ASYMMETRY ENERGY OF NUCLEAR MATTER: TEMPERATURE AND DENSITY DEPENDENCE, AND VALIDITY OF SEMI-EMPIRICAL FORMULA

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In this work, we have done a completely microscopic calculation using a many-body variational method based on the cluster expansion of energy to compute the asymmetry energy of nuclear matter. In our calculations, we have employed the AV_{18} nuclear potential. We have also investigated the temperature and density dependence of asymmetry energy. Our results show that the asymmetry energy of nuclear matter depends on both density and temperature. We have also studied the effects of different terms in the asymmetry energy of nuclear matter. These investigations indicate that at different densities and temperatures, the contribution of parabolic term is very substantial with respect to the other terms. Therefore, we can conclude that the parabolic approximation is a relatively good estimation, and our calculated binding energy of asymmetric nuclear matter is in a relatively good agreement with that of semi-empirical mass formula. However, for the accurate calculations, it is better to consider the effects of other terms.

Key words: Asymmetric nuclear matter, symmetry energy, parabolic approximation, semi-empirical mass formula.

1. INTRODUCTION

Nuclear matter is an extremely large hypothetical system of constrained nucleons which interact through the strong nuclear force, and can be considered as an ideal model for the nuclear matter within the heavy nuclei [1]. The analysis of heavy nucleus behavior in high density and finite temperature is a challenging issue in modern nuclear physics [2]. The nuclear matter asymmetry energy is an important subject in nuclear physics and astrophysics. According to research conducted [3–10], the nuclear matter asymmetry energy plays a crucial role in determining many important nuclear properties such as the neutron skin of nuclear system, structure of nuclei near the drip line, and neutron star mass and radius. They are especial important constraints on the nuclear matter equation of state and the density dependence of the asymmetry energy [9, 11, 12]. The nuclear matter equation of state plays a central roles in the study of high-energy in nuclear physics, especially in heavy ion collisions. The nuclear matter equation of state at zero temperature governs the structure of cold neutron stars, the equation of state for finite temperatures is necessary for studies of core collapse supernovae, black hole formation and cooling of the neutron stars [13–15].

Due to the strong interaction between the nucleons, we must completely include the nucleon-nucleon potential in the nuclear matter calculations. The nuclear potential is usually made out by fitting the nucleon-nucleon scattering data and deuteron properties. The potential models which obtained only from fitting the np scattering data frequently offer a weak description of pp and nn data [16]. Recently, a nuclear potential has been introduced by Wiringa *et al.* which extracted by fitting np and pp data as well as low energy nn scattering parameters and deuteron properties [17]. This potential is written in an operator format which depends on the value of J, L, S and T as well as T_z of nucleon-nucleon pair. Therefore, in the calculations with this potential, a completely microscopic calculation in which the third component of isospin (T_z) would be explicitly considered is required. This is a very important point in the calculations for asymmetric nuclear matter consisting of Z protons and N neutrons, where in general $Z \neq N$. The microscopic studies on asymmetric nuclear matter are very limited, and usually the asymmetry energy of nuclear matter is computed employing a parabolic approximation. Therefore, it is interesting to study the validity of this estimation for the asymmetric nuclear matter.

One of the powerful microscopic methods in the nuclear matter calculations is the variational method based on the cluster expansion of energy. In recent years, we have done several works on the nucleonic matter using this method [18–25]. The purpose of present work is to compute the asymmetry energy of nuclear matter using AV_{18} potential at finite temperature employing the mentioned microscopic technique. We also verify the validity of semi-empirical mass formula at different temperatures and densities. The dependence of asymmetry energy of asymmetric nuclear matter on density and temperature is also investigated.

2. FORMALISM

Here, a system with A interacting nucleons (including N neutrons and Z protons) is considered in order to calculate the binding energy of asymmetric nuclear matter with AV_{18} potential. As it was mentioned in the preceding section, we use the variational method based on the cluster expansion of the energy. In this method, at first we choose a trial wave function as follows,

$$\psi = F\Phi, \tag{1}$$

where Φ is the antisymmetric wave function of A non-interacting nucleons and $F = F(1 \dots A)$ is the inter-nucleon correlation function included by the interaction between the nucleons. We use the Jastrow ansatz for the correlation function,

