+\bar{H}_\mathrm{u}\bar{H}_\mathrm{d})$ while the other mass terms remain unchanged. The negative quartic terms can similarly traced back to the re-appearance of $-3|W|^2$.

The above expression becomes much simpler when written in the diagonal mass basis, given in Equation (6.5). After integrating out T the basis is the same but the mass eigenvalues are different. We find, instead of (7.6),

$$\begin{aligned} V = \frac{1}{8st_0^3}\Big[&s\mu(3W_0 + 2s\mu)h^2 + (2W_0^2 + s\mu W_0 + 2s^2\mu^2)H^2 \\ &- \frac{3}{4}s^2\mu^2h^4 - \frac{1}{4}s\mu(20W_0 + 11s\mu)H^4 + \frac{1}{2}s\mu(2W_0 - s\mu)h^2H^2\Big] + \dots . \end{aligned} \tag{7.19}$$

What we have obtained in (7.19) is, in a way, two copies of (7.13) for which each mass eigenstate has soft mass terms and a dominant quartic term suppressed by one power of μ/W_0. This is exactly the same as in the single-field model, and a very intuitive result. It implies that, also in the two-field case, we must require $W_0 \gg \mu$ to guarantee moduli stabilization. Remember that the mass of the modulus is still $m_t \sim W_0$. In the limit $W_0 \gg \mu$, implying $G \approx S$ to high accuracy, H is stabilized at the origin with a mass $m_H \sim W_0$, at roughly the same scale as the modulus. The inflaton is h and the term proportional to μW_0 drives inflation, just as in Section 7.2.1, the single-field example. Moreover, this implies that the scale of inflation is suppressed compared to the mass scale of both T and H by one power of μ/W_0, making this a consistent limit. The precise constraint on the superpotential parameters is

$$\frac{W_0}{\mu} > \frac{sh_\star^2}{2}\,, \tag{7.20}$$

in one-to-one correspondence with the constraint (7.15) in the single-field model. If the bound is satisfied, we obtain single-field inflation to very high accuracy. If not, multi-field inflation is nearly impossible with T dynamical and potentially running away to infinity. Note that again a slightly weaker bound arises from requiring that $\delta t(H_\mathrm{u}, H_\mathrm{d}) < t_0$, which would be the two-field analog of (7.12). Setting $H = 0$ indeed yields the same effective scalar potential for h as in the single-field example,

$$V_\mathrm{eff}(h) = \frac{1}{4t_0^3}\left(s\mu^2h^2 + \frac{3}{2}\mu W_0 h^2 - \frac{3}{8}s\mu^2 h^4\right) + \dots\,. \tag{7.21}$$

We defined the ratio of fluxes A in Equation (6.6) and, equivalently, in terms of the superpotential parameters after Equation (7.8). In Chapter 6 we considered this to be an essentially free parameter but we can now see that there are constraints on its value imposed by the consistency of the theory after moduli stabilization is taken into account. In particular, due to the backreaction of T the value of A has slightly changed,

$$A = \frac{W_0(W_0 - s\mu)}{W_0^2 + 2s\mu W_0 + 2s^2\mu^2}\,. \tag{7.22}$$

This implies that $A = 0$ is no longer a consistent option. Both $W_0 = 0$ and $W_0 = s\mu$ violate (7.20) when $h > 1$ in large-field inflation. On the other hand, while (7.20) implies that A is very close to one, it is never exactly one. With moduli stabilization taken into account, we must always be in a regime where h drives inflation. If h were to become massless and H were the inflaton, the inflaton and T would have the same mass and T would be immediately destabilized during inflation, as soon as $H \gtrsim 1$. Instead, we may choose μ very small while $\sqrt{\mu W_0}$ is the physical mass of the inflaton which is constrained by COBE normalization of the primordial scalar perturbations. This means that, with this mechanism of moduli stabilization the parameter regime leading to interesting multi-field dynamics, as considered in Chapter 6, is excluded. In light of this, the analysis of the previous chapter does not directly apply to the setup we consider here: Higgs-otic inflation with moduli stabilization using the KKLT mechanism. However, it is not completely ruled out that adding more ingredients or using other stabilization mechanisms creates a mass hierarchy between the heavy Higgs and the Kähler modulus thus opening the region of parameter space that allows for two-field inflation.

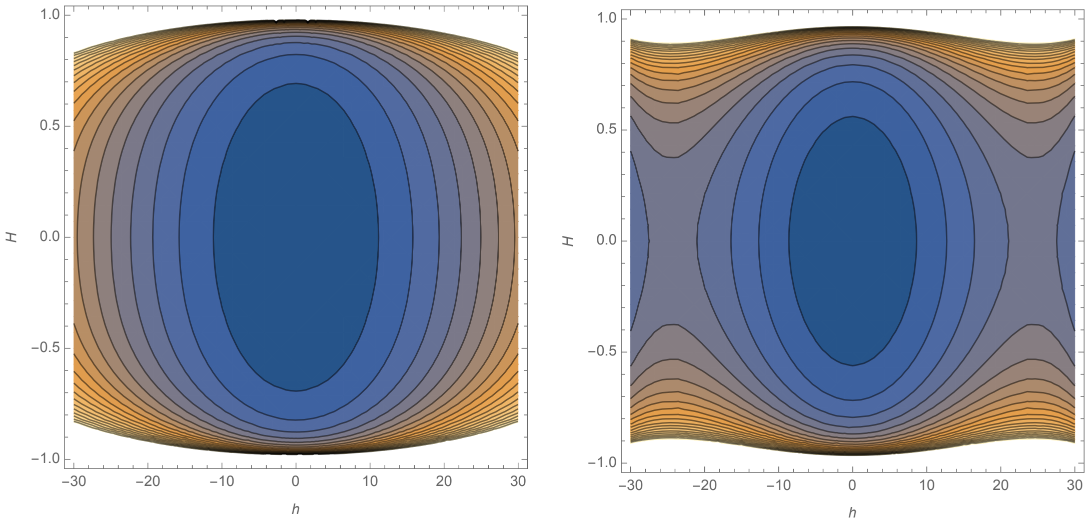

Figure 7.3: Contour plot of the original scalar potential $V(h, H)$ from Section 6.1 (left panel) compared to effective scalar potential after moduli stabilization (right panel) for the parameter choice (7.23). Warmer color means a larger value of V. The darkest blue is the local minimum at $h = H = V = 0$. As expected, the direction H is much steeper than the direction h, which is the inflaton direction. In the right panel local maxima are visible at $H = 0$ and $|h_c| \approx 23$, the point at which the effective theory breaks down and the modulus is destabilized. We have plotted the effective potential (7.19) to all orders in αt_0 and H, and up to fourth order in h. In the case presented here, 60 e-folds of slow-roll inflation are possible along the trajectory $H = 0$. The single-field inflaton potential in that slice is identical to the orange line of Figure 7.1.

A parameter example

Finally, let us consider a parameter example to illustrate our findings. The $h - H$ plane of the potential is displayed in Figure 7.3 for the same parameter values as in the single-field example,

$$W_0 = 0.005\,, \quad \mu = W_0/400\,, \quad s = 1\,, \quad t_0 = 10\,, \quad \alpha = 2\pi/5\,. \tag{7.23}$$

As expected, the field H has a much steeper potential than h so that $H = 0$ can be a viable inflationary trajectory in this example. In fact the effective theory defined by (7.4) only describes the potential correctly for $H < 1$ because of a branch cut in the Kähler potential, in this case at $H = 1$. To show this behavior, the potential in Figure 7.3 is evaluated to all orders in H. In any case, single-field inflation on the trajectory $H = 0$ is a very good approximation. Again, there is a critical field value of the inflaton at which (7.20) is violated and T is destabilized. It is again $h_c \approx 23$ for the chosen parameter values. After integrating out H at the origin, h is thus identical to φ in our single-field model.

7.2.3 DBI-induced flattening and CMB observables

So far we have neglected the non-trivial kinetic term of ϕ discussed at length in Chapter 5. As discussed in that chapter we can, in principle, include this correction by adding higher-derivative operators to the supergravity ansatz (7.1) or (7.4). Unfortunately, using the operator (5.39) in the single-field case, which corresponds to

$$\Delta K = a|\mathbf{H}_\mathrm{u} + \bar{\mathbf{H}}_\mathrm{d}|^2 \left(\partial_\mu \mathbf{H}_\mathrm{u} \partial^\mu \bar{\mathbf{H}}_\mathrm{u} + \partial_\mu \mathbf{H}_\mathrm{d} \partial^\mu \bar{\mathbf{H}}_\mathrm{d}\right) \,, \tag{7.24}$$

in the two-field case, does not capture the full result. By stabilizing the Kähler modulus non-perturbatively we break the no-scale symmetry of the effective theory and new couplings involving T and the open-string modulus ϕ appear. We have seen that these couplings modify the effective scalar potential of the inflaton after properly integrating out T. We could then expect that the kinetic term of the inflaton is also modified at higher orders in α'. However, the higher-derivative operator which captures the correct kinetic term in the $\mathcal{N} = 1$ supergravity picture should also include the multiplet $\mathbf{T}$, not only Φ or $\mathbf{H}_\mathrm{u}$ and $\mathbf{H}_\mathrm{d}$.

However, we do not have to consider modifications to the Kähler potential but rather only to the Lagrangian. In particular we can compare the flux Lagrangian (7.9) to the supergravity Lagrangian (7.13) and read off the correct kinetic term. Comparing the potential in both Lagrangians leads to the identification

$$G = \frac{ZW_0}{\sqrt{4g_\mathrm{s}st_0^3}} \,, \quad S = -\frac{Z(W_0 + 2s\mu)}{\sqrt{4g_\mathrm{s}st_0^3}} \,, \quad D = -\frac{6Zs\mu}{\sqrt{4g_\mathrm{s}st_0^3}} \,. \tag{7.25}$$

Note that the solutions for G and S are the same as the ones obtained before adding the IASD flux and moduli stabilization in Section 6.1. Inserting the solutions (7.25) in (7.9) yields the following leading-order kinetic term for the inflaton field,

$$\mathcal{L}_\mathrm{kin} = -\left(\frac{1}{2} + 3a\frac{\mu W_0 \varphi^2}{16t_0^3}\right)(\partial_\mu \varphi)^2 \,, \tag{7.26}$$

where φ is either the imaginary part of ϕ in the single-field model, or is the same as h in the two-field model and a is given in Equation (5.33).[2] Notice the difference between (7.26) and the result prior to moduli stabilization obtained from the corrected Kähler potential, given by (5.50). The dominant non-trivial piece in the kinetic term is now proportional to μW_0 instead of μ^2. Thus, it is enhanced by a factor of $W_0/\mu \gg 1$. This is quite intuitive, as the same happened in the scalar potential in our analysis in Section 7.2.1. After taking moduli stabilization into account, the dominant term in the potential is proportional to μW_0 instead of μ^2. In the kinetic term this can be understood in the following way: the gaugino condensate responsible for stabilizing T sources the additional IASD flux D. As we have seen in Equation (7.9a), the kinetic term of the D7-brane modulus gains additional D-dependent terms. Since the gaugino condensate term is proportional to W_0 according to the first equality in Equation (3.55), the new kinetic term must be proportional to W_0 as well. Thus, if we discarded the condensate term and switched off D, we would recover the result given in Equation (5.50) and a destabilized Kähler modulus. In total, the leading-order effective action relevant for inflation is given by

$$\mathcal{L}_\mathrm{eff} = -\left(\frac{1}{2} + 3a\frac{\mu W_0 \varphi^2}{16t_0^3}\right)(\partial_\mu \varphi)^2 - \frac{3\mu W_0 \varphi^2}{8t_0^3} + \frac{3s\mu^2\varphi^4}{32t_0^3} \,, \tag{7.27}$$

ignoring the subleading μ-mass term of φ. Let us now evaluate this result for a few reasonable parameter choices to extract the observables predicted by the combined model.

[2]Note that in the two-field case $H = 0$ always, so that both the potential and the kinetic term of the heavy mass eigenstate are irrelevant during inflation.

Parameter examples and CMB observables

In order to find realistic parameter values, consider the coefficient a in terms of the parameters of the string theory setup in Planck units, (5.33). Let us treat $g_\mathrm{s} = 0.1$ as a constant and allow ourselves to vary the string scale M_s within certain bounds. The most important lower bound is that we must require

$$M_\mathrm{s}^4 \gg 3H_\star^2\,, \tag{7.28}$$

so that string excitations are negligible during inflation. We must also keep in mind that the compactification scale M_KK has to fit between the string scale and the inflationary energy scale so that excitations of Kaluza-Klein modes are negligible. In the quadratic approximation we have [51]

$$V_{\mathrm{inf},\star} = 3H_\star^2 = 2\cdot 10^{-11}\cdot 15^2 = 4.5\cdot 10^{-9} \tag{7.29}$$

in Planck units, so that

$$M_\mathrm{s} \gg V_{\mathrm{inf},\star}^{1/4} = 8.2\cdot 10^{-3} M_\mathrm{p} = 1.64\cdot 10^{16}\,\mathrm{GeV}\,. \tag{7.30}$$

To be safe we may choose M_s to be larger than $V_{\mathrm{inf},\star}^{1/4}$ by a factor of 10. We thus consider the parameter example

$$M_\mathrm{s} = 0.082\,,\quad \alpha_G = \frac{1}{24}\,,\quad W_0 = 0.008\,,\quad \mu = \frac{W_0}{400}\,,\quad s = 1\,,\quad t_0 = 15\,,\quad \alpha = \frac{2}{5}\pi\,, \tag{7.31}$$

where the value of t_0 is determined by the relation $M_\mathrm{s}^2 = M_\mathrm{p}^2/(8g_\mathrm{s}^{1/2}t_0^{3/2})$, cf. Equation (3.16). Note that we have assumed that α_G is independent of t_0, i.e., that the gauge theory does not live on the same stack of branes that supports the non-perturbative term in W. We can then perform the canonical normalization of the Lagrangian (7.27) numerically and plot the resulting potential. The result is given in Figure 7.4. The plot contains the quadratic potential before moduli stabilization (blue line), the effective scalar potential in terms of the variable φ after properly integrating out T (orange line), and a numerical plot of the scalar potential in terms of the canonical variable (green dotted line). Apparently, the flattening induced by the non-trivial kinetic term is a very small effect. The CMB observables on the green dotted line are

$$n_\mathrm{s} \approx 0.964\,,\qquad r \approx 0.087\,, \tag{7.32}$$

for 60 e-folds of slow-roll inflation at $\varphi_\star \approx 14.6$. To illustrate the effect the non-trivial kinetic term may have, we can choose a more extreme parameter example. In order to increase a we may increase α_G, s, and μ compared to W_0, as well as decrease M_s. Taking the factor between M_s and $V_{\mathrm{inf},\star}^{1/4}$ to be 3 instead of 10, we consider the modified parameter set

$$M_\mathrm{s} = 0.025\,,\quad \alpha_G = \frac{1}{10}\,,\quad W_0 = 0.08\,,\quad \mu = \frac{W_0}{300}\,,\quad s = 2\,,\quad t_0 = 75\,,\quad \alpha = \frac{2}{5}\pi\,. \tag{7.33}$$

The relevant potentials can be found in Figure 7.5. As expected, the additional flattening is now much stronger. For the green dotted trajectory one obtains

$$n_\mathrm{s} \approx 0.961\,,\qquad r \approx 0.041\,, \tag{7.34}$$

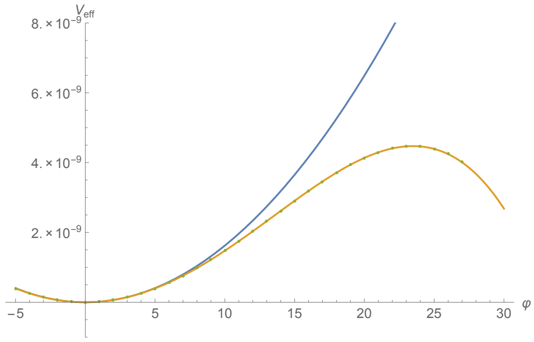

Figure 7.4: Effective potential for the parameter choice (7.31). Naive quadratic potential (blue line) in comparison with effective inflaton potential for φ (orange line) and numerical effective potential for the canonical variable in (7.27). The string scale is chosen too large for the DBI-induced flattening to have an effect.

