

# THE GENERATION OF HIGHLY INTENSE THz RADIATION BASED ON SMITH-PURCELL RADIATION \*

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## Abstract

A photocathode RF gun can generate trains of THz picosecond electron bunches by illuminating the cathode with trains of laser pulses. Let this electron bunches pass close to the surface of a lamellar grating, THz radiation will be emitted, which is the so-called Smith-Purcell Radiation (SPR). If the lamellar grating has a narrow groove, this radiation will be narrow band. By choosing suitable parameters, the SPR frequency can be resonant with the electron bunches frequency, and then generate highly intense, narrow band THz coherent radiation.

## COHERENT SMITH-PURCELL RADIATION

When an electron beam passes close to the surface of a periodic structure (such as a metallic grating), radiation is emitted because of the interaction of the particles with the periodic structure, which is the so-called Smith-Purcell radiation (SPR)[1]. The intensity of radiation is proportional to the number of periods of the grating ( $N_g$ ), hence it is strongly compared to other coherent radiation generation techniques such as synchrotron, transition and diffraction radiation.

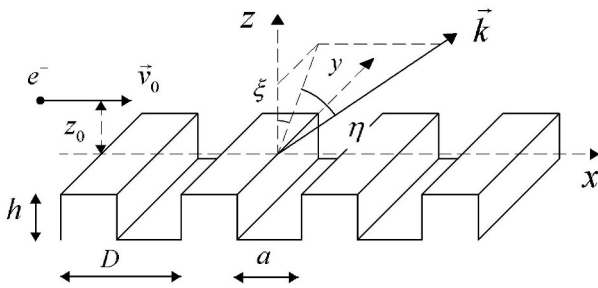


Figure 1: Definition of the geometry. The electron beam moves with constant reduced velocity  $v_0 = \beta_0 c$  at a distance  $z = z_0$  parallel to the grating surface in x-direction.

According to the theory of di Francia[2], the emission mechanism of SP radiation can be interpreted in analogy to the diffraction of light as the diffraction of the field of the electrons (virtual photons) which pass the grating at a distance  $z_0$  away from its surface by the grating grooves. One characteristic signature of SPR is that it must fulfill

the dispersion relation

$$\lambda = \frac{2\pi c}{\omega} = \frac{D}{|n|} \left( \frac{1}{\beta_0} - \sin \eta \right) = \frac{2\pi}{k_0}, \quad (1)$$

where  $\lambda$  is the wavelength of the radiation,  $n$  is the spectral order,  $\beta_0$  is the ratio of the electron velocity to the speed of light  $c$ ,  $D$  is the period of the grating, and  $k_0$  is the wave number. For one electron, the radiated energy density per unit solid angle in direction  $(\eta, \xi)$  of the SPR of order  $n$  can be written as[2]

$$\frac{dW_n}{d\Omega} = \frac{Le^2}{2D^2\epsilon_0} \frac{n^2 \cos^2 \eta \cos^2 \xi}{(1/\beta_0 - \sin \eta)^3} |R_n|^2 \exp\left(\frac{-2z_0}{\lambda_e}\right), \quad (2)$$

Where  $|R_n|^2$  is radiation factor,  $L = N_g D$  is the total length of grating,  $N_g$  is the number of grating periods,  $\eta$  and  $\xi$  are the emission angles as introduced in Fig. 1, and  $\lambda_e$  is the so-called “evanescent wavelength”. For  $N_e$  electrons in a bunch, coherent radiation is produced for wavelengths longer than the bunch length. The total energy density becomes

$$\left(\frac{dW_n}{d\Omega}\right)_{N_e} = \frac{dW_n}{d\Omega} (N_e S_{inc} + N_e(N_e - 1)S_{coh}), \quad (3)$$

where  $S_{inc}$  and  $S_{coh}$  are the incoherent and the coherent form factors, respectively.

## RADIATION FACTOR: NARROW BAND SMITH-PURCELL RADIATION

To calculate the radiation energy, the key problem is to calculate the radiation factors. Van den Berg model[3, 4] is a rigorous solution under infinitely long and wide grating assumption. The radiated energy is calculated by solving two separated integral equations, each having a periodic Green’s function, excited by the charge wake fields. Kesar and co-workers developed EFIE[5] and FDTD[6] model which extend Van den Berg’s model for a finite grating size, and the two models agree well with each other. However they are more complex and need a much longer computation time. For the low relativistic electron energies ( $\sim KeV$ ), Van den Berg’s model ensures a reasonable agreement with experiments[7, 8]. The experiment at 15 MeV have demonstrated good agreement between measured power and the predictions of the EFIE model[9]. The simulated results at 18 MeV with 20 period length showed that in the plane  $(x, z)$ , the angular radiation intensity per groove is greater than that of Van den Berg’s model and their intensity shapes are close to each other[5]. As the number of periods increases, the simulated radiation intensity will be more and more close to that of Van den Berg’s model[5, 6, 10].

