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Gravitational time advancement under gravity's rainbow

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ABSTRACT

Under gravity's rainbow, we investigate its effects on the gravitational time advancement, which is a natural consequence of measuring proper time span for a photon's round trip. This time advancement can be complementary to the time delay for testing the gravity's rainbow, because they are sensitive to different modified dispersion relations (MDRs). Its observability on ranging a spacecraft far from the Earth by two radio and a laser links is estimated at superior conjunction (SC) and inferior conjunction (IC). We find that (1) the IC is more favorable than the SC for measurement on the advancement caused by the rainbow; (2) a specific type of MDR has a significantly larger effect on the advancement than others in both SC and IC cases; and (3) a combination of available optical clocks and the realization of planetary laser ranging in the future will benefit distinguishing the gravity's rainbow from GR by measuring the gravitational time advancement.

1. Introduction

Einstein's general relativity (GR) has been the most successful theory of gravitation when it faces all of astronomical observations and physical experiments [1,2]. However, it seems that GR might be incomplete since it still cannot be rigorously unified with quantum mechanics. If GR is indeed the classical limit of a theory of quantum gravity, there should also have a semiclassical limit or an effective field theory [3,4]. The leading order of such an effective theory can go back to GR and the next-to-leading-order might phenomenologically have corrections depending on the Planck energy E_p or the Planck length l_p [5,6]. In the present work, we focus on those corrections associated with E_p, which can yield modified dispersion relations (MDRs).

In order to incorporate MDRs into curved spacetime, an approach called *gravity's rainbow* was proposed [7]. It is assumed that the geometry of spacetime is also determined by the ratio of the energy of a test particle to E_p, which leads to a rainbow metric. Cosmology in the gravity's rainbow scenario has been intensively studied [8–19]. In the rainbow spacetime, black holes and neutron stars [20–36], thermodynamics and Hawking radiation [37–49], its quantum properties [50–54] and its application in modified gravity [55–59] are also widely discussed. Dynamics of massive and massless particles in the gravity's rainbow is as well investigated [60–63]. In [64], a proposal for testing gravity's rainbow in the Solar System is raised and upper bounds on the parameters of the rainbow functions are obtained based on the experiments on light deflection, photon time delay, gravitational redshift and the weak equivalence principle.

Recently, a new type observable of the Solar System experiments, which is called gravitational time advancement, has been proposed and studied [65,66]. The gravitational time advancement is a natural consequence of a curved spacetime if an observer, who is located at a stronger gravitational field, measures the proper time span for the round trip of a photon passing through a weaker field [65]. It was found [66] that dark energy and dark matter can affect the gravitational time advancement, whose magnitude is small but in principle observable.

In this work, as an extension of the previous works [65,66], we will investigate the gravitational time advancement under the gravity's rainbow and examine its possible observables. In Sect. 2, the rainbow metric we adopt is briefly reviewed for completeness. We detailedly investigate the gravitational time advancement under this rainbow spacetime in Sect. 3, in which two generic configurations for the observer and the turning point of the round trip of a photon are considered. Observability of the time advancement within the rainbow scenario is discussed in Sect. 4. Finally, in Sect. 5, we summarize our results.

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2. Rainbow spacetime

For the observations and experiments conducted in the Solar System, the dominance of the Sun ensures that a spherically symmetric spacetime is a sufficiently good approximation. In the framework of the gravity's rainbow, the Sun's Schwarzschild metric is extended as [7]

$$\mathrm{d}s^2 = -B(r)\mathrm{d}t^2 + A(r)\mathrm{d}r^2 + C(r)\mathrm{d}\Omega^2, \tag{1}$$

where r is the radial distance from the origin and $\mathrm{d}\Omega^2 = \mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\phi^2$ and the metric coefficients are

$$B(r) = f(E)^{-2}\left(1 - \frac{2GM}{r}\right), \tag{2}$$

$$A(r) = g(E)^{-2}\left(1 - \frac{2GM}{r}\right)^{-1}, \tag{3}$$

$$C(r) = r^2 g(E)^{-2}. \tag{4}$$

Here, $f(E)$ and $g(E)$ are the rainbow functions determined by MDRs and their forms are based on phenomenological motivations:

1. Originated from loop quantum gravity and noncommutative spacetime, the rainbow functions are [5,67]

$$f(E/E_p) = 1, \quad g(E/E_p) = \sqrt{1 - \eta E/E_p}, \tag{5}$$

where η is a model parameter. Following the notation of [64], we denote it as MDR1 for short.