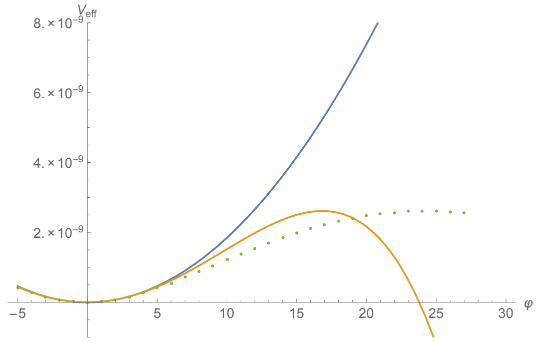

Figure 7.5: Effective potential for the parameter choice (7.33). Naive quadratic potential (blue line) in comparison with effective inflaton potential for φ (orange line) and numerical effective potential for the canonical variable in (7.27). In this case the additional flattening from the kinetic term is clearly visible.

for 60 e-folds of slow-roll inflation at $\varphi_\star \approx 12.6$. While this last example illustrates how strongly the kinetic term can effect the CMB observables, it is questionable whether an appropriate hierarchy $M_\text{s} > M_\text{KK} > V_{\text{inf},\star}^{1/4}$ can be maintained with a value of M_s this low and at what point the effective theory loses its validity.

7.3 Complex structure moduli stabilization and backreaction

So far we have neglected the stabilization of the complex structure moduli and the dilaton. We have assumed that they are stabilized by fluxes at a high scale in a GKP-like setup [24]. We assumed that states that are stabilized supersymmetrically well above the Hubble scale decouple from the dynamics of inflation [149]. However, in certain examples of Type II compactifications the backreaction of the complex structure moduli influences the canonical field range of the inflaton [151, 152, 156, 157]. This can happen whenever the field space metric of the inflaton, in our case

$$\varphi = \int \mathrm{d}\phi \sqrt{K_{\Phi\bar{\Phi}}}\ , \tag{7.35}$$

with φ denoting the canonical distance in field space, K given in Equation (7.1a) and $\phi = \text{Im}(\Phi)$, depends on the heavy moduli which are displaced during inflation. In our case this applies to S and U, but not to T, since T does not appear in the field space metric. It is easy to see that $K_{\Phi\bar{\Phi}}$ is a function of S and U, $K_{\Phi\bar{\Phi}} \sim (\text{Re}(S)\text{Re}(U))^{-1}$ to leading order. Whenever we are in a situation where the moduli S and U are stabilized such that their expectation values depend on ϕ, the canonical distance in field space is modified. The dependence of the moduli on ϕ is expected to occur in most scenarios of moduli stabilization. For the Kähler modulus we computed this dependence explicitly in Equation (7.11).

In [151] it was argued that in many Type II compactifications involving closed-string moduli only, one indeed has schematically

$$U = u_0 + \delta u(\phi)\,, \quad S = s_0 + \delta s(\phi)\,, \tag{7.36}$$

where the inflaton candidate ϕ is a light linear combination of the closed-string fields. In the setup of [151], the backreaction on the kinetic term becomes important beyond some critical inflaton field value where $\delta u(\phi) \gg u_0$. Because of the functional dependence of the kinetic term on the moduli, beyond the ϕ_c the canonical field distance only increases logarithmically. Depending on the numerical value of $\phi_{\rm c}$, defined by $\delta u(\phi_{\rm c}) = u_0$, this can make large-field inflation impossible, since generally for $\phi > \phi_c$ the theory becomes unphysical. However, even if the critical field value, and thus the point where the logarithmic dependence dominates, can be tuned large by a flux choice, in the setups of [151] large-field inflation is under pressure: By tuning $\phi_{\rm c}$ large, u_0 and s_0 are tuned large as well. Due to the inverse dependence in the Kähler metric, this leads to a suppression of the canonical field distance as well. If both $\phi_{\rm c}$ and s_0, u_0 parametrically depend on the fluxes in the same way, both effects cancel each other and, for the setups discussed in [151], the canonical field range cannot be larger than the Planck scale.

In this section we demonstrate that these problems are less severe in Higgs-otic inflation. We stabilize S and U via G_3 fluxes as in Chapter 3 and compute the minimum values (7.36) explicitly. We show that the fluxes allow for enough freedom to tune $\phi_{\rm c}$ large while, at the same time, leaving u_0 and s_0 approximately unchanged, so the backreaction effects can be delayed in field space. This flexibility, coming from the introduction of open-string fields, was also discussed in [152]. Here we aim to make the qualitative arguments given in [152] more explicit and analyze in detail the resulting backreacted field space metric in Higgs-otic inflation. First, we present an analytic study of a useful simplified model which is, however, phenomenologically incomplete due to the presence of a flat direction. Afterwards, we turn to a numerical study of a more complete model with full moduli stabilization.

7.3.1 A model with a flat direction

The model we consider is based on a simple toy model taken from [89]. We propose the following Kähler potential and flux superpotential for a toroidal compactification with a single mobile D7-brane,

$$K = -2\log(U+\bar{U}) - \log\left[(U+\bar{U})(S+\bar{S}) - \frac{1}{2}(\Phi+\bar{\Phi})^2\right] - 3\log(T+\bar{T})\,, \tag{7.37}$$

$$W = \mu\Phi^2 + \int G_3 \wedge \Omega = \mu\Phi^2 + e_0 + imU^3 + ih_0 S + \bar{h}_0 S U^3\,, \tag{7.38}$$

with integer flux quanta e_0, h_0, $\bar{h}_0$, and m. Notice the additional logarithm in the Kähler potential compared to (7.1a). While this does not change the analysis of Section 7.2, it accounts for the fact that the compact orientifold in this case is isotropic: we have taken the two-fold X to be a four-torus and have identified the three complex structure moduli of the two-tori. For the moment, we assume that the D7-brane position moduli are stabilized by the presence of (2,1)-fluxes as explained in Section 6.1, inducing a superpotential term parameterized by μ. Here we treat μ as an independent parameter and discuss its microscopic origin in terms of NS fluxes in Appendix C. Furthermore, we consider only the imaginary self-dual piece of G_3, so that $D_S W = D_U W = 0$ in the vacuum. On the other hand, the flux potential is non-vanishing in the vacuum, so that supersymmetry is broken and $D_T W \neq 0$. After no-scale cancellation we are thus interested in vacua of the scalar potential

$$V = e^K K^{a\bar{b}} D_a W \overline{D_b W} \,, \tag{7.39}$$

where a and b label the fields Φ, S, and U. We assume that all Kähler moduli are stabilized by a KKLT or LVS mechanism as in Section 7.2. In the remainder of this section we neglect the explicit stabilization and backreaction of the Kähler modulus, which has been analyzed in detail above. It only affects the scalar potential and is irrelevant for the backreaction in the kinetic terms. This is because, on the one hand, (7.35) does not explicitly depend on T. On the other hand, the backreaction of T does not affect the backreaction of the complex structure, since the latter only depends on the superpotential and not on the effective scalar potential.

The fluxes in (7.38) are only sufficient to stabilize three out of the four real scalar directions. Decomposing the moduli in real and imaginary parts $S = s_1 + is_2$ and $U = u_1 + iu_2$, we find the following solutions to the F-term constraints in the vacuum at $\mathrm{Im}(\Phi) \equiv \phi = 0$,

$$u_{2,0} = 0 \,, \quad s_{1,0} = \frac{(e_0 \bar{h}_0 + h_0 m) u_1^3}{h_0^2 + \bar{h}_0^2 u_1^6} \,, \quad s_{2,0} = \frac{e_0 h_0 - \bar{h}_0 m u_1^6}{h_0^2 + \bar{h}_0^2 u_1^6} \,, \tag{7.40}$$

and u_1 is a free parameter. During inflation, there is again an interaction between the inflaton ϕ and the complex structure moduli. This leads to a modification of the solutions (7.40) during inflation,

$$s_1 = s_{1,0} - \mu\phi^2 \frac{\bar{h}_0 u_1^3}{2h_0^2 + 2\bar{h}_0^2 u_1^6} \,, \quad s_2 = s_{2,0} - \mu\phi^2 \frac{h_0}{2h_0^2 + 2\bar{h}_0^2 u_1^6} \,, \tag{7.41}$$

whereas u_1 remains unfixed. These expressions are quite analogous to the displacement of the Kähler modulus in (7.11). The inflationary correction is proportional to the Hubble scale, which is determined by μ, divided by the mass of the modulus in question. In particular,

$$s_1 = s_{1,0} - \frac{\bar{h}_0 m_\phi}{8 s_{1,0} m_s^2} \phi^2 \,, \tag{7.42}$$

where $m_s^2 = (h_0^2 + \bar{h}_0^2 u_1^6)/(8u_1^3 s_{1,0})$. Thus, by introducing a hierarchy between the mass of the inflaton and the masses of the moduli, we can suppress the displacement compared to the vev of the field. In other words, we can increase the critical value ϕ_c. This can be achieved by tuning μ to small values compared to h_0 and $\bar{h}_0$. At the same time, this tuning of fluxes does not necessarily affect the vacuum expectation values in Equation (7.40) in the same way. μ does not enter in (7.40), so we can achieve such a mass hierarchy without changing $s_{1,0}$. This is different than in the setups considered in [151], where ϕ_c and $s_{1,0}$ had the same parametric dependence on the fluxes.

Let us now consider the backreaction on the canonical metric in field space during inflation. The metric for the inflaton in field space is given by

$$K_{\Phi\bar{\Phi}} = \frac{1}{4u_1 s_1} \,, \tag{7.43}$$

which indeed decreases for large values of ϕ due to the backreaction coming from (7.42). This implies that, for large-field values, the canonical field distance only grows logarithmically with the inflaton field, after performing the integral in (7.35). However, the point at which the backreaction dominates is flux-dependent, and the canonical field distance travelled before that point,

$$\varphi_{\rm c} \approx \frac{\phi_{\rm c}}{4u_1 s_{1,0}} = \frac{s_{1,0} m_s}{\sqrt{\bar{h}_0 m_\phi}}\,, \tag{7.44}$$

can be tuned larger than $M_{\rm p}$ by generating a mass hierarchy between s_1 and ϕ, as we explained above.

In summary, this model produces the following result. While the integration of the field space metric does lead to a logarithmic dependence of φ on ϕ for large-field values, the point where the logarithm is relevant can be moved far out in field space by a tuning of fluxes. Of course, the fact that u_1 is not stabilized in this setup is a big caveat: we have no means of evaluating the displacement and backreaction of u_1 on the possible field space. The above arguments only apply to s_1. Furthermore we have not considered the microscopic origin of μ, which could also affect the closed-string fields. This is why, in the following, we use additional fluxes to stabilize u_1 and analyze all backreactions on the kinetic term simultaneously. A numerical analysis reveals results that are qualitatively the same as in this simple model, suggesting that the intuition of this example still holds in a more complicated setup.

7.3.2 Stabilizing complex structure in Higgs-otic inflation

To obtain an inflationary theory in which all complex structure moduli are stabilized, we must allow for a general G_3 flux. We also need to identify the microscopic origin of the μ-term yielding a mass for the D7-brane position moduli. In Appendix C we argue through the Type IIA dual theory that the D7-brane position modulus is stabilized by an NS flux. This is shown by considering Type IIA involving a wrapped D6-brane with a geometric flux that sources the term $W \supset a_1 SU$. This geometric flux also induces a supersymmetric mass for the open-string modulus of the D6-brane. The geometric flux in IIA corresponds to an NS flux on the Type IIB side, and the complexified D6 open-string modulus is mapped to the D7 position modulus in the transverse torus. For more details we refer to Appendix C.

Following the model of the previous section we consider the following effective theory

$$K = -2\log\left[(U+\bar{U})\right] - \log\left[(U+\bar{U})(S+\bar{S}) - \frac{1}{2}(\Phi+\bar{\Phi})^2\right] - 3\log(T+\bar{T})\,, \tag{7.45}$$

$$W = \mu\Phi^2 + e_0 + ie_1 U + imU^3 + ih_0 S + \mu SU + \bar{h}_0 SU^3\,, \tag{7.46}$$

where we have also allowed for a linear term in U in the superpotential in addition to the bilinear term SU. This assures that the above superpotential can be written in terms of complexified fluxes pairing (e_0, h_0), (e_1, μ) and $(m, \bar{h}_0)$. For the sake of simplicity, we have assumed that the terms Φ^2 and SU have exactly the same coefficient sourced by the same NS flux, μ. However, in more elaborate examples with different complex structure moduli for the three two-tori, $U_1 \neq U_2 \neq U_3$, this is not necessarily true. We will come back to this issue in Section 7.3.3.

