Institute of Physics Publishing Journal of Physics: Conference Series **31** (2006) 115–122
doi:10.1088/1742-6596/31/1/020 The Third 21COE Symposium: Astrophysics as Interdisciplinary Science

Dark Energy

Varun Sahni
Inter-University Centre for Astronomy & Astrophysics,
Puné 411 007, India
E-mail: varun@iucaa.ernet.in

Abstract. This talk briefly reviews observational and theoretical issues associated with the cosmological constant and dark energy.

1. Introduction

One of the most tantalising observational discoveries is that almost 96% of the matter density in the universe is in an invisible and possibly non-baryonic form. Evidence from rotation curves of galaxies and application of the virial theorem to clusters (of galaxies) reveals that about a third of the closure density is in the form of 'dark matter' which, for most practical purposes can be regarded as being pressureless, while observations of high redshift type Ia supernovae together with observations of the cosmic microwave background and the large scale clustering of galaxies, provide compelling evidence for two-third of the density being in 'dark energy'. Dark Enegy has large negative pressure and its most famous example is the cosmological constant 'Λ', introduced by Einstein in 1917.

Einstein introduced the cosmological constant in an attempt to create a universe that was static and closed and which complied with Ernst Mach's ideas on inertia, in which Einstein then believed [1]. However, within about ten years, observations by Hubble revolutionised cosmological thinking by demonstrating that the universe was expanding. Work by Friedmann and others demonstrated that an expanding universe could be accomodated within the general relativistic framework without the presence of the cosmological constant and soon Einstein recanted his idea of Λ, even calling it "the biggest blunder of my life".

Although discarded by Einstein, the cosmological constant has nevertheless made several 'comebacks', the latest being associated with recent observations of an accelerating universe. The Einstein field equations

$$R_{ik} - \frac{1}{2}g_{ik}R = \frac{8\pi G}{c^4}T_{ik} \tag{1}$$

in a homogeneous and isotropic setting lead to the following equation governing the acceleration/deceleration of the universe

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\sum_i(\rho_i + 3p_i) = -\frac{4\pi G}{3}\sum_i \rho_i(1+3w_i)\ , \tag{2}$$

where the summation is over all forms of matter present in the universe with equation of state

© 2006 IOP Publishing Ltd

$w_i = p_i/\rho_i$. Eqn. (2) together with its companion equation

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\sum_i \rho_i - \frac{k}{a^2} \ , \tag{3}$$

completely describes the dynamics of a FRW universe (k/a^2 is the Gaussian curvature of space). From (2) we find that a universe consisting of a single component will accelerate if $w < -1/3$. Fluids satisfying $\rho + 3p \geq 0$ or $w \geq -1/3$ are said to satisfy the 'strong energy condition' (SEC). We therefore find that, in order to accelerate, 'dark energy' must violate the SEC.

The cosmological constant with $p = -\rho$ ($w = -1$) provides us with a specific example of 'fluid' which violates the SEC. From the conservation equation

$$\dot{\rho} + 3\frac{\dot{a}}{a}(\rho + p) = 0 \ , \tag{4}$$

one immediately finds that the density in the cosmological constant does not evolve with time, since $\dot{\rho} = 0$. Indeed, the energy momentum tensor describing the cosmological constant has the beautifully simple form $T_i^k = \Lambda\delta_i^k$.

In a spatially flat universe one can obtain, with the help of eqn. (3), the following closed form expression for the Hubble parameter $H \equiv \dot{a}/a$ in terms of the cosmological redshift $z = a_0/a(t) - 1$:

$$H(z) = H_0\left[\Omega_m(1+z)^3 + \Omega_X(1+z)^{3(1+w)}\right]^{1/2} , \tag{5}$$

where $H_0 = H(z = 0)$ is the present value of the Hubble parameter, $\Omega_m = 8\pi G\rho_{0m}/3H_0^2$, $\Omega_X = 8\pi G\rho_{0\mathrm{DE}}/3H_0^2$, describe the dimensionless density of matter and dark energy (DE) respectively, ($w \equiv w_{\mathrm{DE}}$).