2. In order to explain the hard spectra of gamma-ray bursts at cosmological distances, the rainbow functions are proposed to be [68]

$$f(E/E_p) = \frac{e^{\alpha E/E_p} - 1}{\alpha E/E_p}, \quad g(E/E_p) = 1, \tag{6}$$

where α is a model parameter. Following the notation of [64], we denote it as MDR2.

3. Providing a constant speed of light and a solution to the horizon problem [7], the rainbow functions are proposed to be [69]

$$f(E/E_p) = g(E/E_p) = \frac{1}{1 - \lambda E/E_p}, \tag{7}$$

where λ is a model parameter. Following the notation of [64], we denote also it as MDR3.

Based on the rainbow metric (1) and MDRs (5)–(7), we can calculate the gravitational time advancement under the gravity's rainbow.

3. Gravitational time advancement under rainbow

The gravitational time delay is the fourth test of GR by measuring the time delay between transmission of radar pulses towards either Venus or Mercury and detection of the echoes [70]. This delay is caused by the dependence of the (average) speed of a light ray on the strength of the gravitational potential along its path. However, if we consider that an observer is located at a place closer to the Sun and the turning point of the round trip of the light ray is farther to the Sun, then measurement of the proper time span of the light's round trip can give the gravitational time advancement in GR [65] and in the presence of dark energy and dark matter [66].

For a photon moving in the gravity's rainbow spacetime (1), its null worldline leads to [71]

$$0 = -B(r)\dot{t}^2 + A(r)\dot{r}^2 + C(r)\dot{\phi}^2, \tag{8}$$

where the dot mean derivative against an affine parameter. Because of the spherical symmetry of the gravitational field, the orbit of the photon is confined to the equatorial plane $\theta = \pi/2$. Along the light trajectory, we have two conserved quantities [71]:

$$\mathcal{E} = B(r)\dot{t} \quad \text{and} \quad \mathcal{L} = C(r)\dot{\phi}. \tag{9}$$

We can have [71]

$$\frac{\mathrm{d}\phi}{\mathrm{d}r} = \pm\frac{1}{C(r)}\left[\frac{1}{A(r)B(r)}\left(\frac{1}{b^2} - \frac{B(r)}{C(r)}\right)\right]^{-1/2}, \tag{10}$$

where $b \equiv \mathcal{L}/\mathcal{E}$. At the closest approach d, $\mathrm{d}r/\mathrm{d}\phi = 0$ gives

$$b = \sqrt{\frac{C(d)}{B(d)}}. \tag{11}$$

Then, the relationship between t and r for light can be obtained as [71]

$$\frac{\mathrm{d}t}{\mathrm{d}r} = \pm\frac{1}{b}\left[\frac{B(r)}{A(r)}\left(\frac{1}{b^2} - \frac{B(r)}{C(r)}\right)\right]^{-1/2}, \tag{12}$$

which leads to a generic expression for the time span of a photon from d to r under the gravity's rainbow as [71,64]

$$\begin{aligned} t(r,d) &\equiv \int_d^r \frac{1}{b}\left[\frac{B(r)}{A(r)}\left(\frac{1}{b^2} - \frac{B(r)}{C(r)}\right)\right]^{-1/2}\mathrm{d}r \\ &= \frac{f(E)}{g(E)}\left[\sqrt{r^2-d^2} + GM\sqrt{\frac{r-d}{r+d}} \right. \\ &\quad \left. + 2GM\ln\left(\frac{r+\sqrt{r^2-d^2}}{d}\right)\right] + \mathcal{O}(G^2). \end{aligned} \tag{13}$$

Since there is a plus sign in the front of the logarithmic correction in Eq. (13), the photon is always delayed with respect to the one in absence of the Sun, i.e., $f(E)[g(E)]^{-1}\sqrt{r^2-d^2}$. When $f(E) = g(E)$, Eq. (13) can effectively return to the one in GR [71]. It also means that the gravity's rainbow with MDR3 does not affect the gravitational time delay [64].

