

Dynamical study of viscous modified Chaplygin gas and confrontation with recent observational data

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Abstract. A cosmological Model of viscous modified Chaplygin gas in classical and loop quantum cosmology (LQC) is proposed and a dynamical stability study is investigated. It is shown that the model is consistent with the recent observational data and gives good predictions for the deceleration and state parameters. The model can also predict the time crossing and gives a solution to the coincidence problem. Furthermore, in LQC background, the big bang singularity found in classical cosmology cease to exist and is replaced by a bounce when the Hubble parameter vanishes at the LQC critical energy density.

1. Introduction

Recently, Type Ia Supernovae observational data [1–3] with cosmic microwave background anisotropies [4–6] and large galaxy surveys [7,8] have shown that the universe is undergoing an accelerated expansion phase. The existence of an exotic kind of energy, called dark energy, with negative pressure that drives the universe to expand was proposed. The mysterious force or energy leading to the accelerated expansion was attributed to:

- a) a vacuum energy
- b) a theory of a modified Newtonian dynamic (MOND) that can solve the problem of the velocity anomalies without the need of a concept of dark matter or dark energy ,
- c) $f(R)$ theory
- d) signature of extra-dimensions
- e) exotic kind of energy called dark energy with negative pressure that drives the universe to expand and is modeled by several candidates: The cosmological constant where the dark energy is a perfect fluid and a dynamical dark energy with a Chaplygin gas models (CG) namely,
 - i) the ordinary CG which has as an equation of a state (EoS) $p = -B/\rho$ and it turns out that it does not fit with the observational data,
 - ii) the generalized CG with an EoS $p = -B/\rho^\alpha$ suffering from perturbative instabilities
 - iii) the modified CG denoted by MCG which is considered as one of the successful dark energy candidate model with an EoS of the form $p = A\rho - B/\rho^\alpha$ where A , B and α are real constant parameters. It is a combined model that unifies both dark energy and dark matter and gives a suitable negative pressure that drives the acceleration of the universe. The MCG EoS parameters were constrained using different observational data. [10–12] and it is preferred because of its small minimum chi square(χ^2) value[9].



The goal of this paper is to make a dynamical study of VMCG and confrontations with recent observational data. In section2, we give a brief review on the viscous modified Chaplygin gas (VMCG) model and solve analytically the conservation equation, check the behavior of the solutions at early , present and late time dominated universe. In section3, we show how the model is constrained using the χ^2 method and recent observational data with the help of Mathematica to calculate the best fit values of EoS parameters and draw the contour plots of some confidence levels. We derive the cosmological parameters taking into account these best fit values. The behavior of the model is then probed at small and present scale using the time evolution of cosmological parameters. In section4, and in the loop quantum cosmology (LQC) framework, a dynamical analysis of our VMCG is conducted. Finally, in section5, we draw our conclusions

2. Classical VMCG model

The VMCG is a generalization of the modified Chaplygin gas model with an EoS of the form [13]

$$p_{eff} = A\rho_{MCG} - \frac{B}{\rho_{MCG}^\alpha} - 3\xi_0 H \rho_{MCG}^{1/2} \quad (1)$$

where ρ_{MCG} is the energy density of MCG, A and B are constants, α is a positive constant, ξ_0 a positive bulk viscosity coefficient, and $H = \dot{a}/a$ is the Hubble expansion parameter. The dot stands for the cosmic time derivative. This model was investigated in Ref. [14] and it assumes that the expansion process is a collection of states out of thermal equilibrium that gives rise to a bulk viscosity. The interest in the VMCG comes from the fact that: First of all, MCG was preferred among other models according to its concordance with the observational data and because of its negative pressure that derives the universe acceleration at late time as well as its effective coupling which unifies dark energy and dark matter fluids. Second, the universe is filled with imperfect fluid (Bulk viscosity). In what follows, we consider a flat space Friedmann–Robertson–Walker (FRW) universe filled with VMCG, the conservation equation and the Friedmann equation are given by

$$\dot{\rho}_{MCG} + 3(\rho_{MCG} + p_{eff}) = 0 \quad , \quad H^2 = \frac{\rho_{MCG}}{3} \quad (2)$$