2. Observations of Dark Matter and Dark Energy

The presence of dark matter in galaxies can be inferred by applying Kepler's famous third law

$$v(r) = \sqrt{\frac{GM(r)}{r}} \tag{6}$$

to a galaxy. This leads to the 'rotation curve' $v(r)$ being determined at a given radial distance from the galactic center. In the absence of any dark matter component, the visible matter in galaxies would lead to a rotation curve which declined with distance as $v \propto r^{-1/2}$, in regions where the matter density was negligible. Very different behaviour is seen for velocity curves which have been compiled for over 1000 spiral galaxies usually by measuring the 21 cm emission line from neutral hydrogen (HI). These curves flatten out to $v \simeq$ constant implying $M(r) \propto r$ on sufficiently large scales (see fig 1). These results translate to a mass-to-light ratio of $M/L = (10 - 20)M_\odot/L_\odot$ in spiral galaxies and in ellipticals, while this ratio can increase to $M/L \simeq (200 - 600)M_\odot/L_\odot$ in low surface brightness galaxies (LSB's) and in dwarfs. ($M_\odot/L_\odot$ is the solar value.)

To determine the extent of dark matter in clusters of galaxies one frequently turns to the virial theorem

$$K + \frac{U}{2} = 0 \ , \tag{7}$$

where $U \simeq -GM^2/R$ is the potential energy of a cluster of radius R, $K \simeq 3M\langle v_r^2\rangle/2$ is the kinetic energy and $\langle v_r^2\rangle^{1/2}$ is the dispersion in the line-of-sight velocity of cluster galaxies. (Clusters in the Abell catalogue typically have $R \simeq 1.5h^{-1}$ Mpc.) Zwicky (1933) was the first to

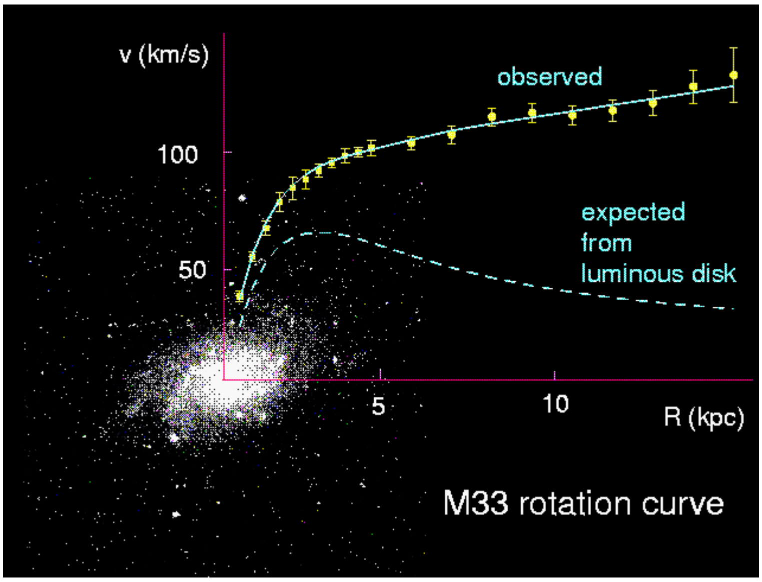

Figure 1. The rotation curve of the dwarf spiral galaxy M33 is shown superimposed on its optical image; from Roy [10].

realise that the Coma cluster would be gravitationally bound only if its total mass substantially exceeded the sum of the masses of its component galaxies. More recent observations support Zwicky's hypothesis and indicate that the mass-to-light ratio in clusters can be as large as $M/L \simeq 300 M_\odot/L_\odot$.

The dependence of the Hubble parameter on the dark energy density – demonstrated in (5) – is intimately related to how the presence of dark energy was discovered using high redshift Type Ia supernovae. Consider the following relation between the light flux F received from a distant supernova, its absolute luminosity $\mathcal{L}$ and its 'luminosity distance' d_L

$$F = \frac{\mathcal{L}}{4\pi d_L^2}. \tag{8}$$

In Newtonian theory $d_L = \sqrt{x^2 + y^2 + z^2}$ since the geometry of space is Euclidean. In general relativity, on the other hand, the geometry of space can be non-Euclidean, and the luminosity distance to an object located at redshift z depends both upon the geometry of space as well as the expansion history of the universe through the relation