The extra terms in W allow us to lift the flat direction but, unfortunately, they also lead to much more complicated equations following from the F-term constraints $D_S W = D_U W = 0$. The solutions can only be studied numerically. We have to ensure that the F-term solution we pick gives a physical vacuum. Luckily the F-term

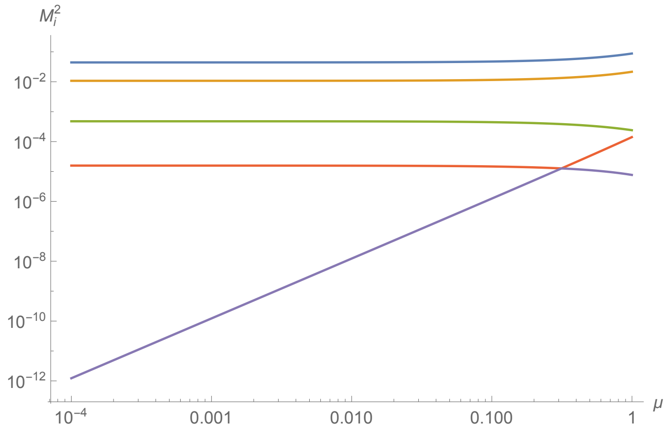

Figure 7.6: Canonically normalized mass eigenvalues of the Lagrangian for the flux choice in (7.47) as well as $t_0 = 30$. The plot is double-logarithmic. The masses are given in units of M_{p}. Evidently, one mass eigenstate—the would-be inflaton—scales with μ while the others do not. Making μ small compared to the other flux parameters introduces a hierarchy between the inflaton and moduli mass scales.

constraints admit a unique Minkowski solution with positive definite Hessian in the vacuum at $\phi = 0$, and at the same time positive vacuum expectation values of the dilaton s_1 and the complex structure modulus u_1. However, this vacuum is not a deformation of the vacuum of the model of the previous section. The Hessian is in general a function of the fluxes and in particular of μ. We have to show that its eigenvalues exhibit a hierarchy of masses as a function of μ. In particular, when μ is much smaller than the other flux parameters, the inflaton, is significantly lighter than all other states. In addition, the masses of all four real scalars contained in S and U are mostly independent of μ. To illustrate this, we can plot the five mass eigenvalues of interest numerically as a function of μ.

We fix all flux parameters except μ to a set of $\mathcal{O}(1-10)$ numbers, and vary μ between 10^{-4} and 1. We choose

$$e_0 = -20\,, \quad e_1 = 20\,, \quad m = 20\,, \quad h_0 = 5\,, \quad \bar{h}_0 = -10\,, \tag{7.47}$$

as a parameter example. Note that this is equivalent to fixing μ to an $\mathcal{O}(1)$ number and varying the remaining parameters to be much larger. What counts is the relative size of μ compared to the rest of the flux quanta. The result is displayed in Figure 7.6. The lightest eigenstate has a mass which scales as μ, while the other four eigenstates are much heavier and do not depend on μ as long as there is a moderate hierarchy between μ and the remaining flux quanta. This means that, within the flux setup (7.46) we can tune the masses of the moduli and the inflaton independently.

However, this is not sufficient to show that the dangerous backreaction is under control. We also need to check the behaviour of the vacuum expectation values of the moduli as a function of μ. We display the four vacuum expectation values in Figure 7.7, for the same set of flux quanta and the same parameter range of μ. It is clear that the vevs are almost independent of μ. These results are thus completely analogous to those of the simple model we discussed analytically, we can tune the fluxes to obtain a mass hierarchy between the inflaton and the closed-string fields while barely modifying the vacuum expectation values of the latter.

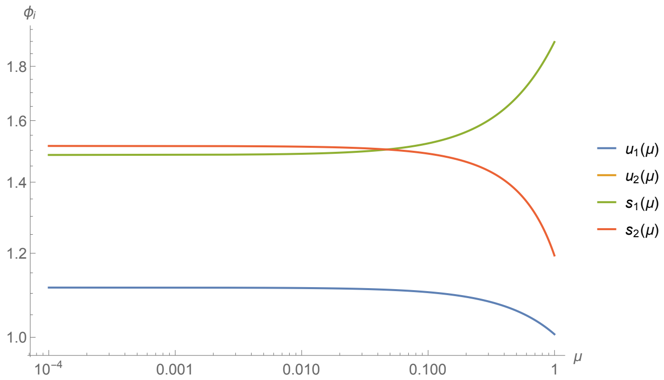

Figure 7.7: Vacuum expectation values of the moduli and axions as a function of μ with the parameters in (7.47), as well as $t_0 = 30$ and $\phi = 0$. The plot is double-logarithmic. The value of the axion of U is negative in this example, and therefore is invisible in the logarithmic plot. For small values of μ the expectation values are independent of μ, contrary to the findings in [151].

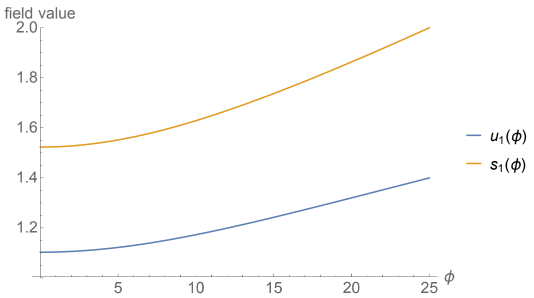

Figure 7.8: Values of the two moduli as a function of ϕ for $\mu = 10^{-1}$, $t_0 = 30$, and the flux choice (7.47).

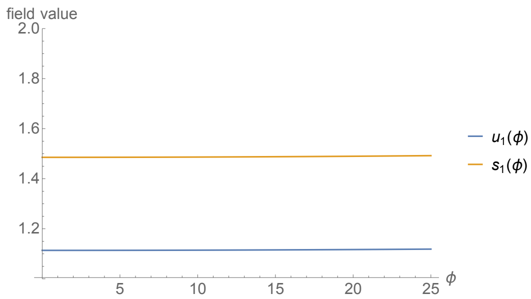

Figure 7.9: Values of the two moduli as a function of ϕ for $\mu = 10^{-3}$, $t_0 = 30$, and the flux choice (7.47). With a smaller value of μ the two fields are almost constant.

7.3.3 Backreaction in the Kähler metric, flux tuning, and large-field excursions

Let us now proceed and study what happens during inflation, that is when $\phi \neq 0$. In Figures 7.8 and 7.9 we have displayed the expectation values of s_1 and u_1, the only two fields entering the Kähler metric of the inflaton, as a function of ϕ for two different values of μ. Decreasing μ (and thus increasing the mass hierarchy) weakens the dependence of s_1 and u_1 on ϕ. After what we learned from the model in Section 7.3.1, this is no surprise. It is the same effect as in that model and also in our study of the backreaction of T, increasing the mass hierarchy reduces the field displacements of the rest of the moduli during inflation. As in the simple model of the previous section, let us consider what happens to the effective field range

$$\varphi = \int \mathrm{d}\phi \sqrt{K_{\Phi\bar{\Phi}}} = \int \mathrm{d}\phi \sqrt{\frac{1}{4 s_1(\phi) u_1(\phi)}} \,. \tag{7.48}$$

The important plot is given in Figure 7.10, for four different values of μ. The result is quite interesting. For large values of μ, as on the red and green curves, we clearly

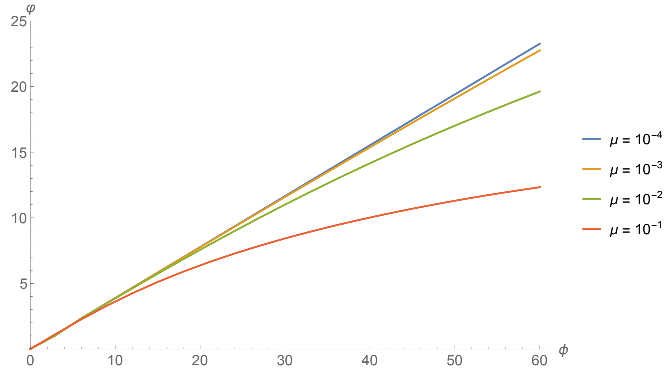

Figure 7.10: Canonically normalized field value as a function of the original variable φ, for four different values of μ. The logarithmic regime for large values of φ is evident in the red and green curves. This is the regime found in [151]. But evidently, for a large hierarchy between μ and the remaining flux quanta, the beginning of the logarithmic regime is pushed to very large-field values so that the backreaction is negligible.

see the reduced value of φ for large ϕ. This is similar to the examples of [151], the ϕ-dependence of the canonical field distance is logarithmic for large-field values. Therefore the backreaction from the closed-string moduli on the field metric of the inflaton makes it difficult to obtain parametrically large displacements. However, we can push out the critical field value where the logarithmic behavior becomes relevant by decreasing μ relative to the other flux parameters. This is possible because, unlike in [151], the closed-string expectation values $s_{1,0}$ and $u_{1,0}$ do not scale with the fluxes in the same way as the critical value ϕ_c. Therefore, as long as a tuning of the different flux parameters is possible, the dangerous effect observed in [151] can be made irrelevant during inflation.

Notice that the flux tuning needed to allow for 60 e-folds of inflation, that is to have $\varphi = \varphi_\star \approx 15$ in the nearly-linear regime is $\mathcal{O}(100 - 1000)$. This is of the same order as the tuning in the Kähler sector in Section 7.2. In fact, the tuning is not just of the same order of magnitude, it is the same tuning. Choosing all flux quanta large compared to μ leads to a large value of $W_0 = \langle \int G_3 \wedge \Omega \rangle$ compared to μ. In KKLT and related mechanisms, this tuning between μ and W_0 is exactly what is needed to make T heavy compared to the inflaton. Therefore, in both cases we must ask how μ can be a number as small as 10^{-4} or 10^{-5}. This seems impossible since all flux quanta are quantized and e_0, e_i, m, h_0, and $\bar{h}_0$ are indeed integers. If μ is also a flux quantum, a value of 10^{-4} is not allowed. However, the μ-term for the open-string modulus in the superpotential can receive contributions from different sources and fluxes in the compactification. By requiring some fine-tuning between the different contributions one can obtain smaller values of μ. This is analogous to the rationale behind the KKLT mechanism and the fine-tuning arguments to obtain a small value of W_0, cf. the discussions in [74, 158] and also [159]. For instance, in the case of a tilted D7-brane in a toroidal compactification, the position modulus feels the presence of NS flux in the two two-tori that are only partially wrapped by the brane. Another possibility is to consider compactifications beyond toroidal models, with a large number of three-cycles leading to many contributions to the μ-term. From the perspective of the brane worldvolume action, the brane is only sensitive to the local background and the flux densities around the brane. These are a combination of many different internal fluxes as well as distant sources back-reacting on the local background. Therefore, a priori there is no restriction to a small value of μ. However, increasing the number of three-cycles implies a larger number of complex structure moduli. Even if this does not change the results in Section 7.2, it might make the analysis in this section intractable. Nevertheless, the leading source of mass terms for the complex structure moduli can, as in our example above, come from a set of fluxes which do not affect the D7 position moduli and thus do not contribute to the μ-term. Then the expectation values of the complex structure moduli are still approximately independent of the value of μ. In that case, we expect the conclusions of this section to be qualitatively unchanged in more complicated models.

Another possibility would be not to decrease μ, but to increase the value of the other fluxes. Not the absolute values but the ratios to μ are what matters in suppressing the backreaction. However, large fluxes are problematic for several reasons. They can easily lead to moduli masses heavier than the KK scale and, furthermore, yield a big RR tadpole which has to be cancelled by a large number of sources with negative D3-brane charge. The backreaction of these fluxes and additional sources in the global compactification might force us to work beyond the validity of our effective theory.

A complete model

In principle we should repeat the analysis of the cosmological observables in Higgs-otic inflation after including the correction to the kinetic term coming from backreaction of S and U. However, as long as the aforementioned flux tuning is possible, this backreaction in the kinetic term is negligible compared to the DBI correction studied in Section 7.2.3, so the numerical results found there do not substantially change. It is interesting to note that the DBI action leads to an alternative to suppress the backreaction in the Kähler metric. The α' corrections from the DBI action lead to a non-trivial kinetic term of the form (7.27), which essentially adds an extra term proportional to the DBI scalar potential in the inflaton Kähler metric. The resulting effective field range depends on the balance between the DBI correction and the backreacted saxion expectation values. At the very least, the DBI correction will always help to delay the suppression of the canonical distance and disconnect it from the Planck scale. A full analysis combining the two previous section would be needed.

The question is then if the models we have presented in this section and Section 7.2 are compatible. The answer seems to be non-trivial, since our example in this section implies, mostly independent of the value of μ, a large expectation value of the flux superpotential, of the order of

$$|W_0| = \left|\langle \int G_3 \wedge \Omega \rangle\right| \approx 60\,, \tag{7.49}$$

in Planck units. The KKLT mechanism used in Section 7.2, on the other hand, requires W_0 to be small compared to $M_{\rm p}^3$. There are two ways out of this predicament. First, in a more complicated compactification with more flux parameters, W_0 may be small even though all flux parameters are integer. This is very similar to the way μ can be made small, as discussed above. Thus, there may be a scenario in which both $W_0 \ll 1$ and $\mu \ll 1$ while the mass hierarchy between inflaton and complex structure moduli is unchanged. In this case T can be stabilized by the KKLT mechanism as in Section 7.2, and the inflaton potential $V \sim \mu W_0 \varphi^2$ may have the correct normalization.

The second possibility is to look for a viable mechanism for Kähler moduli stabilization, even if W_0 is $\mathcal{O}(1-10)$. Both the Large Volume Scenario [31,160] and Kähler

Uplifting [30, 161] do not require a tuning of W_0. In fact, for both mechanisms the interaction with open-string large-field inflation has been studied in [33]. The results are very similar to the KKLT scenario. Inflation is mostly driven by an inflaton mass term proportional to $\mu W_0 \varphi^2$, and $W_0 \gg \mu$ is required to guarantee stability of all Kähler moduli in the inflationary phase. Also in this case there is a certain range of parameter examples that lead to 60 e-folds of slow-roll inflation in accordance with CMB observations. Thus, the two tunings in Section 7.2 and 7.3 are indeed compatible, and all moduli can be stabilized during Higgs-otic inflation.

Running of the Higgs masses

The results obtained in Sections 7.2 and 7.3 not only apply to Higgs-otic inflation, but to any inflationary model in which the inflaton is identified with a D7-brane position modulus in a similar background. In fact, in order to keep the discussion as generic as possible, we have only imposed constraints from cosmological data and not from particle physics. Let us discuss now if the results obtained in the previous sections are still compatible with particle physics phenomenology, if we identify the inflaton with the SM Higgs boson as in [70].

In order to keep one mass eigenstate—the SM Higgs boson—light at low energies, there must be an almost massless field at the supersymmetry breaking scale $M_{\rm SS}$, below which the supersymmetric spectrum decouples. This happens when the running of the soft mass parameters from the compactification scale M_{KK} down to $M_{\rm SS}$ gives rise to a zero eigenvalue in the Higgs mass matrix, i.e., $\det(M_H^2) = m_{H_{\rm u}}^2 m_{H_{\rm d}}^2 - m_3^4 \approx 0$ at $M_{\rm SS}$. For a given value of $M_{\rm SS}$ this imposes a constraint on the mass ratio $A = m_3^2/m_{H_{\rm u}}^2$ at $M_{\rm c}$. It was shown in [162] that, if such a fine-tuned Higgs survives, one necessarily gets $m_h = 126 \pm 3$ GeV for $M_{\rm SS} = 10^9 - 10^{13}$ GeV and standard unification conditions $m_{H_{\rm u}} = m_{H_{\rm d}}$ at $M_{\rm c}$. The question is whether this constraint on the mass ratio A is compatible with the mass hierarchy required to get moduli stability during inflation.