Using Eqs. (1)–(2) we obtain the energy density in terms of the scale factor a that is:

$$\rho_{MCG} = \left(\frac{K}{a^{3(\alpha+1)(1+A-\sqrt{3}\xi_0)}} + \frac{B}{1+A-\sqrt{3}\xi_0} \right)^{\frac{1}{1+\alpha}} \quad (3)$$

where K is an integration constant. As the energy density varies with its parameters, we use the bifurcation theorem in studying the behavior of the solution of the VMCG conservation equation knowing that the dynamics of Eq. (2) depends on its equilibrium and stability. In fact, the equilibrium point reads:

$$\rho_{(MCG)eq} = \left(\frac{B}{1+A-\sqrt{3}\xi_0} \right)^{\frac{1}{1+\alpha}} \quad (4)$$

This result indicates that at large scale ($a \rightarrow \infty$), the energy density is only stable if $\alpha > -1$, $1 + A - \sqrt{3}\xi_0 > 0$ and $B > 0$ corresponding to a dark energy dominated universe. The effective state, deceleration and adiabatic sound speed parameters have as expressions (in terms of the redshift):

$$\omega_{eff} = A - \frac{B}{\rho_{MCG}^{\alpha+1}} - 3\xi_0 H(z) \rho_{MCG}^{-1/2} \quad , \quad q(z) = -1 + \frac{1}{2} \left(3 + \frac{\rho_{MCG} \omega_{eff}}{H(z)^2} \right)$$

and

$$c^2 = A + \alpha \frac{B}{\rho_{MCG}^{\alpha+1}} - \frac{1}{2} \xi_0 \frac{\rho_{MCG}^{1/2}}{H(z)} - \frac{3}{2} \xi_0 H(z) \rho_{MCG}^{-1/2} \quad (5)$$

The conservation equation in Eq. (2) can be rewritten as

$$(1+z) \frac{d\Omega_{MCG}(z)}{dz} = 3 \left\{ (1+A)\Omega_{MCG}(z) - B'\Omega_{MCG}^{-\alpha} - \sqrt{3}\xi_0\Omega_{MCG}^{\frac{1}{2}}(z)[(1+z)^3\Omega_{0M} + \Omega_{MCG}(z)]^{1/2} \right\} \quad (6)$$

with

$$\Omega_{MCG}(z) = \frac{\rho_{MCG}}{3H_0^2} \quad (7)$$

where Ω_{0M} is the present value of the baryonic matter density, z is the redshift parameter, H_0 is the present Hubble parameter and $B' = \frac{B}{(3H_0^2)^{\alpha+1}}$. The Hubble parameter $H(z)$ has as an expression in terms of the redshift parameter :

$$H(z) = H_0 [(1+z)^3\Omega_{0M} + \Omega_{MCG}(z)]^{1/2} \quad (8)$$

3. Constraining VMCG and best fit values of the EoS and Cosmological parameters

We constrain the EoS parameters (H_0, A, B, α, ξ_0) of the VMCG model using the Supernovae Type Ia observational data that consists of 580 data points and belong to Union 2.1 (2012) data. The best fit values of the parameters are obtained by the minimization of the χ^2 function of the distance modulus μ . To reduce the number of the free parameters of the model, we marginalize assuming a constant prior over H_0 by constructing a probability density function for the parameters. As the number of the free parameters is still large, we first fix the viscous coefficient that is assumed to be positive, and then we constrain the EoS parameters (A, B, α). We find that only small values of ξ_0 corresponding to $\omega \approx -1$ are consistent with the observational data. The best fit values of the EoS parameters are listed in Table 1, where B and α have approximately the same values for different choices of ξ_0 . The contour plots of the confidence levels 68.27%, 90% and 95.45% for both A and B are shown in fig. 1.

Table 1. . Summary of the best estimates of the EoS parameters for the VMCG and their 1σ error using Union 2.1 SNe Ia data, and *d.o.f* denotes the degrees of freedom.

