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Abstract. The Morse molecular potential is used for the first time as an effective potential for the overall interaction in charmonium. This procedure allows the calculation of the rotational contributions of P states, the radii of five S states, and an absolute threshold for bound states. The calculation of the latter provides important information on the character of the recently found levels $X(3915)$, $X(3940)$, $\Psi(4040)$, $X(4050)$, $X(4140)$, $\Psi(4160)$, $X(4160)$, $X(4250)$, $X(4260)$, $X(4350)$, $\Psi(4415)$, $X(4430)$, and $X(4660)$.

1. Introduction

The pioneering work of Eichten *et al.* [1], and many other subsequent works, have shown that the static potential plays an important role in the description of heavy mesons. Despite efforts lasting more than three decades, some important features of the static interaction are not yet fully understood. That is why several approximate methods and effective potentials have been proposed for the description of the overall interaction in quarkonia. A partial list of the articles on these two subjects is found in reference [2].

It has been established in the literature that for heavy mesons non-relativistic quantum mechanics with constituent masses for the quark and antiquark can be applied. In this work we use the Morse molecular potential for the first time to describe some low energy states of charmonium and to find its effective potential parameters. This unique description allows us to calculate the contribution of the rotational energy of P states, and the radii of five S states. It also permits the calculation of the absolute threshold for the bound states of charmonium that sheds light on the character of the recently found levels [3] $X(3915)$, $X(3940)$, $\Psi(4040)$, $X(4050)$, $X(4140)$, $\Psi(4160)$, $X(4160)$, $X(4250)$, $X(4260)$, $X(4350)$, $\Psi(4415)$, $X(4430)$, and $X(4660)$.

It is not necessary to consider confinement because we only deal with low energy levels. Confinement is not well understood and there are models of hadrons that do not take it into account. For example, the original MIT bag model treats confinement only at the wall by making the vector current null at it [4]. In the case of the chiral bag model [5], confinement is treated by means of the continuity of the axial vector current at the wall. All the experimental data used below for the energies of charmonium states and the quark c mass were taken from the Particle Data Group [3]. This work is based on the recently published article by de Souza [6] on bottomonium.

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2. The Morse Molecular Potential

The Morse molecular potential [7]

$$V(x) = D\left(e^{-2\alpha x} - 2e^{-\alpha x}\right) \tag{1}$$

is widely used in the description of diatomic molecules where $x = (r-a)/a$, and α is a parameter to be found from the fitting. For $r = a$ the potential has its minimum value, $-D$. The first term in the above expression takes care of the well-known repulsion of the strong force for very small distances [8] and the second term takes care of the overall attraction due to QCD forces. For $|x| < 1$ this potential can be expanded about the minimum up to order 3 in x and the expression

$$V(x) = -D + \frac{1}{2}ka^2x^2 - \lambda k a^3 x^3 \tag{2}$$

is produced, where $\lambda = \alpha/2a$, and $k = m\omega^2$, in which m is the reduced mass of the constituent quark and antiquark. That is, $m = (1/2)M_c$ in which M_c is the mass of the c quark. The constant ω is defined below and is found from the fitting. For this potential the solution of the Schrödinger equation yields the expression [7, 9]

$$\begin{aligned} E_{vL} &= \hbar\omega\left(v+\frac{1}{2}\right) - A\left(v+\frac{1}{2}\right)^2 + B_L L(L+1) \\ &\quad -D_L L^2(L+1)^2 - C_{vL}\left(v+\frac{1}{2}\right)L(L+1) + \ldots \end{aligned} \tag{3}$$

for the vibrational and rotational levels above the minimum of the potential, where the quantum numbers v and L are equal to $0, 1, 2, 3, ...$ and A, B_L, C_L, D_L and C_{vL} are constants to be found from the fitting. In this last equation the first term describes harmonic vibrations, the second term takes into account the anharmonicity of the potential, the third term describes rotations with constant moment of inertia, the fourth term represents the centrifugal distortion and the fifth term represents the coupling between vibration and rotation. This expression can also be written as [9]