$$d_L(z) = (1+z)\int_0^z \frac{dz'}{H(z')}\ , \tag{9}$$

where $H(z)$ is given by (5) for dark energy models in which the equation of state does not evolve with time (*ie* $w =$ constant). Fig. 1 shows the luminosity distance for cosmological models with varying amounts of dark matter (Ω_m) and dark energy (Ω_Λ). The limiting case $\Omega_m = 1$, $\Omega_\Lambda = 0$ corresponds to standard cold dark matter (SCDM) in which the universe decelerates as a weak power law $a(t) \propto t^{2/3}$. The other extreme example $\Omega_\Lambda = 1$, $\Omega_m = 0$ describes the de Sitter universe (also known as steady state cosmology) which accelerates at the steady rate $a(t) \propto \exp\sqrt{\frac{\Lambda}{3}}t$. From figure 1 we see that a supernova at redshift $z = 3$ will appear 9 times brighter in SCDM than it will in de Sitter space !

Historically, the current acceleration of the universe was first deduced from observations of type Ia supernovae which were found to be dimmer than could be accounted for in a flat matter

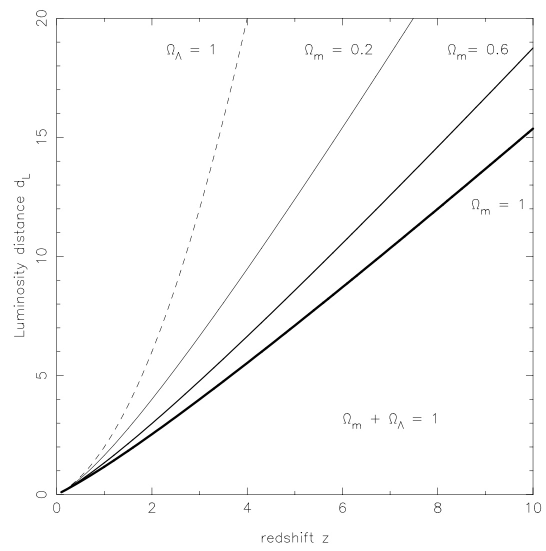

Figure 2. The luminosity distance d_L (in units of H_0^{-1}) is shown as a function of cosmological redshift z for spatially flat cosmological models with $\Omega_m + \Omega_\Lambda = 1$. Heavier lines correspond to larger values of Ω_m. The dashed line shows the luminosity distance in the spatially flat de Sitter universe ($\Omega_\Lambda = 1$). From Sahni and Starobinsky [9].

dominated universe [2, 4, 3, 5, 6]. It may be recalled that, in addition to being extremely bright objects, ($M_B \simeq -19.5$) type Ia supernovae are excellent stadardized candles: the scatter in their luminosities being only $\sim 12\%$. More recently, the acceleration hypothesis has received independent support from observations of the cosmic microwave background and large scale structure [7, 8] both of which, when taken together, suggest that about 2/3rd of the matter density in the universe consists of dark energy, while the remainder is in dark matter + baryons ($\sim 4\%$).

It is important to note that the degeneracy in parameter space $\{\Omega_m, \Omega_\Lambda\}$ arising from Sn observations is almost orthogonal to the degeneracy which arises from CMB measurements, as shown in figure 3.

An important epoch in cosmology is when the universe ceased to decelerate and began accelerating. Assuming for simplicity that the equation of state of dark energy is a constant, one finds for this epoch, the redshift

$$(1 + z_{\rm a})^{-3w} = -(1 + 3w)\frac{\Omega_X}{\Omega_m} \; w < 0. \tag{10}$$

Substituting $\Omega_\Lambda = 0.7, \Omega_m = 0.3$ and $w = -1$ one finds $z_{\rm a} \simeq 0.7$. In other words, the currently accelerating epoch is a fairly recent phenomenon ! Observational evidence for the deceleration epoch comes from studies of high redshift supernovae [5, 12, 13].

The fact that the universe began to accelerate fairly recently leads to the 'cosmic coincidence' puzzle according to which we appear to live during a special epoch when the densities in dark energy and in dark matter are almost equal. If dark energy is in the form of the cosmological constant then one is confronted with an 'initial conditions' problem since the small current value $\rho_\Lambda = \Lambda/8\pi G \simeq 10^{-47}$ GeV^4 implies $\rho_\Lambda/\rho_r \simeq 10^{-123}$ at the Planck time (when the temperature of the universe was $T \sim 10^{19}$ GeV), or $\rho_\Lambda/\rho_r \simeq 10^{-55}$ at the time of the electroweak phase transition ($T \sim 100$ GeV). An extreme fine-tuning of initial conditions therefore appears necessary in order to ensure $\rho_\Lambda/\rho_m \sim 1$ today !

