Generic model of heavy new physics for the kaon systems

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Abstract. Various precision observables, such as flavour changing decays, are loop induced in the standard model and their renormalisation involves cancellations between purely bosonic and fermionic interactions. Here, I will show how perturbative unitarity constraints can be used to derive renormalised matching conditions for generic theories. These general results comprise the matching conditions for specific models that address current flavour anomalies and can be used for phenomenological analyses for rare and CP violating kaon decays.

1. Introduction

Semi-leptonic and leptonic CP violating rare kaon decays are a very sensitive probe of short-distance physics, both within the standard model (SM) and beyond. The $K^+ \to \pi^+ \nu\bar{\nu}$ process is among the most encouraging modes to search for nonstandard signals in flavour physics, and it tests heavier mass scales than other rare meson decays. The recently updated SM prediction for $\mathcal{B}(K^+ \to \pi^+ \nu\bar{\nu}) = 7.73(61) \times 10^{-11}$ [1], and the experimental measurement is $\mathcal{B}(K^+ \to \pi^+ \nu\bar{\nu}) = (10.6^{+4.0}_{-3.4}|_{\rm stat} \pm 0.9_{\rm syst}) \times 10^{-11}$ at 68% CL [2].

The SM contribution to $K \to \pi\nu\bar{\nu}$ decays starts at the one-loop level; hence, it is reasonable to consider that contributions of physics beyond the SM, if present, are also loop-induced. In the SM, K-meson decays are well described by the weak effective theory (WET). Similarly, the SM extensions with heavy particles, whose masses are around/above the electroweak scale, can be studied in the WET framework. However, matching of a new model onto WET in a proper way, ensuring its gauge-invariance and finiteness, might be tedious work to do.

This work is based on Ref. [3] where we consider an extension of the SM in an arbitrary number of bosons and fermions under the assumption that the theory satisfies perturbative unitarity and, thus, renormalisability. Once the particle content is defined, the resulting WET Lagrangian can immediately be expressed. In our setup, we apply Slavnov-Taylor Identities (STIs), which are derived in Ref. [4], to ensure that the Wilson coefficients are free from UV divergencies and gauge-fixing parameters.

The structure of the paper is as follows: In Sec. 2, we present the relevant effective Lagrangian together with generic interaction Lagrangian of the new fields. Here, we also discuss briefly about our model constraints. Then, we give analytic expressions for the four-fermion Wilson coefficients in Sec. 3. To illustrate the application of our setup, we do simple analysis for the Littlest Higgs model with T-parity in Sec. 4.

2. Generic model and effective Lagrangian

In this work, we are aimed to provide a form of the WET Lagrangian that describes leptonic, semileptonic and radiative $B_{d/s}$, and K meson decays for a generic perturbatively unitary model. The five-flavour weak effective Lagrangian for the $d_j \to d_i$ transition is read

$$\begin{aligned}\delta\mathcal{L}_{\Delta F=1} = \frac{1}{16\pi^2} &\sum_{\substack{\ell\in\{e,\mu,\tau\}\\ \sigma,\sigma'\in\{L,R\}}} C^{ij\ell}_{\sigma\sigma'} \left(\bar{d}_i\gamma^\mu P_\sigma d_j\right)\left(\bar{\ell}\gamma_\mu P_{\sigma'}\ell\right) \\ &+ \frac{1}{16\pi^2}\sum_{\sigma\in\{L,R\}} D^{ij}_\sigma \bar{d}_i\sigma^{\mu\nu}P_\sigma d_j F_{\mu\nu} + \text{h.c.}\,,\end{aligned} \tag{1}$$

where $d_i = d, s, b$ denote the down-type quark fields and ℓ the lepton fields, and $F_{\mu\nu}$ is the electromagnetic tensor. $P_{L/R} \equiv (1 \pm \gamma_5)/2$ are the chirality projection operators, and σ/σ' denote the chiralities of the incoming fermions.

