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Oblique projectors in image morphology

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Abstract. At the previous ICPPA (2015) we presented the report [1] where we discussed the results of the solution of the problem of estimation of statistical reliability of linear point structures, obtained from the experiments at the FOBOS spectrometer [2] dedicated to study of the spontaneous fission of the 252Cf nucleus in the mass correlation distribution of fission fragments. These new unusual structures bounded by magic clusters were interpreted as a manifestation of a new exotic decay called collinear cluster tri-partition (CCT) [3]. The reliability of these structures was estimated on the basis of methods of morphological image analysis [1], [4], [5]. To improve the quality of revealing and further estimation of linear structures statistical reliability in the mass correlation distribution of fission fragments we used the formalism of oblique projecting [6] and subjective modeling [7], [8].

1. Introduction

The mathematical methods and models for the morphological image analysis considered in this paper are developed as applied to the analysis of actual scenes as based on their images obtained under uncontrollable recording conditions, such as illumination conditions, spectral distribution, the characteristics of recording instruments, the optical characteristics of scenes, etc. [1], [4], [5]. Methods and models for the morphological image analysis are usually computer-based and(or) mathematical interpretations of subjective analysis results obtained by a so-called researcher-modeler (r-m) concerning the goal of researchment and methods of it realisation (mathematical, computational etc.) In this paper we discuss the mathematical formalism for subjective modeling which can be used by a r-m to describe both formalized and non-formalized incomplete uncertain data in various situations from "absolute ignorance" to "complete knowledge" of the model of the research object. These data are based on a r-m's scientific experience and intuition [7]. Also we discuss the subjective modeling for the morphological image analysis, oblique projecting methods [6] and at the end we discuss an example of subjective morphological methods application in the problem of linear structures revealing and estimation of their statistical reliability in the mass correlation distribution of fission fragments [1].

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2. Elements of morphological analysis [4]

As a mathematical object, an image is a vector-valued function $\mathbf{f}(\cdot)\colon Y \to \mathcal{R}^N$, where $\mathcal{R}^N$ is the N-dimensional Euclidean space, while Y is a bounded closed domain in the plane $\mathcal{R}^2$ or its discrete representation by the subset $\{1, 2, \dots\}^2$. The domain Y is called the field of view. The norm $\|\mathbf{f}(y)\|_N$ is called the brightness of the image $\mathbf{f}(\cdot)$ at the point $y \in Y$, where $\|\mathbf{z}\|_N^2 = z_1^2 + \cdots + z_N^2$ and $\mathbf{z} = (z_1, \dots, z_N) \in \mathcal{R}^N$; and $\mathbf{f}(y)/\|\mathbf{f}(y)\|_N$ is called the color of $\mathbf{f}(\cdot)$ at this point $y \in Y$. The linear operations over images are defined as

$$(\mathbf{f}_1 + \mathbf{f}_2)(y) \stackrel{\text{def}}{=} \mathbf{f}_1(y) + \mathbf{f}_2(y), \qquad (a \cdot \mathbf{f})(y) \stackrel{\text{def}}{=} a \cdot \mathbf{f}(y), \qquad y \in Y. \tag{1}$$

Any image $\mathbf{f}(\cdot)$ (including Y) is μ-measurable and the function $\|\mathbf{f}(\cdot)\|_N^2$ is μ-integrable (a measure μ is defined on the Borel class of $\mathcal{R}^2$). The set of images, which is denoted by $L^2_\mu(Y)$, is a Euclidean space in which $\|\mathbf{f}\|_\infty^2 = \int_Y \|\mathbf{f}(x)\|_N^2\, \mu(dx) < \infty$ and $(\mathbf{f}, \mathbf{g})_\infty = \int_Y (\mathbf{f}(x), \mathbf{g}(x))\, \mu(dx)$ are the norm of the image $\mathbf{f}(\cdot)$ and the scalar product of the images $\mathbf{f}(\cdot)$ and $\mathbf{g}(\cdot)$, respectively.