EoS parameter	ξ_0	α	A	B	χ^2	$\chi^2/d.o.f$
Best	0.01	$0.551^{+0.283}_{-0.218}$	$-0.167^{+0.175}_{-0.191}$	$0.543^{+0.214}_{-0.232}$	562.191	0.974
Fit	0.02	$0.548^{+0.285}_{-0.219}$	$-0.149^{+0.182}_{-0.188}$	$0.543^{+0.215}_{-0.232}$	562.191	0.974
Values	0.0001	$0.549^{+0.283}_{-0.218}$	$-0.186^{+0.168}_{-0.199}$	$0.543^{+0.214}_{-0.232}$	562.191	0.974

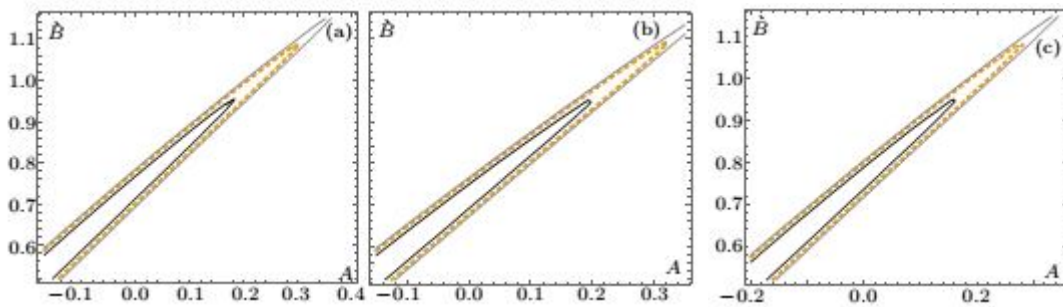


Figure 1. Contour plot of 68.27% CL (black), 90% CL (dashed) and 95.45% CL (gray) regions for VMCG parameters A and B' when (a) $\xi_0 = 0.01$, (b) $\xi_0 = 0.02$ and (c) $\xi_0 = 0.0001$

In fig. 2, the sound speed is plotted in terms of the redshift parameter z using the best fit data listed in Table 1. In the early universe, the sound speed has negative values introducing a fast exponential growth of instabilities that can be explained by the fact that VMCG is an effective coupled dark energy/dark matter fluid and in such models instabilities can occur when the coupling strength is

strong enough compared with the gravitational one.[15]. Moreover, when the coupling becomes moderate in the transition from a matter to a dark energy dominated universe, the sound speed c^2 changes the sign to take positive values and the perturbations grow much slower until the universe is dominated by dark energy. At large scale, the sound speed takes a positive value near zero leading to stable oscillating perturbations and structure predictions consistent with observations. Fig.3 shows respectively the variation of the effective state parameter ω_{eff} and the deceleration parameter q with respect to the redshift z at the best fit values of Table 1. It is obvious that the current value of ω_{eff}

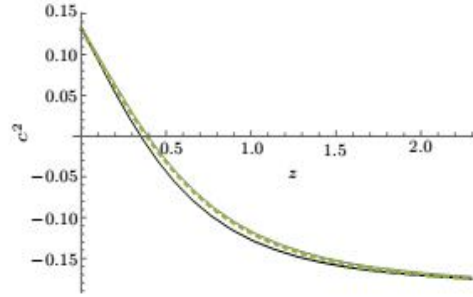


Figure 2. The sound speed c^2 as a function of the redshift z at best fit values of Table 1 for $\xi_0 = 0.01$ (gray line), $\xi_0 = 0.02$ (solid black line) and $\xi_0 = 0.0001$ (dashed line).

varies between -0.76 and -0.74 for different values of ξ_0 admitting an accelerated universe. At matter dominated era, ω_{eff} takes values in the range $\omega_{eff} > -1/3$ allowing a deceleration phase. When the deceleration parameter crosses the zero to negative values, ω_{eff} takes values less than -0.33 and the VMCG behaves like quintessence scalar field. Notice that, for all best values of Table 1, the current deceleration parameter varies between -0.60 and -0.57 , which is consistent with $q_0 \in [-0.7, -0.4]$