The two parameter examples of Higgs-otic inflation considered in Section 7.2 correspond to $A = 0.993$ with $M_{\rm SS} \approx 9 \cdot 10^{13}$ GeV and $A = 0.990$ with $M_{\rm SS} \approx 5 \cdot 10^{13}$ GeV, respectively. As already mentioned in Section 7.2, the mass hierarchy required to get moduli stability and suppress the backreaction of T implies a value of A very close to one. This, in turn corresponds to an almost massless state already at $M_{\rm c}$. Unfortunately, the above values of A are too large and very little running is required to make the Higgs determinant vanish. Therefore, the massless eigenstate will appear close to $M_{\rm c} \approx 10^{16}$ GeV, implying that the Higgs boson at $M_{\rm SS}$ is already tachyonic and triggers electroweak symmetry breaking at a too high-energy scale.

Let us remark that we have assumed no additional physics until $M_{\rm SS}$, and only the MSSM spectrum beyond it. Additional states at high energies could modify the renormalization group equations for the soft mass parameters, leading to less stringent constraints on the value of A. Furthermore, the above tension arises from the fact that the mass scale for the heavy Higgs H coincides with the mass scale of the Kähler modulus, parameterized by W_0. If one finds a scenario where both scales are decoupled, one could decrease the mass of H while maintaining $\mu/W_0 \ll 1$ and moduli stability. That, in turn, would lead to a lower value of A.

Chapter 8

Conclusions

In this thesis we have studied different aspects of inflation with open strings within the framework of Type II string theory. In particular, we focused on Type IIB string theory with D7-branes compactified on an orientifold in the presence of background three-form fluxes. The benchmark model we considered was Higgs-otic inflation, but most of our results hold more generally. Specifically, Chapters 4 and 5 are independent of inflation and deal with general Type II flux compactifications.

In Chapter 4 we have studied the role of Minkowski four-forms in flux compactifications of Type II string theory. We showed that the flux scalar potential of the RR and NS closed-string axions can always be written in terms of these four-forms. Gauge invariance of the four-forms in combination with internal symmetries of the compactification, forces the axion scalar potential to be expressible in an expansion in powers of the four-forms. We showed in Section 4.6, that similar results hold for open-string fields in a Type IIB compactification with D7-branes. We studied the effective action of the position modulus of Dp-branes in toroidal orientifold compactifications of Type IIB string theory in Chapter 5. We found that, for primitive ISD fluxes, the effective action takes a particular form in which the kinetic term becomes non-canonical. We also obtained a correction to the Kähler potential of $\mathcal{N} = 1$ supersymmetric theories, both global and local, that captures the leading-order α' correction including the corrected kinetic terms.

Chapters 6 and 7 focus on the embedding of inflation in Type IIB string theory where the position modulus of a D7-brane is the inflaton candidate. As remarked above, the benchmark model we considered was Higgs-otic inflation. However, we allowed more general flux choices than considered in the simplest incarnation of this model. In Chapter 6 we also studied the multi-field dynamics of this class of models. We found that the effects of the two-field dynamics depend on the fluxes and that the tensor-to-scalar ratio is reduced significantly compared to the single-field approximation, yielding $r = 0.08 - 0.12$. In Chapter 7 we studied moduli stabilization in this class of models. We found that, provided a certain tuning of the fluxes, large-field inflation and moduli stabilization can be combined further reducing the tensor-to-scalar ratio to a lowest value of $r \approx 0.04$. We discussed both the effects of the backreaction coming from integrating out the Kähler moduli and the complex structure moduli on the inflationary sector of the theory. For both classes of moduli, we found that the flux responsible for the mass of the inflaton μ must be tuned much smaller than the Gukov-Vafa-Witten superpotential W_0.

Embedding large-field inflation in string theory in a consistent manner is very challenging. In this thesis, we have taken a few steps in the direction of a realistic embedding However, there is still a number of steps to be taken. As an example of these steps, it will be interesting to find more general sources for the μ-term in flux compactifications and study its dependence on several different fluxes. This will be necessary to study whether the flux hierarchy that is needed to have a parametrically light inflaton is allowed in a more realistic compactification. A bigger challenge will be to take into consideration the backreaction of the flux background on the global geometry in combination with tadpole cancellation conditions and explicit sources for the non-perturbative effects and uplift. However, we think that the work we have done in this thesis is a necessary step in the construction of a fully consistent global model.

Chapter 9

Conclusiones

En esta tésis hemos estudiado diferentes aspectos de inflación con cuerdas abiertas dentro del marco de la teoría de cuerdas de Tipo II. En particular, nos hemos centrado en la teoría de cuerdas de tipo IIB con D7-branas compactificadas en un orientifold en la presencia flujos de fondo. El modelo de referencia que consideramos es la inflación de Higgs-otic, pero la mayoría de nuestros resultados son más generales. Específicamente, los capítulos 4 y 5 son independientes de la inflación y se ocupan de compactificaciones generales de flujo de tipo II.

En el capítulo 4 hemos estudiado el papel de las 4-formas de Minkowski en las compactificaciones de flujo de la teoría de cuerdas de tipo II. Hemos demostrado que el potencial escalar de flujo de los axiones de cuerdas cerradas RR y NS siempre se puede escribir en términos de estas 4-formas. La invariancia del campo gauge de las 4-formas en combinación con las simetrías internas de la compactificación, obliga al potencial escalar del axión a ser expresable en una desarrollo en las potencias de las 4-formas. En sección 4.6, hemos demostrado que resultados similares se obtienen para los campos de cuerda abierta en una compactificación tipo IIB con D7-branas. En el capítulo 5, hemos estudiado la acción efectiva del módulo de posición de Dp-branas en compactificaciones toroidales de orientifolds de la teoría de cuerdas de tipo IIB. Encontramos que, para los flujos de ISD primitivos, la acción efectiva toma una forma concreta y obtuvimos una corrección al potencial Kähler de las teorías supersimétricas $\mathcal{N} = 1$, tanto globales como locales, que captura la corrección α' de orden dominante.

En los capítulos 6 y 7 se estudia la inclusión de la inflación en la teoría de cuerdas de tipo IIB donde el módulo de posición de una D7-brana es el inflatón. Como se ha señalado anteriormente, el modelo de referencia que consideramos es la inflación de Higgs. Sin embargo, permitimos opciones de flujo más generales que las permitidas en la encarnación más simple de este modelo. En el capítulo 6 estudiamos la dinámica multi-campo de esta clase de modelos. Descubrimos que los efectos de la dinámica de dos campos dependen de los flujos y que la relación tensor-escalar se reduce significativamente en comparación con la aproximación de un solo campo, obteniendose $r = 0,08 - 0,12$. En el capítulo 7 estudiamos la estabilización de módulos en esta clase de modelos. Se encuentra que, para un cierto ajuste fino de los flujos, se podría combinar la inflación de grandes campos y la estabilización de los módulos. En particular, discutimos tanto los efectos de la retroacción que provienen de la integración de los módulos de Kähler como los módulos de estructura compleja en el sector inflacionario de la teoría. Para las dos clases de módulos, encontramos que el flujo responsable de la masa del inflatón μ debe ser sintonizado para que sea mucho más pequeño que el superpotencial W_0 de Gukov-Vafa-Witten.

La inclusión de la inflación de gran campo en la teoría de cuerdas de una manera consistente es un desafío. En esta tésis, hemos dado algunos pasos en la dirección de una inserción realista. Sin embargo, todavía hay una serie de pasos que hay que seguir. Como ejemplo de estos pasos, será interesante encontrar fuentes más generales para el término μ en las compactificaciones de flujo y estudiar su dependencia de varios flujos diferentes. Esto será necesario para estudiar si la jerarquía de flujos que se necesita para tener un inflaton parametricamente ligero es posible en una compactificación más realista. Un desafío aun mayor será tomar en consideración la retroacción del fondo del flujo en la geometría global, en combinación con las condiciones de cancelación de "Tadpole" y fuentes explícitas para los efectos no perturbativos y el "up-lift". Pensamos, sin embargo, que el trabajo llevado acabo en este tésis es un paso necesavio para la obtención de un modelo global consistente.

Appendix A

The expansion of the DBI action

In this appendix we give more details on the computation of the effective action of Chapter 5 and the action of the D7-brane position modulus of Chapter 7. We first focus only on the DBI action for general branes before specializing to the DBI and CS action for a D7-brane with imaginary self-dual (ISD) and imaginary anti-self dual (IASD) fluxes. The DBI action in Einstein frame is given by [27, 28]

$$S_{\text{DBI}} = -\mu_p \int d^{p+1} e^{-\phi} \sqrt{-\det\left(P\left[E_{MN} + E_{Mi}(Q^{-1}-\delta)^{ij} E_{jN}\right] + \sigma F_{MN}\right) \det(Q_{ij})} \ , \tag{A.1}$$

with the tensors

$$E_{MN} = g_s^{1/2} G_{MN} - G_{MN} \ , \qquad Q_{ij} = \delta_{ij} + i\sigma[\varphi_i, \varphi_k] E_{kj} \ . \tag{A.2}$$

The integral runs over the world-volume of the brane, M and N are indices of the Dp-brane world-volume, P denotes the pullback onto the world-volume, μ_p denotes the brane tension and is given by $\mu_p = (2\pi)^{-p}(\alpha')^{-(p+1)/2}$, $\sigma = 2\pi\alpha'$, the ten-dimensional spacetime metric is denoted by G_{MN}, the NS two-form by B_{MN} and the field strength of the gauge fields living on the brane by F_{MN}. The two-forms can be grouped in the gauge-invariant combination $\mathcal{F}_2 = B_{MN} - \sigma F_{MN}$.

The four-dimensional action for the scalars, which are the position moduli of the brane, can be obtained by performing the pullback of the metric, expanding the determinant, and integrating over the compact four-cycle wrapped by the brane. In order to do this we assume that the local world-volume fields only feel the local closed-string background around the brane, so we can expand the metric, axio-dilaton and three-form flux in terms of the fluctuations of the transverse real fields $y^m = \sigma\varphi^m$, where y^m are normal coordinates, as follows

$$\begin{aligned} \mathrm{d}s^2 &= Z^{-1/2}\eta_{\mu\nu}\mathrm{d}\hat{x}^\mu \mathrm{d}\hat{x}^\nu + Z^{1/2}\mathrm{d}s^2_{\text{CY}} \ , \\ \tau &= \tau_0 + \frac{1}{2}\sigma^2 \tau_{ij}\varphi^i\varphi^j + \dots \ , \\ G_3 &= \frac{1}{3!} G_{lmn}(x^p)\mathrm{d}x^l \wedge \mathrm{d}x^m \wedge \mathrm{d}x^n \ , \quad G_{lmn}(x^p) = G_{lmn} + \dots \ , \end{aligned} \tag{A.3}$$

to yield the desired action. Here Z denotes the warp factor that is allowed to vary on the internal space, $\tau = C_0 + ie^{-\phi}$ is the complex axio-dilaton, the brane world-volume is parameterized by $\{x^\mu, x^a\}$ and $G_3 = F_3 - \tau H_3$ in terms of the usual RR and NS flux.

We perform the pullback and split the world-volume determinant in the absence of mixed Minkowski-internal tensors, i.e., $g_{\mu a} = B_{\mu a} = 0$, and consider a constant internal profile for the position moduli, $\partial_a \phi = 0$. This leads to

$$\begin{aligned}&\det(P[E_{MN} + \sigma F_{MN}]) \\ &\quad = \det(g_s^{1/2} Z^{-1/2} \eta_{\mu\nu} + g_s^{1/2} Z^{1/2} \sigma^2 \partial_\mu \varphi_m \partial_\nu \varphi_n) \det(g_s^{1/2} g_{ab} + \sigma F_{ab} - B_{ab}) \,. \end{aligned} \tag{A.4}$$

This factorization of Minkowski and internal indices is exact in toroidal compactifications. However, in a Calabi-Yau compactification the internal profile of the scalar fields is in general not constant. This implies that one has to solve an eigenstate equation for the internal space, which is usually non-trivial. Besides, the zero eigenmodes might correspond to mixings between the original position moduli and Wilson lines, making the computation technically much more involved. Therefore we restrict to the simplest cases in which the above factorization can be performed. For a D3-brane all world-volume indices are in Minkowski spacetime so there are no subtleties regarding the compactification.

Moreover, taking into account the contribution from the transverse coordinates, the quantity inside the square root in the DBI action is composed of three factorized determinants,

$$\det(g_s^{1/2} Z^{-1/2} \eta_{\mu\nu} + g_s^{1/2} Z^{1/2} \sigma^2 \partial_\mu \varphi_m \partial_\nu \varphi_m) \,, \tag{A.5a}$$

$$\det(g_s^{1/2} g_{ab} + \sigma F_{ab} - B_{ab}) \,, \tag{A.5b}$$

$$\det(g_{mn} + i\sigma[\varphi_m, \varphi_p](g_s^{1/2} g_{pn} - B_{pn})) \,. \tag{A.5c}$$

For a Dp-brane these three matrices have dimension 4, $(p-3)$ and $(9-p)$, respectively. After rearranging the real fields φ_m in a complex basis denoted by ϕ_i, the first determinant becomes

$$\begin{aligned}&-\det(g_s^{1/2} Z^{-1/2} \eta_{\mu\nu} + g_s^{1/2} Z^{1/2} \sigma^2 \partial_\mu \varphi_m \partial_\nu \varphi_m) \\ &\quad = \frac{g_s^2}{Z^2} \Big(1 + 2Z\sigma^2 \partial^\mu \phi_i \partial_\mu \bar{\phi}_i + Z^2 \sigma^4 \big[2(\partial^\mu \phi_i \partial_\mu \bar{\phi}_i)^2 \\ &\qquad - (\partial^\mu \phi_i \partial_\mu \bar{\phi}_j)(\partial^\nu \phi_j \partial_\nu \bar{\phi}_i) - (\partial^\mu \phi_i \partial_\mu \phi_j)(\partial^\nu \bar{\phi}_i \partial_\nu \bar{\phi}_j) \big] \Big) \,. \end{aligned} \tag{A.6}$$

We can now Taylor-expand the square root in powers of spacetime derivatives of the ϕ_i. This expansion is in accordance with the slow-roll approximation during inflation. This gives

$$\begin{aligned}\mathcal{L} = -\frac{\mu_p V_{p-3}}{Z} f(\phi) \Bigg(1 + Z\sigma^2 \sum_i \partial_\mu \phi_i \partial^\mu \bar{\phi}_i - \frac{1}{2} Z^2 \sigma^4 \Bigg[&\sum_{i \neq j} (\partial^\mu \phi_i \partial_\mu \bar{\phi}_j)(\partial^\nu \phi_j \partial_\nu \bar{\phi}_i) \\ &+ \sum_{i,j} (\partial_\mu \phi_i \partial^\mu \phi_j)(\partial_\nu \bar{\phi}_i \partial^\nu \bar{\phi}_j) \Bigg] + \dots \Bigg) \,, \end{aligned} \tag{A.7}$$

where

$$f(\phi) = \sqrt{\det(g_s^{1/2} g_{ab} + \sigma F_{ab} - B_{ab}) \det(g_{mn} + i\sigma[\varphi_m, \varphi_p](g_s^{1/2} g_{pn} - B_{pn}))} \,. \tag{A.8}$$

Here μ_p is the brane tension, Z is the warp factor and V_{p-3} denotes the volume of the internal cycle wrapped by the brane. This is the action used in Chapter 5.