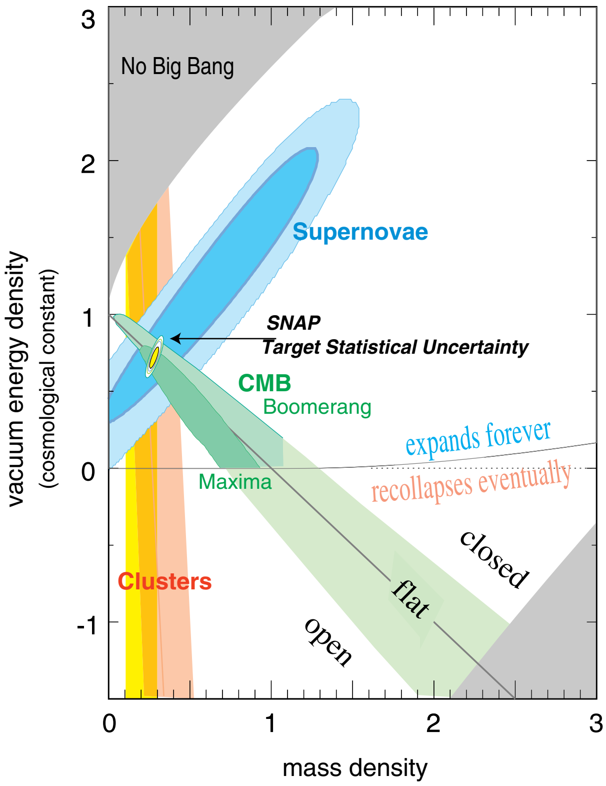

Figure 3. Current results for the energy density in the cosmological constant (vertical-axis) and the matter density (horizontal-axis) from supernova, CMB and LSS observations. Also shown is the target statistical uncertainty of the planned SNAP experiment (overlayed). From Aldering [11].

3. Evolving Dark Energy

3.1. Quintessence

The problems faced by Λ can be made less acute if the equation of state of dark energy evolves with time. The simplest such (quintessence) models are inspired by Inflation and assume that dark energy is a scalar field which couples minimally to gravity and has the Lagrangian density

$$\mathcal{L} = \frac{1}{2}\dot{\phi}^2 - V(\phi) \ , \tag{11}$$

so that the associated energy momentum tensor is

$$\rho \equiv T_0^0 = \frac{1}{2}\dot{\phi}^2 + V(\phi), \ p \equiv -T_\alpha^\alpha = \frac{1}{2}\dot{\phi}^2 - V(\phi) \ , \tag{12}$$

where, for simplicity, the field is assumed to be spatially homogeneous. Scalar field which roll down steep potentials ($\Gamma \equiv V''V/(V')^2 \geq 1$) approach a common evolutionary path from a wide range of initial conditions [14]. An important feature of these so-called ‘tracker’ models is that the scalar field density (and its equation of state) remains close to that of the dominant background matter during most of cosmological evolution. An example of a tracker potential is $V(\phi) = V_0 \exp\left(-\sqrt{8\pi}\lambda\phi/M_p\right)$ [15, 16], where $M_p = 1/\sqrt{G}$ is the Planck mass. In this case

$$\frac{\rho_\phi}{\rho_B + \rho_\phi} = \frac{3(1+w_B)}{\lambda^2} = \text{constant} \ , \tag{13}$$

ρ_B is the background energy density while w_B is the associated background equation of state. Another example is provided by $V(\phi) = V_0/\phi^\alpha$ [15]. In this case the density in the tracker gradually overtakes the density in matter allowing the universe to accelerate at the present epoch. Other quintessence potentials include [17]

$$V(\phi) = V_0[\cosh \lambda\phi - 1]^p, \tag{14}$$

which provides an example of oscillating quintessence having a time-averaged equation of state $\langle w_\phi \rangle = (p-1)/(p+1)$, and [9]

$$V(\phi) \propto \sinh^{\frac{2(1+w)}{w}}(C\phi + D) \ , \tag{15}$$

for which the equation of state w remains constant with time. A more complete list of potentials may be found in [17].