Now, we consider two points A and B in the spacetime (1). Either A or B can be set as the location of the observer and the other will be the location of the turning point of the round trip of a photon. Without loss of generality, we assume r_A is always larger than r_B, i.e. $r_A > r_B$, where r_A and r_B are respectively the radial coordinates of the points A and B. There are two cases: (i) as seen from the point B, the point A is on the opposite side of the Sun, which is denoted as "A-⊙-B", and (ii) the points A and B are on the same side of the Sun, denoted as "A-B-⊙". See Fig. 1 for details.

3.1. A-⊙-B: opposite sides case

According to Eq. (13), the total coordinate time required for the time duration of a photon travelling from the point A to the point B and back to A is given by

$$\begin{aligned} \Delta t_{A\odot B} &= 2t(r_A, d) + 2t(r_B, d) \\ &= \frac{f(E)}{g(E)}\left[2\sqrt{r_A^2 - d^2} + 2\sqrt{r_B^2 - d^2}\right. \end{aligned}$$

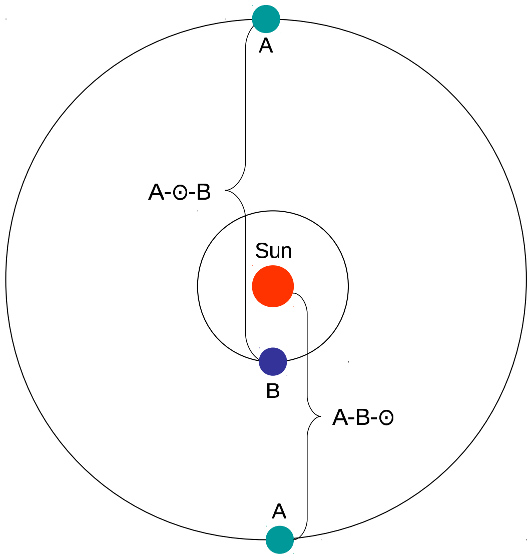

Fig. 1. Schematic diagram (not to scale) of configurations "A-⊙-B" and "A-B-⊙".

$$+2GM\sqrt{\frac{r_A-d}{r_A+d}}+2GM\sqrt{\frac{r_B-d}{r_B+d}}$$
$$+4GM\ln\left(\frac{r_A+\sqrt{r_A^2-d^2}}{d}\right)$$
$$+4GM\ln\left(\frac{r_B+\sqrt{r_B^2-d^2}}{d}\right)\Bigg]+\mathcal{O}(G^2), \tag{14}$$

and its proper time span measured by an observer at the point A is

$$\begin{aligned}\Delta\tau_{A\odot B} &= \frac{1}{f(E)}\left(1-\frac{GM}{r_A}\right)\Delta t_{A\odot B}\\ &= \frac{1}{g(E)}\Bigg[2\sqrt{r_A^2-d^2}+2\sqrt{r_B^2-d^2}\\ &\quad +2GM\sqrt{\frac{r_A-d}{r_A+d}}+2GM\sqrt{\frac{r_B-d}{r_B+d}}\\ &\quad +4GM\ln\left(\frac{r_A+\sqrt{r_A^2-d^2}}{d}\right)\\ &\quad +4GM\ln\left(\frac{r_B+\sqrt{r_B^2-d^2}}{d}\right)\\ &\quad -2GM\frac{\sqrt{r_A^2-d^2}}{r_A}-2GM\frac{\sqrt{r_B^2-d^2}}{r_A}\Bigg]\\ &\quad +\mathcal{O}(G^2).\end{aligned} \tag{15}$$

The signal takes more time for the round trip than the one in absence of the Sun, i.e. $\Delta\tau_{A\odot B}(M=0)$, and its delay is positive for $r_A > r_B \geq d$. Unlike the case of coordinate time span (14) which is immune to MDR3, the proper time span under the gravity's rainbow (15) depends only on the rainbow function $g(E)$, which means that it cannot be influenced by MDR2.