$$\begin{aligned} E_{vL} &= \hbar\omega\left(v+\frac{1}{2}\right) - \frac{(\hbar\omega)^2}{4D}\left(v+\frac{1}{2}\right)^2 + B_L L(L+1) \\ &\quad -D_L L^2(L+1)^2 - C_{vL}\left(v+\frac{1}{2}\right)L(L+1) + \ldots \end{aligned} \tag{4}$$

where ω and D are related to α, a and m by

$$\omega^2 = \frac{2\alpha^2 D}{ma^2}. \tag{5}$$

The constant B_L is given by $B_L = \hbar^2/2ma^2$. In the fitting below we disregarded the last term of Eqs. (3) and (4).

3. The Fitting

The first two levels, $\eta_c(1S)$ and $J/\Psi(1S)$, are a hyperfine doublet due to the spin-spin interaction, but as our Hamiltonian does not depend on spin, we can use the S states $J/\Psi(1S)$ and $\Psi(2S) \equiv \Psi(3686)$ for the fitting. We consider now the P states $\chi_{c0}(1P)$, $\chi_{c1}(1P)$ and $\chi_{c2}(1P)$ which is a triplet due to the spin-orbit interaction whose contribution is given by

$$\Delta E_{SL} = \Delta\left[J(J+1) - L(L+1) - S(S+1)\right], \tag{6}$$

Table 1. The levels considered in the fitting of charmonium.

(υ, L)	Particle	Mass (MeV/c^2)
$(0,0)$	$J/\Psi(1S)$	3096.916 ± 0.011
$(0,1)$	$\chi_c(1P)$	3549.7 ± 37.9
$(1,0)$	$\Psi(3686)$	3686.09 ± 0.04
$(0,2)$	$\Psi(3770)$	3778.1 ± 1.2

where $J = \left|\vec{J}\right| = \left|\vec{L} + \vec{S}\right|$. For the states $\chi_{c0}(1P)$, $\chi_{c1}(1P)$ and $\chi_{c2}(1P)$ we have $S = 1$ and $L = 1$, and thus, $J = 0, 1$ and 2, respectively. Using these values in Eq. (6) we obtain the average values $\Delta_{1P} = (27.7 \pm 13.6)$ Mev/c^2, and $E_{1P} = (3549.7 \pm 37.9)$ Mev/c^2 for the degenerate level, $\chi_c(1P)$. The state $\Psi(3770)$, which is classified as 1^3D_1, is a very well established state, experimentally found with a mass of (3778.1 ± 1.2) Mev/c^2. Table 1 presents a summary of the levels used in the fitting with the corresponding values of υ and L.

4. Results and discussion

4.1. Results for the potential parameters

Fitting the levels of table 1 to Eq. (3) gives the following values for its parameters: $\hbar\omega = (8062.0 \pm 0.1)$ Mev/c^2, $A = (3736.4 \pm 0.1)$ Mev/c^2, $B_L = (282.8 \pm 58.3)$ Mev/c^2, $D_L = (28.3 \pm 5.1)$ Mev/c^2. From Eqs. (3) and (4) we obtain that $A = (\hbar\omega)^2/4D$, and thus $D = (4348 \pm 0.5)$ Mev/c^2.

Using the values of D and $\hbar\omega$ above in Eq. (5) we obtain $a/\alpha = (13.15 \pm 0.05) \times 10^{-2}$ fm, and from the value of B_L we find that $a = (0.28 \pm 0.05)$ fm which is a very reasonable figure since the Compton wavelength of charmonium is about 0.69 fm if we use a constituent mass of 1.7 GeV/c^2 . Using the above values of a and a/α we obtain $\alpha = 2.15 \pm 0.39$.