3. Form of an image and of a class of scene images [4]

Let S denote a scene; let K be a set of recording conditions for scene images, such as weather conditions, illumination characteristics, the reflectivity of the elements of S, etc.; and let

$$\mathbf{V}_S = (\mathbf{f}(\cdot, k) \in L^2_\mu(Y),\ k \in K) \tag{2}$$

denote the class of images of S that can be obtained under all recording conditions $k \in K$. The class $\mathbf{V}_S$ contains all data on the geometric characteristics of S and on the forms of its elements, which are represented by its images $\mathbf{f}(\cdot, k)$, $k \in K$. Choosing an image $f(\cdot) \in \mathbf{V}_S$, we define the class

$$\mathbf{V}_S(f) = \{\mathbf{g}(\cdot) \in \mathbf{V}_S,\ \mathbf{g}(\cdot) \preceq \mathbf{f}(\cdot)\} \tag{3}$$

of images $\mathbf{g}(\cdot) \in \mathbf{V}_S$ that represent the geometric characteristics of elements of S in no more detail than $\mathbf{f}(\cdot) \in \mathbf{V}_S$. In paper [4] the class $\mathbf{V}_S(f)$ is called *the form of images of* $\mathbf{f}(\cdot)$. An image $\mathbf{g}(\cdot)$ is comparable in form with $\mathbf{f}(\cdot)$, but may be simpler than or as complicated in form as $\mathbf{f}(\cdot)$ and possibly doesn't represent all the details of the form of S that are represented in $\mathbf{f}(\cdot)$. In equation (3) the relation $\preceq$ defines a partial quasi-order on $\mathbf{V}_S$: it is reflexive and transitive.

If $\mathbf{g}(\cdot) \preceq \mathbf{f}(\cdot)$ and $\mathbf{f}(\cdot) \preceq \mathbf{g}(\cdot)$, then the images $\mathbf{f}(\cdot)$ and $\mathbf{g}(\cdot)$ are equivalent in form; i.e., they represent the form of S to an identical degree of detail (written as $\mathbf{f}(\cdot) \sim \mathbf{g}(\cdot)$). Therefore, each image $f(\cdot) \in \mathbf{V}_S$ is associated with the class $E_S(\mathbf{f}) = \{\mathbf{g}(\cdot) \in \mathbf{V}_S, \mathbf{g}(\cdot) \sim \mathbf{f}(\cdot)\}$, $\mathbf{f}(\cdot) \in \mathbf{V}_S$, of images that are equivalent in form to $\mathbf{f}(\cdot)$. The class $\mathbf{V}_S(\mathbf{f})$ is a convex cone in $L^2_\mu(Y)$ and is usually a closed set in $L^2_\mu(Y)$ for any image $\mathbf{f}(\cdot) \in \mathbf{V}_S$. Under these conditions, there exists an operator $\Pi_\mathbf{f}\colon L^2_\mu(Y) \to L^2_\mu(Y)$ defined by the condition

$$\|\mathbf{g}(\cdot) - \Pi_\mathbf{f}\mathbf{g}(\cdot)\| = \inf_{\mathbf{h}(\cdot) \in \mathbf{V}_S(\mathbf{f})} \|\mathbf{g}(\cdot) - \mathbf{h}(\cdot)\|, \qquad \mathbf{g}(\cdot) \in L^2_\mu(Y), \tag{4}$$

which is called the projection operator onto $\mathbf{V}_S(\mathbf{f})$. This operator $\Pi_\mathbf{f}$ is also called *the form of the image* $\mathbf{f}(\cdot)$.

4. Oblique projection [6]

Let $L^2_\mu(Y) = L^2_{\mu,1}(Y) \oplus L^2_{2,\mu}(Y)$ be the decomposition of $L^2_\mu(Y)$ into a direct generally not orthogonal sum of subspaces $L^2_{\mu,1}(Y)$ and $L^2_{\mu,2}(Y)$. Accordingly, for any image $\mathbf{f}(\cdot) \in L^2_\mu(Y)$ let $\mathbf{f}(\cdot) = \mathbf{f}_1(\cdot) + \mathbf{f}_2(\cdot)$ be its unique representation as the sum of images $\mathbf{f}_1(\cdot) \in L^2_\mu(Y)$ and $\mathbf{f}_2(\cdot) \in L^2_\mu(Y)$. The operator $\mathrm{S}\colon L^2_\mu(Y) \to L^2_\mu(Y)$, defined for any $\mathbf{f}(\cdot) \in L^2_\mu(Y)$ by the relation $\mathrm{S}f(\cdot) = \mathbf{f}_1(\cdot)$, is called the oblique projector onto $L^2_{\mu,1}(Y)$ parallel to (along) $L^2_{2,\mu}(Y)$. In the