3.2. Chaplygin Gas

A somewhat different route to constructing a dynamical dark energy model is through the Born-Infeld form of the Lagrangian density

$$\mathcal{L} = -V_0\sqrt{1 - \phi_{,\mu}\phi^{,\mu}} \ , \tag{16}$$

where $\phi_{,\mu} \equiv \partial\phi/\partial x^\mu$.

One can show that in this case the equation of state is

$$p = -A/\rho \ , \tag{17}$$

and the conservation equation $dE = -pdV \Rightarrow d(\rho a^3) = -pd(a^3)$ immediately gives

$$\rho_c = \sqrt{A + \frac{B}{a^6}} = \sqrt{A + B(1+z)^6} \ , \tag{18}$$

where B is a constant of integration. Thus the Chaplygin gas behaves like pressureless dust at early times and like a cosmological constant during very late times [18].

The Hubble parameter for a universe containing cold dark matter and the Chaplygin gas is given by

$$H(z) = H_0\left[\Omega_m(1+z)^3 + \frac{\Omega_m}{\kappa}\sqrt{\frac{A}{B} + (1+z)^6}\right]^{1/2} \ , \tag{19}$$

where κ defines the ratio between matter and the Chaplygin gas energy densities at the commencement of the matter-dominated stage.

3.3. Braneworld dark energy

An important property of our universe is that it appears to have accelerated twice: once at the very beginning during Inflation, and again ∼13 billion years later, during the present epoch. It is meaningful to ask whether these two accelerating epochs are completely distinct, or whether there could be a single unifying mechanism describing them. At first glance this does not appear to be an easy task since, in order for the field to 'slow-roll', the inflaton potential must be flat, whereas dark energy requires a steep potential so as to satisfy the 'tracker' criteria. It is therefore interesting that models of 'quintessential inflation'[19] are possible to construct within the Randall-Sundrum (RS) framework [20] wherein our 3+1 dimensional universe is a 'brane' embedded in a higher dimensional 'bulk' space-time. Within the RS setting, the equation of motion of a scalar field propogating on the brane is

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0 \ , \tag{20}$$

where [21]

$$\begin{aligned} H^2 &= \frac{8\pi}{3m^2}\rho(1+\frac{\rho}{2\sigma}) + \frac{\Lambda_4}{3} + \frac{\mathcal{E}}{a^4} \ , \\ \rho &= \frac{1}{2}\dot{\phi}^2 + V(\phi) \ . \end{aligned} \tag{21}$$

$\mathcal{E}$ is an integration constant which transmits bulk graviton influence onto the brane and σ is the brane tension. Equation (21) contains an additional term ρ^2/σ whose effect is to significantly increase the damping experienced by the scalar field as it rolls down its potential. As a result *Inflation can proceed even in steep potentials* commonly associated with dark energy [22, 23, 24, 25]. An important feature of quintessential inflation on the brane is the production of relic gravity waves, whose specrum allows us to obtain a glimpse into the early universe [24, 26]. Another aspect of braneworld cosmology [27] is the possibility of fundamentally new cosmological behaviour (*loitering* [28] & *mimicry* [29]) at moderately high redshifts. In these models, the age of the universe can be larger than in LCDM at *intermediate* redshifts ($z \sim$ few) while the reionization redshift can be lower. Both properties are appealing in view of recent observations [30, 31, 32, 33] which may prove problematic for LCDM.

4. Conclusions

The nature of dark matter and dark energy is clearly one of the biggest problems confronting astrophysics today. I end by listing five questions to which answers are awaited: (1) Is dark energy a Cosmological Constant or is it dynamically evolving ? (2) Is dark energy a classical field or a quantum entity ? (3) Does the acceleration of the Universe arise because of amendments to the *matter* sector or to the *gravity* sector of general relativity ? (4) Is late-time acceleration (dark energy) related to early-time acceleration (Inflation) ? (5) Are dark matter and dark energy connected ?

The answers to these questions will help shed light on the nature of the accelerating universe.