As we showed above, a molecular potential is harmonic about its minimum, and thus we can calculate the value for the constant $k = m\omega^2$ which can be written as $k = mc^2(\hbar\omega)^2/(\hbar c)^2$. Using the above values we obtain $k \approx 1.44 \times 10^3$ GeV/fm^2 $\approx 2.31 \times 10^{23}$N/m which is a quite fair number. For a distance of 0.3 fm it produces a force $F \approx 10^8$N. The following simple calculation shows that this is a reasonable number. If we calculate the average force necessary for producing a work of 4 GeV in 0.3 fm we obtain a force of about 10^6N.

4.2. Number of S and P states

As we see in table 1 the quantum number n of QCD models corresponds to $\upsilon + 1$ in the present model. The maximum number of υ can be calculated for a molecular potential by using the simple following reasoning. In Eq.(3) the second term is always smaller than the first term, and thus we obtain $\upsilon < \frac{\hbar\omega}{A} - \frac{1}{2} = 1.65792 \pm 0.00004$ and thus the possible values of υ are only 0 and 1. Therefore, the possible values for n are only 1 and 2.

We do a similar reasoning for calculating the maximum number of L. In Eq.(3) the fourth term is always smaller than the third term, and so we obtain $L(L+1) < \frac{B_L}{D_L} = 9.99 \pm 2.06$ which is satisfied for $L = 0, 1, 2, 3$. This means, respectively, S, P, D and F states.

4.3. Discussion on the recently found levels

We found above that $D = (4348 \pm 0.5)$ Mev/c^2 which is a very important result and states that there is no charmonium bound state above (4348.8 ± 0.5) Mev/c^2. Therefore, the states $\Psi(4415)$, $X(4430)$, and $X(4660)$ are not bound states of charmonium. On the other hand this

Table 2. Radii of five states of charmonium calculated with the use of Eq. (7).

(υ, L)	Particle	Radius (fm)
$(0,0)$	$J/\Psi(1S)$, $\eta_c(1S)$	0.35 ± 0.06
$(1,0)$	$\Psi(3686)$, $\eta_c(2S)$	0.49 ± 0.09
$(2,0)$	$\Psi(3S)$	0.63 ± 0.11

result shows that the recently found states $X(3915)$, $X(3940)$, $\Psi(4040)$, $X(4050)$, $X(4140)$, $\Psi(4160)$, $X(4160)$, $X(4250)$, $X(4260)$, $X(4350)$ may be bound states of charmonium. But as we saw in section *4.2* above the maxium number of n is two and, thus, if they are charmonium states, none of them is a $3S$ state. They have to be $2P$, $2D$, and $2F$ states.

4.4. Radii of some S states

As it was shown above the Morse potential, when expanded about its minimum, yields Eq. (2). For such a potential Robinett [10] obtained the following equation for the average value of position for S states

$$\langle r\rangle_\upsilon \;=\; a + \frac{3\alpha\hbar\omega}{2m\omega^2 a}\left(\upsilon + \frac{1}{2}\right) = a + \frac{3a\hbar\omega}{4\alpha D}\left(\upsilon + \frac{1}{2}\right), \tag{7}$$

where we have taken into account Eq. (5). We can identify these average values with the radii of charmonium states. We only calculate the radii of the states $J/\Psi(1S)$, $\eta_c(1S)$, $\Psi(3686)$, $\eta_c(2S)$, and $\Psi(3S)$ because the other upper states are far from the bottom of the potential. Using the above values for the constants we obtain the results shown in table 2 for the radii of these five states of charmonium.

5. Conclusion

The fitting of some energy levels of charmonium to the Morse molecular potential makes possible the calculation of parameters of the effective molecular potential, prediction of the radii of five states and sheds some light on the character of the states $X(3915)$, $X(3940)$, $\Psi(4040)$, $X(4050)$, $X(4140)$, $\Psi(4160)$, $X(4160)$, $X(4250)$, $X(4260)$, $X(4350)$, $\Psi(4415)$, $X(4430)$, and $X(4660)$. Therefore, the above results add important information for the understanding of charmonium and complement information obtained from QCD models.

6. References

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