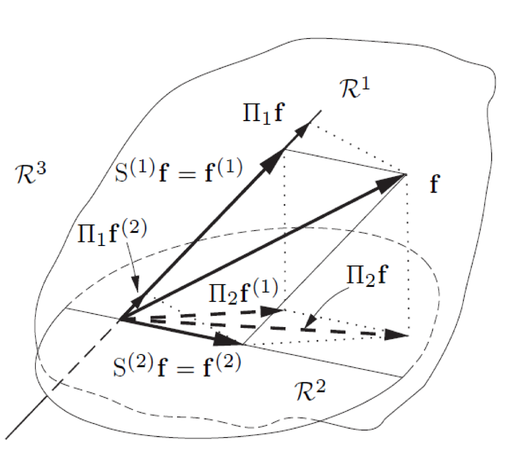

Figure 1. An example of oblique projecting: $\mathcal{R}^3 = \mathcal{R}^1 \oplus \mathcal{R}^2$, oblique projector $\mathrm{S}^{(1)}$ projects onto $\mathcal{R}^1$ along $\mathcal{R}^2$, $\mathrm{S}^{(2)} = I - \mathrm{S}^{(1)}$ projects onto $\mathcal{R}^2$ along $\mathcal{R}^1$, $\mathbf{f} = \mathbf{f}^{(1)} + \mathbf{f}^{(2)}$, the space $\mathcal{R}^1$ can be interpreted as the form of the signal (image) $\mathbf{f}^{(1)}$, $\mathcal{R}^2$ is the form of the disturbance $\mathbf{f}^{(2)}$, $\mathcal{R}^1 \oplus \mathcal{R}^2 = \mathcal{R}^3$, $\mathcal{R}^1 \cap \mathcal{R}^2 = \varnothing$, – the form of the image $\mathbf{f}$. In this case the oblique projecting reveals from the signal $\mathbf{f}$ the signals $\mathbf{f}^{(1)}$ and $\mathbf{f}^{(2)}$, while $\Pi_{\mathbf{f}^{(1)}}\mathbf{f} = \Pi_{\mathbf{f}^{(1)}}\mathbf{f}^{(1)} + \Pi_{\mathbf{f}^{(1)}}\mathbf{f}^{(2)} = \mathbf{f}^{(1)} + \Pi_{\mathbf{f}^{(1)}}\mathbf{f}^{(2)}$ and $\Pi_{\mathbf{f}^{(2)}}\mathbf{f} = \Pi_{\mathbf{f}^{(2)}}\mathbf{f}^{(1)} + \mathbf{f}^{(2)}$ are the most accurate approximations of $\mathbf{f}$ by signals $\mathbf{f}^{(1)} \in \mathcal{R}^1$ and $\mathbf{f}^{(2)} \in \mathcal{R}^2$.

case of a finite-dimensional Euclidean space, for example, $\mathcal{R}^N$, any of its subspaces, say, $\mathcal{R}^k$, has infinitely many complements $\mathcal{R}^{N-k}$ of $\mathcal{R}^k$, $\mathcal{R}^N$=$\mathcal{R}^k \oplus \mathcal{R}^{N-k}$, and oblique projectors onto $\mathcal{R}^k$ along $\mathcal{R}^{N-k}$ representing them (see figure 1).

5. Subjective modeling [7]

Methods were considered in [7] for the mathematic modeling of incomplete and unreliable knowledge about the model $M(x)$ of the research object expressed in the form of subjective judgments made by the r-m about the possible values of the unknown parameter $x \in X$ defining the model. The mathematical model of subjective judgments is defined as the space $(X, \mathcal{P}(X), \mathrm{Pl}^{\tilde{x}}, \mathrm{Bel}^{\tilde{x}})$ with the plausibility measure $\mathrm{Pl}^{\tilde{x}}(\cdot)\colon \mathcal{P}(X) \to \mathcal{L}$ and belief measure $\mathrm{Bel}^{\tilde{x}}(\cdot)\colon \mathcal{P}(X) \to \hat{\mathcal{L}}$, where X is the set of all possible values of the unknown parameter x defining the model $M(x)$, $\mathcal{P}(X)$ is the class of all the subsets of X, the indeterminate element (i.el.) $\tilde{x} \in X$ characterizes (as an undefined propositional variable) the subjective judgments made by the r-m about the validity of each value $x \in X$ by the values of measures such as the plausibility $\mathrm{Pl}^{\tilde{x}}(\tilde{x} = x)$ of the equality $\tilde{x} = x$, and belief $\mathrm{Bel}^{\tilde{x}}(\tilde{x} \neq x)$ in the inequality $\tilde{x} \neq x$. If there are observational data on the subject, available to the r-m he can use them to construct an empirical estimate of the i.el. $\tilde{x}$ and an empirical model $(X, \mathcal{P}(X), \mathrm{Pl}^{\tilde{x}}, \mathrm{Bel}^{\tilde{x}})$ of the subjective judgments about possible values of $x \in X$.