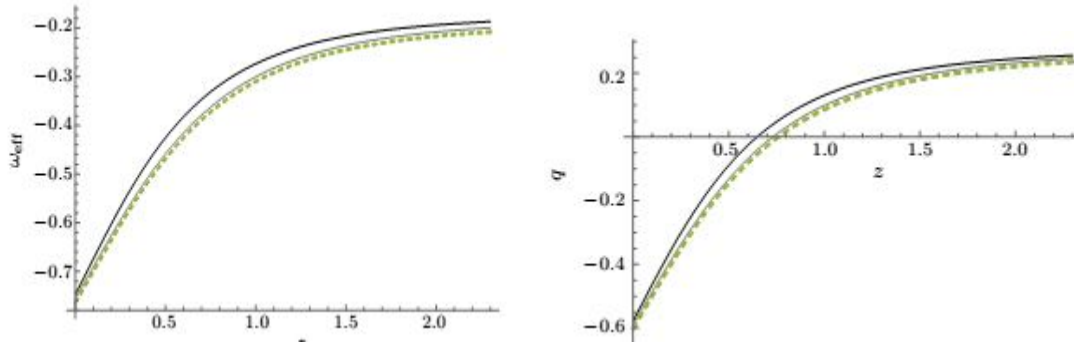


Figure 3. The evolution of the effective state parameter ω_{eff} and deceleration parameter q at best fit values of Table 1 for $\xi_0 = 0.01$ (gray line), $\xi_0 = 0.02$ (solid black line) and $\xi_0 = 0.0001$ (dashed line).

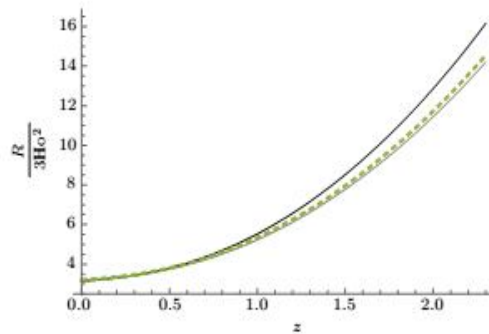


Figure 4. The evolution of the curvature scalar at best fit values of Table 1 for $\xi_0 = 0.01$ (gray line), $\xi_0 = 0.02$ (solid black line) and $\xi_0 = 0.0001$ (dashed line).

given by the standard Λ CDM cosmology. Moreover, a transition from a decelerated $q < 1/2$ to an accelerated $q < 0$ universe is realized when q crosses the zero, and thus the universe passes from matter to dark-energy-dominated universe where $\rho_{DE} \approx \rho_{matter}$ and undergoes an accelerated phase. The crossing happened at approximately $z = 0.75$ for both $\xi_0 = 0.01$ and $\xi_0 = 0.0001$ and at $z = 0.65$ for $\xi_0 = 0.02$. To probe the behavior of the model in the early universe, where $a \rightarrow 0$, we calculate the scalar curvature R in a flat universe, and get:

$$R = H(z)^2 - p_{MCG} + 3\xi_0 H(z) \rho_{MCG}^{1/2} \quad (9)$$

In Fig. 4, the scalar curvature evolution is plotted in terms of the redshift parameter z at the best values of Table 1. As $t \rightarrow 0$, $R \rightarrow \infty$ which indicates the presence of a Big Bang singularity

4.VMCG in LQG

we study the VMCG dynamical behavior when coupled to the baryonic matter in the framework of loop quantum cosmology (LQC), [16–19]. The latter is a non perturbative and background-independent type of quantization of gravity [20,21] used to probe some cosmological problems. In addition to predicting an inflationary phase of the early universe[22–25]and late time cosmic acceleration,[26]. LQC is proved to be very successful in avoiding Big Bang and Big Rip singularities[27] and the semi-classical approximation in LQC formalism can be validly used at late time and at large scale[28]. The modified Friedman equation reads:

$$H^2 = \frac{\rho}{3} \left(1 - \frac{\rho}{\rho_c}\right) \quad (10)$$

where ρ is the total energy density, $\rho_c = \frac{\sqrt{3}}{16\pi^2 \gamma^3 G^2 \hbar}$ is the critical density in LQC and γ is the dimensionless Barbero–Immirzi parameter. It is worth to mention that the quantum correction is negligible when $\rho \ll \rho_c \sim \rho_P$ (ρ_P is the energy density at the Planck scale), but it dominates the dynamics when $\rho \sim \rho_c$. In what follows, we assume a universe filled with VMCG and baryonic matter. To make the dynamical analysis, we introduce the following dimensionless variables:

$$x = \rho_{MCG}/3H^2, \quad y = p_{MCG}/3H^2, \quad z = \rho/\rho_c \quad (11)$$

where the phase space is bounded by $0 \leq x \leq 1$, $0 \leq z \leq 1$ and a negative y (a negative pressure is needed to generate an accelerated expansion). The modified Friedman equation and the effective state parameter can be expressed in terms of the dimensionless variables as