References

[1] Einstein, A. (1917) Sitz. Preuss. Akad. d. Wiss. Phys.-Math **142**.
[2] Perlmutter, S.J. et al., Nature **391**, 51 (1998).
[3] Riess, A.G. et al., Astron. J. **116**, 1009 (1998).
[4] Perlmutter, S.J. et al., Astrophys. J. **517**, 565 (1999).
[5] Riess, A.G. *et al.*, Astrophys.J. **607**, 665 (2005) [astro-ph/0402512]
[6] Astier, P. *et al.*, *The Supernova Legacy Survey: Measurement of Ω_M, Ω_Λ and w from the First Year Data Set*, astro-ph/0510447.
[7] Spergel, D.N., et al, Astrophys.J.Suppl. **148**, 175 (2003) [astro-ph/0302209]
[8] Tegmark, M. et al, Phys. Rev. D **69**, 103501 (2004) [astro-ph/0310723]
[9] Sahni, V. and Starobinsky, A.A. *IJMP D* **9**, 373 (2000) [astro-ph/9904398]; Sahni, V. *Dark Matter and Dark Energy*, Lectures given at the 2nd Aegean Summer School on the Early Universe, Ermoupoli, Island of Syros, Greece, astro-ph/0403324; Sahni V, *Cosmological surprises from Braneworld models of dark energy*, astro-ph/0502032.
[10] Roy, D.P. physics/0007025; also see E. Corbelli and P. Salucci, astro-ph/9909252
[11] Aldering, G., astro-ph/0209550
[12] Alam, U. Sahni, V., Saini, T.D. and Starobinsky, A.A., 2004, Mon. Not.@ Roy. Ast. Soc. **354**, 275.
[13] Alam, U. Sahni, V. and Starobinsky, A.A., 2004 a, JCAP **0406** 008.
[14] Zlatev, I., Wang, L. and Steinhardt, P.J., Phys. Rev. Lett. **82**, 896 (1999).
[15] Ratra, B. and Peebels, P.J.E., Phys. Rev. D **37**, 3406 (1988);

[16] Wetterich, C., Nuclear Physics B **302**, 668 (1988).
[17] Sahni, V. and Wang, L., Phys. Rev. D **62**, 103517 (2000);
[18] Kamenshchik, A., Moschella, U. and Pasquier, V., Phys. Lett. B **511** 265 (2001) [gr-qc/0103004].
[19] Peebles, P.J.E. and Vilenkin, A., Phys. Rev. D **59** 063505 (1999).
[20] Randall, L. and Sundrum, R., Phys. Rev. Lett. **83**, 3370 (1999), [hep-ph/9905221]; L. Randall and R. Sundrum, Phys. Rev. Lett. **83**, 4690 (1999), [hep-th/9906064].
[21] Maartens A, 2004 *Living Rev. Rel.* **7** 7 [gr-qc/0312059]
[22] Copeland, E.J., Liddle, A.R. and Lidsey, J.E. Phys. Rev. D **64** 023509 (2001).
[23] Huey, G. Huey Lidsey, J. Phys. Lett. B **514**, 217 (2001).
[24] Sahni, V., Sami, M. and Souradeep, T., Phys. Rev. D **65** 023518 (2002).
[25] Majumdar, A. S. Phys. Rev. D **64**, 083503 (2001).
[26] Sami, M. and Sahni, V., Phys. Rev. D **70**, 083513 (2004).
[27] Sahni, V. and Shtanov, Yu.V., JCAP 0311,014, (2003) [astro-ph/0202346].
[28] Sahni V and Shtanov Yu, Phys. Rev. D **71** 084018 (2005) [astro-ph/0410221]
[29] Sahni V and Shtanov Yu and Viznyuk, A., *Cosmic Mimicry: Is LCDM a Braneworld in disguise ?*, JCAP (in press) astro-ph/0505004.
[30] Richards G T *et al.*, *Astron. J.* **127** 1305 (2004) [astro-ph/0309274]
[31] Haiman Z and Quataert E, *The Formation and Evolution of the First Massive Black Holes*, 2004, astro-ph/0403225.
[32] Barkana R and Loeb A, *In the Beginning: The First Sources of Light and the Reionization of the universe*, astro-ph/0010468; Ciardi B and Ferrara A, *The First Cosmic Structures and their Effects*, astro-ph/0409018; O. Elgaroy, D.F. Mota and T. Multamaki, Phys. Rev. D **D71**, 067301 (2005) [astro-ph/0501342].
[33] Friaca, A., Alcaniz, J.S. and Lima, J.A.S., MNRAS **362** 1295 (2005) [astro-ph/0504031]; Jain, D. and Dev, A., *Age of High Redshift Objects - a Litmus Test for the Dark Energy Models*, astro-ph/0509212.