Nevertheless, if the observer is located at the point B which is closer to the Sun than the point A, the coordinate time delay for the round trip from B to A and back to B will remain the same as Eq. (14) but the proper time span measured by the observer at B will become

$$\begin{aligned}\Delta\tau_{A\odot B} &= \frac{1}{f(E)}\left(1-\frac{GM}{r_B}\right)\Delta t_{A\odot B}\\ &= \frac{1}{g(E)}\Bigg[2\sqrt{r_A^2-d^2}+2\sqrt{r_B^2-d^2}\\ &\quad +2GM\sqrt{\frac{r_A-d}{r_A+d}}+2GM\sqrt{\frac{r_B-d}{r_B+d}}\\ &\quad +4GM\ln\left(\frac{r_A+\sqrt{r_A^2-d^2}}{d}\right)\\ &\quad +4GM\ln\left(\frac{r_B+\sqrt{r_B^2-d^2}}{d}\right)\\ &\quad -2GM\frac{\sqrt{r_A^2-d^2}}{r_B}-2GM\frac{\sqrt{r_B^2-d^2}}{r_B}\Bigg]\\ &\quad +\mathcal{O}(G^2).\end{aligned} \tag{16}$$

Since the last two terms in Eq. (16) dominate those terms proportional to GM due to their dependence of r_B^{-1}, this proper time span can be effectively decreased and even be less than the proper time span in the absence of the Sun, i.e. $\Delta\tau_{A\odot B}(M=0)$, if r_A is sufficiently larger than r_B by a specific value depending on r_B and d. This effect is the gravitational time advancement (negative time delay) under the gravity's rainbow, which is caused by the fact that clocks run differently at different positions in the gravitational field [65]. When gravity's rainbow vanishes, i.e., $g(E)=1$, our result (16) can return to the one in GR given by [65].

If we consider the configuration of superior conjunction (SC) that the points A and B are on the opposite sides of the Sun and $r_A > r_B \gg d$ which might happen in radio tracking a spacecraft, the gravitational time advancement (16) can be reduced to a simpler form as

$$\begin{aligned}\Delta\tau_{A\odot B}^{\mathrm{SC}} &= \frac{1}{g(E)}\Bigg[2(r_A+r_B)+2GM+4GM\ln\frac{4r_Ar_B}{d^2}\\ &\quad -2GM\frac{r_A}{r_B}\Bigg]+\mathcal{O}\left(G^2,\frac{d}{r_A},\frac{d}{r_B}\right).\end{aligned} \tag{17}$$

We can see that, from Eqs. (16) and (17), MDR1 and MDR3 can affect the gravitational time advancement but MDR2 cannot. If $\eta > 0$ and $g(E) < 1$, MDR1 can amplify the advancement and make it larger than the one in GR. On the contrary to the behavior of MDR1, MDR3 can make the advancement smaller than the one in GR when $\lambda > 0$ and $g(E) > 1$. However, the time delay (14) cannot distinguish MDR3 from the others. It suggests that measurements on the gravitational time delay and advancement can be complementary to each other for constraining MDRs (see Table 1 for a summary).

Table 1
Detectability on MDRs for gravitational time delay and advancement.