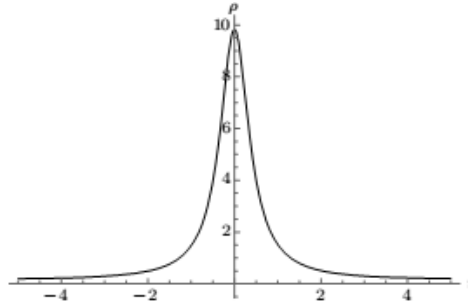
$$\begin{aligned} x' &= 3[x(1-2z) - 1](y - \sqrt{3}\xi_0 x^{1/2}) - 3x \left(\frac{z}{1-z} \right) \\ y' &= -3[A(\alpha + 1)x - \alpha y] - 3[A(1 + \alpha) - y(1-2z + \alpha x)] (y - \sqrt{3}\xi_0 x^{1/2}) + 3y \left(1 - \frac{2z}{1-z} \right) \\ z' &= -3z - 3(1-z)(y - \sqrt{3}\xi_0 x^{1/2}) \end{aligned} \quad (12)$$

where the prime denotes the derivative with respect to the e-folding number $N = \ln a$. Notice that this autonomous system does not depend on the EoS parameter B , and its critical points (x_c, y_c, z_c) are found numerically at the best values of Table 1. Their properties are determined by the sign and nature of the eigenvalues λ_i , $i = 1, 3$ of the Jacobian matrix. When we fix the values of both ξ_0 and A , the critical points are the same and independent of the choice of α as listed in Table 2. For $(\xi_0 = 0.01, A = -0.167, \alpha = 0.551)$ and $(\xi_0 = 0.02, A = -0.149, \alpha = 0.548)$ the only physical and stable critical points

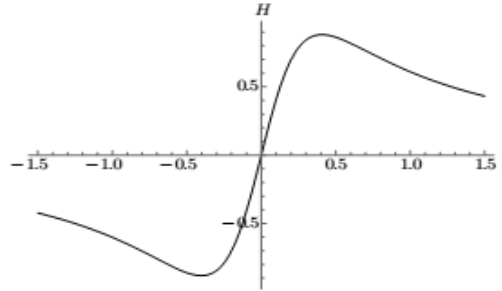
Table 2. The eigenvalues of the Jacobian matrix around critical points P_i for the autonomous system Eq. (33).

	Critical points	Eigenvalues	ω_{eff}
$\xi_0 = 0.01, A=-167$	$P_1(1, -0.98, 0)$	$(-2.99, -2.43(1+\alpha), -0.008)$	$-1\backslash$
	$P_2(1, -0.167, 0)$	$(-0.55, -2.44(1+\alpha), -2.44)$	-0.184
$\xi_0 = 0.02, A=-149$	$P_1(1, -0.96, 0)$	$(-2.98, -2.36(1+\alpha), -0.016)$	-1
	$P_2(1, -0.149, 0)$	$(-0.55, -2.5(1+\alpha), -2.44)$	-0.184
$\xi_0 = 0.01, A1$	$P_1(1, -0.98, 0)$	$(-2.99, -5.93(1+\alpha), -0.008)$	-1
	$P_2(1, 0.0003, 0)$	$(-3, -3(1+\alpha), -1.5)$	0
	$P_3(1, 1, 0)$	$(-2.49, 5.94(1+\alpha), -5.94)$	0.99

P_1 with negative eigenvalues describing an accelerated VMCG-dominated universe with $\omega_{eff} \approx -1$ exactly as predicted in the classical case. Moreover, the values of the critical points corresponding to an accelerated-VMCG-dominated universe change only with ξ_0 .