In order to derive explicit expressions for the Wilson coefficients, $C^{ij\ell}_{\sigma\sigma'}$ and D^{ij}_σ, we consider a generic interaction Lagrangian of fermions (ψ), physical scalars (h), and vector bosons (V_μ) in the following form

$$\begin{aligned}\mathcal{L}_{\text{int}} = &\sum_{f_1f_2s_1\sigma} y^\sigma_{s_1\bar{f}_1f_2} h_{s_1}\bar{\psi}_{f_1}P_\sigma\psi_{f_2} + \sum_{f_1f_2v_1\sigma} g^\sigma_{v_1\bar{f}_1f_2}V_{v_1,\mu}\bar{\psi}_{f_1}\gamma^\mu P_\sigma\psi_{f_2} \\ &+\frac{i}{6}\sum_{v_1v_2v_3} g_{v_1v_2v_3}\Big(V_{v_1,\mu}V_{v_2,\nu}\,\partial^{[\mu}V^{\nu]}_{v_3} + V_{v_3,\mu}V_{v_1,\nu}\,\partial^{[\mu}V^{\nu]}_{v_2} + V_{v_2,\mu}V_{v_3,\nu}\,\partial^{[\mu}V^{\nu]}_{v_1}\Big) \\ &+\frac{1}{2}\sum_{v_1v_2s_1} g_{v_1v_2s_1}\,V_{v_1,\mu}V^\mu_{v_2}h_{s_1} - \frac{i}{2}\sum_{v_1s_1s_2} g_{v_1s_1s_2}\,V^\mu_{v_1}\Big(h_{s_1}\,\partial_\mu h_{s_2} - \left(\partial_\mu h_{s_1}\right)h_{s_2}\Big).\end{aligned} \tag{2}$$

The indices f_i, s_i, and v_i are used to indicate fermion, scalar, and vector fields, correspondingly, and run over all particles in a given multiplet of the gauge group $SU(3)_{\text{color}} \times U(1)_{\text{EM}}$. We suppressed spinor indices. Here, we do not present the non-interacting part of the Lagrangian, which include the standard kinetic terms and gauge-fixing terms. In our setup, massive vectors are taken in the renormalisable gauge, R_ξ, while for the photon we use 't Hooft-Feynman gauge.

In order to build a renormalisable field theory with the Lagrangian of Eq. (2), we impose constraints from the *Slavnov-Taylor Identities* (STIs) derived in Ref. [4] from the vanishing Becchi-Rouet-Stora-Tyutin symmetry. In our one-loop calculation, we use STI relations to cancel UV divergencies and to combine appropriate diagrams that leads to the cancellation of the gauge-fixing parameters in the final result. Moreover, with the help of the STIs we determine the would-be Goldstone boson couplings in terms of the physical coupling constants.

3. Neutral-Current Wilson Coefficients

The Wilson coefficients in Eq. (1) are defined as functions of the coupling constants of the interaction Lagrangian in Eq.(2) and the related masses. In order to determine them we calculate appropriate Green's functions: The dipole coefficients D^{ij}_σ receive contributions only from the photon-penguin diagrams, while the photon-penguin (via equation of motion), Z-penguin and box diagrams contribute to the current-current coefficients $C^{ij\ell}_{\sigma\sigma'}$.

Here, we focus only on the $C^{ij\ell}_{\sigma\sigma'}$ coefficients with $\sigma = L$ that is

$$C^{ij\ell}_{L\sigma} = v^{ij\ell}_{L\sigma} + m^{ij\ell}_{L\sigma} + s^{ij\ell}_{L\sigma},$$

where a sum of diagrams that in the loop comprise only heavy vectors and fermions, expressed by $v^{ij\ell}_{L\sigma}$, heavy vectors, heavy scalars and fermions, expressed by $m^{ij\ell}_{L\sigma}$, and heavy scalars and fermions, expressed by $s^{ij\ell}_{L\sigma}$.