2: Photon Sources and Electron Accelerators

A24 - Other Linac-Based Photon Sources

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Haerberlé et al.[11] reviewed the theoretical studies and concluded that the modal expansion method[12] is a simpler and more time saving way to calculate SPR than other approaches based on Van den Berg's model. So the modal expansion method is applied for the calculation in this paper. Here we assume the grating is enough large to satisfy the infinitely long and wide grating assumption.

For lamellar gratings with narrow groove width and normal depth which can be treated as an array of periodically arranged open resonators, narrow-band SPR can be excited by electron beam passes close to the surface[13]. Starting from the theory of Van den Berg, we deduce a approximately equation of the wave number  $k_p$  corresponding to the peak frequency of the narrow-band radiation and the parameters of grating:

$$\tan(k_p h) \approx \frac{-1 + \sqrt{1 + \frac{4k_p D}{\gamma \beta_0}}}{2ak_p}, \quad (4)$$

where  $\gamma$  is the Lorentz factor.

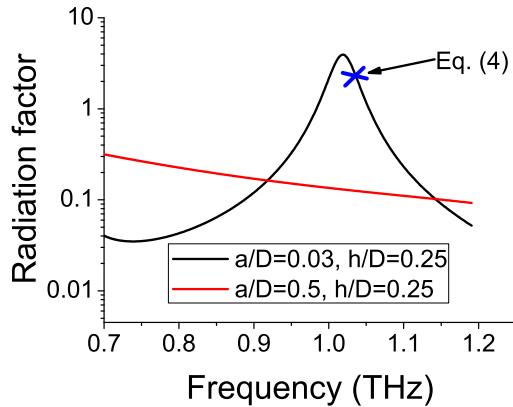


Figure 2: Radiation factor  $|R_n|^2$  as a function of frequency, the period of the grating  $D$  is 0.23 mm.

By using the modal expansion method, we calculate numerically the radiation factors of a ordinary lamellar grating and a narrow groove grating, as shown in Fig. (2). With our approximately function Eq. (4), the predicted frequency corresponding to the peak of radiation factor is also indicated in the figure. One can find that narrow-band SPR can be obtained when a lamellar grating with narrow groove width and normal depth is used, and the radiation factor is extremely higher. Fig. (3) shows the radiation factors for gratings with different groove widths. A grating with smaller groove width can lead to a narrower radiation spectrum.

## ELECTRON BUNCH TRAINS IN A PHOTO-CATHODE RF GUN

In a photocathode rf gun, since the photoelectron emission from the cathode is prompt with respect to the laser

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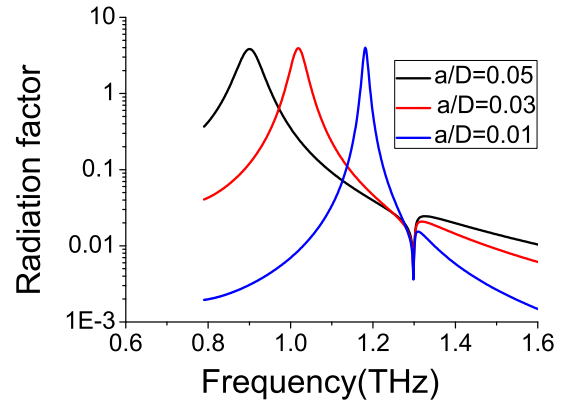


Figure 3: Radiation factors for gratings with different groove widths, where the period of the grating  $D$  is 0.23 m, and  $h/D$  is 0.25.

light, the longitudinal distribution of the electron bunch is primarily determined by the temporal characteristics of the laser pulse. To illuminate the cathode with a train of laser pulses with a periodicity of ps, the THz electron bunch train can be generated (The sketch map is shown in Fig.4). For microbunches with Gaussian distribution state, the bunching factor (temporal coherent form factor) of bunch train can be written as:

$$B(\omega) = \frac{1}{N_b} \left| \frac{\sin \frac{N_b \Delta t}{2} \omega}{\sin \frac{\Delta t}{2} \omega} \right| \exp\left(-\frac{\omega^2 \sigma_t^2}{2}\right), \quad (5)$$

where  $N_b$  is the number of microbunches,  $\Delta t$  is the space between the adjacent microbunches,  $\sigma_t$  is the rms length of microbunches, and  $\omega = 2\pi f$  is the frequency. In this