MDR	Delay	Advancement
1	Yes	Yes
2	Yes	No
3	No	Yes

3.2. A-B-⊙: the same side case

If the points A and B are on the same side of the Sun and $r_A > r_B$, the coordinate time span for a photon travelling from A to B and back to A can be worked out based on Eq. (13) as

$$\begin{aligned}\Delta t_{AB\odot} &= 2t(r_A,d) - 2t(r_B,d)\\ &= \frac{f(E)}{g(E)}\Bigg[2\sqrt{r_A^2-d^2} - 2\sqrt{r_B^2-d^2}\\ &\quad + 2GM\sqrt{\frac{r_A-d}{r_A+d}} - 2GM\sqrt{\frac{r_B-d}{r_B+d}}\\ &\quad + 4GM\ln\left(\frac{r_A+\sqrt{r_A^2-d^2}}{d}\right)\\ &\quad - 4GM\ln\left(\frac{r_B+\sqrt{r_B^2-d^2}}{d}\right)\Bigg] + \mathcal{O}(G^2), \qquad (18)\end{aligned}$$

where the minus sign on the right hand side at the first line is physically caused by such a configuration. When an observer is at the point A, the measured proper time span is

$$\begin{aligned}\Delta\tau_{AB\odot} &= \frac{1}{f(E)}\left(1-\frac{GM}{r_A}\right)\Delta t_{AB\odot}\\ &= \frac{1}{g(E)}\Bigg[2\sqrt{r_A^2-d^2} - 2\sqrt{r_B^2-d^2}\\ &\quad + 2GM\sqrt{\frac{r_A-d}{r_A+d}} - 2GM\sqrt{\frac{r_B-d}{r_B+d}}\\ &\quad + 4GM\ln\left(\frac{r_A+\sqrt{r_A^2-d^2}}{d}\right)\\ &\quad - 4GM\ln\left(\frac{r_B+\sqrt{r_B^2-d^2}}{d}\right)\\ &\quad - 2GM\frac{\sqrt{r_A^2-d^2}}{r_A} + 2GM\frac{\sqrt{r_B^2-d^2}}{r_A}\Bigg]\\ &\quad + \mathcal{O}(G^2). \qquad (19)\end{aligned}$$

Like one of the A-⊙-B cases that the observer is at the point A [see Eq. (15)], the signal takes more time for the round trip and the delay is positive for $r_A > r_B \geq d$.

If an observer in the A-B-⊙ is at the point B instead of A, the coordinate time delay for the round trip from B to A and back to B is the same as Eq. (18) and the proper time span measured by the observer at B reads as

$$\begin{aligned}\Delta\tau_{AB\odot} &= \frac{1}{f(E)}\left(1-\frac{GM}{r_B}\right)\Delta t_{AB\odot}\\ &= \frac{1}{g(E)}\Bigg[2\sqrt{r_A^2-d^2} - 2\sqrt{r_B^2-d^2}\\ &\quad + 2GM\sqrt{\frac{r_A-d}{r_A+d}} - 2GM\sqrt{\frac{r_B-d}{r_B+d}}\\ &\quad + 4GM\ln\left(\frac{r_A+\sqrt{r_A^2-d^2}}{d}\right)\\ &\quad - 4GM\ln\left(\frac{r_B+\sqrt{r_B^2-d^2}}{d}\right)\\ &\quad - 2GM\frac{\sqrt{r_A^2-d^2}}{r_B} + 2GM\frac{\sqrt{r_B^2-d^2}}{r_B}\Bigg]\\ &\quad + \mathcal{O}(G^2). \qquad (20)\end{aligned}$$

Like the situation of Eq. (16) for A-⊙-B, those terms depending on r_B^{-1} dominate others proportional to GM in the above equation so that this proper time span can be smaller than the one in the absence of the Sun if r_A is sufficiently larger than r_B. It can be easily checked that, when we consider a special case that $r_A = r_B + \Delta R$, $\Delta R \ll r_B$, $d = 0$ and $g(E) = 1$, Eq. (20) can give the equation for the "small distance travel" in GR discussed in [65].

Another interesting case, which was not discussed in [65,66], is the configuration of inferior conjunction (IC) of A-B-⊙ where $r_A > r_B \gg d$ so that the gravitational time advancement (20) becomes to

$$\begin{aligned}\Delta\tau^{\mathrm{IC}}_{AB\odot} &= \frac{1}{g(E)}\Bigg[2(r_A-r_B) + 2GM + 4GM\ln\frac{r_A}{r_B}\\ &\quad - 2GM\frac{r_A}{r_B}\Bigg] + \mathcal{O}\left(G^2, \frac{d}{r_A}, \frac{d}{r_B}\right). \qquad (21)\end{aligned}$$

Unlike the case of small distance travel that r_A is comparable with r_B, r_A in the IC condition of Eq. (21) can be much larger than r_B, which can be used to describe ranging measurement on a spacecraft in deep space far beyond the Earth orbit.