We now specialize to a D7-brane with ISD and IASD fluxes. We define the single complex scalar field Φ in accordance with the notation in Chapter 6 and we turn on $(0,3)$-form ISD flux $G = G_{\bar{1}\bar{2}\bar{3}}$, $(2,1)$-form ISD flux $S = \epsilon_{\bar{3}\bar{j}\bar{k}} G_{\bar{3}jk}$ and $(1,2)$-form IASD flux $D = \epsilon_{3jk} G_{3\bar{j}\bar{k}}$. The effective action can then be written as follows[1]

$$\mathcal{L}_{\rm DBI} = -\mu_7 V_4 e^{\phi} f(\phi)(1 + \sigma^2 Z \partial_\mu \phi \partial^\mu \bar{\phi} + ...) \ , \tag{A.9}$$

with the internal function f given by the first determinant in Equation A.8, ignoring the gauge field strength,

$$f(\phi)^2 = 1 + \frac{1}{2Z} e^{-\phi} B_{ab} B^{ab} - \frac{1}{4Z^2} e^{-2\phi} B_{ab} B^{bc} B_{cd} B^{da} + \frac{1}{8Z^2} e^{-2\phi} (B_{ab} B^{ab})^2 \ . \tag{A.10}$$

It contains the scalar potential contribution to all orders in $\alpha' \phi$, just like in [70]. Turning on G, S and D fluxes does not introduce off-diagonal components in B, so we find that $f(\phi)$ completes to a perfect square, yielding

$$f(B) = 1 + \frac{1}{4Z} e^{-\phi} B_{ab} B^{ab} \tag{A.11}$$

after taking the square root. The ten-dimensional Type IIB supergravity equations of motion relate the dilaton and the three-form fluxes of the global compactification. In particular, in the presence of both ISD and IASD fluxes, one obtains [113]

$$\mathrm{Im}(\tau_{3\bar{3}}) = -\frac{g_{\rm s}}{4Z}(SD + \bar{S}\bar{D}) \, , \tag{A.12}$$

where we have again performed a local expansion of the dilaton field around the brane following (A.3),

$$e^{-\phi} = g_{\rm s}^{-1} \left(1 + \sigma^2 \mathrm{Im}(\tau_{3\bar{3}}) |\phi|^2 + \frac{1}{2} \sigma^2 \mathrm{Im}(\tau_{33}) \phi^2 + \frac{1}{2} \sigma^2 \mathrm{Im}(\tau_{\bar{3}\bar{3}}) \bar{\phi}^2 + \dots \right) \ . \tag{A.13}$$

Notice that τ_{33} and $\tau_{\bar{3}\bar{3}}$ are not related to the fluxes, so they can consistently be set to zero. In a similar way we can use the equations of motion for the NS- and RR-forms,

$$\begin{aligned} \mathrm{d}B_2 &= -\frac{\mathrm{Im}(G_3)}{\mathrm{Im}(\tau)} \, , \\ \mathrm{d}C_6 &= H_3 \wedge C_4 - *_{10} \mathrm{Re}(G_3) \, , \\ \mathrm{d}C_8 &= H_3 \wedge C_6 - *_{10} \mathrm{Re}(\tau) \, , \end{aligned} \tag{A.14}$$

to write the NS- and RR-forms in terms of the position modulus and the fluxes. The non-vanishing components are

$$B_{12} = \frac{g_{\rm s}\sigma}{2i} \left[\bar{G}\phi - (S - \bar{D})\bar{\phi} \right] \ , \tag{A.15}$$

$$(C)_{12} = -\frac{g_{\rm s}\sigma}{2iZ} \left[\bar{G}\phi - (S + \bar{D})\bar{\phi} \right] \ , \tag{A.16}$$

$$(C)_{1\bar{1}2\bar{2}} = \frac{g_{\rm s}^2\sigma^2}{4Z} \left(|G|^2 + |S|^2 - |D|^2] \right) |\phi|^2 - \frac{g_{\rm s}^2\sigma^2}{4Z} \left(GS\bar{\phi}^2 + \bar{G}\bar{S}\phi^2 \right) \ . \tag{A.17}$$

[1]The difference in the power of the string coupling and warp factor with respect to the earlier more general formula is in the definition of $f(\Phi)$. In the end this does not matter.

Plugging this into the action gives

$$\mathcal{L}_{\text{DBI}} = -V_4\mu_7 g_{\text{s}}(1+\sigma^2 Z\partial_\mu\phi\partial^\mu\bar{\phi})\bigg[1+\frac{g_{\text{s}}\sigma^2}{4Z}\Big((|G|^2+|S|^2+|D|^2)|\phi|^2 - \bar{G}(\bar{S}-D)\phi^2 - G(S-\bar{D})\bar{\phi}^2\Big)\bigg]\,. \tag{A.18}$$

Note that the negative constant contribution is cancelled by the orientifold contribution. Next there is a contribution to the scalar potential coming from the CS action. For completeness, for a D7-brane it is given by

$$S_{\text{CS}} = \mu_7 \text{STr}\left(\int \mathrm{d}^8x P[-C_6\wedge\mathcal{F}_2 + C_8]\right)\,, \tag{A.19}$$

From which we find, after a similar computation as for the DBI action,

$$\mathcal{L}_{\text{CS}} = \frac{V_4\mu_7 g_{\text{s}}^2\sigma^2}{4Z}\left(-|\bar{G}\phi - S\bar{\phi}|^2 + |D|^2|\phi|^2\right)\,. \tag{A.20}$$

Let us redefine $\phi\to(V_4\mu_7 g_{\text{s}} Z\sigma^2)^{-1/2}\phi$ to obtain a canonical kinetic term at leading order in α'. After combining the CS and DBI contributions we find the following kinetic terms and potential for ϕ,

$$\mathcal{L}_{\text{kin}} = -\partial_\mu\phi\partial^\mu\bar{\phi}\left\{1+\frac{1}{4Z^2V_4\mu_7}\left[(|G|^2+|S|^2+|D|^2)|\phi|^2 - \bar{G}(\bar{S}-D)\phi^2 + \text{c.c.}\right]\right\} \tag{A.21a}$$

$$V = \frac{g_{\text{s}}}{4Z^2}\left(2|G^*\phi - S\bar{\phi}|^2 + \bar{G}D\phi^2 + G\bar{D}\bar{\phi}^2\right)\,, \tag{A.21b}$$

which is the action given in Equation (7.9). Finally, note that we have ignored $\det(Q_{mn})$ here since we consider a model with a D-flat direction.

Appendix B

Flattening of potentials

In this appendix we analyze the effect of canonically normalizing an effective Lagrangian similar to the one obtained in Section 5.1 from the DBI action. This is inspired by the desire to understand the effect of such a type of non-canonical kinetic term on inflationary dynamics. Similar Lagrangians were discussed in [163]. Here, we give general analytic formulae for the slow-roll parameters modified by the non-canonical kinetic terms in Equation (5.1) with monomial inflaton potentials. In all cases the non-canonical kinetic term leads to a flattening of the potential at large field values. This causes a substantial reduction of the tensor-to-scalar ratio r, bringing chaotic inflation models to better agreement with the recent Planck and BICEP data. The strength of the flattening is given by the parameter a, which in the main text is given by Equation (5.33). The relation between a and the string scale implies that for a strong flattening the string scale has to be low. The limit for $a \to \infty$ that we consider in this appendix can therefore not be consistently reached from the type of models that we consider in the main text.

We assume a single complex field and, in addition, make the simplifying assumption that only one of the components of ϕ is the inflaton field, which has a potential suitable for slow-roll, while the other component does not play a role. Moreover, we work in the slow-roll regime and thus neglect the fourth-order derivative terms of ϕ that are generally present. What we study is therefore a version of the Lagrangian (5.1) with a single real scalar field φ,

$$\mathcal{L} = -\frac{1}{2} f(\varphi)\partial_\mu\varphi\partial^\mu\varphi - V(\varphi)\,, \tag{B.1}$$

where

$$f(\varphi) = 1 + aV(\varphi)\,. \tag{B.2}$$

The effect of taking both degrees of freedom of the complex field ϕ into account was studied in Chapter 6 for a quadratic scalar potential.

As emphasized above, the DBI action yields a non-canonical kinetic term for the inflaton. In general, for Lagrangians with more than one scalar field it is not possible to do a global transformation to canonical frame. However, in single-field inflation models one can always recast the Lagrangian into canonical form, via the transformation

$$\frac{d\varphi}{d\psi} = \frac{1}{f^{1/2}(\varphi)} = \frac{1}{\sqrt{1 + aV(\varphi)}}\,, \tag{B.3}$$

which yields the following canonical variable

$$\psi = g(\varphi) = \int d\varphi f^{1/2}(\varphi)\,. \tag{B.4}$$

The Lagrangian, when written in terms of the canonically normalized field ψ, reads

$$\mathcal{L} = -\frac{1}{2}\partial_\mu\psi\partial^\mu\psi - V(g^{-1}(\psi))\,, \tag{B.5}$$

so that V implicitly depends on a. Interestingly, this process leads to a flatter potential. Specifically,

$$\frac{\partial V}{\partial\psi} = \frac{1}{f^{1/2}}\frac{\partial V}{\partial\varphi}\,. \tag{B.6}$$

Since $f > 1$ if $a > 0$ the potential in canonical variables has a smaller first derivative, i.e., a flattened slope, than the original one. A similar flattening from non-canonical kinetic terms has been discussed in the past in the context of string cosmology, for example in [163].

Provided $f > 0$, i.e., the scalar field is not a ghost, the study of the vacua can be performed by analyzing $V(\varphi)$ and neglecting the non-canonical nature of the field. The dynamics of the theory, however, crucially depend on the redefinition of the kinetic term. To quantify this effect we compute the CMB observables in terms of the canonically normalised field, first as general as possible and later applied to monomial potentials. We define the potential slow-roll parameters as usual,

$$\epsilon = \frac{1}{2}\left(\frac{V_\psi}{V}\right)^2\,, \qquad \eta = \frac{V_{\psi\psi}}{V}\,, \tag{B.7}$$

where subscripts denote derivatives. These can be rewritten in terms of φ as follows,

$$\epsilon = \frac{1}{2f}\left(\frac{V_\varphi}{V}\right)^2\,, \qquad \eta = \frac{1}{f}\frac{V_{\varphi\varphi}}{V} - \frac{aV}{f}\epsilon\,. \tag{B.8}$$

Evidently, the effect of the non-canonical kinetic terms is to reduce the slow-roll parameters. The scalar spectral index of the curvature perturbations is

$$\begin{aligned} n_{\rm s} &= 1 - 6\epsilon + 2\eta\,, \\ &= 1 - \frac{3}{f}\left(\frac{V_\varphi}{V}\right)^2 + \frac{2}{f}\frac{V_{\varphi\varphi}}{V} - \frac{aV}{f^2}\left(\frac{V_\varphi}{V}\right)^2\,, \\ &= \frac{1}{f}(1 - 6\epsilon|_{a=0} + 2\eta|_{a=0}) + \frac{aV}{f}(1-2\epsilon)\,, \end{aligned} \tag{B.9}$$

where in the last line only the second piece depends on a. The tensor-to-scalar ratio becomes

$$r = 16\,\epsilon = \frac{8}{f}\left(\frac{V_\varphi}{V}\right)^2\,. \tag{B.10}$$

Both $n_{\rm s}$ and r are to be evaluated at horizon exit, with field values denoted by ψ_* and φ_*. For N_e e-folds of exponential expansion one has

$$N_e = \int_{\psi_{\rm end}}^{\psi_*}\frac{1}{\sqrt{2\epsilon}}d\psi = \int_{\varphi_{\rm end}}^{\varphi_*} f\frac{V}{V_\varphi}d\varphi\,, \tag{B.11}$$

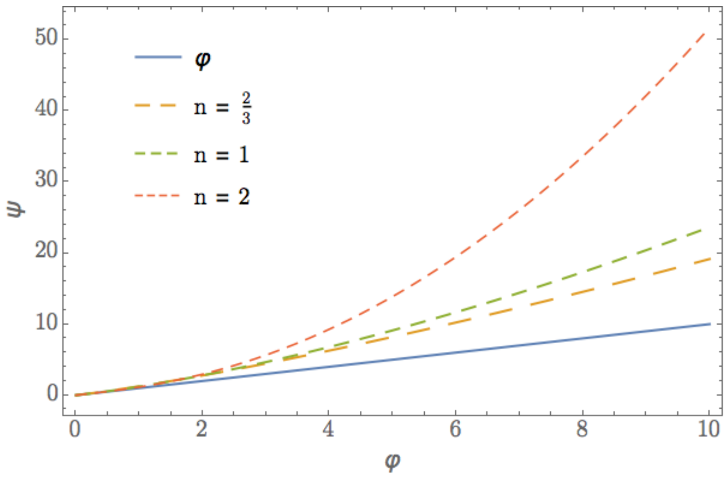

Figure B.1: $\psi(\varphi)$ for monomial potentials of various powers n.

which defines φ_* and ψ_*. The difference between φ_* and ψ_* and φ_{end} and ψ_{end}, respectively, is model-dependent. Therefore, in the following, we study simple examples and quantify the effect of the non-canonical normalization numerically. As discussed in Section 5.1, world-volume and background fluxes generate monomial potentials for Dp-brane position moduli. We therefore consider potentials of the type

$$V_n(\varphi) = v_0 \varphi^n \,, \tag{B.12}$$

with $n \in \mathbb{R}_+$. In this case we can specify $g(\varphi)$ in (B.4),

$$\psi = \frac{\varphi \left[2\sqrt{1 + a v_0 \varphi^n} + n \; {}_2F_1(\frac{1}{2}, \frac{1}{n}; 1 + \frac{1}{n}; -a v_0 \varphi^n)\right]}{2 + n} \,, \tag{B.13}$$

where ${}_2F_1(a, b; c; d)$ is the ordinary hypergeometric function. Note that ψ is real only when $-a v_0 \varphi^n < 1$ which is equivalent to the no-ghost regime. We illustrate the functional dependence of (B.13) in Figure B.1 for representative values of n. The crucial feature of this plot is that all curves lie above the $\psi = \varphi$ reference line, implying that one may schematically write $\psi = \varphi^{m(\varphi)}$ for some $m(\varphi) > 1$, or equivalently $\varphi = \psi^{1/m(\psi)}$. Since $m(\psi) > 1$ the change to a canonically normalized inflaton results in a monomial potential with suppressed power. The schematic form is $V \sim \psi^{n/m(\psi)}$, demonstrating that the effect of the non-canonical coupling in (B.2) is to cause a flattening of the monomial potential. Given the monotonicity of the scalar potentials we thus expect n_s to increase while r decreases.