$$F = S \prod_{i>j} f(ij), \tag{2}$$

where S is the symmetrizing operator and $f(ij)$ is the two-body correlation function. Using the above trial wave function included by this correlation function, we get a cluster expansion for the energy which up to the two-body term is as follows,

$$E([f]) = \frac{1}{A}\frac{<\psi|H|\psi>}{<\psi|\psi>} = E_1 + E_2. \tag{3}$$

At finite temperature, the one-body term E_1 for an asymmetric nuclear matter consisting of Z protons and N neutrons is as follows,

$$E_1 = E_1^{(p)} + E_1^{(n)}. \tag{4}$$

Labels p and n are used for the protons and neutrons, and $E_1^{(i)}$ is,

$$E_1^{(i)} = \sum_k \frac{\hbar^2 k^2}{2m^{(i)}} n^{(i)}(k,T,\rho^{(i)}), \tag{5}$$

where $n^{(i)}(k,T,\rho^{(i)})$ is Fermi-Dirac distribution function,

$$n^{(i)}(k,T,\rho^{(i)}) = \frac{1}{Exp\{\beta[\varepsilon^{(i)}(k) - \mu^{(i)}(T,\rho^{(i)})]\} + 1}. \tag{6}$$

In the above equation, $\mu^{(i)}$ is the chemical potential, $\rho^{(i)}$ is the number density and $\varepsilon^{(i)} = \frac{\hbar^2 k^2}{2m^{(i)}}$ is the single particle energy of nucleon i, and $\beta = \frac{1}{k_B T}$ (T is the temperature). The total density of system is,

$$\rho = \rho^{(p)} + \rho^{(n)}. \tag{7}$$

For the asymmetric nuclear matter, we can also define the asymmetry parameter as follows,

$$\xi = \frac{N-Z}{N+Z} = \frac{\rho^{(n)} - \rho^{(p)}}{\rho^{(n)} + \rho^{(p)}}. \tag{8}$$

This definition leads to the following relations for the nucleon density,

$$\rho^{(p)} = (1-\xi)\rho, \qquad \rho^{(n)} = (1+\xi)\rho. \tag{9}$$

By comparing Eqs. (9), (5) and (4), we can conclude that the energy of asymmetric nuclear matter is different for various asymmetry parameter.

The two-body energy has the following form,

$$E_2 = \frac{1}{2A}\sum_{ij} < ij \mid \upsilon(12) \mid ij - ji >, \tag{10}$$

where

$$v(12) = -\frac{\hbar^2}{2m}[f(12),[\nabla_{12}{}^2, f(12)]] + f(12)V(12)f(12). \quad (11)$$

$V(12)$ is the nucleon-nucleon pair potential. The two-body correlation function, $f(12)$, is considered as follows,

$$f(12) = \sum_{p=1}^{3} f^{(p)}(r_{12})O^{(p)}(12), \quad (12)$$

where operators $O^{(p)}(12)$ are,

$$O^{(p=1,3)}(12) = (\frac{1}{4} - \frac{1}{4}\sigma_1.\sigma_2),\ (\frac{1}{2} + \frac{1}{6}\sigma_1.\sigma_2 + \frac{1}{6}S_{12}),\ (\frac{1}{4} + \frac{1}{12}\sigma_1.\sigma_2 - \frac{1}{6}S_{12}). \quad (13)$$

In above equation, $S_{12} = 3(\sigma_1.\hat{r})(\sigma_2.\hat{r}) - \sigma_1.\sigma_2$ is the tensor operator. We employ the AV_{18} nuclear potential in our calculations. This potential has the following form [17],

$$V(12) = \sum_{p=1}^{18} V^{(p)}(r_{12})O_{12}^{(p)}. \quad (14)$$

The first fourteen operators are,

$$O_{12}^{p=1-14} = 1, \sigma_1.\sigma_2, \tau_1.\tau_2, (\sigma_1.\sigma_2)(\tau_1.\tau_2), S_{12}, S_{12}(\tau_1.\tau_2), \mathbf{L}.\mathbf{S}, \mathbf{L}.\mathbf{S}(\tau_1.\tau_2), \mathbf{L}^2, \mathbf{L}^2(\sigma_1.\sigma_2), \mathbf{L}^2(\tau_1.\tau_2), \mathbf{L}^2(\sigma_1.\sigma_2)(\tau_1.\tau_2), (\mathbf{L}.\mathbf{S})^2, (\mathbf{L}.\mathbf{S})^2(\tau_1.\tau_2), \quad (15)$$

and four additional operators which break the charge independence of nuclear force are,

$$O_{12}^{p=15-18} = \mathbf{T}_{12}, \mathbf{T}_{12}(\sigma_1.\sigma_2), S_{12}\mathbf{T}_{12}, (\tau_{1z} + \tau_{2z}). \quad (16)$$

$\mathbf{T}_{12} = 3(\tau_1.\hat{r})(\tau_2.\hat{r}) - \tau_1.\tau_2$ is the isotensor operator.