The scales of measures of plausibility $\mathrm{Pl}^{\tilde{x}}$ and belief $\mathrm{Bel}^{\tilde{x}}$ are $L = ([0,1], \leq, +, \times)$ and $\hat{L} = ([0,1], \geq, \widehat{+}, \widehat{\times})$, respectively, where operations of addition $+$, $\widehat{+}$ and multiplication $\times$, $\widehat{\times}$ are determined by the equalities

$$\begin{aligned} a + b &= \max\{a, b\}, & a \widehat{+} b &= \min\{a, b\}, \\ a \times b &= \min\{a, b\}, & a \widehat{\times} b &= \max\{a, b\}, \end{aligned} \qquad a, b \in [0, 1]. \tag{5}$$

For each set $E \in \mathcal{P}(X)$ measures $\mathrm{Pl}^{\tilde{x}}$ and $\mathrm{Bel}^{\tilde{x}}$ are determined by the equalities

$$\begin{aligned} &\mathrm{Pl}^{\tilde{x}}(E) \equiv \mathrm{Pl}^{\tilde{x}}(\tilde{x} \in E) = \sup_{x \in E} t^{\tilde{x}}(x), && E \neq \varnothing; && \mathrm{Pl}^{\tilde{x}}(\varnothing) \equiv 0, \\ &\mathrm{Bel}^{\tilde{x}}(E) \equiv \mathrm{Bel}^{\tilde{x}}(\tilde{x} \in E) = \inf_{x \in X \setminus E} \hat{t}^{\tilde{x}}(x), && E \neq X; && \mathrm{Bel}^{\tilde{x}}(X) = 1, \end{aligned} \tag{6}$$

where $t^{\tilde{x}}(x) = \mathrm{Pl}^{\tilde{x}}(\tilde{x} = x)$, $\hat{t}^{\tilde{x}}(x) = \mathrm{Bel}^{\tilde{x}}(\tilde{x} \neq x)$, $x \in X$. The functions $t^{\tilde{x}}(\cdot)\colon X \to L$ and $\hat{t}^{\tilde{x}}(\cdot)\colon X \to \hat{L}$ are called the distributions of plausibilities and beliefs of the values x, their values

$t^{\tilde{x}}(x)$ and $\hat{t}^{\tilde{x}}(x)$ in equation (6) determine the plausibility of the equality $\tilde{x} = x$ and, accordingly, the belief of the inequality $\tilde{x} \neq x$, $x \in X$, the values $\mathrm{Pl}^{\tilde{x}}(E)$ and $\mathrm{Bel}^{\tilde{x}}(E)$ in equation (6) are the plausibility and belief of the inclusion $x \in E \in \mathcal{P}(X)$. The space $(X, \mathcal{P}(X), \mathrm{Pl}^{\tilde{x}}, \mathrm{Bel}^{\tilde{x}})$, in turn, completely characterizes the i.el. $\tilde{x}$ and is further referred to as its model.

6. The subjective morphological model [8]

As it is shown in [6], [8], the oblique projector can be obtained by solving the problem of optimal estimation of the signal $\mathbf{f}^{(1)}$ within the subjective morphological model of observations defined by r-m, in which the signal $\mathbf{f}^{(1)}$ and the disturbance $\mathbf{f}^{(2)}$ are realizations of independent indeterminate vectors $\tilde{\mathbf{f}}^{(1)}$ and $\tilde{\mathbf{f}}^{(2)}$, respectively. Their values are observed according to the scheme $\tilde{\mathbf{f}} = \tilde{\mathbf{f}}^{(1)} + \tilde{\mathbf{f}}^{(2)}$ and they are defined by their distributions

$$t^{\tilde{\mathbf{f}}^{(i)}}(\mathbf{f}^{(i)}) = \begin{cases} 1, & \mathbf{f}^{(i)} \in R^i, \\ 0, & \mathbf{f}^{(i)} \notin R^i, \end{cases} \qquad i = 1, 2, \quad \mathcal{R}^1 \oplus \mathcal{R}^2 = \mathcal{R}^3, \tag{7}$$

of plausibilities of their values. The optimal estimate $\mathbf{f}_*^{(1)}$ minimizing the error plausibility [7] in the estimation of $\tilde{f}^{(1)} = f^{(1)}$ is determined by the condition $\mathbf{f}_*^{(1)} = \mathrm{S}^{(1)}\mathbf{f}$ (see figure 1).