While a proper field redefinition exists for all n there are only a few values for which we can use functional identities to rewrite (B.13) in a more familiar form,

$$n = 0: \quad \psi = \sqrt{1 + a v_0}\, \varphi + C \,, \tag{B.14}$$

$$n = 1: \quad \psi = \frac{2}{3 a v_0} \left[(1 + a v_0 \varphi)^{3/2} - 1\right] + C \,, \tag{B.15}$$

$$n = 2: \quad \psi = \frac{1}{2} \varphi \left[\sqrt{1 + a v_0 \varphi^2} + \frac{1}{\sqrt{a v_0} \varphi} \operatorname{arsinh}(\sqrt{a v_0} \varphi)\right] + C \,, \tag{B.16}$$

with $C = 0$ fixed by the requirement $V(0) = 0$, i.e., demanding the cosmological constant to vanish in the vacuum. Notice that, as expected, in the first case of a trivial

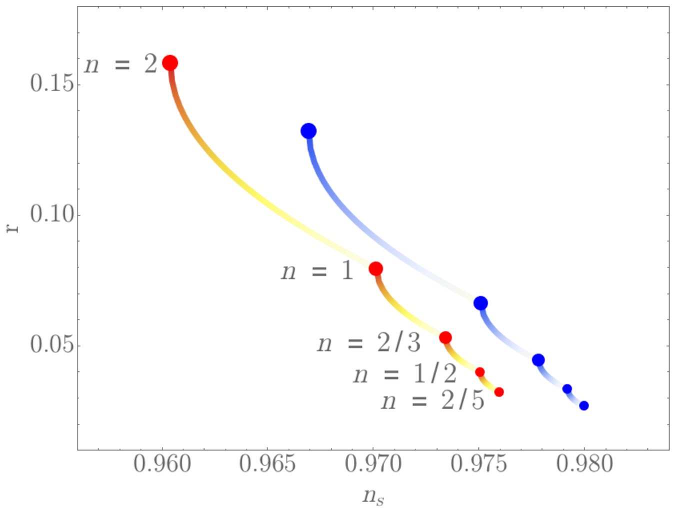

Figure B.2: CMB observables as predicted by the canonically normalized theory, with initial values $n = 2$, $n = 1$, $n = \frac{2}{3}$, and $n = \frac{2}{3}$. Darker color means lower values of av_0. For small av_0 the effect of the additional kinetic term is negligible, while for large av_0 the potential $V(\psi)$ approaches a monomial with power 1 for $n = 2$, $\frac{2}{3}$ for $n = 1$, $\frac{1}{2}$ for $n = \frac{2}{3}$, and so on. The two distinct lines correspond to $N_e = 50$ and $N_e = 60$, respectively.

potential the field redefinition is simply a rescaling of φ. But because V is constant this is also the most uninteresting case. On the other hand, the Lagrangian for $n = 2$ is an example of single-field Higgs-otic inflation for which there is no analytical form for the inverse $\varphi(\psi)$. The inverse exists only for $n = 1$. This means that, to study the implications in the most interesting cases, we must resort to either approximations or numerics. In the remainder of this appendix we use a combination of both.

We present the results of a numerical analysis of the CMB observables in Figure B.2, using $n = 2, 1, \frac{2}{3}, \frac{2}{5}$ as examples. We vary the value of av_0 to study the strength of the flattening effect. Remember that increasing av_0 means power suppression in the monomial potential of the canonically normalized inflaton. To better understand the numerical results let us first consider the limit of small av_0, so that $aV \ll 1$ and $f \simeq 1$ in (B.2). A first-order Taylor expansion in av_0 leads to simplified expressions for the slow-roll parameters,

$$\epsilon = \frac{1}{2}(1 - av_0\varphi^n)\frac{n^2}{\varphi^2}\,, \tag{B.17}$$

$$\eta = (1 - av_0\varphi^n)\left(\frac{n(n-1)}{\varphi^2} - \frac{n^2}{2}av_0\varphi^{n-2}\right)\,. \tag{B.18}$$

For φ_{end}, defined by $\epsilon(\varphi_{\text{end}}) = 1$, we find

$$\varphi_{\text{end}} = \frac{n}{\sqrt{2}}\left(1 - \frac{av_0}{2}\left(\frac{n}{\sqrt{2}}\right)^n\right)\,. \tag{B.19}$$

Furthermore, the observable modes of the fluctuations leave the horizon at

$$\varphi_* = \sqrt{x} - \frac{av_0}{n+2}\left(x^{\frac{n+1}{2}} + (n+1)\left(\frac{n}{\sqrt{2}}\right)^n\sqrt{x}\right)\,, \tag{B.20}$$

where we have introduced $x = 2nN_e + \frac{1}{2}n^2$. Using this value in the expanded slow-roll parameters leads to

$$\epsilon_* = \frac{n^2}{2x} + av_0 n^2 \left(\frac{n+1}{n+2} \left(\frac{n}{\sqrt{2}} \right)^{n+2} x^{-2} - \frac{n}{2n+4} x^{\frac{1}{2}n-1} \right) , \tag{B.21}$$

$$\eta_* = \frac{n^2 - n}{x} + av_0 n \left(\frac{2n^2+2}{n+2} \left(\frac{n}{\sqrt{2}} \right)^{n+2} x^{-2} - \frac{3n^2}{2n+4} x^{\frac{1}{2}n-1} \right) , \tag{B.22}$$

at horizon exit. The structure is remarkably similar in both cases, which can be traced back to the term proportional to aV/ϵ in the integral that determines N_e. We expect the term proportional to $x^{\frac{1}{2}n-1}$ to dominate in the brackets because $x \sim \mathcal{O}(100)$. Hence, both functions decrease as av_0 increases. In the limit of small av_0 this explains why the observables move towards the bottom-right in the n_s-r plane as the non-trivial kinetic term is amplified.

The limit of large av_0 is even more illuminating, cf. the related analyses in [133,163]. Assuming $f \simeq aV$ leads to

$$\varphi = \left(\frac{n+2}{2\sqrt{av_0}} \right)^{\frac{2}{n+2}} \psi^{\frac{2}{n+2}} , \tag{B.23}$$

as the inverse of (B.13). The corresponding scalar potential becomes

$$V(\psi) = v_0 \left(\frac{n+2}{2\sqrt{av_0}} \right)^{\frac{2n}{n+2}} \psi^{\frac{2n}{n+2}} . \tag{B.24}$$

Thus, we obtain an analytic result for the canonically normalized theory for any value of n. In particular, starting with a power of n in φ we obtain a power of $\frac{2n}{n+2} < n$ in the canonical field ψ. This explains another feature in Figure B.2: in the regime of large a, starting with $n = 2$ yields a monomial potential of power 1, $n = 1$ yields power $\frac{2}{3}$, $n = \frac{2}{3}$ leads to $V \sim \psi^{\frac{1}{2}}$, and so on. This is why the curves in the figure connect.

Appendix C

Fluxes and μ-terms

In order to motivate our choice for the superpotential given in Equation (7.1b), it is important to understand the microscopic origin of the μ-term. We give an example of the origin of the μ-term and we do not consider the most general case. In order to do this we provide here a toroidal type IIB orientifold example which we dualize to the corresponding type IIA model. This is the easiest path to show how certain closed-string fluxes not only contribute to the moduli superpotential but also generate μ-terms for charged matter fields. To be concrete, we consider a toroidal setting $T^2 \times T^2 \times T^2$ with the standard $O(3)$ orientifold projection with NS fluxes,

$$H_3 = -\sum_{i=1}^{3} a_i a_i \,, \tag{C.1}$$

which are expanded in the standard basis of three-forms on the torus, following the notation around Equation (3.40). The corresponding term in the Gukov-Vafa-Witten superpotential is

$$W = -\sum_{i} a_i S U_i \,, \tag{C.2}$$

where S is the complex dilaton and U_i are the complex structure moduli of the three tori. Consider now a D7-brane wrapping the first two tori and transverse to the third. We want to show that the same NS fluxes induce a μ-term for the adjoint position modulus Φ_3 which parametrizes the position of this D7-brane in the transverse torus.

The mirror of this model is a type IIA toroidal orientifold with an $O(6)$ projection and a D6-brane wrapping, for example, the cycle

$$\Pi_3 = (0,1)_1 \times (0,-1)_2 \times (1,0)_3 \,, \tag{C.3}$$

where (n, m) means that the D6-brane wraps n times around the x direction and m times around y. In the IIA mirror the NS fluxes of the IIB setup map into geometric fluxes, which we consider to be for simplicity $\omega^3_{45} = a_3$, in the notation of [15]. Let us consider now the Chern-Simons coupling on the world-volume of the D6-brane,

$$\int_{\Pi_3 \times M_4} C_3 \wedge F \wedge F \,. \tag{C.4}$$

In the presence of geometric fluxes ω^i_{ab} or, equivalently, on a twisted torus one replaces

$$F_{ab} \to F_{ab} + \omega^i_{ab} A_i = F_{ab} + (\omega.A)_{ab} \,, \tag{C.5}$$

so that, after putting the legs of C_3 in the Minkowski direction and integrating by parts, we get

$$F_4^0 \int_{\Pi_3} (\omega.A) \wedge A \,, \tag{C.6}$$

where F_4^0 is a type IIA Minkowski four-form. We thus see that there is a coupling of this four-form to a Wilson line bilinear controlled by the background ω. In particular, for the three-cycle above we find the action

$$a_3 F_4^0 \mathrm{Tr}(A_3)^2 = a_3 F_4^0 \mathrm{Tr}(\theta_3)^2 \,, \tag{C.7}$$

where θ_3 is a Wilson line scalar on the D6-brane. As discussed in Chapter 4, in type IIA F_4^0 couples to the real part of the superpotential, i.e.,

$$\mathcal{L} \subset F_4^0 \, \mathrm{Re}(W) \,, \tag{C.8}$$

so we can identify a contribution to the superpotential

$$\mathrm{Re}(W_{a_3}) = a_3 \mathrm{Tr}(\theta_3)^2 \,. \tag{C.9}$$

Holomorphicity allows us to complete the form of the induced superpotential. Along the third torus the D6-brane open-string modulus is a combination of the Wilson line θ_3 and position modulus ϕ_3, which parameterizes the motion in the direction perpendicular to the D6-brane in that complex plane,

$$\Phi_3 = \theta_3 + T_3 \phi_3 \,. \tag{C.10}$$

Hence, the piece of the superpotential proportional to a_3 is

$$W_{a_3} = a_3 \Phi_3^2 \,, \tag{C.11}$$

which is a μ-term. Let us check for completeness that the cross term in Φ_3^2 involving the coupling $\theta_3 \mathrm{Re}(T_3) \phi_3$ also appears in the action. In addition to the above CS coupling, there is a coupling on the twisted torus of the form

$$\int_{\Pi_3 \times M_4} C_3 \wedge F \wedge [\omega.B]_P \,, \tag{C.12}$$

where the subscript P indicates the pullback and B is the NS two-form. After partial integration we find a coupling for the above choice of D6-brane,

$$F_4^0 \int_{\Pi_3} A \wedge [\omega.B] \phi_3 = F_4^0 a_3 \mathrm{Tr}(\theta_3 b_3 \phi_3) \,, \tag{C.13}$$

where $T_3 = b_3 + iJ_3$ and ϕ_3 is the position modulus transverse to the D6-brane in the third complex plane. We observe that the required term is indeed present.

Going back to IIB, dualizing along the three horizontal directions of the torus, we end up with a D7-brane which is localized on the third torus and wraps the other two. The field Φ_3 is now mapped to a complex scalar which parameterizes the position on the third torus. We conclude that in IIB the standard NS flux a_3 gives rise not only to a moduli superpotential piece $W \sim a_3 S U_3$ but also to a contribution to the μ-term for the adjoint Φ_3.

Bibliography

[1] M. Planck "About irreversible radiation procresses," *S.-B Preuss. Akad. Wiss.* **479** (1899).

[2] J. Polchinski "String theory to the rescue," [1512.02477].

[3] E. Hubble "A relation between distance and radial velocity among extra-galactic nebulae," *Proc. Nat. Acad. Sci.* **15** (1929) 168–173.

[4] A. A. Penzias and R. W. Wilson "A Measurement of excess antenna temperature at 4080-Mc/s," *Astrophys. J.* **142** (1965) 419–421.

[5] **Supernova Search Team** Collaboration A. G. Riess *et al.* "Observational evidence from supernovae for an accelerating universe and a cosmological constant," *Astron. J.* **116** (1998) 1009–1038 [astro-ph/9805201].

[6] **Supernova Cosmology Project** Collaboration S. Perlmutter *et al.* "Measurements of Omega and Lambda from 42 high redshift supernovae," *Astrophys. J.* **517** (1999) 565–586 [astro-ph/9812133].

[7] C. L. Bennett, A. Banday, K. M. Gorski, G. Hinshaw, P. Jackson, P. Keegstra, A. Kogut, G. F. Smoot, D. T. Wilkinson, and E. L. Wright "Four year COBE DMR cosmic microwave background observations: Maps and basic results," *Astrophys. J.* **464** (1996) L1–L4 [astro-ph/9601067].

[8] **WMAP** Collaboration G. Hinshaw *et al.* "Nine-Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Cosmological Parameter Results," *Astrophys. J. Suppl.* **208** (2013) 19 [1212.5226].

[9] **Planck** Collaboration P. A. R. Ade *et al.* "Planck 2015 results. XIII. Cosmological parameters," *Astron. Astrophys.* **594** (2016) A13 [1502.01589].

[10] **BICEP2, Keck Array** Collaboration P. A. R. Ade *et al.* "BICEP2 / Keck Array VIII: Measurement of gravitational lensing from large-scale B-mode polarization," *Astrophys. J.* **833** (2016) no. 2, 228 [1606.01968].