The Z-penguin diagrams in the generically extended SM are calculated in Ref. [4], where authors use STI sum-rules for the renormalisation. In Ref. [3], we show that STIs can also be used to combine the penguin and box contributions which leads to the gauge-invariance of the $C^{ij\ell}_{\sigma\sigma'}$. Specifically, we use the Eq. (3.8) from Ref. [4] to the interaction of leptons with vectors,

$$\sum_Z g^{\sigma}_{Z\bar{\ell}\ell} g_{Zv_2\bar{v}_1} = -\delta_{\bar{v}_1 v_2} g^{\sigma}_{\gamma\bar{\ell}\ell} g_{\gamma v_2 \bar{v}_1} - \sum_{f_3} \left(g^{\sigma}_{\bar{v}_1 \bar{\ell} f_3} g^{\sigma}_{v_2 \bar{f}_3 \ell} - g^{\sigma}_{v_2 \bar{\ell} f_3} g^{\sigma}_{\bar{v}_1 \bar{f}_3 \ell} \right),$$

where the sum over Z can be understood as a summation over heavy neutral vectors, e.g., Z'-models. The above equation allows to combine Z-penguin results with the photon-penguin and box diagrams. Hence, we obtain generalised forms of the penguin-box functions - X,Y,Z of. [5].

4. Kaon Decay in the Littlest Higgs model with T-parity

To exemplify our formalism we apply it to the Littlest Higgs model with T-parity (LHT) [6], and we do naive analysis on the $K \to \pi\nu\bar{\nu}$ decays in this model.

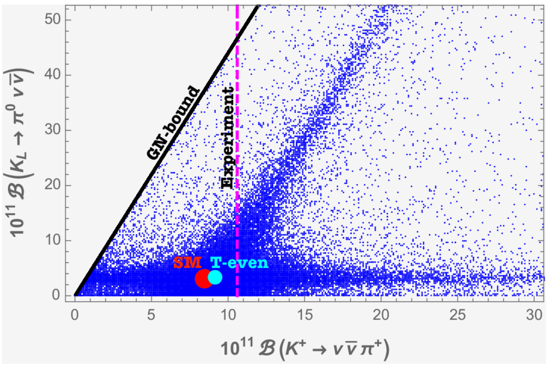

Figure 1: Correlation between branching ratios of $K^+ \to \pi^+\nu\bar{\nu}$ and $K_L \to \pi^0\nu\bar{\nu}$ constraint by the $|\epsilon_K|$ value. The experimental measurement is taken from Ref. [2], and the solid black line indicates the Grossman-Nir bound.

Firstly, we note that to obtain the relevant Wilson coefficients we need to specify only a few Feynman rules, and our results are in agreement with Ref. [6]. For the analysis we use a new physics scale $f = 1$ TeV and the mixing parameter $x_L = 0.5$. This choice implies that the mass of the $T-$even heavy top partner is ~ 1.4 TeV which seems still fine with the current LHC bounds [7]. We scan randomly masses of the mirror quarks in the range $1.6 < m^q_{Hi} < 4.5$ TeV, where $i = 1, 2, 3$ denotes the fermion generation. The three angles and three CP violating phases of the flavour mixing matrix, V_{Hd}, are taken as $0 \leq \theta^d_{12}, \theta^d_{13}, \theta^d_{23} \leq \pi/4$ and $0 \leq \delta^d_{12}, \delta^d_{13}, \delta^d_{23} \leq 2\pi$.

Previously, it was reported that the branching ratio of $K_L \to \pi^0\nu\bar{\nu}$ could be enhanced as high as $5 \cdot 10^{-10}$, while being in agreement with the measured value for $\mathcal{B}(K^+ \to \pi^+\nu\bar{\nu})$ [8]. However, it can be seen from Fig. 1 that with the current measurement results [2] the experimental line is significantly shifted to the left.

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