**Figure 5.** The evolution of the total energy density ρ with time. Parameters are set at the best fit values of Table 1 for $\xi_0 = 0.01$ with $\rho_c = 10$.

However, those describing a decelerated matter-dominated universe and a decelerated VMCG dominated universe depend on both (ξ_0, A) . For $(\xi_0 = 0.01, A = 1)$ the critical points are $P_1(1, -0.98, 0)$ a stable critical point because it has negative eigenvalues as α is a positive constant and it corresponds to an accelerated-VMCG-dominated universe and $P_2(0.0003, 0.0003, 0)$ and $P_3(1, 1, 0)$ unstable saddle points due to the opposite signs of their eigenvalues corresponding respectively to a decelerated matter-dominated universe and a decelerated-VMCG dominated universe.

**Figure 6.** The evolution of the Hubble parameter H with time. Parameters are set at the best fit values of Table 1 for $\xi_0 = 0.01$ with $\rho_c = 10$ and $\rho_{0MCG} + \rho_{0M} = 10$.

From fig. 6 the universe undergoes an accelerated expansion till a final de Sitter universe. In classical cosmology, the model suffers from Big Bang singularity. This problem does not occur in the loop quantum cosmology scenario. From Fig. (5) and (6), when $\rho_{tot} \approx 12\rho_c$, the Hubble parameter takes a maximum value and when ρ_{tot} takes its maximum value ρ_c , the Hubble parameter vanishes, thus the universe undergoes a contraction then enters the bounce.

5. Main results and conclusions

We have considered a model of VMCG where we have shown that the observational data of Union 2.1 constrain the viscous coefficient ξ_0 to values much smaller than one, otherwise the perturbation instabilities at the present time will grow exponentially leading to a non-consistent model. In fact, with small values of ξ_0 , the model is found to be suitable to describe the current universe and gives good predictions at the present time for both state and deceleration parameters $\omega_{\text{eff}} = \in (-0.76, -0.74)$ and $q_0 = \in (-0.60, -0.57)$. It is worth to mention that the obtained value of the state parameter is in agreement with $q_0 = -0.53(+0.17)(-0.13)$ at (68% C.L.; SN Ia+SALT2 fitter+ BAO/CMB) given by Ref. [29] and $q_0 = -0.54(+0.05)(-0.07)$ at (68% C.L.; SNIa+BAO/CMB+H(z)+uniform prior with $q_f = -1$) given by Ref. [30]. The present value of the effective state parameter of VMCG is also consistent with $\omega_0 = -1.04(+0.72)(-0.69)$ at (95% C.L.; Planck+WP+BAO) for a dynamical state parameter estimated in Ref. [31] and $\omega_0 = -0.91(+0.16)(-0.20)$ (SNLS3 team) of Refs. [32-33]. The perturbation instabilities, at the matter-dominated era are dropped down in present and late time as the coupling between dark energy and dark matter is decreasing. At large scale, the VMCG has no future singularities and its equation of state is nearly equivalent to the cosmological constant with $\omega_{\text{eff}} = -1$, while the sound speed parameter takes a constant value different from zero as a difference between a dynamical fluid model and an inert cosmological constant model. Thus, the VMCG discussed here reproduces the main results of the standard model without assuming a priori the existence of a cosmological constant [13]. Moreover, the problems related to fine-tuning and coincidence problem are solved and the value of the redshift where ($\rho_{\text{DE}} \approx \rho_{\text{matter}}$) for both $\xi_0 = 0.01$ and $\xi_0 = 0.0001$ is $z \approx 0.75$. This value is in agreement with $z_t = 0.64(+0.13)(-0.07)$ given by Ref. [29] for models with the final de Sitter phase, $t = 0.71 \pm 0.03$ of the Λ CDM model of Ref. [11], $z_t = 0.74 \pm 0.05$ given by Ref. [34] and z_t at (more than 68% C.L.; SN Ia + BAO/CMB(WMAP9)+H(z)+uniform prior with $q_f = -1$) of Ref. [30]. At LQC background and at small scale the Big Bang singularity problem is solved and replaced by a bounce. At a large scale the stability of the model does not depend on the EoS parameter B and VMCG solutions depend only on ξ_0 .

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