[11] Z. Kermish *et al.* "The POLARBEAR Experiment," *Proc. SPIE Int. Soc. Opt. Eng.* **8452** (2012) 1C [1210.7768].

[12] A. Kogut *et al.* "The Primordial Inflation Explorer (PIXIE): A Nulling Polarimeter for Cosmic Microwave Background Observations," *JCAP* **1107** (2011) 025 [1105.2044].

[13] J. Polchinski *String theory. Vol. 1: An introduction to the bosonic string.* Cambridge University Press 2007.

[14] J. Polchinski *String theory. Vol. 2: Superstring theory and beyond.* Cambridge University Press 2007.

[15] L. E. Ibáñez and A. M. Uranga *String theory and particle physics: An introduction to string phenomenology.* Cambridge University Press 2012.

[16] E. Bergshoeff, R. Kallosh, T. Ortin, D. Roest, and A. Van Proeyen "New formulations of $D = 10$ supersymmetry and D8 - O8 domain walls," *Class. Quant. Grav.* **18** (2001) 3359–3382 [hep-th/0103233].

[17] E. Bergshoeff, H. J. Boonstra, and T. Ortin "S duality and dyonic p-brane solutions in Type II string theory," *Phys. Rev.* **D53** (1996) 7206–7212 [hep-th/9508091].

[18] M. Fukuma, T. Oota, and H. Tanaka "Comments on T dualities of Ramond-Ramond potentials on tori," *Prog. Theor. Phys.* **103** (2000) 425–446 [hep-th/9907132].

[19] T. W. Grimm and J. Louis "The Effective action of Type IIA Calabi-Yau orientifolds," *Nucl. Phys.* **B718** (2005) 153–202 [hep-th/0412277].

[20] T. W. Grimm and J. Louis "The Effective action of $\mathcal{N} = 1$ Calabi-Yau orientifolds," *Nucl. Phys.* **B699** (2004) 387–426 [hep-th/0403067].

[21] J.-P. Derendinger, C. Kounnas, P. M. Petropoulos, and F. Zwirner "Superpotentials in IIA compactifications with general fluxes," *Nucl. Phys.* **B715** (2005) 211–233 [hep-th/0411276].

[22] G. Villadoro and F. Zwirner "$\mathcal{N} = 1$ effective potential from dual Type IIA D6/O6 orientifolds with general fluxes," *JHEP* **06** (2005) 047 [hep-th/0503169].

[23] S. Gukov, C. Vafa, and E. Witten "CFT's from Calabi-Yau four folds," *Nucl. Phys.* **B584** (2000) 69–108 [hep-th/9906070]. [Erratum: Nucl. Phys.B608,477(2001)].

[24] S. B. Giddings, S. Kachru, and J. Polchinski "Hierarchies from fluxes in string compactifications," *Phys. Rev.* **D66** (2002) 106006 [hep-th/0105097].

[25] P. Candelas and X. de la Ossa "Moduli Space of Calabi-Yau Manifolds," *Nucl. Phys.* **B355** (1991) 455–481.

[26] J. Polchinski "Dirichlet Branes and Ramond-Ramond charges," *Phys. Rev. Lett.* **75** (1995) 4724–4727 [hep-th/9510017].

[27] R. C. Myers "Dielectric branes," *JHEP* **12** (1999) 022 [hep-th/9910053].

[28] A. A. Tseytlin "On non-Abelian generalization of Born-Infeld action in string theory," *Nucl. Phys.* **B501** (1997) 41–52 [hep-th/9701125].

[29] S. Kachru, R. Kallosh, A. D. Linde, and S. P. Trivedi "De Sitter vacua in string theory," *Phys. Rev.* **D68** (2003) 046005 [hep-th/0301240].

[30] V. Balasubramanian and P. Berglund "Stringy corrections to Kähler potentials, SUSY breaking, and the cosmological constant problem," *JHEP* **11** (2004) 085 [hep-th/0408054].

[31] J. P. Conlon, F. Quevedo, and K. Suruliz "Large-volume flux compactifications: Moduli spectrum and D3/D7 soft supersymmetry breaking," *JHEP* **08** (2005) 007 [hep-th/0505076].

[32] R. Kallosh and A. D. Linde "Landscape, the scale of SUSY breaking, and inflation," *JHEP* **12** (2004) 004 [hep-th/0411011].

[33] W. Buchmuller, E. Dudas, L. Heurtier, A. Westphal, C. Wieck, and M. W. Winkler "Challenges for Large-Field Inflation and Moduli Stabilization," *JHEP* **04** (2015) 058 [1501.05812].

[34] D. Baumann and L. McAllister *Inflation and String Theory.* Cambridge University Press 2015.

[35] A. H. Guth "The Inflationary Universe: A Possible Solution to the Horizon and Flatness Problems," *Phys. Rev.* **D23** (1981) 347–356.

[36] A. D. Linde "A New Inflationary Universe Scenario: A Possible Solution of the Horizon, Flatness, Homogeneity, Isotropy and Primordial Monopole Problems," *Phys. Lett.* **B108** (1982) 389–393.

[37] A. Albrecht and P. J. Steinhardt "Cosmology for Grand Unified Theories with Radiatively Induced Symmetry Breaking," *Phys. Rev. Lett.* **48** (1982) 1220–1223.

[38] A. R. Liddle and S. M. Leach "How long before the end of inflation were observable perturbations produced?," *Phys. Rev.* **D68** (2003) 103503 [astro-ph/0305263].

[39] J. M. Bardeen "Gauge Invariant Cosmological Perturbations," *Phys. Rev.* **D22** (1980) 1882–1905.

[40] J. M. Bardeen, P. J. Steinhardt, and M. S. Turner "Spontaneous Creation of Almost Scale - Free Density Perturbations in an Inflationary Universe," *Phys. Rev.* **D28** (1983) 679.

[41] D. Wands, K. A. Malik, D. H. Lyth, and A. R. Liddle "A New approach to the evolution of cosmological perturbations on large scales," *Phys. Rev.* **D62** (2000) 043527 [astro-ph/0003278].

[42] S. Weinberg "Adiabatic modes in cosmology," *Phys. Rev.* **D67** (2003) 123504 [astro-ph/0302326].

[43] R. L. Arnowitt, S. Deser, and C. W. Misner "The Dynamics of general relativity," *Gen. Rel. Grav.* **40** (2008) 1997–2027 [gr-qc/0405109].

[44] M. Sasaki "Large Scale Quantum Fluctuations in the Inflationary Universe," *Prog. Theor. Phys.* **76** (1986) 1036.

[45] V. F. Mukhanov "Quantum Theory of Gauge Invariant Cosmological Perturbations," *Sov. Phys. JETP* **67** (1988) 1297–1302. [Zh. Eksp. Teor. Fiz.94N7,1(1988)].

[46] S. Groot Nibbelink and B. J. W. van Tent "Density perturbations arising from multiple field slow roll inflation," [hep-ph/0011325].

[47] S. Groot Nibbelink and B. J. W. van Tent "Scalar perturbations during multiple field slow-roll inflation," *Class. Quant. Grav.* **19** (2002) 613–640 [hep-ph/0107272].

[48] Z. Lalak, D. Langlois, S. Pokorski, and K. Turzynski "Curvature and isocurvature perturbations in two-field inflation," *JCAP* **0707** (2007) 014 [0704.0212].

[49] A. Achucarro, J.-O. Gong, S. Hardeman, G. A. Palma, and S. P. Patil "Features of heavy physics in the CMB power spectrum," *JCAP* **1101** (2011) 030 [1010.3693].

[50] M. Sasaki and E. D. Stewart "A General analytic formula for the spectral index of the density perturbations produced during inflation," *Prog. Theor. Phys.* **95** (1996) 71–78 [astro-ph/9507001].

[51] **Planck** Collaboration P. A. R. Ade *et al.* "Planck 2015 results. XX. Constraints on inflation," *Astron. Astrophys.* **594** (2016) A20 [1502.02114].

[52] F. Vernizzi and D. Wands "Non-gaussianities in two-field inflation," *JCAP* **0605** (2006) 019 [astro-ph/0603799].

[53] S. Yokoyama, T. Suyama, and T. Tanaka "Primordial Non-Gaussianity in Multi-Scalar Slow-Roll Inflation," *JCAP* **0707** (2007) 013 [0705.3178].

[54] C. P. Burgess, M. Cicoli, and F. Quevedo "String Inflation After Planck 2013," *JCAP* **1311** (2013) 003 [1306.3512].

[55] M. Cicoli and F. Quevedo "String moduli inflation: An overview," *Class. Quant. Grav.* **28** (2011) 204001 [1108.2659].

[56] C. P. Burgess and L. McAllister "Challenges for String Cosmology," *Class. Quant. Grav.* **28** (2011) 204002 [1108.2660].

[57] A. Westphal "String Cosmology Large-Field Inflation in String Theory," *Adv. Ser. Direct. High Energy Phys.* **22** (2015) 351–384.

[58] D. H. Lyth "What would we learn by detecting a gravitational wave signal in the cosmic microwave background anisotropy?," *Phys. Rev. Lett.* **78** (1997) 1861–1863 [hep-ph/9606387].

[59] J. P. Conlon and F. Quevedo "Kähler moduli inflation," *JHEP* **01** (2006) 146 [hep-th/0509012].

[60] J. R. Bond, L. Kofman, S. Prokushkin, and P. M. Vaudrevange "Roulette inflation with Kähler moduli and their axions," *Phys. Rev.* **D75** (2007) 123511 [hep-th/0612197].

[61] M. Cicoli, C. P. Burgess, and F. Quevedo "Fibre Inflation: Observable Gravity Waves from IIB String Compactifications," *JCAP* **0903** (2009) 013 [0808.0691].

[62] J. J. Blanco-Pillado, C. P. Burgess, J. M. Cline, C. Escoda, M. Gomez-Reino, R. Kallosh, A. D. Linde, and F. Quevedo "Racetrack inflation," *JHEP* **11** (2004) 063 [hep-th/0406230].

[63] S. Dimopoulos, S. Kachru, J. McGreevy, and J. G. Wacker "N-flation," *JCAP* **0808** (2008) 003 [hep-th/0507205].

[64] E. Silverstein and A. Westphal "Monodromy in the CMB: Gravity Waves and String Inflation," *Phys. Rev.* **D78** (2008) 106003 [0803.3085].

[65] L. McAllister, E. Silverstein, and A. Westphal "Gravity Waves and Linear Inflation from Axion Monodromy," *Phys. Rev.* **D82** (2010) 046003 [0808.0706].

[66] G. R. Dvali, Q. Shafi, and S. Solganik "D-brane inflation," in *4th European Meeting From the Planck Scale to the Electroweak Scale (Planck 2001) La Londe les Maures, Toulon, France, May 11-16, 2001.* 2001. [hep-th/0105203].

[67] C. P. Burgess, M. Majumdar, D. Nolte, F. Quevedo, G. Rajesh, and R.-J. Zhang "The Inflationary brane anti-brane universe," *JHEP* **07** (2001) 047 [hep-th/0105204].

[68] A. Avgoustidis, D. Cremades, and F. Quevedo "Wilson line inflation," *Gen. Rel. Grav.* **39** (2007) 1203–1234 [hep-th/0606031].

[69] K. Dasgupta, C. Herdeiro, S. Hirano, and R. Kallosh "D3 / D7 inflationary model and M theory," *Phys. Rev.* **D65** (2002) 126002 [hep-th/0203019].

[70] L. E. Ibáñez, F. Marchesano, and I. Valenzuela "Higgs-otic Inflation and String Theory," *JHEP* **01** (2015) 128 [1411.5380].

[71] D. Escobar, A. Landete, F. Marchesano, and D. Regalado "D6-branes and axion monodromy inflation," *JHEP* **03** (2016) 113 [1511.08820].

[72] A. Landete, F. Marchesano, and C. Wieck "Challenges for D-brane large-field inflation with stabilizer fields," *JHEP* **09** (2016) 119 [1607.01680].

[73] J. D. Brown and C. Teitelboim "Neutralization of the Cosmological Constant by Membrane Creation," *Nucl. Phys.* **B297** (1988) 787–836.

[74] R. Bousso and J. Polchinski "Quantization of four form fluxes and dynamical neutralization of the cosmological constant," *JHEP* **06** (2000) 006 [hep-th/0004134].

[75] N. Kaloper and L. Sorbo "A Natural Framework for Chaotic Inflation," *Phys. Rev. Lett.* **102** (2009) 121301 [0811.1989].

[76] N. Kaloper, A. Lawrence, and L. Sorbo "An Ignoble Approach to Large Field Inflation," *JCAP* **1103** (2011) 023 [1101.0026].

[77] E. Dudas "Three-form multiplet and Inflation," *JHEP* **12** (2014) 014 [1407.5688].

[78] F. Marchesano, G. Shiu, and A. M. Uranga "F-term Axion Monodromy Inflation," *JHEP* **09** (2014) 184 [1404.3040].

[79] M. Berg, E. Pajer, and S. Sjors "Dante's Inferno," *Phys. Rev.* **D81** (2010) 103535 [0912.1341].

[80] L. E. Ibáñez and I. Valenzuela "BICEP2, the Higgs Mass and the SUSY-breaking Scale," *Phys. Lett.* **B734** (2014) 354–357 [1403.6081].

[81] E. Palti and T. Weigand "Towards large r from [p, q]-inflation," *JHEP* **04** (2014) 155 [1403.7507].

[82] R. Blumenhagen and E. Plauschinn "Towards Universal Axion Inflation and Reheating in String Theory," *Phys. Lett.* **B736** (2014) 482–487 [1404.3542].

[83] N. Kaloper and A. Lawrence "Natural chaotic inflation and ultraviolet sensitivity," *Phys. Rev.* **D90** (2014) no. 2, 023506 [1404.2912].

[84] L. E. Ibáñez and I. Valenzuela "The inflaton as an MSSM Higgs and open string modulus monodromy inflation," *Phys. Lett.* **B736** (2014) 226–230 [1404.5235].

[85] A. Hebecker, S. C. Kraus, and L. T. Witkowski "D7-Brane Chaotic Inflation," *Phys. Lett.* **B737** (2014) 16–22 [1404.3711].

[86] M. Arends, A. Hebecker, K. Heimpel, S. C. Kraus, D. Lust, C. Mayrhofer, C. Schick, and T. Weigand "D7-Brane Moduli Space in Axion Monodromy and Fluxbrane Inflation," *Fortsch. Phys.* **62** (2014) 647–702 [1405.0283].

[87] L. McAllister, E. Silverstein, A. Westphal, and T. Wrase "The Powers of Monodromy," *JHEP* **09** (2014) 123 [1405.3652].