4. Observability of time advancement under gravity's rainbow

After working out the equations for the gravitational time advancement under gravity's rainbow, we discuss its observability in this section.

4.1. A-⊙-B

In the A-⊙-B configuration, a SC condition is favorable for measurement on the time advancement due to the smallness of d. According to Eq. (17), we can find that the time advancement caused by the gravity's rainbow is given by

$$\delta\tau^{\mathrm{SC}}_{A\odot B} \equiv \Delta\tau^{\mathrm{SC}}_{A\odot B} - \Delta\tau^{\mathrm{SC}}_{A\odot B}\Big|_{M=0}, \qquad (22)$$

the time advancement in GR is

$$\bar{\delta}\tau^{\mathrm{SC}}_{A\odot B} \equiv \Delta\tau^{\mathrm{SC}}_{A\odot B}\Big|_{g=1} - \Delta\tau^{\mathrm{SC}}_{A\odot B}\Big|_{\substack{g=1\\ M=0}}, \qquad (23)$$

and their relative deviation is defined as

$$r^{\mathrm{SC}}_{A\odot B} \equiv \frac{\delta\tau^{\mathrm{SC}}_{A\odot B} - \bar{\delta}\tau^{\mathrm{SC}}_{A\odot B}}{\Delta\tau^{\mathrm{SC}}_{A\odot B}}. \qquad (24)$$

Since the time advancement is defined as negative time delay, $\delta\tau^{\mathrm{SC}}_{A\odot B} - \bar{\delta}\tau^{\mathrm{SC}}_{A\odot B} > 0$ means that the advancement caused by the gravity's rainbow is smaller than the one in GR, and vice versa. $r^{\mathrm{SC}}_{A\odot B}$ represents the theoretical resolution for time measurement required to distinguish the gravity's rainbow from GR.

We consider a SC condition that an observer on the Earth with $r_B = 1$ au[1] conducts two radio-tracking measurements on X-band (7.2 GHz) and Ka-band (34.3 GHz) to range a spacecraft at a distance of 40 au from the Sun, $r_A = 40$ au, which is close to the

[1] We use lower-case 'au' to represent the astronomical unit, according to International Astronomical Union 2012 Resolution B2: http://www.iau.org/static/resolutions/IAU2012_English.pdf.

Table 2
Estimation of observability on the gravitational time advancement in SC condition with links of X-band, Ka-band and visible laser where $r_A = 40$ au, $r_B = 1$ au and $d = 1.5$ $R_\odot$. The parameters in the rainbow functions are taken as $\eta = 1.3 \times 10^{20}$ and $\lambda = 8.5 \times 10^{21}$ [64] and their uncertainties are set as 10%.

MDR	Band	Frequency (GHz)	$\delta\tau^{SC}_{A\odot B}$ (µs)	$\bar{\delta}\tau^{SC}_{A\odot B}$ (µs)	$\delta\tau^{SC}_{A\odot B} - \bar{\delta}\tau^{SC}_{A\odot B}$ (s)	$r^{SC}_{A\odot B}$
1	X	7.2	$-88.5348375693597 \pm (1.4 \times 10^{-12})$	-88.5348375693464	$-(1.4 \pm 0.1) \times 10^{-17}$	$-(3.4 \pm 0.3) \times 10^{-22}$
1	Ka	34.3	$-88.5348375694125 \pm (6.7 \times 10^{-12})$	ibid.	$-(6.7 \pm 0.7) \times 10^{-17}$	$-(1.6 \pm 0.2) \times 10^{-21}$
1	Visible	6×10^5	$-88.5348387388309 \pm (1.16949 \times 10^{-7})$	ibid.	$-(1.2 \pm 0.1) \times 10^{-12}$	$-(2.9 \pm 0.3) \times 10^{-17}$
3	X	7.2	$-88.534837568 \pm (1.8 \times 10^{-8})$	ibid.	$(1.8 \pm 0.2) \times 10^{-15}$	$(4.5 \pm 0.5) \times 10^{-20}$
3	Ka	34.3	$-88.534837561 \pm (8.7 \times 10^{-8})$	ibid.	$(8.7 \pm 0.9) \times 10^{-15}$	$(2.1 \pm 0.2) \times 10^{-19}$
3	Visible	6×10^5	$-88.534684637 \pm (1.529 \times 10^{-5})$	ibid.	$(1.5 \pm 0.2) \times 10^{-10}$	$(3.7 \pm 0.4) \times 10^{-15}$