Using the above formalism, we can compute the binding energy of asymmetric nuclear matter. The procedure of these calculations has been fully presented in our previous articles [18–25].

3. RESULTS AND DISCUSSIONS

We have calculated the binding energy of asymmetric nuclear matter using the method discussed in the previous section at various densities and temperatures. Our results for the binding energy *versus* asymmetry parameter ($\xi = \frac{N-Z}{N+Z}$) have been shown in Fig. 1 for different densities and temperatures. For all densities and temperatures, it is seen that the binding energy increases as the asymmetry parameter

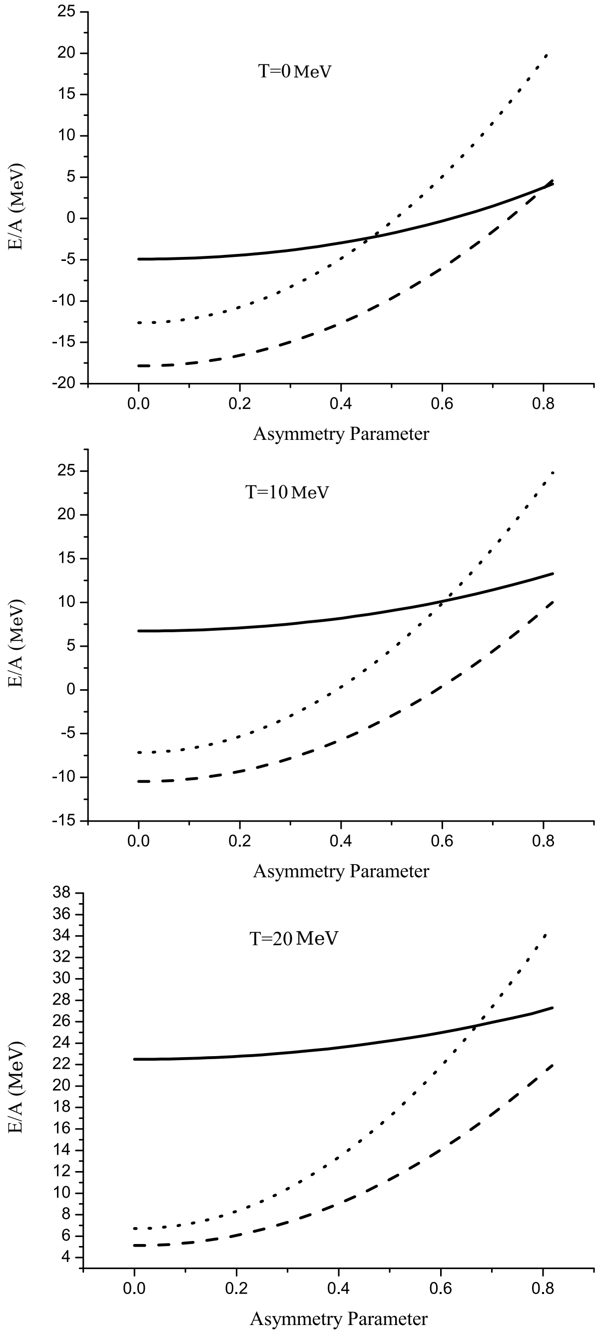

Fig. 1 – The binding energy per nucleon of asymmetric nuclear matter *versus* asymmetry parameter (ξ) for densities $\rho = 0.05$ (full curve), 0.25 (dashed curve) and 0.5 fm^{-3} (dotted curve) at different temperatures (T).

increases. We have found that for each value of temperature and asymmetry parameter, below a specific value of density, the binding energy decreases by increasing density, while above this density, it increases as density increases. This specific density is called the saturation density which depends on both temperature and asymmetry parameter. For each density, by comparison of the energy curves, we observe that by increasing the temperature, the slope of energy curve *versus* asymmetry parameter decreases. Fig. 1 shows that for each temperature, the increasing rate of binding energy *versus* asymmetry parameter has different values for different densities.