[88] S. Franco, D. Galloni, A. Retolaza, and A. Uranga "On axion monodromy inflation in warped throats," *JHEP* **02** (2015) 086 [1405.7044].

[89] R. Blumenhagen, D. Herschmann, and E. Plauschinn "The Challenge of Realizing F-term Axion Monodromy Inflation in String Theory," *JHEP* **01** (2015) 007 [1409.7075].

[90] H. Hayashi, R. Matsuda, and T. Watari "Issues in Complex Structure Moduli Inflation," [1410.7522].

[91] A. Hebecker, P. Mangat, F. Rompineve, and L. T. Witkowski "Tuning and Backreaction in F-term Axion Monodromy Inflation," *Nucl. Phys.* **B894** (2015) 456–495 [1411.2032].

[92] I. Garca-Etxebarria, T. W. Grimm, and I. Valenzuela "Special Points of Inflation in Flux Compactifications," *Nucl. Phys.* **B899** (2015) 414–443 [1412.5537].

[93] A. Retolaza, A. M. Uranga, and A. Westphal "Bifid Throats for Axion Monodromy Inflation," *JHEP* **07** (2015) 099 [1504.02103].

[94] D. Escobar, A. Landete, F. Marchesano, and D. Regalado "Large field inflation from D-branes," *Phys. Rev.* **D93** (2016) no. 8, 081301 [1505.07871].

[95] S. Bielleman, L. E. Ibáñez, F. G. Pedro, and I. Valenzuela "Multifield Dynamics in Higgs-otic Inflation," *JHEP* **01** (2016) 128 [1505.00221].

[96] J. Louis and A. Micu "Type 2 theories compactified on Calabi-Yau threefolds in the presence of background fluxes," *Nucl. Phys.* **B635** (2002) 395–431 [hep-th/0202168].

[97] O. DeWolfe, A. Giryavets, S. Kachru, and W. Taylor "Type IIA moduli stabilization," *JHEP* **07** (2005) 066 [hep-th/0505160].

[98] P. G. Cámara, A. Font, and L. E. Ibáñez "Fluxes, moduli fixing and MSSM-like vacua in a simple IIA orientifold," *JHEP* **09** (2005) 013 [hep-th/0506066].

[99] P. Binetruy, F. Pillon, G. Girardi, and R. Grimm "The Three form multiplet in supergravity," *Nucl. Phys.* **B477** (1996) 175–202 [hep-th/9603181].

[100] K. Groh, J. Louis, and J. Sommerfeld "Duality and Couplings of 3-Form-Multiplets in $\mathcal{N}=1$ Supersymmetry," *JHEP* **05** (2013) 001 [1212.4639].

[101] D. G. Cerdeno, A. Knauf, and J. Louis "A Note on effective $\mathcal{N}=1$ super Yang-Mills theories versus lattice results," *Eur. Phys. J.* **C31** (2003) 415–420 [hep-th/0307198].

[102] H. Jockers and J. Louis "The Effective action of D7-branes in $\mathcal{N}=1$ Calabi-Yau orientifolds," *Nucl. Phys.* **B705** (2005) 167–211 [hep-th/0409098].

[103] J. Shelton, W. Taylor, and B. Wecht "Nongeometric flux compactifications," *JHEP* **10** (2005) 085 [hep-th/0508133].

[104] G. Aldazabal, P. G. Cámara, A. Font, and L. E. Ibáñez "More dual fluxes and moduli fixing," *JHEP* **05** (2006) 070 [hep-th/0602089].

[105] S. J. Gates, Jr. "Super p-form gauge superfields," *Nucl. Phys.* **B184** (1981) 381–390.

[106] S. J. Gates, Jr. and W. Siegel "Variant superfield representations," *Nucl. Phys.* **B187** (1981) 389–396.

[107] B. A. Ovrut and D. Waldram "Membranes and three-form supergravity," *Nucl. Phys.* **B506** (1997) 236–266 [hep-th/9704045].

[108] P. Binetruy, G. Girardi, and R. Grimm "Supergravity couplings: A Geometric formulation," *Phys. Rept.* **343** (2001) 255–462 [hep-th/0005225].

[109] G. Girardi, R. Grimm, B. Labonne, and J. Orloff "Correspondence between 3-form and non-minimal multiplet in supersymmetry," *Eur. Phys. J.* **C55** (2008) 95–99 [0712.1923].

[110] B. B. Deo and S. J. Gates "Comments on nonminimal $\mathcal{N}=1$ scalar multiplets," *Nucl. Phys.* **B254** (1985) 187–200.

[111] I. A. Bandos and C. Meliveo "Three form potential in (special) minimal supergravity superspace and supermembrane supercurrent," *J. Phys. Conf. Ser.* **343** (2012) 012012 [1107.3232].

[112] H. Nishino and S. Rajpoot "Alternative auxiliary fields for chiral multiplets," *Phys. Rev.* **D80** (2009) 127701.

[113] P. G. Cámara, L. E. Ibáñez, and I. Valenzuela "Flux-induced Soft Terms on Type IIB/F-theory Matter Curves and Hypercharge Dependent Scalar Masses," *JHEP* **06** (2014) 119 [1404.0817].

[114] P. G. Cámara, L. E. Ibáñez, and A. M. Uranga "Flux induced SUSY breaking soft terms," *Nucl. Phys.* **B689** (2004) 195–242 [hep-th/0311241].

[115] M. Grana, T. W. Grimm, H. Jockers, and J. Louis “Soft supersymmetry breaking in Calabi-Yau orientifolds with D-branes and fluxes,” *Nucl. Phys.* **B690** (2004) 21–61 [hep-th/0312232].

[116] D. Lust, S. Reffert, and S. Stieberger “Flux-induced soft supersymmetry breaking in chiral Type IIB orientifolds with D3 / D7-branes,” *Nucl. Phys.* **B706** (2005) 3–52 [hep-th/0406092].

[117] P. G. Cámara, L. E. Ibáñez, and A. M. Uranga “Flux-induced SUSY-breaking soft terms on D7-D3 brane systems,” *Nucl. Phys.* **B708** (2005) 268–316 [hep-th/0408036].

[118] F. Carta, F. Marchesano, W. Staessens, and G. Zoccarato “Open string multi-branched and Kähler potentials,” *JHEP* **09** (2016) 062 [1606.00508].

[119] J. Wess and J. Bagger *Supersymmetry and supergravity.* 1992.

[120] M. Grana “D3-brane action in a supergravity background: The Fermionic story,” *Phys. Rev.* **D66** (2002) 045014 [hep-th/0202118].

[121] L. Aparicio, D. G. Cerdeno, and L. E. Ibáñez “Modulus-dominated SUSY-breaking soft terms in F-theory and their test at LHC,” *JHEP* **07** (2008) 099 [0805.2943].

[122] S. Cecotti, S. Ferrara, and L. Girardello “Structure of the Scalar Potential in General $\mathcal{N} = 1$ Higher Derivative Supergravity in Four-dimensions,” *Phys. Lett.* **B187** (1987) 321–326.

[123] S. Cecotti, S. Ferrara, and L. Girardello “Flat Potentials in Higher Derivative Supergravity,” *Phys. Lett.* **B187** (1987) 327–334.

[124] I. Antoniadis, E. Dudas, and D. M. Ghilencea “Supersymmetric Models with Higher Dimensional Operators,” *JHEP* **03** (2008) 045 [0708.0383].

[125] J. Khoury, J.-L. Lehners, and B. Ovrut “Supersymmetric P(X,ϕ) and the Ghost Condensate,” *Phys. Rev.* **D83** (2011) 125031 [1012.3748].

[126] D. Baumann and D. Green “Supergravity for Effective Theories,” *JHEP* **03** (2012) 001 [1109.0293].

[127] M. Koehn, J.-L. Lehners, and B. A. Ovrut “Higher-Derivative Chiral Superfield Actions Coupled to $\mathcal{N} = 1$ Supergravity,” *Phys. Rev.* **D86** (2012) 085019 [1207.3798].

[128] F. Farakos and A. Kehagias “Emerging Potentials in Higher-Derivative Gauged Chiral Models Coupled to $\mathcal{N} = 1$ Supergravity,” *JHEP* **11** (2012) 077 [1207.4767].

[129] M. Koehn, J.-L. Lehners, and B. A. Ovrut “DBI Inflation in $\mathcal{N} = 1$ Supergravity,” *Phys. Rev.* **D86** (2012) 123510 [1208.0752].

[130] M. Koehn, J.-L. Lehners, and B. Ovrut “Ghost condensate in $\mathcal{N} = 1$ supergravity,” *Phys. Rev.* **D87** (2013) no. 6, 065022 [1212.2185].

[131] F. Farakos, S. Ferrara, A. Kehagias, and M. Porrati "Supersymmetry Breaking by Higher Dimension Operators," *Nucl. Phys.* **B879** (2014) 348–369 [1309.1476].

[132] R. Gwyn and J.-L. Lehners "Non-Canonical Inflation in Supergravity," *JHEP* **05** (2014) 050 [1402.5120].

[133] S. Aoki and Y. Yamada "Impacts of supersymmetric higher derivative terms on inflation models in supergravity," *JCAP* **1507** (2015) no. 07, 020 [1504.07023].

[134] D. Ciupke, J. Louis, and A. Westphal "Higher-Derivative Supergravity and Moduli Stabilization," *JHEP* **10** (2015) 094 [1505.03092].

[135] E. Dudas and D. M. Ghilencea "Effective operators in SUSY, superfield constraints and searches for a UV completion," *JHEP* **06** (2015) 124 [1503.08319].

[136] G. Lopes Cardoso, D. Lust, and T. Mohaupt "Moduli spaces and target space duality symmetries in (0,2) Z(N) orbifold theories with continuous Wilson lines," *Nucl. Phys.* **B432** (1994) 68–108 [hep-th/9405002].

[137] I. Antoniadis, E. Gava, K. S. Narain, and T. R. Taylor "Effective mu term in superstring theory," *Nucl. Phys.* **B432** (1994) 187–204 [hep-th/9405024].

[138] A. Brignole, L. E. Ibáñez, C. Munoz, and C. Scheich "Some issues in soft SUSY breaking terms from dilaton / moduli sectors," *Z. Phys.* **C74** (1997) 157–170 [hep-ph/9508258].

[139] A. Brignole, L. E. Ibáñez, and C. Munoz "Orbifold induced mu term and electroweak symmetry breaking," *Phys. Lett.* **B387** (1996) 769–774 [hep-ph/9607405].

[140] D. Ciupke "Scalar Potential from Higher Derivative $\mathcal{N} = 1$ Superspace," [1605.00651].

[141] M. Shaposhnikov and C. Wetterich "Asymptotic safety of gravity and the Higgs boson mass," *Phys. Lett.* **B683** (2010) 196–200 [0912.0208].

[142] L. J. Hall and Y. Nomura "A Finely-Predicted Higgs Boson Mass from A Finely-Tuned Weak Scale," *JHEP* **03** (2010) 076 [0910.2235].

[143] C. Liu and Z.-h. Zhao "θ_{13} and the Higgs mass from high scale supersymmetry," *Commun. Theor. Phys.* **59** (2013) 467–471 [1205.3849].

[144] L. J. Hall and Y. Nomura "Grand Unification and Intermediate Scale Supersymmetry," *JHEP* **02** (2014) 129 [1312.6695].

[145] M. Ibe, S. Matsumoto, and T. T. Yanagida "Flat Higgs Potential from Planck Scale Supersymmetry Breaking," *Phys. Lett.* **B732** (2014) 214–217 [1312.7108].

[146] A. Hebecker, A. K. Knochel, and T. Weigand "A Shift Symmetry in the Higgs Sector: Experimental Hints and Stringy Realizations," *JHEP* **06** (2012) 093 [1204.2551].

[147] L. E. Ibáñez, F. Marchesano, D. Regalado, and I. Valenzuela "The Intermediate Scale MSSM, the Higgs Mass and F-theory Unification," *JHEP* **07** (2012) 195 [1206.2655].

[148] A. Hebecker, A. K. Knochel, and T. Weigand "The Higgs mass from a String-Theoretic Perspective," *Nucl. Phys.* **B874** (2013) 1–35 [1304.2767].

[149] W. Buchmuller, C. Wieck, and M. W. Winkler "Supersymmetric Moduli Stabilization and High-Scale Inflation," *Phys. Lett.* **B736** (2014) 237–240 [1404.2275].

[150] E. Dudas and C. Wieck "Moduli backreaction and supersymmetry breaking in string-inspired inflation models," *JHEP* **10** (2015) 062 [1506.01253].

[151] F. Baume and E. Palti "Backreacted Axion Field Ranges in String Theory," *JHEP* **08** (2016) 043 [1602.06517].

[152] I. Valenzuela "Backreaction Issues in Axion Monodromy and Minkowski 4-forms," [1611.00394].

[153] F. Ruehle and C. Wieck "One-loop Pfaffians and large-field inflation in string theory," *Phys. Lett.* **B769** (2017) 289–298 [1702.00420].

[154] D. Baumann, A. Dymarsky, S. Kachru, I. R. Klebanov, and L. McAllister "Compactification Effects in D-brane Inflation," *Phys. Rev. Lett.* **104** (2010) 251602 [0912.4268].

[155] A. Landete, F. Marchesano, G. Shiu, and G. Zoccarato "Flux Flattening in Axion Monodromy Inflation," [1703.09729].

[156] R. Blumenhagen, A. Font, M. Fuchs, D. Herschmann, and E. Plauschinn "Towards Axionic Starobinsky-like Inflation in String Theory," *Phys. Lett.* **B746** (2015) 217–222 [1503.01607].

[157] R. Blumenhagen, I. Valenzuela, and F. Wolf "The Swampland Conjecture and F-term Axion Monodromy Inflation," [1703.05776].

[158] F. Denef and M. R. Douglas "Distributions of flux vacua," *JHEP* **05** (2004) 072 [hep-th/0404116].

[159] M. Cicoli, J. P. Conlon, A. Maharana, and F. Quevedo "A Note on the Magnitude of the Flux Superpotential," *JHEP* **01** (2014) 027 [1310.6694].

[160] V. Balasubramanian, P. Berglund, J. P. Conlon, and F. Quevedo "Systematics of moduli stabilisation in Calabi-Yau flux compactifications," *JHEP* **03** (2005) 007 [hep-th/0502058].

[161] A. Westphal "de Sitter string vacua from Kähler uplifting," *JHEP* **03** (2007) 102 [hep-th/0611332].

[162] L. E. Ibáñez and I. Valenzuela "The Higgs Mass as a Signature of Heavy SUSY," *JHEP* **05** (2013) 064 [1301.5167].

[163] G. Gur-Ari "Brane Inflation and Moduli Stabilization on Twisted Tori," *JHEP* **01** (2014) 179 [1310.6787].