Table 3
Estimation of observability on the gravitational time advancement in IC condition with links of X-band, Ka-band and visible laser where $r_A = 40$ au and $r_B = 1$ au. The parameters in the rainbow functions are taken as $\eta = 1.3 \times 10^{20}$ and $\lambda = 8.5 \times 10^{21}$ [64] and their uncertainties are set as 10%.

MDR	Band	Frequency (GHz)	$\delta\tau^{IC}_{AB\odot}$ (µs)	$\bar{\delta}\tau^{IC}_{AB\odot}$ (µs)	$\delta\tau^{IC}_{AB\odot} - \bar{\delta}\tau^{IC}_{AB\odot}$ (s)	$r^{IC}_{AB\odot}$
1	X	7.2	$-311.4174782731487 \pm (4.9 \times 10^{-12})$	-311.4174782730994	$-(4.9 \pm 0.5) \times 10^{-17}$	$-(1.3 \pm 0.1) \times 10^{-21}$
1	Ka	34.3	$-311.4174782733345 \pm (2.35 \times 10^{-11})$	ibid.	$-(2.4 \pm 0.2) \times 10^{-16}$	$-(6.0 \pm 0.6) \times 10^{-21}$
1	Visible	6×10^5	$-311.4174823867135 \pm (4.11361 \times 10^{-7})$	ibid.	$-(4.1 \pm 0.4) \times 10^{-12}$	$-(1.0 \pm 0.1) \times 10^{-16}$
3	X	7.2	$-311.41747826664 \pm (6.4 \times 10^{-10})$	ibid.	$(6.5 \pm 0.7) \times 10^{-15}$	$(1.7 \pm 0.2) \times 10^{-19}$
3	Ka	34.3	$-311.41747824234 \pm (3.08 \times 10^{-9})$	ibid.	$(3.1 \pm 0.3) \times 10^{-14}$	$(7.9 \pm 0.8) \times 10^{-19}$
3	Visible	6×10^5	$-311.41694033895 \pm (5.37934 \times 10^{-5})$	ibid.	$(5.4 \pm 0.5) \times 10^{-10}$	$(1.4 \pm 0.1) \times 10^{-14}$

semi-major axis of Pluto, and the closest approach of the radio signals is $d = 1.5\ R_\odot$ where $R_\odot$ is radius of the Sun. Following a trend of developing interplanetary laser ranging [72–74], we take a laser link with 600 THz into account. The values of parameters η for MDR1 (5) and λ for MDR3 (7) are respectively taken as 1.3×10^{20} and 8.5×10^{21} based on the results from the time delay and redshift experiments according to [64], because these measurements are of the same kind we discuss here.

Our results of observability for this case are listed in Table 2. It is found that, in this SC condition, the gravitational time advancements under the gravity's rainbow and the one in GR can reach about −88 microsecond (µs). With MDR1, the contribution caused by the gravity's rainbow in the time advancement ranges from -1.4×10^{-17} s to -1.2×10^{-12} s, where the minus signs mean MDR1 enlarge the time advancement in GR and the absolute values depend on the frequency of ranging signal; and the time resolution for distinguishing the gravity's rainbow from GR needs to be from -3.4×10^{-22} to -2.9×10^{-17}. With MDR3, the contribution caused by the gravity's rainbow in the time advancement has values from 1.8×10^{-15} s to 1.5×10^{-10} s, where the positive values mean MDR3 lessen the time advancement in GR; and the time resolution is required to be from 4.5×10^{-20} to 3.7×10^{-15}.