For each value of density at a specific temperature, we can write our results for the binding energy *versus* asymmetry parameter (ξ) as a polynomial function in term of different powers of ξ. By comparing the extracted polynomial function with the semi-empirical mass formula used for the binding energy of asymmetric nuclear matter ($E = a_v + a_{sym}\xi^2$), the requirement of presence of other powers of ξ can be analyzed, and it is determined whether the semi-empirical mass formula would be broken or not. We have found that the best function which is fitted with our results for the binding energy of asymmetric nuclear matter is a polynomial up to the fourth power of ξ,

$$E = a_0 + a_1\xi + a_2\xi^2 + a_3\xi^3 + a_4\xi^4. \qquad (17)$$

The coefficients of various powers of ξ (a_0, a_1, a_2, a_3 and a_4) *versus* density at different temperatures have been given in Tables 1, 2 and 3. By investigating the values of a_0 from these tables, we observe that for each temperature, a_0 decreases by increasing the density up to a specific density, and then above this density, a_0 increases. By comparing these tables, we can find that for each density, with an increase in temperature, an increasing in a_0 is seen. From Tables 1, 2 and 3, by investigating the coefficients a_1, a_3 and a_4, we can conclude that these coefficient are density and temperature dependent. The given values of a_2 in these tables indicate

Table 1

a_0, a_1, a_2, a_3 and a_4 (MeV) *versus* density (ρ) for temperature $T = 0$ MeV.

ρ (fm^{-3})	a_0	a_1	a_2	a_3	a_4
0.05	4.921	-0.05167	12.01	-0.117	2.574
0.10	-10.08	-0.01954	17.64	1.548	0.4468
0.15	-13.76	-0.1445	23.76	0.1695	1.371
0.20	-16.3	-0.2257	28.82	-0.6103	2.067
0.25	-17.85	-0.3245	33.57	-1.974	3.028
0.30	-18.45	-0.4127	37.37	-1.949	3.02
0.35	-18.19	-0.4084	40.68	-1.887	3.007
0.40	-17.1	-0.5143	44.33	-2.762	3.524
0.45	-15.23	-0.6283	47.94	-3.658	3.932
0.50	-12.61	-0.6323	50.63	-2.82	3.355

Table 2

a_0, a_1, a_2, a_3 and a_4 (MeV) *versus* density (ρ) for temperature $T = 10$ MeV.

ρ (fm^{-3})	a_0	a_1	a_2	a_3	a_4
0.05	6.73012	-0.04794	8.94846	0.48205	0.75552
0.10	-0.02839	-0.17483	15.43355	-0.33542	0.77439
0.15	-4.86116	-0.13702	20.64022	0.60392	0.2858
0.20	-8.2671	-0.26539	26.04789	-0.0697	0.79922
0.25	-10.48181	-0.24034	30.19788	0.29485	0.71646
0.30	-11.63058	-0.40422	34.81898	-0.85014	1.4016
0.35	-11.79859	-0.62335	39.13478	-1.82609	1.88743
0.40	-11.07976	-0.47165	41.84846	-0.96744	1.5085
0.45	-9.50737	-0.5509	45.37664	-1.40054	1.66413
0.50	-7.1490	-0.52195	47.92776	0.1053	0.63481

Table 3

a_0, a_1, a_2, a_3 and a_4 (MeV) *versus* density (ρ) for temperature $T = 20$ MeV.

ρ (fm^{-3})	a_0	a_1	a_2	a_3	a_4
0.05	22.50472	-0.21782	8.02506	-2.70781	2.36938
0.10	16.14381	-0.12781	11.66631	0.04014	0.33926
0.15	11.29118	-0.09781	15.88062	1.21584	-0.27871
0.20	7.65327	-0.18837	20.78746	0.18392	0.49755
0.25	5.12343	-0.24998	24.86982	0.36818	0.36781
0.30	3.61406	-0.39247	29.43257	-1.14565	1.30578
0.35	3.07363	-0.34095	32.36919	0.23839	0.38758
0.40	3.45085	-0.56422	36.75817	-1.36493	1.19642
0.45	4.68426	-0.55863	39.84991	-0.73474	0.63449
0.50	6.71498	-0.48627	42.39283	1.09249	-0.69099

that for each temperature, the value of a_2 increases by increasing the density. We have found that at high densities, the increasing rate of a_2 is lower than that at low densities. A comparison between the values of a_2 in different temperatures shows that for all densities, the value of a_2 decreases as the temperature increases. Here, we can conclude that the value of a_2 depends on both density and temperature.