7. Real experimental data

In this section we present the results of application of oblique projecting to the problem of the linear-point structures revealing from mass correlation distribution of 252Cf fission fragments [1]. The mass-mass distribution under analysis is presented at figure 2. Some results of oblique projection application are presented at figure 3. Accordingly to it a researcher can eliminate from the image under analysis the parts that are treated as disturbances. To obtain the estimation of reliability of the linear-point structures revealed one can use the method described at [1].

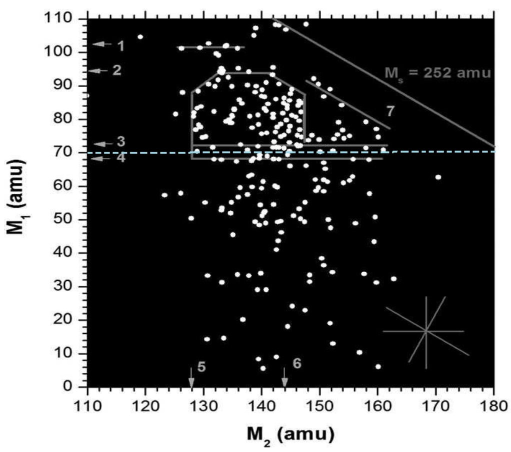

Figure 2. The mass–mass distribution of 252Cf spontaneous fission fragments. At the right lower corner some directions are shown which are of special interest to the researcher. The dashed line bounds the data area which is shown at figure 3a and is used for morphological analysis.

8. Conclusion

The mathematical formalism for subjective modeling which can be used by a r-m to describe both formalized and non-formalized incomplete uncertain data in various situations from "absolute ignorance" to "complete knowledge" of the model of the research object was considered. Also the subjective modeling for the morphological image analysis, oblique projecting methods and an example of subjective morphological methods application in the problem of linear structures revealing in the mass correlation distribution of fission fragments were considered.

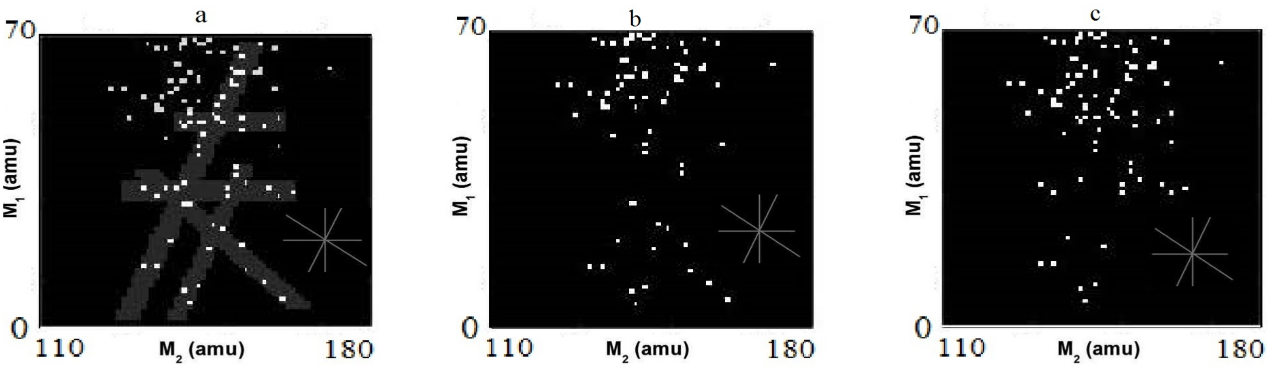

Figure 3. a) The part of the mass-mass distribution of 252Cf fission fragments from figure 2. Accordingly to the method considered in [1] this image is assumed to be the sum of images of three different forms: horizontal strips, strips defined by the condition "sum of mass fragments equals to a constant" and strips with the angle of inclination $\sim 60°$. The parameters of strips such as their width, angle of inclination, their number and length are chosen optimally by means of a procedure analogues to the procedure described at [1]. b) Image (a) without the calculated oblique projection onto the form of horizontal strips. c) Image (a) without the calculated oblique projection onto the form of strips defined by the condition "sum of mass fragments equals to a constant". Thus a researcher can eliminate from the image under analysis the parts that are treated as disturbances by him.

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