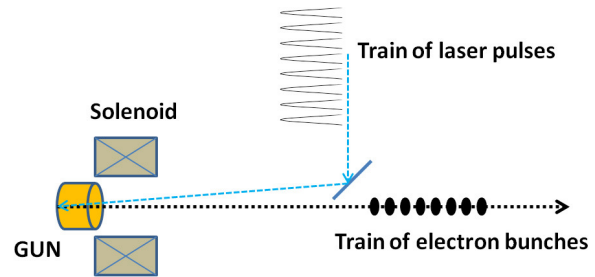


Figure 4: Layout of the photocathode rf gun.

case, the problem arises on how to preserve the modulation for large enough beam currents when longitudinal space-charge effects might contribute to smearing out any structure in the temporal beam profile. To restrain the space charge forces, we proposed in previous paper to use a stacked laser pulse with a greater transverse radius, and a multicell gun is suggested to obtain electron microbunches with higher energies[14]. According to ASTRA[15] simulation, Fig. (5) shows the bunching factors and the distribution of 8 microbunches in the x-z plane at the focal point of

the solenoid (0.96m downstream from the cathode). Where the bunch charge is 400 pC ( $50 \text{ pC} \times 8$  microbunches), and the space between the adjacent microbunches is 1 ps.

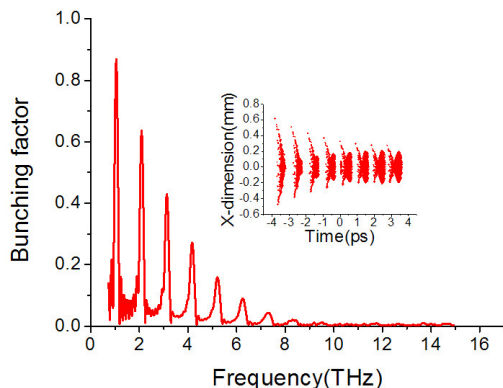


Figure 5: Bunching factor and the distribution of 8 microbunches in the x-z plane.

## RESONANTLY ENHANCED NARROW-BAND COHERENT SMITH-PURCELL RADIATION

When the THz electron bunch train passes close to the surface of the lamellar grating with narrow width and normal depth grooves, the frequencies of the excited narrow-band SPR and the electron microbunches can be resonated, then highly intense THz radiation can be generated. The radiation energy can be evaluated by using Eq. (2) and Eq. (3), and the parameters of the grating and electron bunch are listed in Tab. (1). As shown in Fig. (6), the radiation energy can be tens  $\mu\text{J}$  per solid angle.

Table 1: Parameters of Grating and Electron Bunch

Period (D)	0.23 mm
Width of the grooves (a)	$6.9 \mu\text{m}$
Depth of the grooves (h)	$57.5 \mu\text{m}$
Number of periods ( $N_g$ )	400
Height ( $z_0$ )	1.0 mm
Charge (q)	400 pC
Coherent factor	as shown in Fig. (5)

## SUMMARY

In this paper, we present a scheme to generate resonantly enhanced narrow-band coherent THz radiation. Lamellar grating with narrow width and normal depth grooves is used to generate narrow-band Smith-Purcell radiation, and THz bunch train from a photocathode rf gun is used as an excitor. Carefully choosing the parameters, the frequencies of the excited narrow-band SPR and the electron

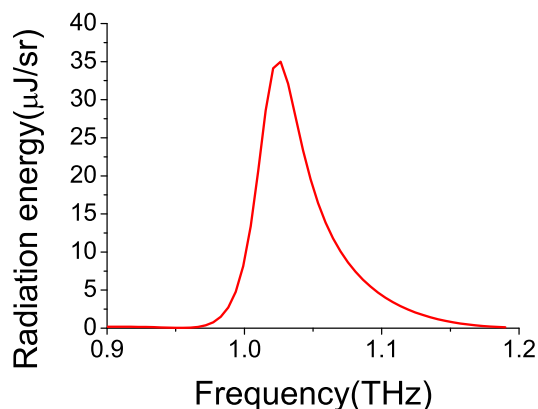


Figure 6: The radiation energy per solid angle excited by electron bunch train.

microbunches can be resonated, then highly intense THz radiation can be obtained. The proposed scheme is in principle able to generate intense narrow-band THz radiation and offers a promising way towards the intense narrow-band THz sources. This powerful compact THz radiation source should have wide applications in THz imaging, non-linear spectroscopy, etc.

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