Now, by more investigating Tables 1, 2 and 3, we see that for all densities and temperatures, with respect to the other coefficients, the coefficients a_0 and a_2 have the main contributions in the binding energy. However, we test the the effects of a_1, a_3 and a_4 relative to a_2 as follows. By obtaining the magnitude of ratio $\frac{a_1}{a_2}$, it is clear that at low densities, this ratio is of order 10^{-3}, therefore the effect of a_1 in the binding energy of asymmetric nuclear matter is negligible. We have found that at high densities, this ratio is of order 10^{-1}, therefore it is better to consider the effect of a_1 for the accurate calculations. By comparing the magnitude of a_3 and a_4, it is seen that for all temperatures and densities, the magnitude of a_4 is always greater than that of a_3. Our results show that the highest order of magnitude of ratio $\frac{a_3}{a_2}$ reaches about 10^{-2}. For the magnitude of $\frac{a_4}{a_2}$, that is of order 10^{-1}. The above results indicate that the dominant term in the asymmetry energy of nuclear matter is related to a_2, and the other terms which are related to a_1, a_3 and a_4 are small with respect to that of a_2. However, in order to do more accurate calculation, these terms could be considered.

Here, for the binding energy of asymmetric nuclear matter, we rewrite Eq. (17) as the following relation

$$E = a_v + E_{asym}, \tag{18}$$

where $a_v = a_0$ is the volume energy and E_{asym} is the asymmetry energy (for the symmetric nuclear matter in which $N = Z$, E_{asym} is zero),

$$E_{asym} = E^{(1)}_{asym} + E^{(2)}_{asym} + E^{(3)}_{asym} + E^{(4)}_{asym}. \tag{19}$$

In fact the contribution of asymmetry energy is considered as four terms. Now, at a specific density and temperature, we compare the effects of different terms of E_{asym} with respect to $E^{(2)}_{asym}$ for the low ($\xi = 0.1$) and high ($\xi = 0.9$) values of asymmetry parameter. Our results indicate that for the low values of asymmetry parameter, the magnitude of ratio $\frac{E^{(1)}_{asym}}{E^{(2)}_{asym}}$ is significant, especially at higher densities, therefore, for this case of asymmetric nuclear matter, it is better to consider the effects of $E^{(1)}_{asym}$. However, by increasing the asymmetry parameter, the effect of this term decreases. The magnitude of ratio $\frac{E^{(3)}_{asym}}{E^{(2)}_{asym}}$ and $\frac{E^{(4)}_{asym}}{E^{(2)}_{asym}}$ at low value of asymmetry parameter, is very small. The magnitude of these ratios at high values of asymmetry parameter becomes more important, but they are yet small.

4. SUMMARY AND CONCLUSIONS

We have considered the asymmetric nuclear matter including N neutrons and Z protons which interact through the nuclear force. For this system, the binding energy has been computed by a fully microscopic variational many-body technique based on the cluster expansion of the energy. In our formalism, we have presented the AV_{18} two-nucleon potential. We have also calculated the asymmetry energy of nuclear matter. It is found that the asymmetry energy is a function of density and temperature. For different densities and temperatures, our results for the binding energy versus asymmetry parameter have been fitted as a polynomial function. It was a polynomial of forth power of asymmetry parameter. We have seen that all the coefficients of this polynomial are density and temperature dependent. By comparing our polynomial function with the semi-empirical mass formula used for the binding energy of asymmetric nuclear matter, the requirement of presence of other powers of asymmetry parameter has been analyzed, and the validity of semi-empirical mass formula has been investigated. According to our results, the square term in the asymmetry energy of nuclear matter is the dominant term, and the other terms compared to this term are small. Therefore, we can ignore the effects of non-square terms. Our study shows that the asymmetry energy of asymmetric nuclear matter has a relatively good agreement with the semi-empirical mass formula. However, according to our calculations, to increase the accuracy of calculations, it is better to include the other terms in the asymmetry energy. For example, the first order term at low values of asymmetry parameter is considerable, especially at low densities.

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