It shows that, the time advancement caused by the gravity's rainbow with MDR3 has a bigger deviation from the one in GR than the advancement due to MDR1 in such a SC case by nearly 2 orders of magnitude. It also suggests that if the planetary laser ranging become available in the future, the measurement on the gravitational advancement might be able to detect the gravity's rainbow and obtain its new constraints, given the fact that optical clocks on the ground have achieved the accuracy and stability at the 10^{-18} level [75–77].

4.2. A-B-⊙

According to Eq. (21), the time advancement at IC condition caused by the gravity's rainbow in the A-B-⊙ configuration is given by

$$\delta\tau^{IC}_{AB\odot} \equiv \Delta\tau^{IC}_{AB\odot} - \Delta\tau^{IC}_{AB\odot}\Big|_{M=0}, \tag{25}$$

the time advancement in GR is

$$\bar{\delta}\tau^{IC}_{AB\odot} \equiv \Delta\tau^{IC}_{AB\odot}\Big|_{g=1} - \Delta\tau^{IC}_{AB\odot}\Big|_{\substack{g=1\\M=0}}, \tag{26}$$

and their relative deviation is defined as

$$r^{IC}_{AB\odot} \equiv \frac{\delta\tau^{IC}_{AB\odot} - \bar{\delta}\tau^{IC}_{AB\odot}}{\Delta\tau^{IC}_{AB\odot}}, \tag{27}$$

which represents the time resolution required for distinguishing the gravity's rainbow from GR in such an IC condition.

We consider an IC condition that an observer is at $r_B = 1$ au who conducts two radio ranging measurements (X-band and Ka-band) and a laser ranging (600 THz) on a spacecraft at $r_A = 40$ au. Our results of observability for this case are listed in Table 3. We find that, in this IC condition, the gravitational time advancements under the gravity's rainbow and the one in GR can reach about −311 µs, which is nearly 3.5 times larger than those in the SC condition we discuss before. It demonstrates that the IC condition is more favorable than the SC condition for measurement on the gravitational advancement. With MDR1, the contribution caused by the gravity's rainbow in the time advancement ranges from -4.9×10^{-17} s to -4.1×10^{-12} s and the time resolution for distinguishing the gravity's rainbow from GR needs to be from -1.3×10^{-21} to -1.1×10^{-16}, which also depend on the frequency. With MDR3, the contribution caused by the gravity's rainbow in the time advancement has values from 6.5×10^{-15} s to 5.4×10^{-10} s and the time resolution is required to be from 1.7×10^{-19} to 1.4×10^{-14}.

Like the case of SC condition, it shows the time advancement caused by the gravity's rainbow with MDR3 has a larger deviation from the one in GR than the advancement due to MDR1 in this IC case by nearly 2 orders of magnitude. It also suggests that the planetary laser ranging will benefit the detection on the gravity's rainbow in the future.

5. Conclusions

Under the gravity's rainbow with three various MDRs, we investigate its effects on the gravitational time advancement. If an observer measures the proper time span for the round trip of a photon passing through a weaker gravitational field, then such a time advancement will be a natural consequence. We find that this time advancement can be complementary to the classical test of Shapiro time delay because they are sensitive to different MDRs (see Table 1).

Considering ranging a spacecraft at a distance of Pluto from the Earth, we estimate its observability on the time advancement under SC and IC configurations (see Fig. 1). We also assume that two radio links (at X-band and Ka-band) and a laser link (600 THz) are used in the ranging. It is found that (1) the IC configuration is more favorable for measuring the time advancement; and (2) the time advancement caused by MDR3 is significantly larger than others (see Tables 2 and 3 for details). We expect that, with a combination of optical clocks and planetary laser ranging, measurements on the gravitational time advancement will benefit detecting the gravity's rainbow in the future.

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